

Transfer of power in non-linear gravitational clustering

J. S. Bagla^{★†} and T. Padmanabhan[†]

Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411 007, India

Accepted 1996 December 15. Received 1996 November 22; in original form 1996 June 11

ABSTRACT

We investigate the transfer of power between different scales and the coupling of modes during the non-linear evolution of gravitational clustering in an expanding universe. We start with a power spectrum of density fluctuations that is exponentially damped outside a narrow range of scales, and use numerical simulations to study the evolution of this power spectrum. Non-linear effects generate power at other scales, with most power flowing from larger to smaller scales. The ‘cascade’ of power leads to equipartition of energy at smaller scales, implying a power spectrum with index $n \approx -1$. We find that such a spectrum is produced in the range $1 < \delta < 200$ for density contrast δ . *This result continues to hold even when small-scale power is added to the initial power spectrum.* Semi-analytic models for gravitational clustering suggest a tendency for the effective index to move towards a critical index $n_c \approx -1$ in this range. For $n < n_c$, power in this range grows faster than linear rate, while if $n > n_c$, it grows at a slower rate – thereby changing the index closer to n_c . At scales larger than the narrow range of scales with initial power, a k^4 tail is produced. We demonstrate that non-linear small scales do not affect the growth of perturbations at larger scales.

Key words: cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

Gravitational instability leads to growth of density inhomogeneities in an expanding universe. In Fourier space, one can study this growth as the evolution of Fourier modes of density contrast, which evolve independently of each other in the linear regime. The actual growth rate depends on the background cosmology and, in a matter-dominated universe with $\Omega=1$, the amplitude of Fourier modes grows as the expansion factor $a(t) \propto t^{2/3}$ (this is the background model considered in this paper).

The situation is quite different if the linear perturbation theory is not applicable and one has to consider the effect of coupling between different modes. The key effect of such a coupling will be the transfer of power between different length-scales. Some aspects of power transfer have been studied by using N -body simulations in the past. For example, Little, Weinberg & Park (1991) conducted a series of N -body experiments with an initial power spectrum that

was truncated at different length-scales. They concluded that the structure and appearance of a non-linear universe is dominated by a small range of scales, centred around the scale that is becoming non-linear. A detailed study of the transfer of power and the evolution of phases was carried out by Soda & Suto (1992) using the Zeldovich approximation, wherein they showed that one-dimensional collapse leads to equipartition of fluctuation power per unit wavenumber. They also carried out a few experiments with N -body simulations for gravitational clustering in three dimensions and noted that a similar trend is indicated in this case as well. Klypin & Melott (1992) studied the evolution of the ratio of kinetic energy at different scales using N -body simulations, and concluded that this ratio tends towards a universal value in the non-linear evolution of clustering and approaches the value for the $n = -1$ power spectrum.

The simplest context in which one can view power transfer is as follows: some amount of power is injected at a given scale at $t = t_1$ and the non-linear coupling is allowed to transfer it to other scales. By studying such an evolution numerically and computing the power spectra at later epochs we will be able to understand how the transfer of power takes place. In this paper, we will demonstrate a few generic features of power transfer between different modes by

[★]Present address: Institute of Astronomy, Madingley Road, Cambridge CB3 0HA.

[†]E-mail: jasjeet@ast.cam.ac.uk (JSB); paddy@iucaa.ernet.in (TP)

studying the evolution of the power spectrum in some such models, and will present theoretical arguments regarding their validity. The theoretical arguments are based on some of the recent approaches to the study of non-linear clustering (briefly discussed in the next section) and are presented in the spirit of a paradigm with which to understand the numerical results. They should, of course, not be thought of as rigorous mathematical proofs.

2 TRANSFER OF POWER

The evolution of the density contrast in Fourier space can be described by an equation of the form (Peebles 1974)

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k = 4\pi G\bar{\rho}\delta_k + Q, \quad (1)$$

where $\delta_k(t)$ is the Fourier transform of the density contrast, $\bar{\rho}$ is the background density and Q is a non-local, non-linear function which couples the mode k to all other modes k' . Coupling between different modes is significant in two cases. (i) An obvious case is one with $\delta_k \geq 1$, i.e. the amplitude of density perturbations at the scale of interest is of order unity or larger. (ii) A more interesting possibility arises for modes with no initial power (or exponentially small power). In this case non-linear coupling provides the only driving terms, represented by Q in equation (1). These generate power at the scale k through mode-coupling, provided initial power exists at some other scale. *Note that the growth of power at the scale k will now be governed purely by non-linear effects even though $\delta_k \ll 1$.* As we shall see, this fact leads to some interesting effects.

The exact solution to equation (1) is, of course, not known. However, it is possible to understand some simple features of non-linear clustering by studying the characteristics of partial differential equations that govern the evolution of the two-point correlation function (see Peebles 1980 and Nityananda & Padmanabhan 1994). One may draw the following conclusions from such a study for hierarchical models (for more details see Padmanabhan 1996a,b).

(i) The power at a scale x , at an epoch a , is related closely to the linearly extrapolated power at a scale l where

$$l = x [1 + \bar{\xi}(a, x)]^{1/3}; \quad \bar{\xi}(a, x) \equiv \frac{3}{x^3} \int_0^x \xi(a, y) y^2 dy. \quad (2)$$

Here ξ is the two-point correlation function and a is the scalefactor. $\bar{\xi}(x, a)$ is the mean correlation averaged up to the scale x at the epoch a (Hamilton et al. 1991).

