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A conformal theory of gravitation

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Certain aspects of the new theory of gravitation proposed in a recent paper are examined in greater detail. It is shown that in the smooth fluid approximation the familiar Einstein equations follow as a result of a specific conformal transformation. The equations of the the theory differ from those of Einstein in the neighbourhood of a particle, however. This is illustrated by means of an explicit solution.

Criticisms of the theory by other authors are considered and discussed.

Introduction

The purpose of the present paper is to explore some aspects of the new theory of gravitation (Hoyle & Narlikar 1964, referred to hereafter as I) in greater detail. For this purpose we begin by giving a brief outline of the theory.

Formally, the equations of the theory are derived from the principle of direct interparticle action. The action function, omitting all interations except the inertia, is written in the form

$$J = \sum_{a<b}\sum \iint \tilde{G}(A,B)\,\mathrm{d}a\,\mathrm{d}b. \tag{1}$$

In the above expression the double sum is over pairs of particles labelled by $a, b, \ldots$. A typical point A on the world line of a has coordinates a^{i_A} ($i_A = 1, 2, 3, 4$). The 'proper time' at A measured along the world line is given by a, where

$$\mathrm{d}a^2 = g_{i_A k_A}\,\mathrm{d}a^{i_A}\,\mathrm{d}a^{k_A}. \tag{2}$$

g_{ik} is the metric of the Riemannian space in which the world lines are embedded. The biscalar $\tilde{G}(A,B)$ denotes the inertial interaction between A and B and satisfies the conformally invariant scalar wave equation

$$g^{i_X k_X}\tilde{G}(X,A);{}_{i_X k_X} + \tfrac{1}{6}R(X)\,\tilde{G}(X,A) = -(-\bar{g})^{-\frac{1}{2}}\,\delta^4(X,A), \tag{3}$$

where R is the scalar curvature, $\delta^{(4)}(X,A)$ the four dimensional delta function and $\bar{g}$ the determinant of the parallel propagators $\bar{g}_{i_A k_X}$. It should be noted that the property of conformal invariance gives a unique form for the scalar wave operator, namely, that used in (3) (Penrose 1963). Further, it is assumed that the elementary interaction between the two members of a pair is symmetric, i.e. $\tilde{G}(A,B) = \tilde{G}(B,A)$.

The action is therefore defined in terms of particle world lines and the space time geometry. The physical equations are to be obtained by (a) variation of world lines, (b) variation of geometry. The former yield the equations of motion (i.e. how the world lines should be alined in the space time continuum) and the latter the equations of gravitation (i.e. the nature of space time geometry).

It is convenient to express these equations in terms of the 'mass' functions. The mass at X due to the world line a is defined by

$$m^{(a)}(X) = -\int \tilde{G}(X, A)\,\mathrm{d}a. \tag{4}$$

The mass of particle a at A is given by

$$m_a(A) = \sum_{b \neq a} m^{(b)}(A). \tag{5}$$

Using these definitions, we can rewrite the action as

$$J = -\tfrac{1}{2}\sum_a \int m_a \mathrm{d}a. \tag{6}$$

Apart from the factor $\frac{1}{2}$ this is formally the same as the usual inertial contribution to the action, suggesting that m_a plays the role of inertial mass. Equations (4) and (5) therefore incorporate Mach's idea that the inertia of a particle arises from the rest of the particles in the universe.

The equations of motion for the world line a are

$$\frac{\mathrm{d}}{\mathrm{d}a}\left(m_a \frac{\mathrm{d}a^{i_A}}{\mathrm{d}a}\right) + m_a \Gamma^{i_A}_{k_A l_A} \frac{\mathrm{d}a^{k_A}}{\mathrm{d}a}\frac{\mathrm{d}a^{l_A}}{\mathrm{d}a} - g^{i_A k_A}\frac{\partial m_a}{\partial a^{k_A}} = 0, \tag{7}$$

