

Estimation of Inflation parameters for Perturbed Power Law model using recent CMB measurements

Suvodip Mukherjee,^a Santanu Das,^a Minu Joy^b and Tarun Souradeep^a

^a*Inter University Centre for Astronomy and Astrophysics,
Post Bag 4, Ganeshkhind, Pune-411007, India*

^b*Dept. of Physics, Alphonsa College, Pala 686574, India*

*E-mail: suvodip@iucaa.ernet.in, santanud@iucaa.ernet.in, minujoy@gmail.com,
tarun@iucaa.ernet.in*

ABSTRACT: Cosmic Microwave Background (CMB) is an important probe for understanding the inflationary era of the Universe. We consider the Perturbed Power Law (PPL) model of inflation which is a soft deviation from Power Law (PL) inflationary model. This model captures the effect of higher order derivative of Hubble parameter during inflation, which in turn leads to a non-zero effective mass m_{eff} for the inflaton field. The higher order derivatives of Hubble parameter at leading order sources constant difference in the spectral index for scalar and tensor perturbation going beyond PL model of inflation. PPL model have two observable independent parameters, namely spectral index for tensor perturbation ν_t and change in spectral index for scalar perturbation ν_{st} to explain the observed features in the scalar and tensor power spectrum of perturbation. From the recent measurements of CMB power spectra by WMAP, Planck and BICEP-2 for temperature and polarization, we estimate the feasibility of PPL model with standard Λ CDM model. With this model, we estimate a non-zero value of tensor spectral index at significance of 5.36 and a non-zero value of effective mass $\frac{m_{\text{eff}}^2}{H^2} = -0.0237 \pm 0.0045$, of the inflaton field.

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1 Introduction

The precision measurement of temperature and polarization of Cosmic Microwave Background (CMB) by several experiments like WMAP, Planck, BICEP etc. have enabled us to constraint several cosmological parameters with unprecedented accuracy. Minimal Λ CDM model explains the observed temperature spectra by Planck [1, 2]. The recent detection of B mode polarization of CMB by BICEP [3] have provided us a window for measuring the primordial gravitational waves which is an important probe to understand the nature of inflation.

Inflation is the rapid accelerated expansion of the Universe postulated at very early time and that also predicts the generation of initial scalar and tensor perturbation from the quantum fluctuations of the early universe. These perturbations lead to anisotropies in CMB temperature field and also provide the initial seed for structure formation of the Universe. CMB power spectra for temperature and polarization is a window to the early era of the Universe and dynamics of the inflationary era can be studied with several observable quantities from CMB power spectra. Many single field inflationary models predict adiabatic, Gaussian and nearly scale independent perturbation and recent measurements from WMAP [4, 5] and Planck [6] are consistent with these prediction. One of such model is Power Law (PL) inflation introduced by Lucchin & Matarrese [7] where scale factor $a(t)$ during inflation evolves as $a(t) \sim t^p$. It predicts a scale invariant power spectrum for both scalar and tensor perturbation. The spectral indices are related by $n_s - 1 = n_t \propto (\frac{d \ln H}{d\phi})^2$. The consistency relation between tensor to scalar ratio r and tensor spectral index n_t is $n_t = -r/6.2$ [8]. However, for the inflation to end and reheating to begin, it is essential to consider the change in spectral index by $n_{run} = dn_s/d \ln k$. Souradeep et al. [9] extended the PL model to a model called Perturbed Power Law (PPL), which considers soft deviation from PL inflationary model by capturing the next higher order derivatives of Hubble parameter.

In this paper, we briefly discuss the PPL model with two main parameters ν_t and ν_{st} to capture the effect of n_s , n_t and n_{run} . These two parameters are related to the inflationary parameters $(\frac{d \ln H}{d\phi})^2$ and $\frac{d^2 \ln H}{d\phi^2}$. The model also predicts similar consistency relation between r and n_t . Considering the power spectra measured by WMAP and Planck and also the B mode polarization recently measured by BICEP-2 [3], we obtain the constraints on these inflationary parameters.

The paper is organized as follows. In Sec. 2, we briefly discuss the basic formalism of PL and PPL model. In Sec. 3, we constrain the inflationary parameters ν_s , ν_t and ν_{st} using the WMAP, Planck and BICEP-2 likelihood. Conclusion and feasibility of PPL model is discussed in Sec. 4.