(ii) N -body simulations of hierarchical models show that the non-linear mean correlation function at x relates to the linear mean correlation function at l in an almost universal manner (Hamilton et al. 1991). The relation can be approximated by the following map (Bagla & Padmanabhan 1995),

$$\bar{\xi}(a, x) = \begin{cases} \bar{\xi}_L(a, l) & (\bar{\xi}_L < 1.2, \bar{\xi} < 1.2) \\ 0.7 \bar{\xi}_L(a, l)^3 & (1.2 < \bar{\xi}_L < 6.5, 1.2 < \bar{\xi} < 195) \\ 11.7 \bar{\xi}_L(a, l)^{3/2} & (6.5 < \bar{\xi}_L, 195 < \bar{\xi}) \end{cases} \quad (3)$$

provided we can assume stable clustering of virialized objects in the non-linear regime.

The relation between $\bar{\xi}(a, x)$ and $\bar{\xi}_L(a, l)$ was originally given by Hamilton et al. (1991) based on the N -body simulation data. Equation (3) is an approximate fit to N -body data that has the advantage of being a power-law fit in each of the three regimes. The three regimes used in this power-law fit have some physical relevance, as has been shown in a recent paper (Padmanabhan 1996a) that also justifies the slopes of the power laws used in each regime. Several authors have provided more exact fitting functions to describe the three regimes in a unified manner (see Hamilton et al. 1991; Peacock & Dodds 1996; Jain, Mo & White 1995). There is also some controversy regarding the actual index in the intermediate regime ($\bar{\xi} \propto \bar{\xi}_L^3$) and whether the relation given above is truly 'universal' (see for example Peacock & Dodds 1996; Jain, Mo & White 1995; Padmanabhan et al. 1996). However, all the models suggested in the literature lead to the following feature: in the quasi-linear regime, there exists a critical index n_c such that the growth of power is faster than a^2 for spectra with $n < n_c$ and slower than a^2 for spectra with $n > n_c$.

With the simple scaling given above, it is easy to show that $n_c = -1$; but if a more accurate fitting function is used then this value may vary around -1 . For the sake of illustration we shall take $n_c = -1$ but the general arguments in this paper do not depend on the particular choice of n_c . The existence of such an index suggests that, during the evolution, there will be a tendency for the power spectra to acquire such an index. This conclusion – which is a direct consequence of the results described above – is worth testing in numerical simulations.

3 NUMERICAL EXPERIMENTS

We shall now try to see what these results might imply for the transfer of power in a more general context, but before describing the results of such an experiment, we would like to spell out the theoretical expectations. This is important because it shows – up front – what the results should be and helps us to understand them. Needless to say, the real justification for the claims made below comes from the results of the simulations.

To begin with, it is well known that the power transfer in gravitational clustering is mostly from large scales to small scales. (A significant exception is the generation of the k^4 tail which we shall discuss towards the end of the paper.) This is clearly borne out by the numerical experiments of Little et al. (1991) that showed that the structure and appearance of non-linear structures in simulations is largely independent of the initial power at small scales. Suppose we start with a power spectrum that is centred at some scale $k_0 = 2\pi/L_0$ and has a small width Δk . The first structures to form in such a system are voids with a typical diameter L_0 . The formation and fragmentation of sheets bounding the voids leads to generation of power at scales $L < L_0$. First-bound structures form at the mass scale corresponding to L_0 . In such a model the linear $\bar{\xi}$ at $L < L_0$ is nearly constant with an effective index of $n \approx -3$. Assuming we can use equation (3) with the local index in this case, we expect the power to grow very rapidly as compared to the linear rate of

a^2 . (The rate of growth is a^6 for $n = -3$ and a^4 for $n = -2.5$.) The different rate of growth for regions with different local indices will lead to a steepening of the power spectrum, with an accompanying slowing down of the rate of growth. This rapid growth is expected in the quasi-linear

regime and should lead to a power spectrum with the critical index n_c as its slope.

Consider next a more complex situation, with initial power concentration around two scales L_0 and $L_1 < L_0$. If we assume that the power at L_1 has a higher amplitude than the

