

Constraints on the shape of the density spectrum from *COBE* and galaxy surveys

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ABSTRACT

Many popular models for galaxy formation are based on power spectra which behave as $P(k) = Ak$ for small k and flatten at large k . The resulting mass fluctuation, $(\delta M/M)_R^2 \equiv \sigma_{\text{gal}}^2(R)$, can be constrained by galaxy surveys up to $R \approx 60 h^{-1}$ Mpc. On the other hand, the recent *COBE* results constrain the dark matter spectrum at very large scales ($R \gtrsim 10^3 h^{-1}$ Mpc). The *COBE* results, $(\Delta T/T)_{\text{rms}} = (1.1 \pm 0.2) \times 10^{-5}$ and $(\Delta T/T)_Q = (0.48 \pm 0.15) \times 10^{-5}$, are consistent with a scale-invariant spectrum, $\sigma_{\text{DM}}(R) = (R_0/R)^2$ at large scales with $R_0 \approx (23.9 \pm 2.1) h^{-1}$ Mpc. This is consistent with APM data and large-scale streaming velocity measurements [both of which require $\sigma_{\text{gal}}(50 h^{-1} \text{ Mpc}) \approx 0.2$], provided that the scale-invariant spectrum is extrapolated from 3000 to $50 h^{-1}$ Mpc. The fact that the extrapolated *COBE* result matches galaxy survey results at $50 h^{-1}$ Mpc suggests that biasing is not significant at $R \gtrsim 50 h^{-1}$ Mpc. On the other hand, a CDM spectrum, normalized to the *COBE* value, will overshoot galaxy survey results at small scales. Such a spectrum can be consistent with observations only if the biasing varies with scale and $b < 1$ at small scales. Comparing the shape of $\sigma(R)$ at $R \lesssim 60 h^{-1}$ Mpc, determined from galaxy surveys (*IRAS*, CfA and APM), with the *COBE* result, we find that a relatively rapid bend in $\sigma(R)$ around $R \approx (40-60) h^{-1}$ Mpc is needed. We show that simple models to describe this bend, based on spectra of the form $P(k) = Ak[1 + (k/k_c)^n]^{-1}$, are severely constrained. The implications of this result are discussed.

Key words: galaxies: formation – cosmic microwave background – dark matter – large-scale structure of Universe.

1 INTRODUCTION

Standard models for galaxy formation assume that the inhomogeneities observed in the Universe grew out of small perturbations in the past, through gravitational instability (see e.g. Peebles 1980; Padmanabhan 1992). Given the initial power spectrum, $P_i(k) = |\delta_k(t_i)|^2$, where $\delta_k(t)$ is the Fourier transform of the density contrast $[\delta_\rho(\mathbf{x}, t)/\rho_b(t)]$, one can – in principle – find the power spectrum of fluctuations today. In practice, however, this task can be reliably achieved only in the linear regime, where $(\delta\rho/\rho) \ll 1$. Using linearized Einstein equations, one can predict the form of $P_R(k)$ at the recombination epoch, given the initial spectrum. If the initial

spectrum is a power law, $P_i(k) \propto k^n$ ($-3 < n \leq 1$), then the spectrum at recombination will behave as k^n for small k (large scales) and will flatten at large k . [We ignore effects due to free streaming, since we shall *not* be concerned with hot dark matter (HDM) models in this paper.] During further evolution, small scales (large k) will go non-linear first and structure will form hierarchically. Numerical simulations show that the non-linear evolution (i) will steepen the spectrum at small scales, but (ii) will not significantly affect the form of the spectrum at large scales. Thus the spectrum evolved by linear theory does contain useful information about the Universe at large scales, say, at $R \gtrsim 20 h^{-1}$ Mpc. Even at small scales, the slope of the linear-theory spectrum provides a lower bound to the actual slope and – as we shall see – offers interesting constraints.

At large scales, the spectrum is constrained by the recent *COBE* observations of microwave background radiation (MBR) anisotropy (Smoot et al. 1992; Wright et al. 1992). If

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we make the natural assumption that the spectrum is scale-invariant [$P(k) \propto k$] at small k , the amplitude of the spectrum is completely determined by the *COBE* result. Combination of the *COBE* results with those from galaxy surveys leads to an interesting overall picture about the spectrum, which we shall discuss in this paper.

