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Direct Particle Formulation of Mach's Principle

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4. Other Formulations of Mach's Principle

Introduction

This chapter illustrates the strikingly different ways in which different people have attempted to implement Mach's Principle. Like Brans and Dicke, Hoyle and Narlikar set out to realize in a systematic manner Einstein's contention (p. 180) that in a Machian approach the *inertial mass* of any body must be determined by a kind of interaction of that body with all the other masses in the universe. Narlikar gives a particularly clear rationale for such an approach at the beginning of his paper.

In contrast, Raine takes as his point of departure Einstein's brief 1918 paper (pp. 185–186) in which he actually coined the expression Mach's Principle and gave a formal definition of it: The metric tensor in a Machian solution of general relativity must be completely determined by the energy–momentum tensor of *matter*, understood in the narrow sense (i.e., gravitational waves are not to contribute). The development of the Green's function approach (or integral formulation) as a way to give rigorous mathematical expression to this idea must represent one of the most remarkable examples of simultaneous discovery in science – it was developed independently by Al'tshuler, Lynden-Bell, Sciama and Waylen, and Gilman, as Raine recounts.

Finally, Bleyer and Liebscher's paper is the most radical attempt in this volume to relate the distribution of matter in the universe as a whole to the deep structure of local physics, in this case the causal (light-cone) structure of Minkowski space. This is work in the spirit of Dicke's 'generalized Mach's Principle,' in accordance with which one seeks systematically for ways in which the universe at large might influence local physics (cf. the remarks of Brill and Brans, pp. 333 and 337).

J.B.B.

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Direct Particle Formulation of Mach's Principle

Jayant V. Narlikar

1. Introduction

There are two ways of measuring the Earth's spin about its polar axis. By observing the rising and setting of stars the astronomer can determine the period of one revolution of the Earth around its axis: the period of $23^h56^m4^s.1$. The second method employs a Foucault pendulum whose plane gradually rotates around a vertical axis as the pendulum swings. Knowing the latitude of the place of the pendulum, it is possible to calculate the Earth's spin period. The two methods give the same answer.

At first sight this does not seem surprising. Closer examination, however, reveals why the result is nontrivial. The first method measures the Earth's spin period against a background of distant stars, while the second employs the standard Newtonian mechanics in a spinning frame of reference. In the latter case, we take note of how Newton's laws of motion get modified when their consequences are measured in a frame of reference spinning relative to the 'absolute space' in which these laws were first stated by Newton.

Thus, implicit in the assumption that equates the two methods is the coincidence of absolute space with the background of distant stars. It was Ernst Mach in the last century who pointed out that this coincidence is nontrivial. He read something deeper in it, arguing that the postulate of absolute space that allows one to write down the laws of motion and arrive at the concept of inertia is somehow intimately related to the background of distant parts of the universe. This argument is known as 'Mach's Principle,' and we will analyze it further.

When expressed in the framework of the absolute space, Newton's second law of motion takes the familar form

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$$\mathbf{P} = m\mathbf{f}. \tag{1}$$

This law states that a body of mass m subjected to an external force $\mathbf{P}$ experiences an acceleration $\mathbf{f}$. Let us denote by S the coordinate system in which $\mathbf{P}$ and $\mathbf{f}$ are measured.

Newton was well aware that his second law has the simple form (1) only with respect to S and those frames that are in uniform motion relative to S. If we choose another frame S' that has an acceleration $\mathbf{a}$ relative to S, the law of motion measured in S' becomes

$$\mathbf{P}' \equiv \mathbf{P} - m\mathbf{a} = m\mathbf{f}'. \tag{2}$$

Although (2) outwardly looks the same as (1), with $\mathbf{f}'$ being the acceleration of the body in S', something new has entered into the force term. This is the term $m\mathbf{a}$, which has nothing to do with the external force but depends solely on the mass m of the body and the acceleration $\mathbf{a}$ of the reference frame relative to the absolute space. Realizing this aspect of the additional force in (2), Newton termed it 'inertial force.' As this name implies, the additional force is proportional to the inertial mass of the body.