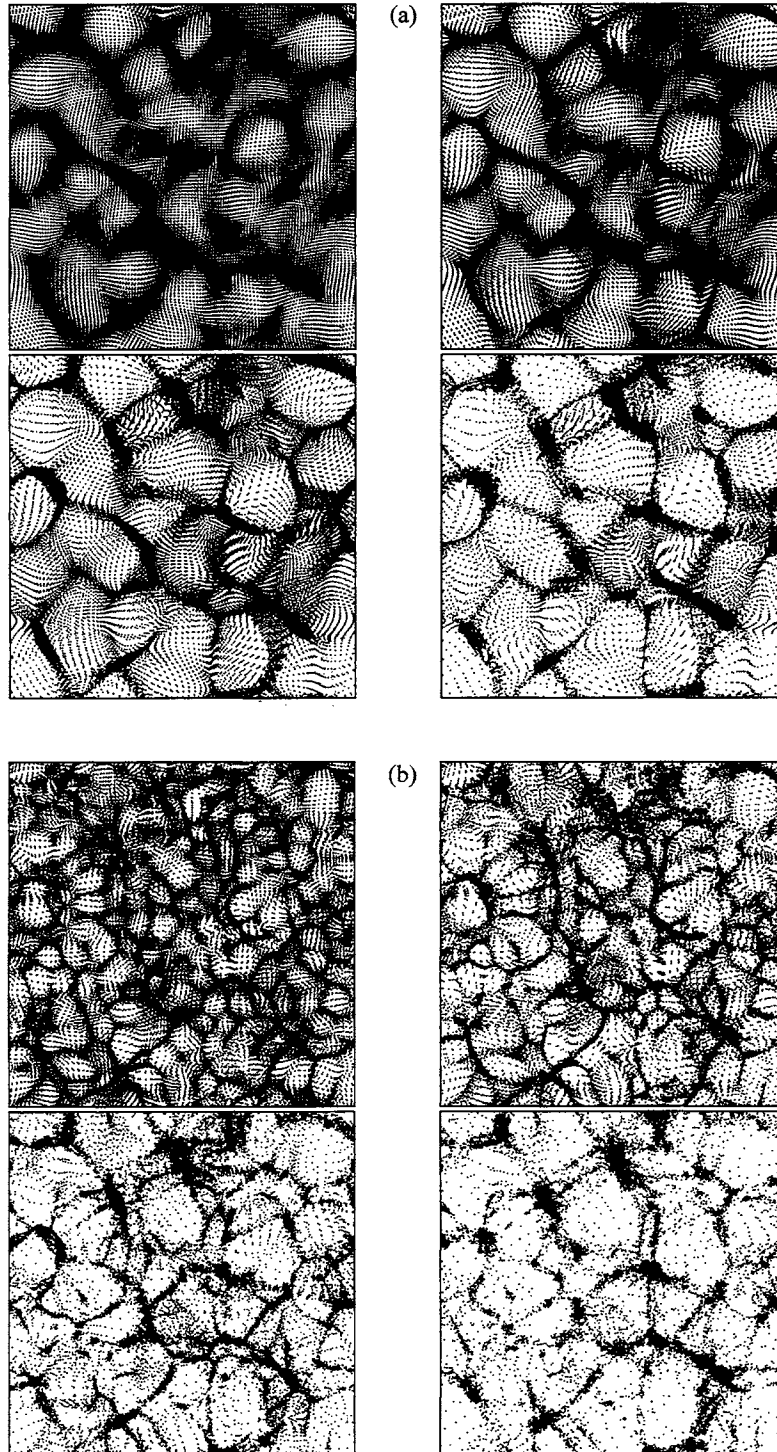


Figure 1. (a) This figure shows a slice from simulations of model I at four epochs. The upper left panel corresponds to $a = 0.25$, the upper right panel to $a = 0.5$, the lower left panel to $a = 1$ and the lower right panel to $a = 2$. The thickness of the slice is $6L$ and the width and height are $128L$, where L is one grid length. (b) This figure shows the corresponding slice from simulations of model II. (c) This figure shows the corresponding slice from simulations of model III.

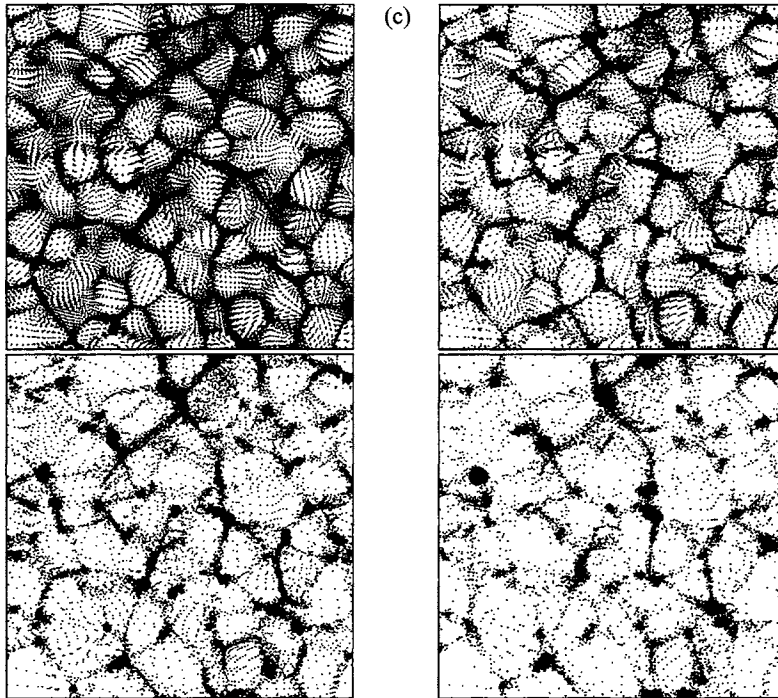


Figure 1 – continued

smaller scale(s) will reach the quasi-linear phase before the larger one(s) and – in the subsequent evolution – will approach the critical index n_c . We again expect the spectrum to have $n = n_c$ at scale $L < L_1$. If the largest non-linear scales dominate the non-linear evolution of perturbations, as shown in a limited context (Little et al. 1991), then the spectrum at scales $L_1 < L < L_0$ should also approach one with an index $n = n_c$ after the larger scale becomes non-linear. However, if the extra small-scale power implies a much steeper spectrum, we should expect the small-scale power to grow at a slower rate in order to match up with the $n = n_c$ power spectrum at larger scales. If this is indeed the case, then the power spectrum is effectively driven by the largest scale which is entering the quasi-linear phase at the epoch of consideration. This scale is expected to influence the scales in the quasi-linear regime.

The question we would like to address in this paper is: to what extent is the above qualitative picture supported by numerical simulations? To answer this question we will use N -body simulations and study the evolution of some toy models. We begin with a brief summary of the N -body code used for these numerical experiments. For greater details of the particle mesh (PM) code used here see Bagla & Padmanabhan (1996).