2 COMPARISON OF MODELS WITH *COBE* AND GALAXY SURVEYS

The theoretical models are best compared with observation using the rms fluctuations in mass, $\sigma(R)$, where

$$\begin{aligned} \sigma^2(R) &= \langle (\delta M/M)_R^2 \rangle \\ &= \int_0^\infty \frac{dk}{k} \left[\frac{k^3 |\delta_k|^2}{2\pi^2} \right] \left[\frac{3 \sin kR - 3kR \cos kR}{(kR)^3} \right]^2 \quad (1) \\ &= \int_0^\infty \frac{dk}{k} \Delta^2(k) W_s(kR)^2. \end{aligned}$$

The quantity $\Delta^2(k)$ represents the contribution from each logarithmic interval in k to the mean-square fluctuations in mass, and W_s is the Fourier transform of the spherical window function.

Let us now consider the constraints on $\sigma(R)$ from observations. The most interesting constraint is from the recent *COBE* measurements (Smoot et al. 1992):

$$\left(\frac{\Delta T}{T} \right)_{\text{rms}} = (1.1 \pm 0.2) \times 10^{-5}, \quad (2)$$

$$\left(\frac{\Delta T}{T} \right)_Q = (0.48 \pm 0.15) \times 10^{-5}. \quad (3)$$

Theoretical estimates for the above quantities can be made from the standard formulae (e.g. Peebles 1982):

$$\left(\frac{\Delta T}{T} \right)_{\text{rms}}^2 = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l \exp\left(-\frac{l^2 \theta_c^2}{2}\right), \quad (4)$$

$$\left(\frac{\Delta T}{T} \right)_Q^2 = \frac{5}{4\pi} C_2 \exp(-2\theta_c^2), \quad (5)$$

where

$$\langle |a_{lm}|^2 \rangle = C_l = \frac{H_0^4}{2\pi} \int_0^\infty dk \frac{|\delta_k|^2}{k^2} |j_l(k\eta)|^2. \quad (6)$$

Here, θ_c is the 'smearing angle' due to the beamsize of *COBE* ($\theta_c \approx 0.425 \theta_{\text{FWHM}} \approx 0.05129$ rad if $\theta_{\text{FWHM}} \approx 7^\circ$); $\eta = 2H_0^{-1} = 6000 h^{-1}$ Mpc and j_l is the spherical Bessel function of order l . In arriving at the above expression, we have assumed that the Sachs-Wolfe term dominates the anisotropy and that $\Omega = 1$. [In models with zero cosmological constant, spatial curvature becomes important at $\theta > \theta_{\text{curv}} = \Omega/(1-\Omega)^{1/2}$. There is no natural definition of scale invariance for $\theta > \theta_{\text{curv}}$. We will see later that *COBE* results are consistent with scale invariance, thereby offering additional support for $\Omega = 1$.]

Since *COBE* is essentially probing large angular scales, it is legitimate to use the asymptotic form $P(k) \approx Ak$ in comput-

ing C_l . Then we find that $C_2 = AH_0^4/24\pi$ and $C_l = [6C_2/l(l+1)]$. Substituting these relations into (4) and (5), we get $(\Delta T/T)_Q^2 = (5.28 \times 10^{-3})(AH_0^4)$ and $(\Delta T/T)^2 = 0.03(AH_0^4)$. The quantity AH_0^4 is directly related to the fluctuations in the gravitational potential $\Phi^2 = (k^3 |\phi_k|^2 / 2\pi^2)$ at large scales. Since

$$\phi_k = (4\pi G \rho_b)(\delta_k/k^2) = (3/2)H^2(\delta_k/k^2),$$

we find that $AH_0^4 = (8\pi^2/9)\Phi^2$ and we can re-express the anisotropies as

$$\left(\frac{\Delta T}{T} \right)_Q \approx 0.22 \Phi, \quad \left(\frac{\Delta T}{T} \right)_{\text{rms}} \approx 0.51 \Phi. \quad (7)$$