and the gravitational equations are

$$\begin{aligned}(R_{ik} - \tfrac{1}{2}g_{ik}R)\,(\sum_{a<b}\sum m^{(a)}m^{(b)}) + 3g_{ip}g_{kq}T^{pq} \\ - \sum_{a<b}\sum [m^{(a)}(g_{ik}g^{pq}m^{(b)}_{;pq} - m^{(b)}_{;ik}) + m^{(b)}(g_{ik}g^{pq}m^{(a)}_{;pq} - m^{(a)}_{;ik})] \\ - 2\sum_{a<b}\sum [m^{(a)}_{;i}m^{(b)}_{;k} + m^{(a)}_{;k} + m^{(b)}_{;i} - \tfrac{1}{2}g_{ik}m^{(a);l}m^{(b)}_{;l}] = 0.\end{aligned} \tag{8}$$

The quantities and indices in (7) and (8) refer to a typical point X in space time. The energy momentum tensor T^{pq} is defined by the usual expression

$$T^{pq}(X) = \sum_a \int \delta^{(4)}(X, A)\,[-\bar{g}(X, A)]^{-\frac{1}{2}}\, m_a \frac{\mathrm{d}a^{i_A}}{\mathrm{d}a}\frac{\mathrm{d}a^{k_A}}{\mathrm{d}a}\bar{g}^p_{i_A}\bar{g}^q_{k_A}\,\mathrm{d}a. \tag{9}$$

From (3) and (4) we see that $m^{(a)}$ satisfies, identically, the equation

$$g^{ik}m^{(a)}_{;ik} + \tfrac{1}{6}Rm^{(a)} = N^{(a)}(X), \tag{10}$$

where

$$N^{(a)}(X) = \int \delta^4(X, A)\,[-\bar{g}(X, A)]^{-\frac{1}{2}}\,\mathrm{d}a. \tag{11}$$

It can be verified that the divergence of (8) leads to the equations of motion.

These are the equations of the theory. In the following two sections we will show how in the smooth-fluid approximation the equations (8) can be reduced to the equations of Einstein by a specific conformal transformation, and how the equations (8) behave in the neighbourhod of a particle worldline. Finally, we shall deal with the criticisms of certain aspects of the theory, as given in (I), which have recently come from McCrea (1965), Hawking (1965) and Pirani & Deser (1965).

Conformal invariance

Consider a general conformal transformation given by

$$g^*_{ik} = \Omega^2 g_{ik}, \tag{12}$$

where Ω is a well defined function of x^i. It is easily verified that the equation (10) is invariant under such a transformation. That is, if we define

$$m^{*(a)} = \Omega^{-1} m^{(a)}, \tag{13}$$

the equation (10), written in terms of starred quantities takes exactly the same form as before. (This applies to both sides of the equation. The transformation law (12) shows that $N^{*(a)} = \Omega^{-3} N^{(a)}$.) This property is characteristic of the particular form of the biscalar $\tilde{G}(A, B)$ chosen to define mass. A biscalar Green function defined by any other form of scalar wave equation will not exhibit this property. The situation here is analogous to the case of electromagnetism where Maxwell's equations are conformally invariant.

With the transformation law (13) for mass functions it is possible to show by a direct, but rather lengthy, calculation that the equations of motion (7) and the gravitational equations (8) are conformally invariant. The same result follows more elegantly simply by noticing that the action J is invariant under conformal transformation. For,

$$\left.\begin{aligned} G^*(A,B) &= G(A,B)\,[\Omega(A)]^{-1}\,[\Omega(B)]^{-1}, \\ \mathrm{d}a^* &= \Omega(A)\,\mathrm{d}a, \quad \mathrm{d}b^* = \Omega(B)\,\mathrm{d}b. \end{aligned}\right\} \tag{14}$$