According to Mach, the Newtonian discussion was incomplete in the sense that the existence of the absolute space was postulated arbitrarily and in an abstract manner. Why does S have a special status in that it does not require the inertial force? How can one physically identify S without recourse to the second law of motion, which is based on it?

To Mach the answers to these questions were contained in the observation of the distant parts of the universe. It is the universe that provides a background reference frame that can be identified with Newton's frame S. Instead of saying that it is an accident that Earth's rotation velocity relative to S agrees with that relative to the distant parts of the universe, Mach took it as proof that the distant parts of the universe somehow enter into the formulation of local laws of mechanics.

One way this could happen is by a direct connection between the property of inertia and the existence of the universal background. To see this point of view, imagine a single body in an otherwise empty universe. In the absence of any forces, (1) becomes

$$m\mathbf{f} = 0.$$

What does this equation imply? Following Newton we would conclude that $\mathbf{f}=0$, that is, that the body moves with uniform velocity. But we now no longer have a background against which to measure velocities. Thus $\mathbf{f}=0$ has no operational significance. Rather, $\mathbf{f}$ should be completely indeterminate. And it is not difficult to see that such a conclusion follows naturally provided we argue that

$$m = 0. \tag{3}$$

In other words, the measure of inertia depends on the existence of the background in such a way that in the absence of the background the measure vanishes! This aspect introduces a new feature into mechanics not considered by Newton. The Newtonian view that inertia is the property of matter has to be augmented to the statement that inertia is the property of matter as well as of the background provided by the rest of the universe.

Einstein, an avid reader of Mach, was impressed by this chain of reasoning and hoped that his theory of gravity would turn out to incorporate Mach's principle. This hope was not realized in the end. There are several anti-Machian solutions in general relativity.

For example, there are empty space solutions that are nontrivially different from the flat spacetime of special relativity. In these solutions $R_{ik}=0$ but $R_{iklm}\neq 0$. What do the timelike geodesics in such spacetime mean? With no 'background' of matter why are these trajectories of 'particles under no force' singled out?

On a second count there are cosmological solutions of Einstein's equations wherein the distant background *rotates* with respect to the local inertial frame. Ironically, the classic paper of Kurt Gödel (1949) which produced one such model appeared in the 70th birthday festschrift for Einstein. By then, however, Einstein himself had lost his enthusiasm for Mach's Principle. In his autobiographical notes he writes (Einstein 1949):

> Mach conjectures that in a truly rational theory inertia would have to depend upon the interaction of the masses, precisely as was true for Newton's other forces, a conception which for a long time I considered as in principle the correct one. It presupposes implicitly, however, that the basic theory should be of the general type of Newton's mechanics: masses and their interaction as the original concepts. The attempt at such a solution does not fit into a consistent field theory, as will be immediately recognized.

Although Einstein himself moved away from Mach's Principle, there were others who felt its impact and sought to give expression to it in quantitative theories of gravity. For example, Dennis Sciama (1953) and Carl Brans and Robert Dicke (1961), among others, proposed alternative theories of gravity. However, these were field theories, since the general belief (shared by these authors with Einstein) was that field theories alone provide a proper description of physics.

Nevertheless, action-at-a-distance theories can also foot the bill if they are properly formulated and applied with the right cosmological boundary conditions. I will discuss this possibility here.

2. The Hoyle-Narlikar Formulation

In 1964, Fred Hoyle and I proposed an action-at-a-distance theory of inertia that directly incorporated Mach's principle. In this theory the inertial mass of ath particle ($a=1, 2, \ldots$) at world point X was given by

$$m_a(X) = \lambda_a \sum_{b \neq a} \lambda_b \int G(X,B)\, ds_b, \tag{4}$$

where ds_b is the element of proper time on the worldline of particle b and λ_b a coupling constant. The action at a distance is through the two-point scalar propagator G satisfying the relation

$$\Box G(X,B) + \frac{1}{6} R G(X,B) = \frac{\delta_4(X,B)}{\sqrt{-g(X)}}, \tag{5}$$

and we define

$$m^{(b)}(X) = \lambda_b \int G(X,B)\, ds_b. \tag{6}$$

[$\Box$ and R in (5) are evaluated at X.]