2 Background of Perturbed Power Law (PPL) model of inflation

Perturbed Power Law (PPL) [9, 10] is an extension to Power Law (PL) inflationary model [7, 11] by considering higher order corrections to Hubble parameter. These corrections to Hubble parameter leads to an effective mass for the inflaton field, which in turn affects the density power spectra $P(\vec{k})$.

For single field inflationary models, Hubble parameter $H(k)$ and its evolution during inflation can be translated into the inflationary potential using Hamilton-Jacobi formulation [12, 13] as,

$$V(\phi) = \frac{3m_p^2 H^2(\phi)}{8\pi} \left[1 - \frac{m_p^2}{12\pi} \left(\frac{\partial \ln H^2}{\partial \phi} \right) \right], \quad (2.1)$$

where, m_p is the Planck mass.

For any given potential, slow-roll parameters (ϵ and δ) are related to the derivatives of the Hubble parameters as

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{m_p^2}{4\pi} \left(\frac{d \ln H}{d\phi} \right)^2 \ll 1, \quad (2.2)$$

$$\delta = \frac{\ddot{\phi}}{\phi H} = -\frac{m_p^2}{4\pi H} \frac{d^2 H}{d\phi^2} \ll 1. \quad (2.3)$$

These parameters are assumed to be less than 1 (slow-roll approximation, $\ddot{\phi} \ll 3H\dot{\phi}$) and are also validated by Planck [2]. These parameters are used to study the dynamics of inflaton field and are related to several observable quantities like scalar spectral index (n_s) and tensor spectral index (n_t) and also higher order corrections to these spectral index.

For PL model, the potential takes the form,

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{4\pi}{p}} \frac{\phi}{m_p}\right). \quad (2.4)$$

This leads to slow-roll parameters $\epsilon = -\delta = 1/p$ and corresponds to spectral index for scalar perturbation (ν_s) and tensor perturbation (ν_t) as

$$\nu_s = \nu_t = \frac{3}{2} - \frac{\nu}{2}. \quad (2.5)$$

where,

$$\nu = \frac{-2\epsilon}{1-\epsilon}. \quad (2.6)$$

The parameter ν is related to the usual definition of spectral index n_s and n_t by

$$n_s - 1 = n_t = \nu. \quad (2.7)$$

This constant spectral index for both scalar and tensor power spectrum suggest its name as 'Power Law' (PL) model of inflation. Tensor to scalar ratio r is related to the spectral index by [8],

$$r = -6.2n_t. \quad (2.8)$$

For a single field inflation model, effective mass m_{eff} of the inflaton field can be defined as

$$\frac{m_{\text{eff}}^2}{H^2} = -(\nu_{st})(\delta + 3) + \frac{\dot{\epsilon} - \dot{\delta}}{H}, \quad (2.9)$$

where, $\nu_{st} = (\epsilon + \delta)$. It can be expressed by higher order derivative of Hubble parameter and the leading order contribution is from the 2^{nd} derivative of Hubble parameter, expressed as

$$\frac{m_{\text{eff}}^2}{\text{H}^2} \approx \frac{3m_p^2}{4\pi} \frac{d^2 \ln \text{H}}{d\phi^2}. \quad (2.10)$$

For the PL model of inflation, $m_{\text{eff}} = 0$ and hence leads to constant spectral index for scalar and tensor perturbation as $n_s - 1 = n_t \approx -2\epsilon$. A non-zero effective mass m_{eff} affects only the scalar perturbation, whereas tensor perturbation continues to be massless excitations.

We consider soft departure from the PL model by accounting for non-zero value of m_{eff}^2 , which leads to change in the slow roll parameters and varying spectral index for scalar perturbation. For PPL model of inflation, spectral index for scalar perturbation ν_s and tensor perturbation ν_t are defined as

$$\nu_s = \frac{3}{2} + \epsilon(k) + \nu_{st}, \quad (2.11)$$

$$\nu_t = \frac{3}{2} + \epsilon, \quad (2.12)$$

where, $\nu_{st} \approx \nu_s - \nu_t$ is the difference between spectral index for scalar and tensor perturbation arising due to non-zero value of $\frac{d^2 \ln \text{H}}{d\phi^2}$. So in the model of PPL we can express the spectral features of primordial power spectrum for both scalar and tensor by two parameters ν_t and ν_{st} .