This property of conformal invariance clears up what at first sight seems a puzzling point in the theory. In analogy to Einstein's theory, equations (7) and (8) might be expected to give complete information about the particle worldlines and about the ten functions g_{ik} that determine the geometry. However, unlike the Einstein equations, the number of independent equations in (8) is only nine. This follows by taking the trace of (8) which vanishes identically, as was pointed out in I. The theory does not give a complete determination of the metric tensor; one function is left undetermined. The above considerations of conformal invariance show that we can regard Ω as the undetermined function. That is if $[g_{ik}, m^{(a)}, m^{(b)}, \ldots]$ is a solution of the equation, so is $[g^*_{ik}, m^{*(a)}, m^{*(b)}, \ldots]$ where Ω is *any* well behaved function. *Our point of view is that all such solutions are physically equivalent.*

Consider now the smooth-fluid approximation in which the particle worldlines lose their identity. Define the total mass at X by

$$m(X) = \sum_a m^{(a)}(X). \tag{15}$$

In this approximation a test particle at X would acquire an inertia measured by $m(X)$. An explicit determination of $m(X)$ is complicated by the fact that the theory is nonlinear: If this were a flat space theory, where geometry is not affected by the basic interactions, a direct addition of individual contributions from different particles could be made. Such a calculation is possible even in curved space in the case of electromagnetism, where the elementary interactions do not significantly modify the geometry. The interference effects between elementary electromagnetic interactions can be worked out without ambiguity and the calculation leads to an absorber theory of radiation along the lines of Wheeler & Feynman (1945, 1949). For details of such calculations cf. Hogarth (1962), Hoyle & Narlikar (1963). A look at the equations (8) is enough to show that such a calculation is meaningless in the present theory, however. We shall return to this point in the next section.

In the smooth-fluid approximation the equations (8) take the form

$$\tfrac{1}{2}m^2(R_{ik}-\tfrac{1}{2}g_{ik}R)+3T_{ik}-m(g_{ik}g^{pq}m_{;pq}-m_{;ik})-2(m_{;i}m_{;k}-\tfrac{1}{4}m_{;l}m^{;l}g_{ik}) = 0. \quad (16)$$

The properties of conformal invariance and zero trace continue to hold even in the smooth fluid approximation. These properties enable a further remarkable simplification to be made in the following way.

Suppose formally a solution of (16) is obtained with some g_{ik} and $m(X)$. If we now construct another solution with

$$\Omega(X) = m(X)/m_0, \quad m_0 \text{ a constant,} \quad (17)$$

we shall have

$$m^*(X) = m_0, \quad g^*_{ik} = \left[\frac{m(X)}{m_0}\right]^2 g_{ik}. \quad (18)$$

(Notice that this is possible quite generally—not only in the smooth-fluid case.) Under this transformation, (16) becomes

$$\tfrac{1}{2}m_0^2(R^*_{ik}-\tfrac{1}{2}g^*_{ik}R^*)+3T^*_{ik} = 0. \quad (19)$$

Dropping stars, we can rewrite (19) in the familiar form

$$R_{ik}-\tfrac{1}{2}g_{ik}R = -\kappa T_{ik}, \quad \kappa = 6/m_0^2. \quad (20)$$

This is the 'Einstein case' discussed in I. From the above reductions we see that the Einstein equations follow from the present theory by the choice of a *special* conformal frame. After the transformation has been made the particular conformal property is of course lost. The gain from a practical point of view is that in this particular conformal frame the equations are simpler in form. This is analogous to the choice of a suitable coordinate frame of reference (e.g. the rest frame or centre of mass frame) in solving problems in special relativity. The choice of the frame destroys Lorentz invariance but makes the equations simpler in form.

It is interesting to compare the equation (16) with those obtained by Gürsey (1963) and by Peres (1962). Gürsey uses a conformal function $\Omega = \phi = (-g)^{-\frac{1}{8}}$, so as to achieve $(-g^*)^{-\frac{1}{8}} = 1$ in the transformed frame. Peres introduces a conformal transformation on mass as well, and arrives at the equation (16). However, in the present formulation, the equation (16), as well as the Einstein equations, are obtained as a smooth-fluid approximation from a discrete direct-particle picture. The approximation is not valid in the neighbourhood of a particle. In the following section we shall consider the nature of the equations (8) in the neighbourhood of a particle and show that they differ from (16) and (20) in a remarkable manner.