We can now compare the theoretical results with the *COBE* observations. To begin with, notice that $(\Delta T_{\text{rms}}/\Delta T_Q) \approx 2.3$ if the spectrum has $n=1$. The *COBE* results allow this ratio to fall between 1.43 and 3.94, with a mean value of 2.29. This is quite consistent with the assumption of $n=1$. We shall hereafter assume that $P(k) \propto k$ for small k . The parameter Φ and the amplitude A can now be determined by comparing (7) with *COBE* results. Within the errors, we obtain $\Phi \approx 2.2 \times 10^{-5}$ and $A = (24 h^{-1} \text{ Mpc})^4$. Also note that the quadrupole result gives

$$C_2^{1/2} = (4\pi/5)^{1/2} (\Delta T/T)_Q = (0.76 \pm 0.24) \times 10^{-5};$$

thus the maximum permitted value for $C_2^{1/2}$ is about 10^{-5} .

A spectrum of the form $P(k) = Ak$ will lead to a mass fluctuation of the form $\sigma_{\text{DM}}(R) = (R_0/R)^2$, provided the spectrum flattens at large k , leading to a convergent integral in (1). [The subscript 'DM' is added to emphasize the fact that the *COBE* (Sachs-Wolfe) result probes the dominant gravitating component, namely the dark matter.] In this case, $R_0 = fA^{1/4}$. For a wide class of spectra which we will consider later, $f \approx 1$ to sufficient accuracy. In this case, the bound on A implies that $\sigma_{\text{DM}}(R) = (R_0/R)^2$ at large scales, with $R_0 \approx 24 h^{-1}$ Mpc. This $\sigma_{\text{DM}}(R)$ is shown at the right-hand bottom edge of Fig. 1. The error bounds are also shown by two adjacent lines. Strictly speaking, the *COBE* result covers only scales which subtend $\theta \gtrsim 7^\circ$ on the sky; the extrapolation of *COBE* results to smaller scales is shown by a dashed line. Since the spectra in most theoretical models turn down at small R , this dashed line represents an upper bound to $\sigma_{\text{DM}}(R)$.

The galaxy survey results constrain the form of $\sigma_{\text{gal}}(R)$ at small scales. (A priori, we cannot expect σ_{gal} to be the same as σ_{DM} .) We have plotted in Fig. 1 the $\sigma_{\text{gal}}(R)$ obtained from three galaxy surveys (CfA, *IRAS* and APM), based on the analysis by Hamilton et al. (Hamilton et al. 1991; the original data are from Huchra et al. 1983; Strauss et al. 1990; Maddox et al. 1990a,b). The CfA and *IRAS* surveys are three-dimensional surveys, and the published results can be directly converted to give $\sigma_{\text{gal}}(R)$. The APM result is published in the form of the angular correlation function, which can be inverted using Limber's equation to give $\sigma_{\text{gal}}(R)$ using the selection function suggested in Maddox et al. (1990a,b). [For more details regarding the determination of $\sigma_{\text{gal}}(R)$, see Hamilton et al. 1991.]

For the sake of completeness, we have marked a few other constraints on $\sigma(R)$ in the same diagram. The POTENT reconstruction of velocity fields (Bertschinger et al. 1990) gives a lower bound to $\sigma_{\text{DM}}(R)$ in the range (40–60) h^{-1}