The propagator G is symmetric with respect to its two points:

$$G(X,B) = G(B,X). \tag{7}$$

The rationale for these formulas will be considered next.

First we notice that the interaction conveys the property of inertia from one particle to another. Next, from (7) we also learn that the interaction works symmetrically between pairs of particles. Finally, the wave equation (5) ensures that the mass interaction propagates with the speed of light.

3. A Digression into Electromagnetic Theory

What are these functions $m^{(b)}(X)$? That they communicate the property of inertia from particles b to any particle placed at the spacetime point X is clear from the context. To arrive at a suitable form for them we take hints from action-at-a-distance electromagnetism, in which it is usual to introduce electromagnetic disturbances that arise specifically from sources, that is, from moving electrical charges. Accordingly, we introduce the 4-potential $A_i^{(b)}(X)$ as denoting the electromagnetic effect at X from the electric charge b. The $A_i^{(b)}(X)$ satisfies the wave equation

$$\Box A_i^{(b)} + R_i^k A_k^{(b)} = 4\pi J_i^{(b)}, \tag{8}$$

where $J_i^{(b)}$ is the 4-current generated by the charge b. The solution of (8) may be written in the integral form

$$A_i^{(b)}(X) = 4\pi \int e_b G_{ik}(X, B)\, db^k, \tag{9}$$

where $G_{ik}(X, B)$ is a Green's function of the wave operator $(g_i^k \Box + R_i^k)$. The well-known Coulomb potential is a special case of (8).

The Green's function is not uniquely fixed from the form of the wave operator alone. Boundary conditions must also be specified. The customary boundary condition is imposed by causality; that is, the influence from B to X must vanish if X lies outside the future light cone of B. The Green's function satisfying this condition is called the *retarded Green's function*. We will denote such a Green's function with a superscript R. Similarly, a Green's function confined to the past light cone of B is called the *advanced Green's function* and is denoted with a superscript A.

These Green's functions have played a key role in action-at-a-distance theories. It was originally believed that action at a distance must be instantaneous and hence inconsistent with the framework of special relativity. However, Schwarzschild (1903), Tetrode (1922), and Fokker (1929a,b; 1932) demonstrated during the first three decades of this century that a relativistically consistent action-at-a-distance theory can indeed be formulated. If we consider two spacetime points A and B with s_{AB}^2 as the invariant square of the relativistic distance between them, then $\delta(s_{AB}^2)$, where δ is the Dirac delta function, is a convenient function for transmitting physical influences between A and B. For, this function acts only when A and B are connectable by a light ray (that is, when $s_{AB}^2=0$). This delta function therefore necessarily occurs as the main component in any Green's function in the action-at-a-distance theory. The action principle, which is the basis of the electromagnetic theory in Riemannian spacetime, is described below. We start with the action

$$\mathcal{A} = -\sum_a \sum_{<b} 4\pi e_a e_b \int \int \bar{G}_{ik}\, da^i db^k \tag{10}$$

where $\bar{G}_{ik}$ is the *symmetric Green's function* given by

$$\bar{G}_{ik}(A, B) \equiv \frac{1}{2}[G_{ik}^R(A, B) + G_{ik}^A(A, B)]. \tag{11}$$

Thus $\bar{G}_{ik}(A, B) = \bar{G}_{ik}(B, A)$ and each term in the action is completely symmetric between each pair of particles. The electromagnetic potential given by (9) is a symmetric half-advanced plus half-retarded combination, rather than the more familiar pure retarded one. However, the action (10) together with suitable cosmological boundary conditions reproduces all the electromagnetic effects of the standard Maxwell field theory. The key issue recognized first by Wheeler and Feynman (1945, 1949) is that no charge is isolated. The motion of a typical charge a invokes a reaction from all other charges in the universe, which we may term the *response of the universe*.

