Slow Magnetoacoustic Waves in Fan-like Coronal Loops

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MHD waves

Magnetohydrodynamics = Magneto + Hydro+ Dynamics

The Field of MHD waves was explored by a legendary electrical engineer Hannes Alfvén and he was given the nobel prize in 1970 for his work on MHD waves.

He described the branch of MHD waves now known as the Alfven waves.

Synopsis

- The present theoretical development is motivated by the observational findings of Wang et al. 2009 appeared in Astronomy and Astrophysics journal.

- In the present work we approach a theoretical MHD wave model to satisfy the observational findings of Wang et al. 2009.

Governing MHD wave equations

$$\frac{D}{Dt} + v.\nabla\rho + \rho\,\nabla.v = 0$$

$$\frac{DV}{Dt} = -\frac{\nabla p}{\rho} + \frac{1}{\mu\rho}(\nabla \times B) \times B + \frac{4}{3}\eta_t \nabla^2 V$$

$$\frac{DP}{Dt} - \gamma\frac{P}{\rho}\frac{D\rho}{Dt} = (\gamma - 1)(\nabla \cdot \kappa \nabla T) = 0$$

$$\frac{\partial B}{\partial t} = \nabla \times (V \times B)$$

$$\nabla \cdot B = 0$$

$$P = \frac{\rho}{\tilde{\mu}} RT$$

$\frac{D}{Dt} = \frac{\partial}{\partial t} + (V \cdot \nabla)$ is the material derivative for time dependent variations with flow.

Then the Above set of equations becomes:

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla\right) \rho + \rho \nabla \cdot V = 0 \qquad (1)$$

$$\rho \left(\frac{\partial}{\partial t} + V \cdot \nabla \right) V = - \nabla p + \frac{1}{\mu} (\nabla \times B) \times B + \rho \eta_t \nabla^2 V \qquad (2)$$

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla \right) p - \gamma \frac{P}{\rho} \left[\frac{\partial}{\partial t} + V \cdot \nabla \right] \rho = (\gamma - 1) (\nabla \cdot \kappa \nabla) T \qquad (3)$$

$$\frac{\partial}{\partial t} = \nabla \times (V \times B) \qquad (4)$$

$$P = \rho R T / \mu \qquad (5)$$

$$\nabla \cdot B = 0 \qquad (6)$$

Linearization

The perturbations are taken as $B = B_0 + B_1$, $P = P_0 + P_1$, $T = T_0 + T_1$ and $\rho = \rho_0 + \rho_1$.
The linearized set of equations become :

The continuity equation:

$$\frac{\partial \rho_1}{\partial t} + (V_1 \cdot \nabla)\rho_0 + \rho_0(\nabla \cdot V_1) = 0 \tag{7}$$

The momentum equation

$$\frac{\partial}{\partial t} V_1 = -\frac{\nabla p_1}{\rho_0} + \frac{1}{\mu \rho_0} \{ (\nabla \times B_1) \times B_0 \} + \frac{4}{3} \eta_t \nabla^2 V_1 \tag{8}$$

The energy equation

$$\frac{\partial P_1}{\partial t} + (V_1 \cdot \nabla) p_0 - \gamma \frac{p_0}{\rho_0} [\frac{\partial \rho_1}{\partial t} + (V_1 \cdot \nabla)\rho_0) = (\gamma - 1)(\nabla \cdot \kappa \nabla T_1) \tag{9}$$

The induction equation

$$\frac{\partial B_1}{\partial t} = \nabla \times (V_1 \times B_0) \quad (10)$$

$$\nabla \cdot B_1 = 0 \quad (11)$$

$$\frac{p_1}{p_0} - \frac{\rho_1}{\rho_0} - \frac{T_1}{T_0} = 0 \quad (12)$$

To take Fourier transform we replace $\frac{\partial}{\partial t}$ With $-i\omega$ and ∇ with $i\vec{K}$ because

the plane wave solutions are taken in form of $e^{(-\omega t + \vec{k} \cdot r)}$

Then the above equations becomes.