Geometry in the neighbourhood of a particle

The situation near a particle differs from the smooth fluid approximation in that it is no longer possible to approximate $\sum_{b\neq a} m^{(b)}$ by the unrestricted sum $\sum_b m^{(b)}$. We again choose a conformal frame in which

$$\sum_b m^{(b)} = m_0. \tag{21}$$

For convenience in writing, we take the particle in question as being labelled by $a = 1$, and we denote $m^{(1)}$ by μ. The restricted summations in (8) can be expressed in terms of m_0, μ, and the derivatives of μ. For example,

$$\sum_{a<b}\sum m^{(a)}m^{(b)} = \mu \sum_{b>1} m^{(b)} + \sum_{1<b<c}\sum m^{(b)}m^{(c)} \approx \mu(m_0-\mu)+\tfrac{1}{2}(m_0-\mu)^2. \tag{22}$$

Here we have used the smooth-fluid approximation in evaluating $\sum_{1<b<c}\sum m^{(b)}m^{(c)}$, the understanding being that a large number of particles is involved in the summations, and that none of these many particles happens to be very close to $a = 1$. Equations (8) become

$$\tfrac{1}{2}(m_0^2-\mu^2)(R_{ik}-\tfrac{1}{2}g_{ik}R) = -3T_{ik}+\mu(\mu_{;ik}-g^{mn}\mu_{;mn}g_{ik}) \\ -2(\mu_{;i}\mu_{;k}-\tfrac{1}{4}\mu^{;l}\mu_{;l}g_{ik}). \tag{23}$$

Provided we can obtain a solution in which $\mu \to 0$ with distance from the particle, equations (23) merge into the Einstein equations.

The solution given below is analogous to the Schwarzschild solution in general relativity. That is to say, in the rest frame of the particle we assume the existence of spherical symmetry. Coordinates can then be chosen such that we have a static metric

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2+\sin^2\theta\, d\phi^2), \tag{24}$$

with ν, λ as functions of r only. Further we take μ to be a function of r only. We proceed to show that a solution exists outside the particle, a solution with $\lambda+\nu = 0$, as in the Schwarzschild case. To obtain this solution we use the form

$$ds^2 = e^{\nu}dt^2 - e^{-\nu}dr^2 - r^2(d\theta^2+\sin^2\theta\, d\phi^2), \tag{25}$$

from the outset.

For a randomly moving particle the above conditions will not in general be satisfied. The present case in which the external world is assumed to possess central symmetry with respect to the particle is obviously highly special.

Summing equation (10) over all particles, we have

$$g^{ik}(\sum_b m^{(b)})_{;ik} + \tfrac{1}{6}R\sum_b m^{(b)} = \sum_b N^{(b)}(X), \tag{10}$$

$$N^{(b)}(X) = \int \delta^4(X,B)\,[-\bar{g}(X,B)]^{-\frac{1}{2}}\, db. \tag{11}$$

The left hand side of (10) is calculated at the field point X. In the conformal frame with $\sum_b m^{(b)} = m_0$, m_0 a constant, we evidently have

$$R(X) = \frac{6}{m_0}\sum_b N^{(b)}(X). \tag{26}$$

It follows that $R = 0$ except at a particle. At a particle R possesses a delta function singularity. It is just these singularities that generate the gravitational field.

Because $R = 0$ outside the particular particle under investigation we must have

$$g^{mn}\mu_{;mn} = 0. \tag{27}$$

With μ a function of r only, this leads to

$$r^2 e^{\nu}\mu' = \text{constant}, \tag{28}$$

in which the prime denotes differentiation with respect to r.