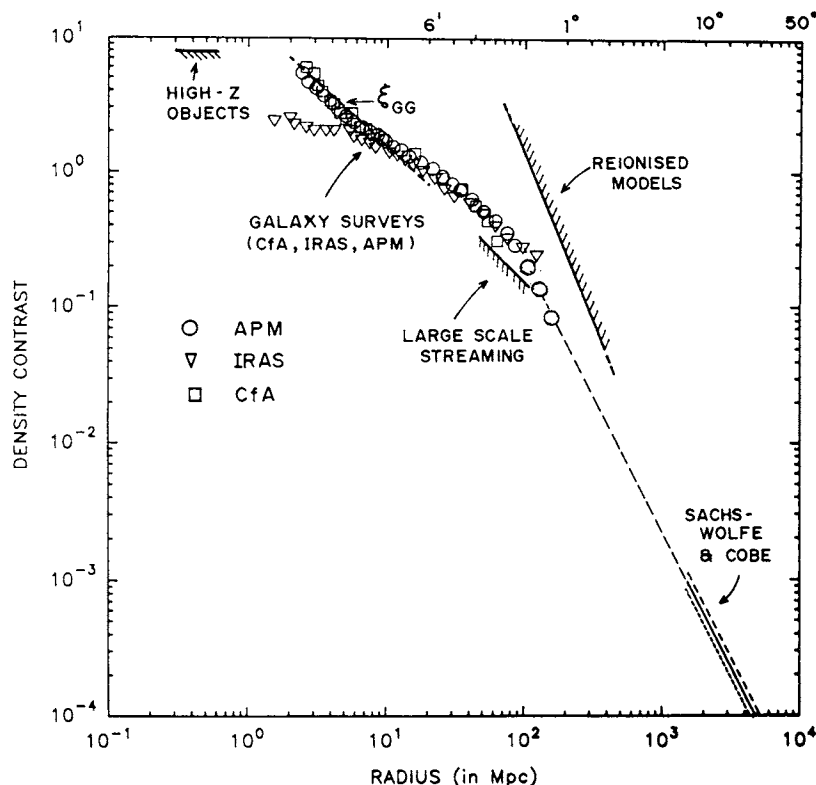


Figure 1. Various constraints on the rms fluctuations in mass, $\sigma(R) = \langle (\delta M/M)_R^2 \rangle^{1/2}$, are plotted against the scale R . Within a factor of order unity, this $\sigma^2(R)$ also represents the power in each octave, $[k^3 P(k)/2\pi^2]$, at $k = R^{-1}$. At the bottom right, we have indicated the result from *COBE* based on $(\Delta T/T)_{\text{rms}}$, assuming a spectrum which is scale-invariant at large scales. The $\sigma(R)$ s determined from CfA, *IRAS* and APM surveys are shown by different symbols. In the case of APM, $W(\theta)$ has been inverted to give $\xi(r)$. The lower bound on $\sigma(R)$ (marked ‘HIGH-Z OBJECTS’) arises from the requirement that mass-scales in the range $(10^{10} - 10^{11}) M_{\odot}$ should go non-linear by $z = 4.5$. The bound marked ‘LARGE SCALE STREAMING’ arises from the *POTENT* reconstruction of velocity fields. We have also shown the (rather weak) upper bound from small-angle anisotropies in the MBR in the reionized models, and the slope of the galaxy-galaxy correlation function ($h = 0.5$).

Mpc. The fact that masses in the range $(10^{10} - 10^{11}) M_{\odot}$ should have gone non-linear by $z \approx 4.5$ leads to another lower bound on $\sigma(R)$ shown near the top-left of Fig. 1 (Kashlinsky & Jones 1991; Padmanabhan 1991). The small-scale anisotropy of MBR in models with reionization provides a (weak) upper bound to $\sigma(R)$. We have also shown by a thick dashed line the slope corresponding to the galaxy-galaxy correlation function.

Several conclusions can be drawn from this constraint diagram. To begin with, notice that the *COBE* line denoting $\sigma_{\text{DM}}(R)$, extrapolated linearly, agrees with the $\sigma_{\text{gal}}(R)$ estimated from APM and large-scale streaming at around $50 h^{-1}$ Mpc. This consistency is easily verified by the following simple estimate. We know from theory that $v_{\text{pec}} \approx (1/3)v_{\text{H}}\sigma_{\text{DM}}$, where v_{H} and v_{pec} are Hubble and peculiar velocities at a given scale. If we take $v_{\text{pec}} \approx 330 \text{ km s}^{-1}$ at $50 h^{-1}$ Mpc, we find that $\sigma_{\text{DM}}(50 h^{-1} \text{ Mpc}) \approx 0.2$. To estimate the APM result, note that the three-dimensional correlation function $\xi(R)$ is related (approximately) to the angular correlation function $W(\theta)$ by $\xi(R) = W(\theta)(D/R)$, where $D \approx 200 h^{-1}$ Mpc is the sample depth and $\theta = (R/D)$. From the published APM result, $W(14^\circ) \approx (1-5) \times 10^{-3}$; since 14° at the depth of $200 h^{-1}$ Mpc corresponds to $R \approx 50$

h^{-1} Mpc, we find

$$\begin{aligned} \xi(50 h^{-1} \text{ Mpc}) &\approx (200 h^{-1} \text{ Mpc}/50 h^{-1} \text{ Mpc}) W(14^\circ) \\ &\approx (0.004-0.020). \end{aligned}$$