The continuity equation

$$\omega\rho_1 - \rho_0(K_z V_{1z}) = 0 \tag{13}$$

The momentum equation

$$\omega\rho_0 V_{1z}\hat{e}_z = +\frac{K_z p_1}{\rho_0}\hat{e}_z - \frac{1}{\mu\rho_0} B_{0z}B_{1z}K_z\hat{e}_z - \frac{4}{3} iK^2\rho_0\eta V_{1z}\hat{e}_z \tag{14}$$

Induction equation

$$\omega B_{1z} = -B_{0x}K_z V_{1z} \tag{15}$$

$$\frac{p_1}{p_0} - \frac{\rho_1}{\rho_0} - \frac{T_1}{T_0} = 0 \tag{16}$$

$$T_1 = \frac{T_0}{p_0} p_1 - \frac{T_0}{\rho_0} \rho_1 \tag{17}$$

And the equation of energy becomes

$$i\omega p_1 + V_{1z}\ K_z\ P_0 - \gamma\ \frac{P_0}{\rho_0}\,[-i\omega\rho_1 + iV_{1z}\ K_z\ \rho_0] = i\,(\gamma - 1)\,\kappa\ k_z^2\ T_1 \qquad (18)$$

Using $T_1 = \frac{T_0}{p_0}\,p_1 - \frac{T_0}{\rho_0}\,\rho_1$ and solving for P_1 we get :

$$P_1 = \frac{i(\gamma - 1)\kappa\ k_z^2\ T_0\ p_0\ \rho_1 + \gamma\ p_0^{\ 2}\ \omega\ \rho_1 + (1 - \gamma)\ V_{1z}\ K_z\ p_0^{\ 2}\ \rho_0}{p_0\ \rho_0\omega + i(\gamma - 1)k_z^2\ T_0\ \rho_0} \qquad (19)$$

Substituting the value of P_1 and ρ_1 in equation (20) and solving in respect of wave no. k we get :

$$k^4\left(\frac{4}{3}\eta\,\kappa\,(1-\gamma)\,T_0\,\rho_0\,\omega + i\,(1-\gamma)\,\kappa\,T_0\,p_0\right)$$
$$+\,k^2\left(\frac{4}{3}i\eta\,p_0\rho_0\,\omega^2 - i\,(1-\gamma)\,\kappa\,T_0\,\rho_0\,\omega^2\right) - p_0^{\;2}\omega\,)$$
$$+\,p_0\rho_0\,\omega^3 = 0 \qquad (20)$$

Here

$$A = \left(\frac{4}{3}\eta\,\kappa\,(1-\gamma)\,T_0\,\rho_0\,\omega + i\,(1-\gamma)\,\kappa\,T_0\,p_0\right)$$

$$B = \left(\frac{4}{3}i\eta\,p_0\rho_0\,\omega^2 - i\,(1-\gamma)\,\kappa\,T_0\,\rho_0\,\omega^2\right) - p_0^{\,2}\omega\,) \tag{21}$$

$$C = p_0\rho_0\,\omega^3$$

Hence the dispersion relation becomes

$Ak^4 + Bk^2 + C = 0$

Here wave number k is considered to be a complex quantity

- The dispersion relation is the equation which relates the wave number (k) and the angular frequency (ω) and explains the propagation and the damping behavior of a wave

- The real part of the wave number k represents the propagation properties of the wave while the imaginary part of wave number k shows the damping of the wave.

- The above equation is derived by taking in account the heat conductivity and the compressive viscosity as the dissipation mechanism.

- The final dispersion is solved numerically .

- Solving the dispersion relation we find four roots in terms of web number k.

- Two of the roots belong to the fast mode while the rest two belong to the slow mode.

- As we are interested in studying the study of the slow magnetoacoustic waves we consider the slow wave mode values for our theoretical MHD wave model.

Formulation

We consider the following values

$$\eta = 10^{-16} T_0^{2.5} (\mathrm{g\ cm^{-1} s^{-1}})$$

(Braginskii 1965)

$$\kappa = 10^{-6} T_0^{2.5} (\mathrm{ergs\ cm^{-1}\ s^{-1}\ K^{-1}})$$

The physical parameters are taken from Wang et al. 2009, i.e. observed periodicities P_1=26.2 min, P_2=13.5 min, P_3=11.2 min.

Observed temperature T=.7 MK

The loop height (Λ = 72 Mm)

The coronal electron density i.e. 10^{10} cm^{-3} is used in the theoretical model.