Next we note that adoption of the special form (25) for the line element requires $R_1^1 = R_4^4$. For this condition to be consistent with the field equations (23) it is necessary that

$$\mu\mu'' = 2\mu'^2. \tag{29}$$

Equations (28) and (29) have the solution

$$\mu = \frac{q}{r-p}, \quad e^{\nu} = \left(1-\frac{p}{r}\right)^2, \tag{30}$$

in which p, q are constants. Substituting (30) in the remaining field equation we find this is satisfied provided p, q are related by $q = m_0 p$, so that

$$\mu = m_0\frac{p}{r-p}, \quad e^{\nu} = \left(1-\frac{p}{r}\right)^2. \tag{31}$$

A relation between m_0 and p can be obtained by considering the equation for μ including the delta function singularity at the particle $a = 1$,

$$g^{ik}\mu_{;ik} + \tfrac{1}{6}R\mu = N^{(1)}(X). \tag{32}$$

The second term on the left is zero except at the particle. Since $\mu \to -m_0$ as $r \to 0$ this term can be written as

$$-\tfrac{1}{6}m_0 R = -\sum_b N^{(b)} = -N^{(1)}, \tag{33}$$

at the particle $a = 1$, where (26) has been used for R. Hence the equation for μ, including the singularity at $r = 0$, is

$$g^{ik}\mu_{;ik} = 2N^{(1)}(X), \tag{34}$$

the right hand side being zero except when the field point X falls on the particle. By integrating this equation invariantly through a four dimensional tube enclosing the particle (the curved surface of the tube had constant r) it is easily seen that†

$$2\pi m_0 p = 1. \tag{35}$$

† *Footnote added* 13 *April* 1966. The reader may prefer to derive (35) from the Schwarzschild solution of general relativity. The contracted field equation of general relativity, $R = 8\pi GT$ takes the form (34) when μ^k is defined by

$$4\pi G m_b \mu^k = \left[0, \frac{2}{r} - e^{\nu}\left(\nu' + \frac{2}{r}\right), 0, 0\right].$$

Since in both cases, μ^k only has an r-component and since $\sqrt{-g} = r^2 \sin\theta$ in both cases, it follows that μ' is the same. In the Schwarzschild case $e^{\nu} = 1 - 2Gm_b/r$, $r \neq 0$, and $\mu' = 1/2\pi r^2$, while in our case $\mu' = m_0 p/r^2$. Equating the two forms of μ' gives (35).

We draw attention to the following features of this solution:

(i) In the Schwarzschild solution e^{ν} is of the form $1-p/r$ not $(1-p/r)^2$. The present solution differs therefore from the Schwarzschild solution in that the t coordinate always remains time-like. The r and t coordinates do not interchange their spacelike and timelike properties at $r = p$. Both solutions have singularities at $r = 0$ and $r = p$.

(ii) The inertial mass m_a of the particle a is $\sum_{b \neq a} m^{(b)}$, as can be seen from the equations of motion, (7).

For our solution, applied to the particle $a = 1$, we have $\sum_{b \neq a} m^{(b)} = m_0 - \mu$ and this is $2m_0$ at the particle. The inertial mass is not singular at the particle.

(iii) For $r \gg p$, the solution (31) can be approximated to by

$$\mu = \frac{1}{2\pi r}, \quad e^{\nu} = 1 - \frac{2p}{r}. \tag{31'}$$

Evidently the quantity p is the 'gravitational mass' of the particle.

It is of interest that had we used the smooth-fluid equations (20), taking m_0 for $\sum_{b \neq 1} m^{(b)}$, we should have obtained

$$e^{\nu} = 1 - \frac{3}{2\pi m_0 r} = 1 - \frac{3p}{r}, \tag{36}$$

when m_0 and p are related by (30). The gravitational mass would have been greater by the factor $\frac{3}{2}$. The difference between (31) and (36) makes it clear that near a particle the smooth-fluid equations cannot be used. The mass field μ of the particle itself plays an important part in the solution of the equations.