All simulations used a PM code and 128^3 particles in a 128^3 box. In the units of length used here, each side of the simulation box measures 128 units. We used the triangular shaped cloud (TSC) for interpolation and the ‘poor man’s’ Poisson solver for solving the Poisson equation in Fourier space. Force was computed in Fourier space from the potential. For more details on PM codes, see Hockney & Eastwood (1980). Numerical integration of the equation of motion was done using the standard leap-frog method. Step size for integration was chosen by enforcing an upper limit

on the maximum displacement of a particle in one step. The ‘woe factor’ ($10\Delta t/t_{\text{dyn}}$) for simulations used here is about 0.2.

In all the figures for power spectrum in this paper, we shall show only that region in the k -space where the uncertainty is small. The smallest scale shown in these figures is $1.5L_{\text{ny}}$ where L_{ny} is the Nyquist scale. A cube of this size, or larger, contains a sufficiently large number of particles for the error in the power spectrum to be fairly small at these scales. The largest scale used in these plots is $L = L_{\text{box}}/2$, thus the averaging for the power spectrum is done over at least eight independent regions in the simulation box.

We used three models for our study. Parameters of these models were chosen as follows.

(i) The initial power spectrum for model I, the ‘reference’ model, was a Gaussian peaked at the scale $k_0 = 2\pi/L_0$; $L_0 = 24$ and having a spread of $\Delta k = 2\pi/128$. The amplitude of the peak was chosen so that $\Delta_{\text{lin}}(k_0 = 2\pi/L_0, a = 0.25) = 1$, where $\Delta^2(k) = k^3 P(k)/(2\pi^2)$ and $P(k)$ was the power spectrum. Needless to say, the simulation started while the peak of the Gaussian was in the linear regime [$\Delta(k_0) \ll 1$].

(ii) Model II had initial power concentrated in two narrow windows in k -space. In addition to power around $L_0 = 24$ as in model I, we added power at $k_1 = 2\pi/L_1$; $L_1 = 8$ using a Gaussian with the same width as that used in model I. Amplitude at L_1 was chosen to be five times higher than that at $L_0 = 24$, thus $\Delta_{\text{lin}}(k_1; a = 0.05) = 1$.

(iii) Model III was similar to model II, with the small-scale peak shifted to $k_1 = 2\pi/L_1$; $L_1 = 12$. The amplitude of the small-scale peak was the same as in model II.

We now describe the results of our simulations of these models. The panels of Fig. 1 show a slice from the simula-

tion volume at four epochs for each model. A comparison of non-linear structures in these panels shows that, at early epochs, the non-linear objects in these models appear to be very different from each other. Dissimilarities between different models slowly disappear, however, and their appearance at later epochs is dominated by the larger wave-mode. This is true even for model III, which has a lot of small-scale power. (If we define an effective index of the power spectrum by joining the peaks of two Gaussians, then the effective index $n_{\text{eff}} \sim 0$ for model II and $n_{\text{eff}} \sim 2$ for model III.) It is clear from these pictures that the large-scale mode takes a longer time to assert itself for the case with greater small-scale power.

To compare the evolution of these models in a more quantitative fashion, we use the power spectrum. The top panel of Fig. 2 shows the evolution of the power spectrum for model I. The y-axis is $\Delta(k)/a$, the power per logarithmic scale divided by the linear growth factor. This is plotted as a function of scale $L = 2\pi/k$ for different values of scalefactor $a(t)$; curves are labelled by the value of a . As we have divided the power spectrum by its linear rate of growth, the change of shape of the spectrum occurs strictly because of non-linear mode coupling. It is clear from this figure that power at small scales grows rapidly and saturates to growth at a rate close to the linear rate (shown by crowding of curves) at later epochs. The effective index for the power spectrum approaches $n = -1$ within the accuracy of the simulations. Thus this figure clearly demonstrates the general features we expect from our understanding of scaling relations.

The other two panels of Fig. 2 show the corresponding curves for models II and III respectively. These models had power at small scale in addition to the power at large scales. The large-scale power is same in all the models and the initial conditions for the relevant region in k -space had the same initial phases, making comparison meaningful. These figures show that when large scales become non-linear, power at small scales grows at a rate *slower* than the linear rate in the quasi-linear regime, until we obtain a power spectrum with $n \approx -1$ for $L < L_0$. *The amplitude of the power spectrum for models II and III at this stage is the same as the corresponding power spectrum in model I.* A comparison of the lower panels also shows that the approach to n_c is slower for the model with more small-scale power (model III).

Fig. 3 shows power spectra of all three models at a late epoch. At this epoch $\Delta_{\text{in}}(k_0) = 4.5$, and it is clear from this figure that the power spectra of these models are very similar to one another.

The growth of power at three different scales, $L = 8, 12$ and 24 , is plotted for the three models in Fig. 4. The thick, dashed lines represent models I, II and III respectively. Curves have been labelled by the length-scale. The thick lines demonstrate power generated by mode coupling at the two smaller length-scales. From the dashed lines for model II, one can see that power at $L = 8$ decreases with respect to the linear rate of growth so as to asymptotically match with the amplitude in the reference model, i.e. model I. The same effect is seen in the dot-dashed lines for model III. This figure also shows that the existence of power at $L = 8$ does not influence the evolution of power at $L = 12$, if power exists at a larger scale. In this sense, gravitational clustering

transfers power from larger to small scales [‘cascades’] but this does not occur in the opposite direction [‘does not inverse cascade’]. Figs 3 and 4 demonstrate this fact very clearly.