Hence $\sigma_{\text{gal}}(50 h^{-1} \text{ Mpc}) \approx \xi^{1/2} \approx (0.06-0.14)$; a more precise analysis gives the slightly larger value of $\sigma_{\text{gal}} \approx 0.2$ seen in Fig. 1, which is consistent with the peculiar velocity estimate. On the other hand, the *COBE* result suggests $\sigma_{\text{DM}}(50 h^{-1} \text{ Mpc}) \approx (24/50)^2 \approx 0.2$. These observations, therefore, seem to suggest that $\sigma_{\text{DM}}(50 h^{-1} \text{ Mpc}) \approx \sigma_{\text{gal}}(50 h^{-1} \text{ Mpc}) \approx 0.2$. Thus, within the error bars, it is consistent to assume that the bias factor $b(R)$ is unity at $R \approx 50 h^{-1}$ Mpc.

Fig. 1 also suggests that the magnitude of the slope of $\sigma_{\text{gal}}(R)$ is lower at small scales [say, $(5-50) h^{-1}$ Mpc] compared to its value at larger scales. A spectrum of the form $P(k) \approx Ak$ at large scales, extrapolated to $50 h^{-1}$ Mpc, has to bend rather sharply at smaller scales to account for the observation, if the biasing factor is not scale-dependent. To make these conclusions more quantitative, we have fitted (in Fig. 2) a series of simple test spectra to the constraints. The simplest type of spectrum, which behaves as $P \propto k$ at small k and exhibits a sharp bend at large k , is given by the family of functions

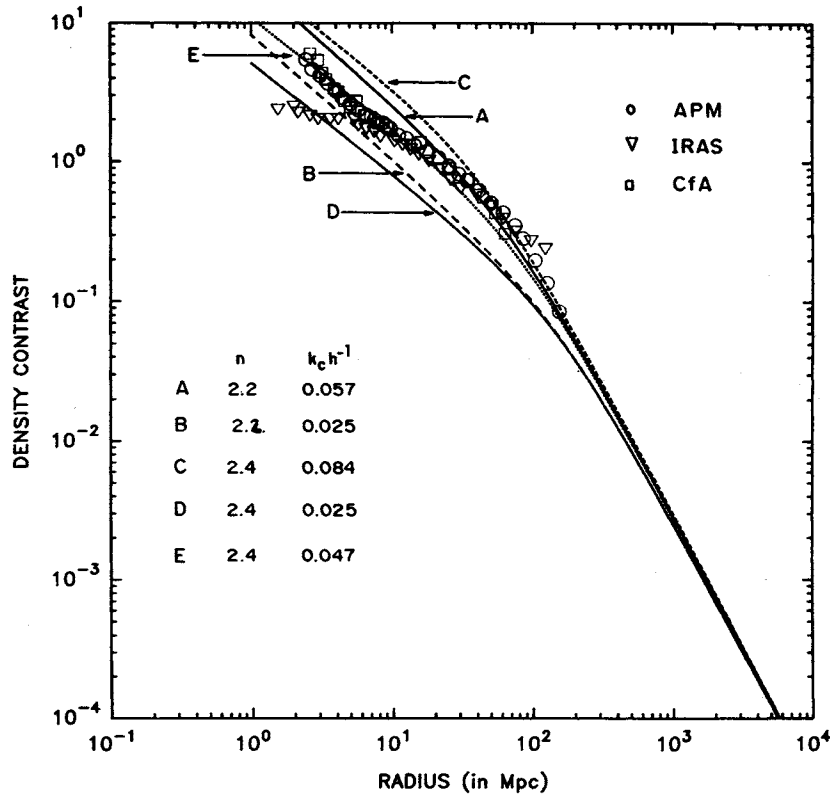


Figure 2. The results of galaxy surveys and *COBE* anisotropy are fitted using simple test spectra of the form $P(k) = Ak[1 + (k/k_c)^n]^{-1}$. All the spectra are normalized using the maximum permitted value of a_2 from *COBE*, namely $a_2 = 10^{-5}$. There is reasonable agreement for $n=2.4$, $k_c = 0.047 h$. (The fit can be marginally improved by choosing $n=2.41$ and $k_c = 0.047 h$.) This suggests a length-scale of $k_c^{-1} = 21 h^{-1}$ Mpc in the theory. We have set $h = 0.5$.