(iv) It is worth noticing the status of the terms in the field equations (23) more explicitly. Outside the singularity $T_{ik} = 0$, $g^{mn}\mu_{;mn} = 0$, and

$$R_{ik} - \tfrac{1}{2} g_{ik} R = \frac{2}{m_0^2 - \mu^2} (\mu\mu_{;ik} - 2\mu_{;i}\mu_{;k} + \tfrac{1}{2}\mu^{;l}\mu_{;l} g_{ik}). \tag{37}$$

A straightforward calculation shows that the (44) component of (37) leads to

$$e^{\nu}\frac{\nu'}{r} + \frac{e^{\nu} - 1}{r^2} = -\frac{p^2}{r^4}, \tag{38}$$

in which the left hand side of (37) is worked out for the line element (25) while the right hand side is worked out for the solution (30). Equation (38) integrates to give

$$e^{\nu} = 1 + \frac{A}{r} + \frac{p^2}{r^2}, \tag{39}$$

in which A is a constant of integration. This brings out the important point that the p^2/r^2 term in μ is generated by the terms in μ and its derivatives on the right hand side of (37). These terms may be regarded as an energy-momentum tensor which is nonzero outside the particle. It is for this reason that we do not obtain the usual Schwarzschild solution.

At the particle itself the situation is different. The strongest singularities lie on the right hand side of (23). This can be seen immediately by contracting (23) to give

$$-(m_0^2 - \mu^2) R = -3T - 3\mu g^{mn}\mu_{;mn}. \tag{40}$$

At the particle $\mu = -m_0$, m_a in (9) is $2m_0$ and $T = 2m_0 N^{(1)}$. Although $R = 6N^{(1)}/m_0$ from (26), the coefficient $m_0^2 - \mu^2 \to 0$ at the particle. Hence (40) leads to

$$g^{mn}\mu_{;mn} = 2N^{(1)},$$

i.e. to the equation (34). We have already emphasized that a contraction of the field equations leads to an equation that is already known from the conformally invariant equation satisfied by the propagator $\tilde{G}$.

Criticisms and comments

In this section we shall consider, and answer, certain criticisms of the above theory. We shall also comment further on the particular solution obtained in the preceding section.

Recently McCrea (1965) has claimed that the sign of gravitation is as much arbitrary as it is in the Einstein formulation. In the latter the action is taken to be

$$J = \frac{1}{16\pi G}\int R\sqrt{(-g)}\,\mathrm{d}^4X - \sum_a m_a \int \mathrm{d}a, \tag{41}$$

the masses m_a of the particles being regarded as positive constants. The essential point is that two terms in the action are required in Einstein's theory, whereas in our theory the action is given by the single term in (1). The overall sign for the action is irrelevant, so that an action consisting of n terms has $n-1$ arbitrary choices in the signs of the coupling constants, one choice in Einstein's theory (the sign of G), no choice in our theory. That gravitation is attractive in our theory is shown by (35), which requires m_0, p to have the same sign, i.e. inertial mass and gravitational mass have the same sign.

In a recent paper, Hawking (1965) has calculated $m^{(a)}(X)$ in Robertson–Walker type cosmological models with $k = 0$, and by carrying through the summation $\sum_a m^{(a)}$ obtains an infinite value. This procedure ignores the delta function bumps on R at each particle (cf. (26)). The functions $m^{(a)}(X)$ cannot be calculated on the basis of a preassigned geometry. Each particle modifies the geometry in its own neighbourhood, and this must be taken into account when the functions $m^{(a)}(X)$ are calculated.

The solution obtained in the preceding section shows there is no possibility of determining the constant $m_0 = \sum_b m^{(b)}$ through the linear superposition method used by Hawking The calculation yielded the form of $\mu = m^{(a)}$, $a = 1$ in the neighbourhood of $a = 1$. The calculation also showed that $\sum_{b\neq 1} m^{(b)}$ was modified in the neighbourhood of $a = 1$, the modifications being always such that $\mu + \sum_{b\neq 1} m^{(b)}$ remained equal to m_0. This shows that any attempt to calculate $\sum_b m^{(b)}$ directly is self-defeating, as it must be from the conformal invariance of the theory. Reference to (17) shows that m_0 can have any value—a change of m_0 represents a scale transformation of the coordinates.