3.1 Critical index

The two panels of Fig. 5 illustrate two features related to the existence of fixed points in a clear manner. In the top panel we have plotted the index of growth $n_a \equiv [\partial \ln \xi(a, x) / \partial \ln a]_x$ as a function of ξ in the quasi-linear regime. Curves correspond to an input spectrum with index $n = -2, -1, 1$. The dashed horizontal line at $n_a = 2$ represents the linear growth rate. An index above the horizontal line will represent a rate of growth faster than the linear growth rate and the one below will represent a rate which is slower than the linear rate. It is clear that, in the quasi-linear regime, the curve for $n = -1$ closely follows the linear growth, while $n = -2$ grows faster and $n = 1$ grows slower; so the critical scale index is $n_c \approx -1$. The curves are based on the fitting formula from Hamilton et al. (1991). Other fitting formulae suggested by Jain et al. (1995) and Peacock & Dodds (1996) give somewhat different curves, but all these models have fixed points close to $n_c = -1$.

The lower panel of Fig. 5 shows the slope $n_x = -3 - (\partial \ln \xi / \partial \ln x)_a$ of ξ for different power-law spectra. It is clear that n_x crowds around $n_c \approx -1$ in the quasi-linear regime.

As an aside, we can derive an upper limit on the index of an arbitrary power spectrum. Consider the pair conservation equation (equation 20 of Nityananda & Padmanabhan 1994) for a set of particles. We can rewrite this equation by using the definition of the index of the power spectrum n_x and the index of the rate of growth n_a ,

$$n_a = h \left(\frac{3}{\xi} - n_x \right). \quad (4)$$

For the growing mode, the amplitude of perturbations is a monotonically increasing quantity, ensuring that the index n_a is always positive. The pair velocity h is also a positive quantity at scales with $\xi > 0$. Therefore the index n_x satisfies the inequality

$$n_x < \frac{3}{\xi}. \quad (5)$$

We have plotted the curve $3/\xi$ in the lower panel of Fig. 5 as a dotted line. This inequality must be satisfied by non-linear structures that have grown out of small inhomogeneities via gravitational instability. Any distribution of mass that does not satisfy this inequality could not have formed as a result of gravitational collapse only.

The index $n_c = -1$ corresponds to the isothermal profile with $\xi(a, x) = a^2 x^{-2}$ and has some interesting features to recommend it as a candidate for fixed point. For example, in the $n = -1$ spectrum each logarithmic scale contributes the same amount of correlation potential energy (for more details, see Padmanabhan 1996a). If the regime is modelled by scale-invariant radial flows, then the kinetic energy will scale in the same way. It is conceivable that the flow of power leads to such an equipartition state as a fixed point,