- The electron density n_e as a function of height like $e^{\frac{-z}{\Lambda}}$, where z is the height and Λ is the loop height.

- The wave propagation speed is calculated by $v_{ph} = \frac{\omega}{k}$

- The wavelengths are calculated by $\lambda = \frac{2\pi}{k_r}$

- The damping lengths are calculated by $d_L = \frac{1}{k_i}$

- The damping per wavelength $D_L = \frac{k_i}{k_r}$

- The energy flux density calculated by the relation $\rho\,[\,(\delta v)^2\,/\,2]\,v$

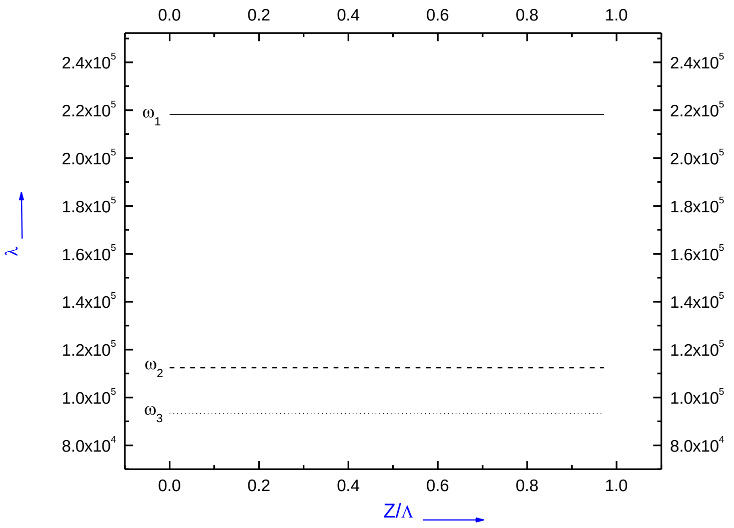
Results and Discussion

- The theoretically estimated wave propagation speed is 138.72 kms^{-1} the phase speed found to be constant corresponding to all three observed periodicities.

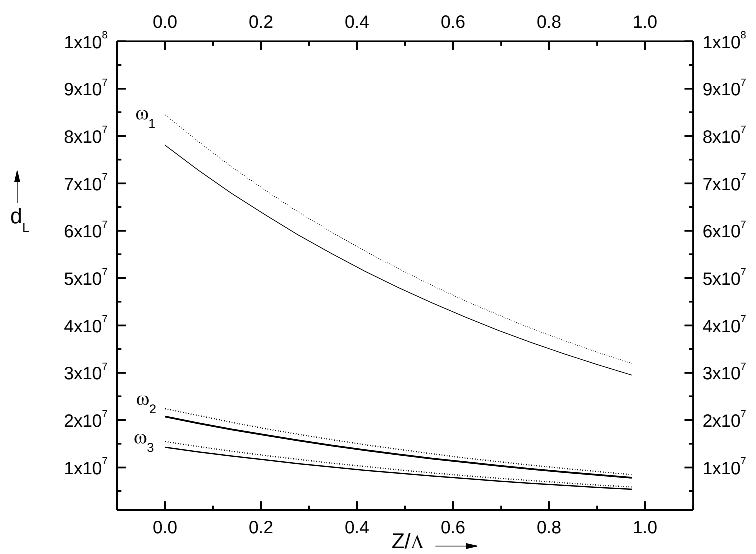
- We consider the same temperature throughout the loop that could be a reason that we found the constant speed correspond to each periodicity.

- The wavelengths corresponding to each height are calculated and are found to be independent of the variation in the loop height.

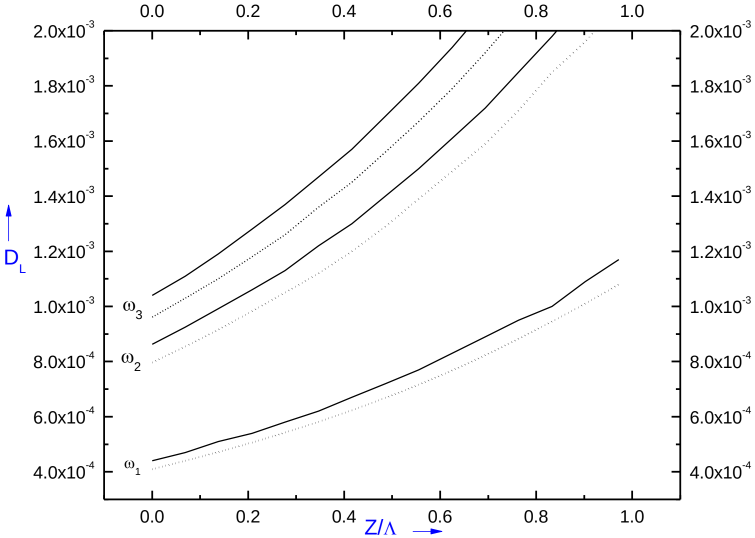
- The waves having long periodicities are found to have longer wavelengths.