$$P(k) = \frac{Ak}{1 + (k/k_c)^n} \quad (8)$$

[These spectra were studied earlier by Peacock (1991) and Padmanabhan (1991).] Fig. 2 shows the behaviour of $\sigma(R)$ for these functions, for different choices of k_c and n . In all the cases, we normalize the spectrum to give $C_2 = 10^{-5}$, which is the maximum value allowed by *COBE* results. A good fit is achieved for the dotted curve, which corresponds to $n=2.4$ and $k_c = 0.047 h \text{ Mpc}^{-1}$. We have also shown in the diagram two curves ($n=2.2$, $k_c = 0.057 h$ and $n=2.4$, $k_c = 0.084 h \text{ Mpc}^{-1}$) which overshoot the best-fitting curve, and two curves ($n=2.2$, $k_c = 0.025 h$ and $n=2.4$, $k_c = 0.025 h \text{ Mpc}^{-1}$) which undershoot the best-fitting curve. This result shows that a sharp bend at $k_c = 0.047 h \text{ Mpc}^{-1}$ (corresponding to $R_c \approx 21 h^{-1} \text{ Mpc}$) is needed even to come anywhere close to fitting the data. [Even in this case, the fit is far from good in detail. To illustrate this point, we have calculated the angular correlation function, scaled to the Lick depth, for the best-fitting spectrum. This is shown in Fig. 3, along with some representative maximum and minimum points based on the APM result. The large errors in the $W(\theta)$ determined by the APM survey make it difficult to judge how good this fit is, but it does not seem to be very satisfactory.] Notice that these spectra are characterized by pure power laws for both large and small k . In contrast, a CDM spectrum *with the same*

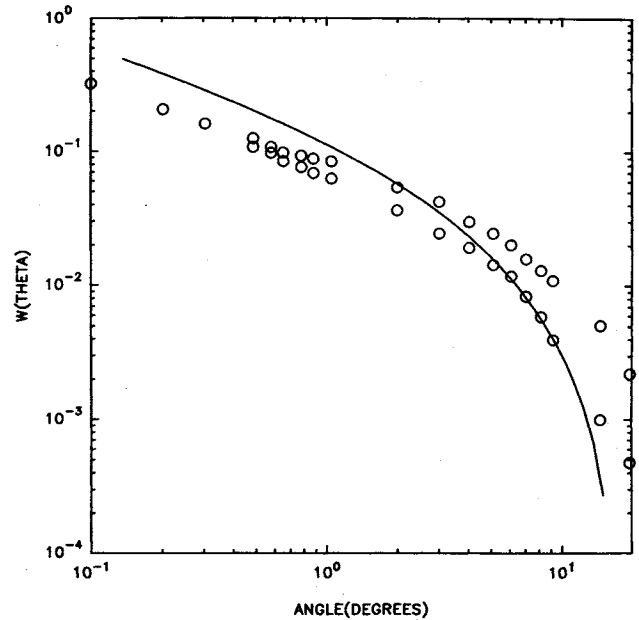


Figure 3. The angular correlation function $W(\theta)$, computed for the best-fitting spectrum ($n=2.4$, $k_c = 0.047 h$), is compared with the $W(\theta)$ from the APM survey. For simplicity, we have only indicated the maximum and minimum points allowed by the APM survey for a representative sample of θ values. All results are scaled to Lick depth and $h = 0.5$.