What is the response of the universe? It was shown by Dirac (1938) that when an electric charge a accelerates, it suffers a force of radiative damping, and that this force can be calculated by evaluating half the difference of the retarded and the advanced fields F of the charge *on its worldline*:

$$Q(a) = \frac{1}{2}[F^R(a) - F^A(a)]. \tag{12}$$

In the Maxwell field theory Dirac's result had remained just a curiosity without a proper reasoning as to why the radiative reaction must be determined by the above formula. In the Wheeler–Feynman theory the 'correct' response from the universe to the motion of a is precisely this!

Moreover, if we add (12) to the basic time-symmetric direct particle field of a, *viz.*

$$F(a) = \frac{1}{2}[F^R(a) + F^A(a)] \tag{13}$$

we get the total effect in the neighborhood of a to be a pure retarded one. A correct response therefore eliminates all advanced effects except those present in the radiation reaction. However, all this works provided we have the correct cosmological boundary conditions, which are spelled out below.

In 1945, Wheeler and Feynman had shown that to get the correct response the universe has to be a perfect absorber. Their work was carried out within the framework of a static universe. When Hogarth (1962) repeated the calculation in an expanding universe, he found that the correct response (12) is possible in a universe that is a perfect absorber in the future but not in the past. The steady-state cosmology fulfills this condition, but all known Friedman models fail to meet it. In 1963, Hoyle and I arrived at the same conclusion with somewhat more general assumptions (Hoyle and Narlikar 1963). Because of the crucial requirement of perfect absorption, this theory is sometimes called the 'absorber theory of radiation.'

4. Inertia and Gravity

Our purpose in the above digression into electromagnetism was to show that a similar approach to inertia leads us to a Machian theory of gravity. In the case of inertia, we note that the functions $m^{(b)}(X)$ are scalars, and so we have to deal with scalar Green's functions. Thus we wrote (6) in analogy to (9), and (7) in analogy to (11), while the inertial action in analogy to (10) becomes

$$\boldsymbol{A} = -\sum_{a}\sum_{<b}\int\int \lambda_a \lambda_b \tilde{G}(A, B)\, ds_a\, ds_b. \tag{14}$$

The analogy continues further. The wave equation (5) is conformally invariant and gives us a conformally invariant Machian theory just as (10) gives us a conformally invariant electromagnetic theory.

The action of HN theory is given by (14), and with the help of definition (6) we may write it as

$$\boldsymbol{A} = -\sum_{a}\int m_a\, ds_a. \tag{15}$$

Written in this form, this action appears to have only the inertial term of Newtonian mechanics. How can such an action yield any gravitational equations?

The answer to this question lies in the fact that the m_a's in (15) are not constants but depend on spacetime coordinates *as well as on spacetime geometry*. For they are defined with the help of Green's functions, which in turn are defined in terms of spacetime geometry. Thus if we make a small variation

$$g_{ik} \rightarrow g_{ik} + \delta g_{ik},$$

the wave equation (5) will change and so will its solution. Thus we will have

$$\tilde{G}(A, B) \rightarrow \tilde{G}(A, B) + \delta\tilde{G}(A, B)$$

and hence $\boldsymbol{A} \rightarrow \boldsymbol{A} + \delta\boldsymbol{A}$. We therefore have a nontrivial problem whose solution may be expressed in the following way. To simplify matters we will take all λ_a to be equal to unity.

Define the following functions:

$$m(X) = \sum_{a} m^{(a)}(X) = \frac{1}{2}[m^R(X) + m^A(X)], \tag{16}$$

$$\phi(X) = m^R(X) m^A(X), \quad m_k \equiv m_{,k}, \ldots, \tag{17}$$

$$N(X) = \sum_{a}\int \delta_4(X, A)[-g(X)]^{-1/2}\, ds_a. \tag{18}$$

As in the electromagnetic case, we have chosen the symmetric (half $R+$

half A) Green's function. The gravitational equations then become

$$R_{ik}-\frac{1}{2}g_{ik}R=-6\phi[T_{ik}-\frac{1}{6}(g_{ik}\Box\phi-\phi_{;ik})$$
$$-\frac{1}{2}(m_i^R m_k^A+m_k^R m_i^A-g_{ik}g^{pq}m_p^R m_q^A)], \tag{19}$$