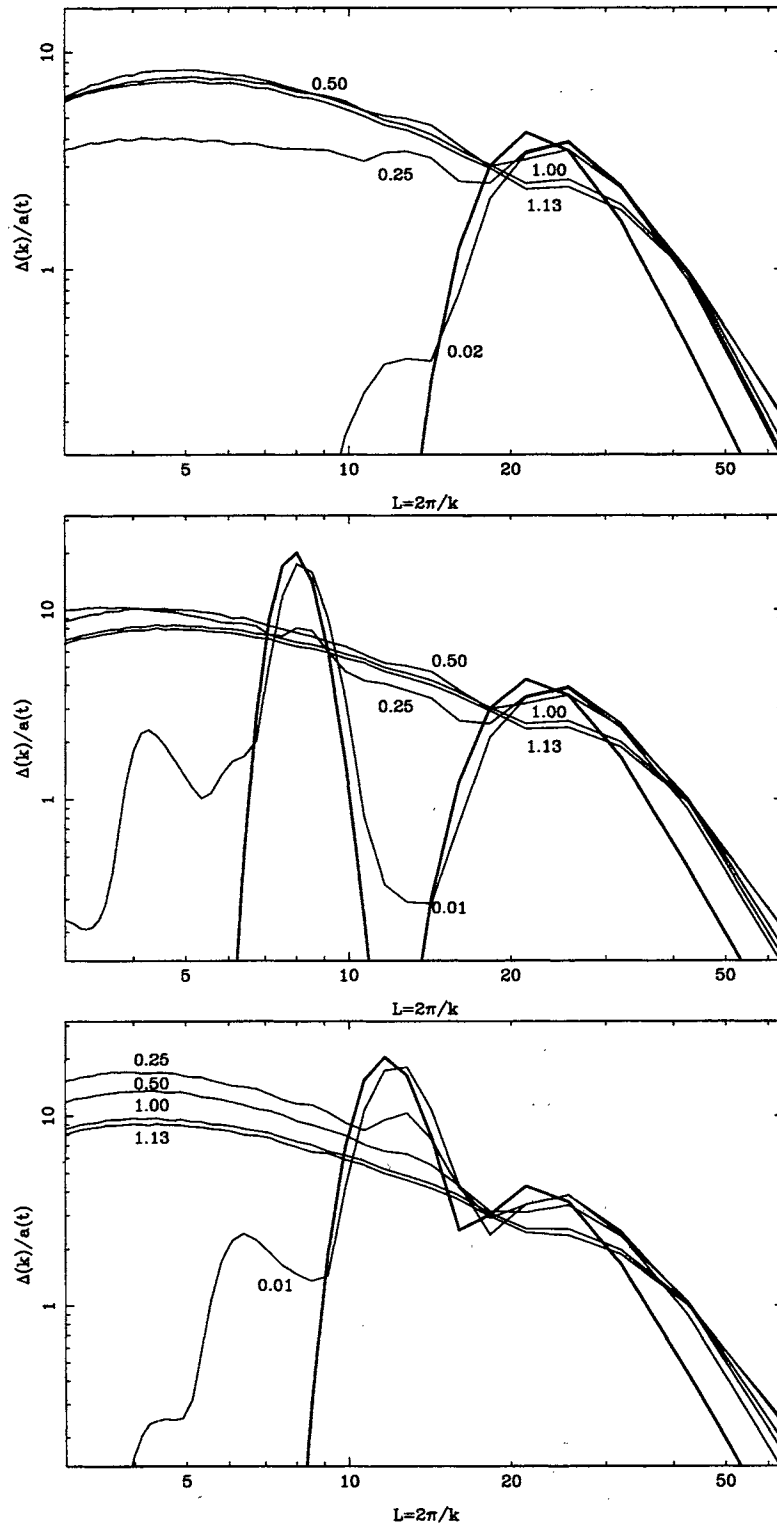


Figure 2. This figure shows the evolution of power spectra for the three models. Thick lines show the linear power spectrum for these models and thin lines show the non-linear power spectrum from N -body simulations. The y-axis is the square root of power per logarithmic scale divided by a . With the linear growth rate divided out, only non-linear evolution can modify the spectrum in this plot. The x-axis is the length-scale. In model I (top panel) the initial power spectrum is a Gaussian peaked at a length-scale of $L_0 = 24$ and amplitude-adjusted to make it reach non-linearity at $a = 0.25$. Power spectra for different epochs are labelled by the scalefactor. This figure demonstrates that power is generated at smaller scales, and at late times this power saturates to a power spectrum with $n \approx -1$. The second panel shows the evolution of model II. In this model there is additional power (apart from what is there in model I) at $L_1 = 8$, with an amplitude five times higher than the Gaussian peaked at $L = 24$. The third panel shows the same curves for model III. In this model there is additional power (apart from what is there in model I) at $L_1 = 12$ with an amplitude five times higher than the Gaussian peaked at $L = 24$. Notice that at late times, power spectra of all three models are very similar.

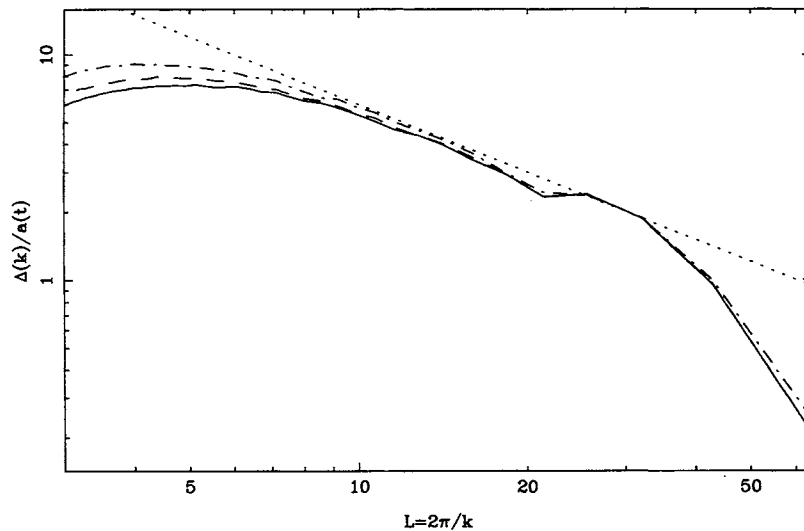


Figure 3. Comparison of power spectra for the three models at a late epoch. This figure shows that non-linear evolution has erased large differences between model I (thick line), model II (dashed line) and model III (dot-dashed line). The dotted straight line corresponds to a power spectrum with index $n = -1$.

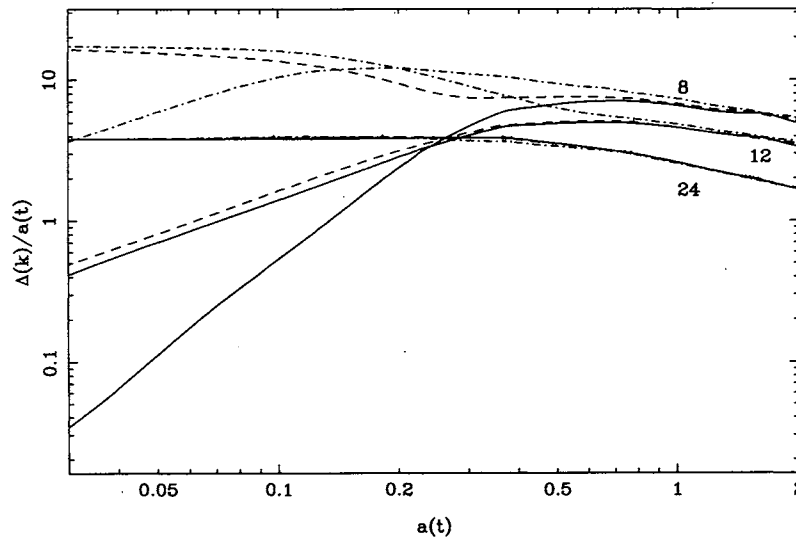


Figure 4. Growth of power at three scales $L = 8, 12, 24$ for model I (thick line), model II (dashed line) and model III (dot-dashed line). This figure shows that at late times the power in all three models approaches the same value. In particular, the growth of power at $L = 24$ is not influenced by additional power present at smaller scales in models II and III, showing that non-linear small scales do not influence larger scales.

though it is difficult to prove such a result in any generality. Equipartition of kinetic energy and the role of $n = -1$ as the transition index have been pointed out previously by Klypin & Melott (1992). They studied the evolution of kinetic energy at different scales for various models to arrive at this conclusion. It would be interesting to see whether the existence of such a fixed point can be derived starting from first principles, say, from the study of equation (1).