In the presence of effective mass, the evolution of slow roll parameter ϵ can be expressed as

$$\dot{\epsilon} = 2\epsilon\nu_{st}\text{H}. \quad (2.13)$$

By solving this equation upto leading order in ϵ , we find $\epsilon(k)$ as

$$\epsilon(k) \propto k^{2\nu_{st}}, \quad (2.14)$$

where ν_{st} is a constant. The power spectrum for scalar perturbation $P_s(k)$ and tensor perturbation $P_t(k)$ becomes,

$$P_s(k) = A(\nu_t, \nu_s) \left(\frac{\text{H}^2(k)}{\epsilon(k)} \right), \quad (2.15)$$

$$P_t(k) = 8A(\nu_t, \nu_t) \text{H}^2(k). \quad (2.16)$$

These above equations can be written in terms of the wave number k as,

$$P_s(k) = A(\nu_t, \nu_s) k^{3-2\nu_s}, \quad (2.17)$$

$$P_t(k) = 8A(\nu_t, \nu_t) k^{3-2\nu_t}, \quad (2.18)$$

where, $A(x, y) = \frac{4^y \Gamma^2(y)}{8\pi^3 (x-1/2)^{2y-1}}$. In PPL, we can understand the evolution of Hubble parameter from ν_t and ν_{st} which are related to the 1st and 2nd order derivative of Hubble parameter and can completely capture the essence of n_s and n_t and $n_{run} = dn_s/d \ln k$.

3 Estimation of Cosmological parameters for Perturbed Power Law

PPL model discussed in the previous section shows that ν_t and ν_{st} are the observables to study the evolution of Hubble parameter during inflation. Using the recent measurements from WP+Planck+BICEP-2, we impose constraints on these parameters which in turn translates to constraining $(\frac{d \ln \text{H}}{d\phi})^2$ and

$\frac{d^2 \ln H}{d\phi^2}$. For this analysis, we use WP+Planck+BICEP-2 likelihood, where we add up the results from commander_v4.1_lm49.clik, lowlike_v222.clik and CAMSpec_v6.2TN_2013_02_26.clik likelihood for WP and Planck [14, 15] and BICEP likelihood [16] to perform parameter estimation using the cosmological parameter estimation code SCoPE [17]. We choose the 7 parameters model as $(\Omega_b h^2, \Omega_m h^2, h, \tau, \nu_{st}, A_s, r)$ for PPL model.