Pirani & Deser (1965) have criticized our claim that the theory requires gravitational forces to be attractive from a point of view that is different from that of

McCrea. They argue that if the coefficient 1/6 on the left hand side of the propagator equation (3) for $\tilde{G}(X, A)$ were taken negative the sign of gravitational forces would be reversed. However, the theory would not then be conformally invariant. Our point is that conformal invariance leads to gravitational forces of the correct sign. A further aspect of the conformal theory is that no term of the type λg_{ik} can be included in the field equations, as it sometimes is in Einstein's theory, because such a term is not consistent with conformal invariance.

In the absence of matter Einstein's equations give $R_{ik} = 0$. Yet in the absence of the matter the action (41) is zero. How can the variation of nothing yield information about the properties of space-time? Pirani & Deser suggest that this paradox could be avoided if the integrand in the first term of (41) were replaced by

$$g^{ab}(\Gamma^d_{ab}\Gamma^c_{dc} - \Gamma^d_{ac}\Gamma^c_{bd})\sqrt{(-g)}. \tag{42}$$

But this quantity is not of the form: invariant $\times \sqrt{(-g)}$. By a special choice of the coordinate system (42) can be made to vanish at any assigned point.

Pirani & Deser have also taken objection to our use of the description 'direct particle theory.' They prefer the term 'nonlocal field theory'. Although the words used to describe the theory are to some extent a matter of taste, we feel that in any field theory, local or non-local, the equations of the theory should survive when there are no particles. Such is not the case for the equations (8). In the absence of particles *there is no theory*. The only quantities not immediately defined in terms of the particles are the g_{ik}, which describe the space-time geometry in which particle interactions propagate. If the g_{ik} were similar to ordinary fields an *unambiguous* quantization of gravitation would have been achieved long ago. In view of disagreements on this question (Mandelstam 1962) it seems premature to us to argue as if the g_{ik} were ordinary fields.

Pirani & Deser have further questioned whether the theory really satisfies the well known practical tests of Einstein's theory. If the Sun were a single particle the theory would certainly encounter the difficulty that the term in p^2/r^2 in $e^\nu = (1-p/r)^2$ would lead to an erroneous value for the rotation of the perihelion of Mercury. But the Sun is a collection of particles, of the order of 10^{57}. The second order term in e^ν for a collection of particles is less than for a single particle of the same total mass by the reciprocal of the number of particles in the collection, by 10^{-57} in the case of the Sun. The situation can be seen immediately from the field equations (23). Outside the Sun, $T_{ik} = 0$. In these equations μ is *the contribution of one particle only*. This is so small that the whole right hand side of (23) can be taken as zero outside the Sun. Hence $R_{ik} = 0$ and the solution takes the usual Schwarzschild form.

It is true that an approximation was used in deriving (23). Thus in (22) we wrote

$$2\sum_{1<b<c}\sum m^{(b)}m^{(c)} = [\sum_{b\neq 1} m^{(b)}]^2 - \sum_{b\neq 1}[m^{(b)}]^2 \approx [\sum_{b\neq 1} m^{(b)}]^2. \tag{43}$$

The approximation consisted in neglecting square terms in comparison with cross product terms. For a collection of n particles there are n square terms but $n(n-1)$ cross product terms, and

$$\sum_{b\neq 1}[m^{(b)}]^2 \approx n^{-1}[\sum_{b\neq 1} m^{(b)}]^2. \tag{44}$$

The same result follows from a linear addition of contributions to e^{ν}. With one particle giving $e^{\nu} = (1-p/r)^2$, a linear addition for n particles gives

$$e^{\nu} = 1-\frac{2np}{r}+\frac{np^2}{r^2} = 1-\frac{2np}{r}+n^{-1}\left(\frac{np}{r}\right)^2. \tag{45}$$

Here np/r is the usual relativistic parameter. The second order term is multiplied by n^{-1}. (N.B. A linear argument, while correct for the first order term, cannot be taken too literally for the second order term, although it does give the correct order of magnitude for this term.)