Equations (3) also show that, in the non-linear regime with $\xi > 200$, the fixed point is $n_{c,NL} = -2$. Speculating along similar lines, we would expect the gravitational clustering to lead to an x^{-1} profile at the non-linear end changing over to x^{-2} in the quasi-linear regime.

3.2 Influence of small scales on large scales

Let us now consider the flow of power to larger scales in gravitational clustering. It is well-known that the motion of particles conserving momentum leads to a k^4 tail to the power spectrum, if the original power was sub-dominant to k^4 at small k (Zeldovich 1965; Peebles 1974). Fig. 6 shows the tail for a simulation which had initial power that peaked around $k_0 = 2\pi/L_0$; $L_0 = 8$. The initial power spectrum was a narrow Gaussian with amplitude adjusted so that the peak reached non-linearity at $a = 0.25$. In the initial stages of evolution, there is evidence for a k^4 tail. The amplitude of the tail grows as a^4 initially, in comparison with the linear rate a^2 , and slows down at later stages. If we assume the

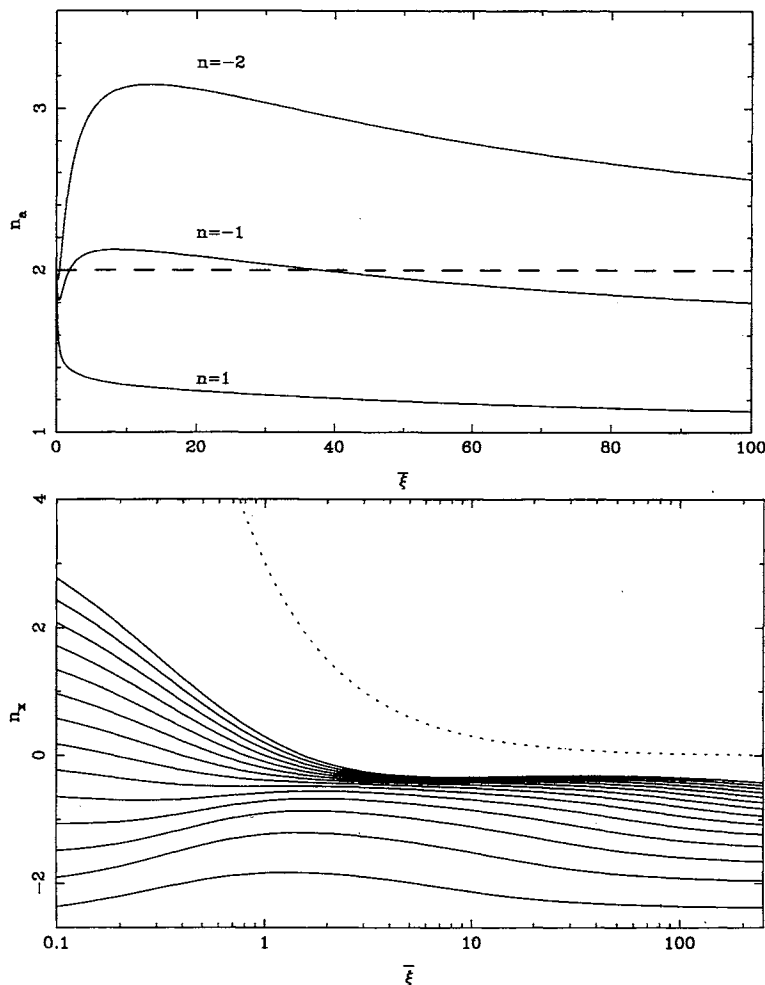


Figure 5. The top panel shows the exponent of the rate of growth of density fluctuations as a function of amplitude. We have plotted the rate of growth for three scale-invariant spectra $n = -2, -1, 1$. The dashed horizontal line indicates the exponent for linear growth. For the range $1 < \delta < 100$, the $n = -1$ spectrum grows as in linear theory; $n < -1$ grows faster and $n > -1$ grows slower. The lower panel shows the evolution of index $n_x = -3 - (\partial \ln \bar{\xi} / \partial \ln x)_a$ with $\bar{\xi}$. Indices vary from $n = -2.5$ to $n = 4.0$ in steps of 0.5. The tendency for n_x to crowd around $n_x = -1$ is apparent in the quasi-linear regime. The dashed curve is a bounding curve for the index ($n_x < 3/\bar{\xi}$) if perturbations grow via gravitational instability.

quasi-linear evolution is governed by equation (3) then the index $n=4$ will change to $n = -3/8$. In Fig. 6 we have plotted a line with this slope for reference. There is some evidence for the slope approaching this value; the evolution of the slope is definitely in the right direction. (It is clear from the lower panel of Fig. 5 that the index of an $n=4$ spectrum evolves rapidly, even when $\bar{\xi} < 1$. For example, the index evolves to less than 3 for $\bar{\xi} = 0.1$.) Shandarin & Melott (1990) have studied the evolution of the k^4 tail in detail using *two-dimensional* numerical simulations. They also note the change of slope and a slow decline in the rate of growth of the k^4 tail.

We have plotted the rate of growth for a few modes in the k^4 tail as a function of scalefactor. This is shown in Fig. 7, which also has two reference lines for growth proportional to a^2 and to $a^{3/8}$. The latter is the rate of growth in the quasi-linear regime obtained from the scaling relation in equations (3). It is clear that the rate of growth for all the modes shown here starts out as a^2 but drops to a rate slower than a at late times.