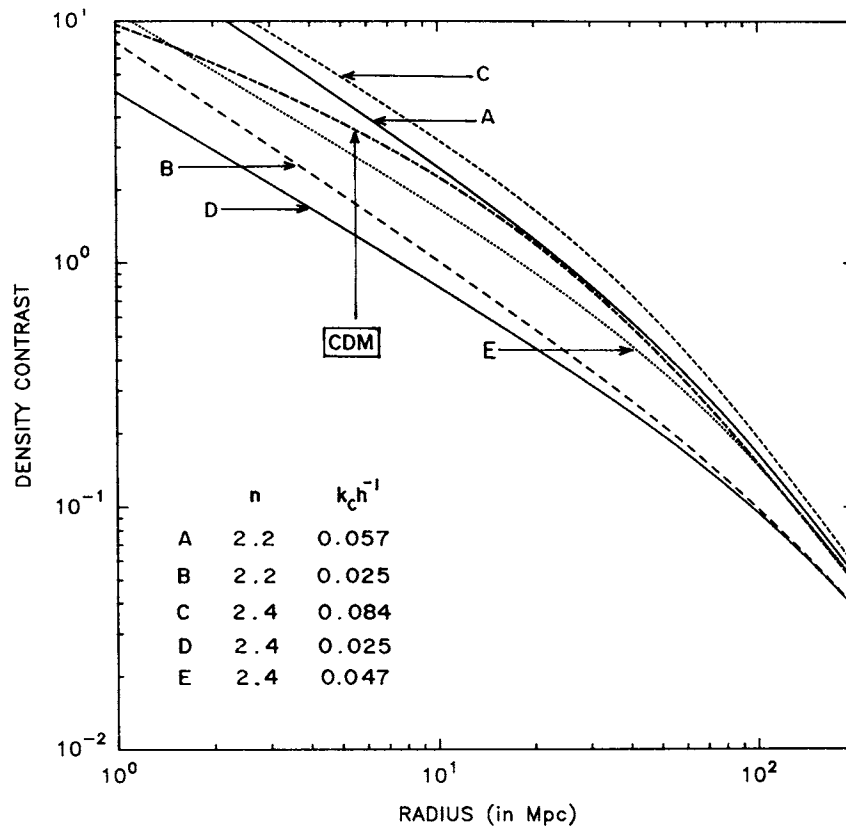


Figure 4. The model spectra in Fig. 2 are shown on an enlarged scale and compared with CDM (labelled dash-dot-dot curve). All curves are normalized by the *COBE* result. Note that the CDM spectrum shows a much more significant curvature compared to the model spectra.

COBE normalization shows much more curvature. We show in Fig. 4 such a CDM spectrum, along with the curves A–E. The CDM spectrum agrees fairly well with the best-fitting curve at large scales, but has a very different shape at small scales. If we normalize the CDM spectrum using the *COBE* results, we will get an amplitude of $A = 5.3 \times 10^6$. [In contrast, notice that, if we normalize the CDM spectrum by setting $\sigma_8 \equiv \sigma(8 h^{-1} \text{ Mpc}) = 1$, we find that $P \approx A_0 k$ at small k , with $A_0 = 4.4 \times 10^6$ for a universe with $\Omega = 1$, $\Omega_{\text{vac}} = 0$, $h = 0.5$.] This will make $\sigma_{\text{DM}}(8 h^{-1} \text{ Mpc})$ overshoot the value of unity. Thus we would require $b(8 h^{-1} \text{ Mpc}) < 1$ to obtain the correct σ_{gal} . In addition, the shape of the spectrum will be highly curved in the range $(2\text{--}20) h^{-1} \text{ Mpc}$, thereby leading to a $\xi_{\text{gg}}(R)$ which is in contradiction with observations. Non-linear effects could steepen the small-scale spectrum, thereby reducing the curvature, but this would also *increase* the amplitude to still higher values.

3 CONCLUSIONS

The above result clearly shows that it is not easy to explain both the large- and small-scale power in a natural fashion. Before the *COBE* results, it was generally felt that all that one required was an enhancement of large-scale power to bring the theory into agreement with APM results. *COBE* has added a new dimension to the problem, namely the shape of the power spectrum. Large-scale streaming motions and APM suggest that $\sigma(50 h^{-1} \text{ Mpc}) \approx 0.2$. The *COBE*

result is consistent with this fact, provided that the scale-invariant spectrum is extended straight down to $50 h^{-1} \text{ Mpc}$. If this is done, however, one requires a relatively sharp bend to account for the small-scale observations, and presumably a new length-scale in the theory.