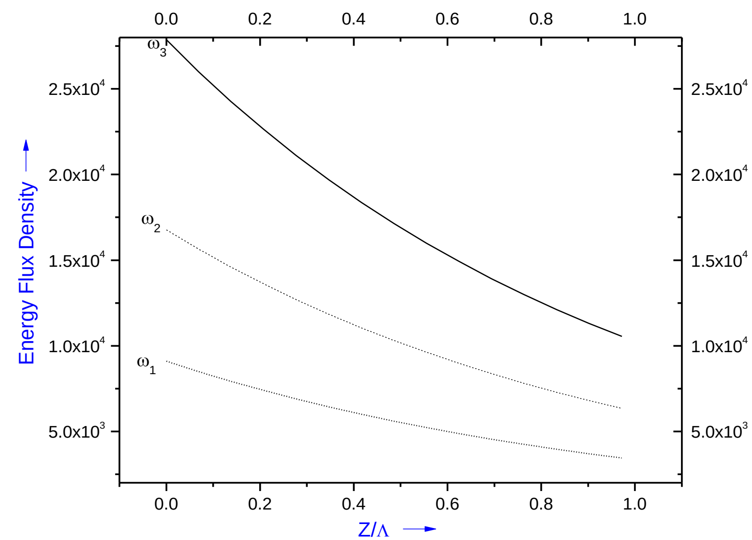
The variation of wavelength with loop height for the three different observed periodicities. The solid line belongs to the first (P_1=26.2 min) periodicity, while the dashed and dotted lines correspond to second (P_2=13.5 min) and third (P_3=11.2 min).



The variation of damping length with loop height for the three different observed periodicities. The solid lines shows the effect of compressive viscosity along with thermal conduction while the dotted lines show the effect of thermal conduction only



The variation of damping per wavelength with loop height for the three different observed periodicities. The solid lines shows the effect of compressive viscosity along with thermal conduction while the dotted lines show the effect of thermal conduction only.



The variation of energy flux density carried by the wave with loop height for the three different observed periodicities..

- The damping lengths were calculated corresponding to each periodicity, For the first case in presence of thermal conduction and compressive viscosity and for the second case in presence of thermal conduction only a decrease in the damping length was noticed with the increasing loop height.

- The damping of the wave was found stronger near the foot point of the loop as compared to the upper part.

- The damping per wavelength was calculated for each three periodicities and found to increase with the increasing loop height.

- The energy flux density carried by the waves are also calculated corresponding to each periodicity.

•The energy flux density was found to vary from 2.7×10^4 to 1.05×10^4 ergs cm^{-2} s^{-1} for the first periodicity, from 1.6×10^4 to 6.3×10^3 ergs cm^{-2} s^{-1} for second periodicity and 9.1×10^3 to 3.4×10^3 ergs cm^{-2} s^{-1} for the third periodicity.

• The magnitude of energy flux density carried by the wave was found larger near the foot point of the loop while it was found to become weaker with increasing loop height.

•The larger frequency waves were found to carry large energy flux.

Conclusion

- The propagation speed of the magnitude 138 km s^{-1}, constant for all the observed periodicities, this result is in good agreement with the observed propagation speed by Wang et al. 2009.

- We estimated that the longer waves having longer periodicities are found to have longer wavelengths.

- The wave corresponding to the higher frequencies are found to carry larger energy flux density.

- The wavelengths and the propagation speed of wave are found to be independent of the variation in loop height.

- The damping lengths were found to decrease with the loop height.

- The damping per wavelengths were found to increases with the increasing loop height.

References

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Thanks