With the form of the metric outside the Sun the same as in Einstein's theory, the discussions of the bending of light and of the gravitational red-shift are the same. While geodesics are also the same, a further investigation is necessary for the planetary motions because the equations of motion (7) are not strictly of geodesic form. For an idealized massless particle we could write $m_a = \sum_{b \neq a} m^{(b)} = m_0$, in which case (7) reduces immediately to the geodesic equations. But since planets are not idealized test particles this reduction is not a sufficient guarantee that the motions will really be geodesic.

The equations of motion (7) can be written in the form

$$\frac{d^2a^i}{da^2}+\Gamma^i_{kl}\frac{da^k}{da}\frac{da^l}{da}+\frac{\partial \ln m_a}{\partial a^k}\left(\frac{da^i}{da}\frac{da^k}{da}-g^{ik}\right) = 0. \tag{7}$$

We can think of the third term on the left hand side as a 'force' arising from the gradient of m_a, noticing that $m_a = m_0-\mu$, where $\mu \equiv m^{(a)}$ is the mass 'field' of the particle itself. The component of this force in the direction of motion is zero since $u_i(u^iu^k-g^{ik}) = 0$, $u^i \equiv da^i/da$. Hence the force is normal to the trajectory of the particle.

The direction of a normal force arising from the particle itself must depend on the curvature of the trajectory. When the trajectory is straight—i.e. geodesic—the direction is indeterminate and the force must be zero, in which case the motion *is* geodesic since the term in $\partial\mu/\partial a^i$ then disappears from (7). The geodesic postulate is therefore self-consistent. If the trajectory is not straight we expect the mass gradient to introduce a term of order p/ρ where ρ is the first curvature of the trajectory. For a proton, the gravitational mass $p \sim 10^{-52}$ cm, whereas ρ for the planets $\sim 10^{13}$ cm. The factor $p/\rho \sim 10^{-65}$, so we expect any such term to be exceedingly small.

A further problem arises, but this is present in the usual theory also. The equation

$$\frac{d^2a^i}{da^2}+\Gamma^i_{kl}\frac{da^k}{da}\frac{da^l}{da} = 0 \tag{46}$$

is not the same for a particle with mass as it is for a massless test particle because in the former case the symbols Γ^i_{kl} must include the geometrical disturbance induced by the particle itself. The case for omitting the self-terms in Γ^i_{kl} is similar to what it is in the usual theory.

For simplicity we regard the Sun as composed of n similar particles each of gravitational mass p. The time T required for an orbital revolution of a planet moving in a circle of radius r is easily shown to be given by

$$\frac{T}{2\pi r} = n^{-\frac{1}{2}}\left(\frac{r}{p}\right)^{\frac{1}{2}}. \tag{47}$$

We here take the velocity of light to be unity, so the left hand side of (47) is simply the ratio of the orbital period to the time required for light to travel round the orbit. Since n can be regarded as known, the theory would be open to explicit test if r/p could be measured. Practical measurement of r unfortunately does not proceed in terms of the gravitational radius of individual particles because gravitational effects on a microscopic scale are obscured by electromagnetic and nuclear effects. A theoretical relation between p and the Bohr radius would suffice to overcome this difficulty, but a more sophisticated theory involving an understanding of electric charge would be needed in order to deduce such a relationship. We conclude that although (47) is in principle open to test the methods used in practice do not provide such a test.

The above considerations show that the predictions of the present theory are the same as those of general relativity in regions away from the source particles. The differences between the two theories become more marked as we approach a particle, as was shown in the previous section. The purpose of the present paper is to emphasize this difference. Further investigation of the situation near a particle in the present theory leads to some remarkable results which we propose to discuss in a future paper.

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