The $\sigma_{\text{gal}}(R)$ in the above analysis has been obtained from the raw data in a fairly simple-minded way. (For example, there could be a systematic shift between infrared and optical selection of samples; the redshift-space distortion can affect the shape of the curve; such effects have not been included in this analysis.) But our basic result seems to be reasonably independent of the *detailed* behaviour of $\sigma_{\text{gal}}(R)$; it only depends on the overall trend. We also stress the fact that most of these conclusions can be reliably based on the linear theory, and that non-linear effects will only evolve the spectrum in a direction which will make the conclusions even stronger. It may be possible to probe the spectrum directly around $(20\text{--}50) h^{-1} \text{ Mpc}$ by concentrating on MBR anisotropies at scales of 6 arcmin to 1° . Such observations may offer an important clue to the shape of the density spectrum.

After this work was completed, we came to know of a preprint (Saunders et al. 1992) which determines the $\sigma_{\text{gal}}(R)$ from QDOT-QIGC data. It turns out that QDOT-QIGC data show evidence for structure that almost corresponds to the k_c we derived. Though it is too early to draw any firm conclusions, it is tempting to identify this scale as the length up to which the effects of the interaction between the already formed smaller scale structures are significant. In principle,

such an hypothesis can be tested if we study the angular distribution function for a small region in the sky corresponding to objects at a narrow range of redshift (say, around 0.3 or 0.5) and test whether the derived k_c is larger. However, it may be difficult to obtain a reliable distribution function from such data. Even if this hypothesis is not totally incorrect, however, it gives an idea of the new problems in the study of the formation and evolution of large-scale structures (length-scales larger than, say, $10 h^{-1}$ Mpc). Initial small- k perturbations are still evolving linearly when new structures appear to form as a result of the non-linear interaction between smaller scale features.

Since this paper was submitted for publication, several papers have appeared which derive constraints from *COBE* for galaxy formation scenarios (see e.g. Efstathiou, Bond & White 1992; Taylor & Rowan-Robinson 1992; Davis, Summers & Schlegel 1992). We shall briefly comment on the difference in perspective between this paper and the others. In the present work, we have put together the observations at various scales and attempted to extract from them two effective parameters, k_c and n . The spectrum in equation (8) is not directly related to the spectra in any of the standard theoretical models. In contrast, the papers cited above all deal with specific models for structure formation (CDM models with or without cosmological constant, mixed dark matter models, etc.) and attempt either to constrain the models or to obtain a 'best-fitting' scenario. In some sense, these two approaches are complementary, and have their own advantages and disadvantages. Use of a specific class of spectrum arising from certain dark matter models has a stronger physical motivation than does fitting of the data to (8). However, such an analysis – by its very nature – is too strongly tied to the models which are being studied. As a consequence, it is somewhat difficult to draw model-independent conclusions which are hiding in the available data. We have attempted to fit the data to a test spectrum of the form in (8) – rather than to any class of dark matter models – to see what general conclusions can be drawn from the observations at various scales. In fact, any specific model which is in good agreement with the data will have an *effective* k_c and n which are close to the best-fitting values we have obtained.

There is another motivation for keeping the analysis model-independent at this stage. It is clear, both from the present work and from the references cited above, that the simplest models (with a single-component dark matter, $\Omega_{\text{vac}} = 0$, $\Omega = 1$) are incapable of accounting for all data *if the bias factor is independent of scale*. Most of the authors cited above attempt to keep the bias factor constant, but introduce more complicated combinations of dark matter. However, if the biasing is due to any sensible physical mechanism then it is very likely to be scale-dependent. In such a case, there is

not much point in comparing a theoretical spectrum (however well motivated the model) with observations at different scales without understanding the various biasing mechanisms which operate at different scales. Unfortunately, our knowledge of biasing processes is quite primitive at present. We therefore feel that it is indeed worthwhile not to tie oneself down to specific theoretical models but to concentrate on more general conclusions which can be drawn from the data. In that sense, any theoretical model which – after the inclusion of biasing effects – leads to an effective k_c and n close to the best-fitting values will be acceptable. Only when the biasing effects are sufficiently well understood will we be able really to constrain the theoretical models. We plan to investigate the evolution of these parameters, due to various biasing mechanisms in different models, in a future publication.

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