together with the 'source' equation for $m(X)$

$$\Box m+\frac{1}{6}Rm=N. \tag{20}$$

The derivation leading to the final set of equations of the theory may appear somewhat long-winded to anybody unfamiliar with the techniques of direct interparticle action. We have followed here the method used by Hoyle and the author, who arrived at this theory via their earlier work on electromagnetism. As in the electromagnetic case, the universe responds to a local event. To ensure causality and to eliminate advanced effects, the correct response should be given by

$$\sum_a m^{(a)A}(X)=\sum_a m^{(a)R}(X)=m(X). \tag{21}$$

Under these conditions the equations (19) further simplify to

$$R_{ik}-\frac{1}{2}g_{ik}R=-\frac{6}{m^2}\left[T_{ik}-\frac{1}{6}(g_{ik}\Box m^2-m^2_{;ik})\right.$$
$$\left.-\left[m_i m_k-\frac{1}{2}g_{ik}m^l m_l\right]\right]. \tag{22}$$

Had we accepted the standard field theoretical approach and introduced a scalar inertia field $m(X)$, we could have arrived at (20) and (22) from an action given by

$$\boldsymbol{A}=\int\left[\frac{1}{12}Rm^2-m^l m_l\right]\sqrt{-g}\,d^4x-\sum_a\int m\,ds_a. \tag{23}$$

The action-at-a-distance approach, although unfamiliar to a typical theoretical physicist, is useful in that it gives a more direct expression to Mach's Principle. The physical interpretation of the field theoretical term (23) is not so easy to see. For this reason, we have discussed the former approach at some length.

Notice that in the former approach our action (15) contained only the last term of (23), but there m was made up of nonlocal two-point functions. Here m is a straightforward field with sources whose dynamical properties are defined through the first term in the above action.

Since the property of conformal invariance was used in the formulation of the theory, we expect the final equations (20) and (22) to exhibit conformal invariance. This expectation is borne out. If (g_{ik}, m) are a solution of these equations, then so are

$$\bar{g}_{ik} = \Omega^2 g_{ik}, \ \bar{m} = \Omega^{-1} m. \tag{24}$$

Thus, apart from coordinate invariance of general relativity, this theory also shows conformal invariance.

The symmetry of conformal invariance of the action leads to a vanishing of trace of the field equations. It may be easily verified that the trace of (22) vanishes in view of (20). The vanishing of trace represents the fact that the problem is underdetermined. Just as the vanishing of $T^{ik}{}_{;k}$ in general relativity shows that more solutions can be generated from any given solution by coordinate transformations, so we can generate more solutions through (24). All these solutions are physically equivalent provided we stick to the rule that Ω does not vanish or become infinite.

5. The Transition to General Relativity

Suppose we are allowed to choose an Ω in the above range that ensures that

$$\bar{m} = \Omega^{-1} m = \text{constant} = m_0. \tag{25}$$

This choice of Ω is possible provided m does not vanish or become infinite. This conformal frame is called the *Einstein frame*, in which we get a simplified form for (22):

$$R_{ik} - \frac{1}{2} g_{ik} R = -\kappa T_{ik}, \tag{26}$$

with the constant κ given by

$$\kappa = \frac{6}{m_0^2}. \tag{27}$$

Thus we have arrived at Einstein's equations! At first sight we don't seem to have gained anything. We have no new theory and hence no new predictions, as in the Brans–Dicke theory. Closer examination, however, reveals several ways in which this theory goes beyond relativity.