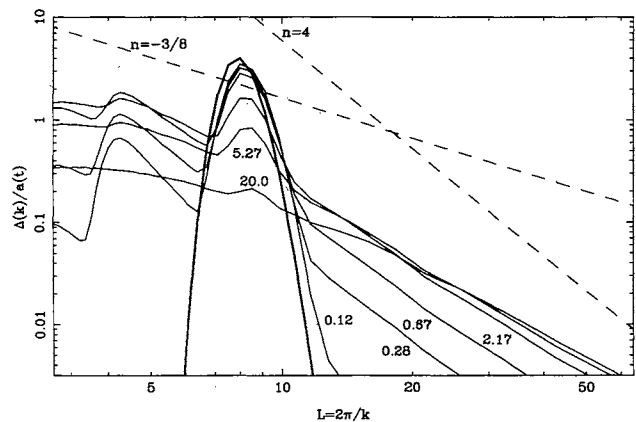


Figure 6. Influence of small scales on large scales. The axes are the same as in Fig. 2. Thick lines shows the initial power spectrum and other curves show the non-linear spectrum at various epochs, labelled by a . This figure shows the generation and evolution of the k^4 tail. It is seen that the slope of this tail changes rapidly and later goes over to the expected quasi-linear index of $n = -3/8$. Reference lines with these slopes are represented as dashed lines.

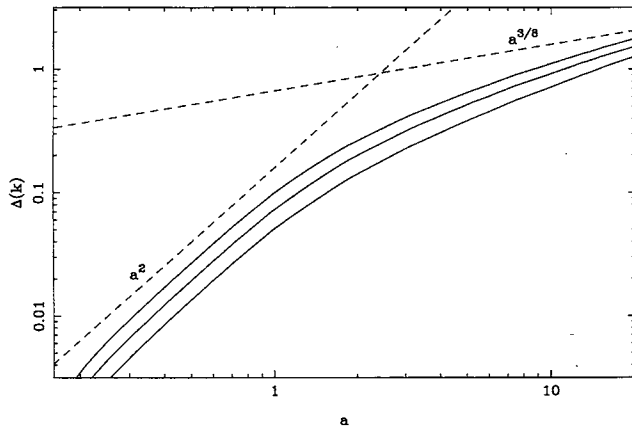


Figure 7. This figure shows the rate of growth for power at some modes in the k^4 tail. For comparison we have plotted a dashed line for growth proportional to a^2 and another for $a^{3/8}$. This figure clearly shows that at early times the square root of the power grows in proportion with a^2 and at late times the rate of growth drops below a . However, it does not reach the expected quasi-linear rate until the end of this simulation.

We stress that the evolution of power outside the band containing initial power is entirely the result of power transfer by non-linear mode coupling. While at smaller scales this transfer is significant and leads to equal amounts of kinetic energy per logarithmic waveband, the flow of power to larger scales is less. One can easily see that a $k^{-3/8}$ spectrum will contribute an amount of energy $k^{5/8}$ per logarithmic band. There is less energy at larger wavelengths, i.e. at smaller k . It is, of course, understandable on general grounds that large scales will not be affected by the strong non-linearities in the small scales (see e.g. the discussion in section 28 of Peebles 1980).

4 CONCLUSIONS

In conclusion, we note that the transfer of power in gravitational clustering shows some generic patterns which are worth exploring further. Fig. 4 demonstrates that small

scales (even if highly non-linear) do not influence larger scales. The dominance of cascading over inverse cascading, as well as the existence of a universal index for the induced power spectrum, is reminiscent of fluid turbulence. It may be possible to use some of the concepts from the study of turbulence to make ideas like critical indices, fixed points, equipartition of energy, etc., sharper, and to build a new paradigm for understanding non-linear gravitational clustering.

ACKNOWLEDGMENTS

This work was completed while TP was visiting the Department of Astrophysics at Princeton University, supported by NASA grant NAG5-2759. JSB thanks CSIR India for continued support. The authors thank J. P. Ostriker and P. J. E. Peebles for useful discussions.

REFERENCES

- Bagla J. S., Padmanabhan T., 1995, IUAA preprint 8/95
 Bagla J. S., Padmanabhan T., 1997, IUAA preprint 39/96 (Pramana, in press)
 Hamilton A. J. S., Kumar P., Edward L., Matthews A., 1991, ApJ, 374, L1
 Hockney R. W., Eastwood J. W., 1988, Computer Simulations using Particles, Institute of Physics Publ.
 Jain B., Mo H. J., White S. D. M., 1995, MNRAS, 276, L25
 Klypin A. A., Melott A. L., 1992, ApJ, 399, 397
 Little B., Weinberg D. H., Park C., 1991, MNRAS, 253, 29
 Nityananda R., Padmanabhan T., 1994, MNRAS, 271, 976
 Padmanabhan T., 1996a, MNRAS, 278, L29
 Padmanabhan T., 1996b, in Lahav O., Terlevich E., Terlevich R., eds, Proc. 36th Herstmonceux Conf. Cambridge Univ. Press, Cambridge, p. 207
 Padmanabhan T., Cen R., Ostriker J. P., Summers F. J., 1996, ApJ, 466, 604
 Peacock J. A., Dodds S. J., 1996, MNRAS, 280, 19
 Peebles P. J. E., 1974, A&A, 32, 391
 Peebles P. J. E., 1980, Large Scale Structure of the Universe. Princeton Univ. Press, Princeton NJ
 Shandarin S. F., Melott A. L., 1990, ApJ, 364, 396
 Soda J., Suto Y., 1992, ApJ, 196, 379
 Zeldovich Ya. B., 1965, Adv. Astron. Astrophys., 3, 241