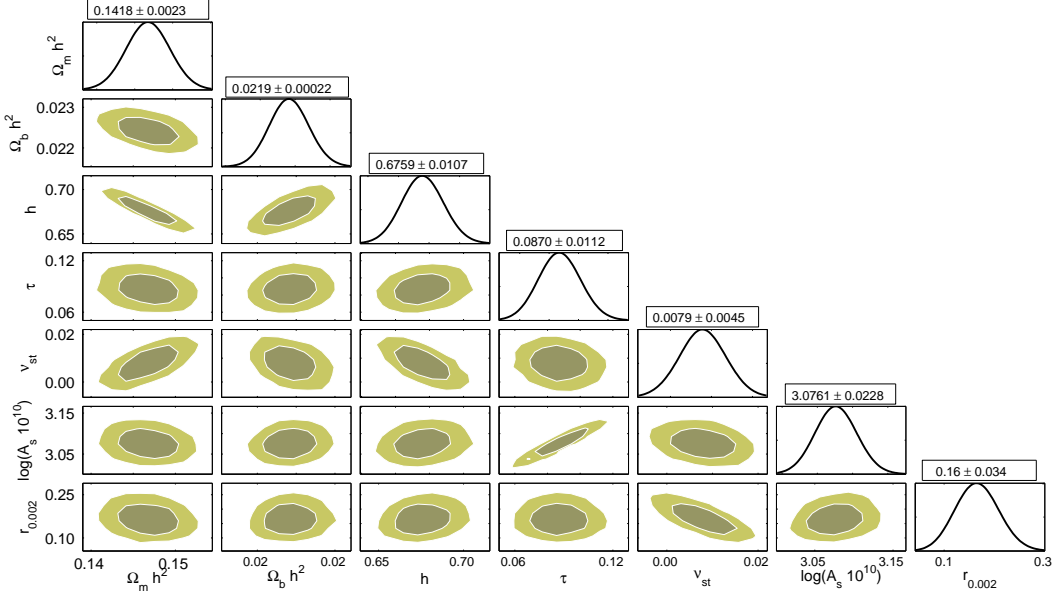


Figure 1. The two dimensional likelihood contours for the 7 parameters with $r_{0.002}$ for WP+Planck+BICEP-2 likelihood are plotted in the lower triangle. The mean and standard deviation are mentioned for each parameters above the one dimensional marginalized probability distribution plot. These values are consistent with the results obtained by Planck [2].

Fig. 1 gives the contour plots for the 7 parameters estimated with PPL model. These parameters are completely consistent with the Λ CDM cosmological model supported by Planck [2]. The mean for the inflationary parameter $\nu_{st} \equiv \nu_s - \nu_t = 0.0079$, which also constraints the effective mass $\frac{m_{\text{eff}}^2}{H^2} = -0.0237$. This measurement indicates that the departure from pure PL inflationary model is not significant (1.76σ). However, the negative value of $\frac{m_{\text{eff}}^2}{H^2}$ (or $\frac{d^2 \ln H}{d\phi^2}$) is more plausible with the data and indicates departure from PL inflation. For the other inflationary parameter ν_t , we plot the two dimensional contour between $n_t = 3 - 2\nu_t$ and ν_{st} given in Fig. 2(a) that also constraints $(\frac{d \ln H}{d\phi})^2$ and $\frac{m_{\text{eff}}^2}{H^2}$ respectively. This plot indicates that the current data set (WP+Planck+BICEP-2) allows only a restricted feasible range of the inflationary parameters to vary. In Fig. 2(b) we plot the one dimensional distribution for n_t with a mean of -0.0268 . This show that for PPL model of inflation spectral index for tensor is non-zero at 5.36σ . This result is expected to improve further from the polarization data of Planck. It is essential to compare the scalar spectral index $n_s = 4 - 2\nu_s$ obtained from PPL model with the value obtained by Planck. As we have used the inflationary parameters r and ν_{st} as free parameters, ν_s is obtained using $\nu_s = \nu_t + \nu_{st}$. We plot the one dimensional distribution for n_s in Fig. 2(c) for the PPL model. The measurement is consistent with the result obtained by Planck ($n_s = 0.9603 \pm 0.0073$) [2] within 1σ . From this estimation we can conclude that the PPL

model is consistent with current data set of WP, Planck and BICEP-2.