1. Our starting point was based on Mach's Principle. It is only in the many-particle approximation, when the response condition (21) is satisfied, that we arrive at the final Einstein-like field equations. An empty universe in relativity is given by

$$R_{ik} = 0,$$

which can have well-defined spacetimes as solutions. Test particles in such spacetimes will have well-defined trajectories. Such trajectories would not make any sense according to Mach, since we no longer have a material background against which to measure the motion of these particles. These solutions in fact correspond to the $\mathbf{f}=0$ solutions of Newtonian theory. In the HN theory, an empty universe corresponds to

$$m=0, \text{ indeterminate } g_{ik},$$

in accord with the Machian $m=0$ solution of (3).

2. The sign of κ is fixed arbitrarily in general relativity. Neither in the heuristic derivation of Einstein nor in the Hilbert action principle is κ required to be positive. It is only when κ is determined by reference to Newtonian gravity in the weak-field approximation that we conclude that $\kappa>0$. In the HN theory (27) shows that κ must necessarily be positive. (This conclusion does not depend on our assumption of $\lambda_a=1$; the result follows whatever sign the λ_a are given.)

3. In the direct interparticle approach, it is not possible to accommodate the λ-term of cosmic repulsion without making the wave equation (5) nonlinear. Thus Occam's razor automatically comes into play. In relativity, the λ-term is still possible.

4. The transition from (22) to (26) is possible provided $0<\Omega<\infty$. What happens if we break this rule? Suppose in the solution of (22) we had a hypersurface on which $m=0$. If we insist on the transformation (25) in a region that contains such a hypersurface, we have to pay the price of $\Omega\to 0$, which in turn produces spacetime singularities. The work of Kembhavi (1978) showed that the well-known cases of spacetime singularities of relativity arise because of the occurrence of zero-mass hypersurfaces in the solution of the equations (22). For a simple example of this conclusion, let us look at the standard Big-Bang singularity of relativity.

Consider the Minkowski line element (with $c=1$)

$$ds^2=d\tau^2-dx^2-dy^2-dz^2 \tag{28}$$

as a solution of (22). It is easily verified that the mass function satisfying both (20) and (22) for a uniform number density N of particles is

$$m\propto\tau^2. \tag{29}$$

This is the simplest possible cosmological solution in this theory.

If we now insist on going over to a frame with constant mass $\tilde{m}$, then from (24) we see that the appropriate Ω must be given by

$$\Omega\propto\tau^2. \tag{30}$$

However, Ω vanishes on the hypersurface $m=0$. The transformation to the Einstein conformal frame is 'illegal.' The price paid for insisting that $\tilde{m}$=constant is that the resulting model has a geometrical singularity at $\tau=0$. In fact, it is easily verified that the new model is none other than the singular Einstein–de Sitter model.

5. It is instructive to see how the phenomenon of Hubble redshift is explained in the flat spacetime model of (28) and (29). Clearly, a photon traveling in Minkowski spacetime does not undergo redshift. Consider, however, what happens to a photon arriving at the observer at the present epoch τ_0 from a galaxy at a distance r. This photon originated in an atomic (or molecular) transition at time τ_0-r.

From atomic physics, the wavelength of a photon so transmitted varies inversely as the mass of the electron (making the atomic transition). From (29) we see that if λ is the wavelength of this photon and λ_0 is the wavelength of a photon emitted in a similar transition at τ_0 at the observer, then

$$1+z \equiv \frac{\lambda}{\lambda_0} = \frac{m(\tau_0)}{m(\tau_0-r)} = \frac{\tau_0^2}{(\tau_0-r)^2}. \tag{31}$$

Thus the redshift in the above HN cosmology arises from the variation of particle masses.

6. Concluding Remarks

This basic theory therefore resembles general relativity in the Einstein frame but has more general implications in the sense that unlike the relativity theory it is conformally invariant. It has the advantage that it starts with the Machian notion of inertia of a particle arising from other particles in the universe.

Further work along these lines has opened up the possibilities of a variable gravitational constant (Hoyle and Narlikar 1974), anomalous redshifts (Arp and Narlikar 1993), and creation of matter (Hoyle, Burbidge, and Narlikar 1993). These investigations lead to observationally testable results, thus bringing the theory scientific respectability.

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The discussion for this paper followed Hoyle's talk and is on p. 272ff.