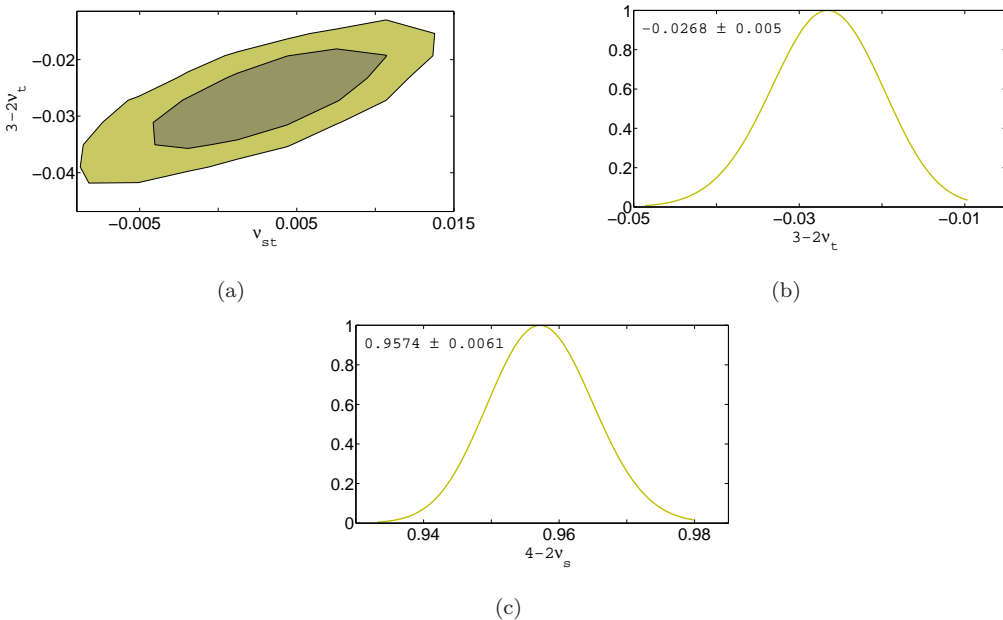


Figure 2. (a): The two dimensional likelihood contours for the $n_t = 3 - 2\nu_t$ and ν_{st} using WP+Planck+BICEP-2 likelihood. This shows a positive correlation between these two parameters and ν_{st} is consistent with zero within 95% confidence limit. (b) One dimensional marginalized probability distribution for n_t is plotted with the mean and the standard deviation. The data indicates non-zero n_t for PPL model with 5.36σ . (c): One dimensional marginalized probability distribution for the derived parameter $n_s = 4 - 2\nu_s$ where $\nu_s = \nu_t + \nu_{st}$ is plotted with the mean and standard deviation to compare with the measurement of scalar spectral index n_s obtained by Planck. This value is consistent within 1σ with the Planck measurement of $n_s = 0.9603 \pm 0.0073$ [2].

4 Discussions and Conclusions

Power Law (PL) inflationary model considers constant spectral index for both scalar and tensor perturbations as shown in Eq. 2.7. This results into only non-zero $(\frac{d \ln H}{d\phi})^2$ and all other higher order terms as zero. Perturbed Power Law (PPL) model of inflation introduced by Souradeep et al. [9], is a soft departure from PL model by considering the higher order derivatives of Hubble parameter $(\frac{d^2 \ln H}{d\phi^2})$ during inflation. This term lead to varying spectral index for scalar perturbations but the constant spectral index for tensor perturbations and hence leads to observable features on CMB power spectra C_l^{XX} for $X \in T, E, B$. In PPL model the feature of n_s, n_t and n_{run} can be incorporated by any two parameters from ν_s, ν_t, ν_{st} . With the precision measurement of CMB temperature and polarization power spectra, we can estimate these inflationary parameters which are important tools for understanding the inflationary era.

Using the recent measurements of CMB temperature and polarization from WMAP, Planck and BICEP-2, we estimate the 7 parameters cosmological model $(\Omega_b h^2, \Omega_m h^2, h, \tau, \nu_{st}, A_s, r)$ for PPL inflation. In Fig. 1, we plot the contour and one dimensional plot for 7 parameters and it shows that all the standard parameters are consistent with the PPL model. The inflationary parameter $\nu_{st} = \nu_s - \nu_t$

peaks at 0.0079 and is consistent with zero at 1.76σ . As shown in Eq. 2.9, this translates to effective mass $\frac{m_{\text{eff}}^2}{H^2} = -0.0237$. Though this value is consistent with zero within 2σ , negative value of $\frac{m_{\text{eff}}^2}{H^2}$ seems more plausible with the data. The two dimensional contour for the inflationary parameters ν_t and ν_{st} is plotted in Fig. 2(a). From the data we get non-zero value of $n_t = 3 - 2\nu_t = 0.0268$ with a 5.36σ significance shown in Fig. 2(b). This estimation can improve further with the future measurement of polarization by Planck and other experiments. We also estimate ν_s which is related to other inflationary parameters by $\nu_s = \nu_t + \nu_{st}$. In Fig. 2(c), we show that $n_s = 4 - 2\nu_s$ peaks at 0.9574 and is consistent within 1σ with $n_s = 0.9603 \pm 0.0073$ measured by Planck [2].

Our estimation shows that PPL model is viable with the recent CMB measurements. With a better constraint on the tensor to scalar ratio r from the polarization measurements by Planck and other future missions, we can estimate the values of m_{eff} and ν_t more precisely, which will help in understanding the exact nature of the inflation.

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