

Two new diagnostics of dark energy

Varun Sahni^a, Arman Shafieloo^a and Alexei A. Starobinsky^b

^a *Inter-University Centre for Astronomy and Astrophysics,
Post Bag 4, Ganeshkhind, Pune 411 007, India and*

^b *Landau Institute for Theoretical Physics, Kosygina 2, Moscow 119334, Russia*

We introduce two new diagnostics of dark energy (DE). The first, Om , is a combination of the Hubble parameter and the cosmological redshift and provides a *null test* of dark energy being a cosmological constant Λ . Namely, if the value of $Om(z)$ is the same at different redshifts, then DE $\equiv \Lambda$, *exactly*. The slope of $Om(z)$ can differentiate between different models of dark energy even if the value of the matter density is not accurately known. For DE with an unevolving equation of state, a positive slope of $Om(z)$ is suggestive of Phantom ($w < -1$) while a negative slope indicates Quintessence ($w > -1$). The second diagnostic – *acceleration probe* \bar{q} – is the mean value of the deceleration parameter over a small redshift range. It can be used to determine the cosmological redshift at which the universe began to accelerate, again without reference to the current value of the matter density. We apply the Om and \bar{q} diagnostics to the Union data set of type Ia supernovae combined with recent data from the cosmic microwave background (WMAP5) and baryon acoustic oscillations.

1. INTRODUCTION

The nature of dark energy (DE) is one of the most intriguing questions facing physics. The fact that DE provides the main contribution to the energy budget of the universe today while remaining subdominant during previous epochs, provides a challenge to model builders attempting to understand the nature of this seemingly all-pervasive ether-like substance.

Theoretical models for DE include the famous cosmological constant, Λ , suggested by Einstein in 1917 [1] and shown to be related to the vacuum energy $\langle T_{ik} \rangle_{\text{vac}} \propto \Lambda g_{ik}$ several decades later [2, 3]. Indeed, the cosmological constant appears to occupy a privileged position amongst other DE models by virtue of the fact that its equation of state $w = -1$ is Lorentz invariant and so appears the same to any inertial observer. However, within the context of cosmology, an explanation of DE in terms of Λ faces one drawback, namely, in order for the universe to accelerate today the ratio of the energy density in the cosmological constant to that in radiation must have been very small at early times, for instance $\rho_\Lambda \simeq 3 \times 10^{-58} \rho_{\text{EW}}$ at the time of the electroweak phase transition. Although vacuum energy may conceivably be associated with small numbers such as the neutrino mass ($\rho_\Lambda \sim m_\nu^4$) or even the fine structure constant [4], a firm theoretical prediction for the value of Λ is currently lacking, allowing room for alternatives including models in which both the DE density and its equation of state (EOS) evolve with time. Alternatives to the cosmological constant include scalar field models called quintessence which have $w > -1$, as well as more exotic ‘phantom’ models with $w < -1$.

Although most recent studies show that a cosmological constant + cold dark matter (LCDM) is in excellent agreement with observational data, dynamical dark energy can explain the data, too [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Indeed, the enormous variety of DE models suggested in the literature (see [16] for reviews) has been partially responsible for the burgeoning industry of model independent techniques aimed at reconstructing the properties of dark energy directly from observations [17]. It is well known that model independent methods must be wary of several pitfalls which can subvert their efficacy. These relate to priors which are sometimes assumed about fundamental cosmological quantities such as the EOS and the matter density. As first pointed out in [18], an incorrect prior for the EOS can lead to gross misrepresentations of reality. The same applies to the value of the matter density. Indeed, as we shall demonstrate later in this paper, an incorrect assumption about the value of Ω_{0m} can lead to dramatically incorrect conclusions being drawn about the nature of dark energy. Clearly the need of the hour, then, is a diagnostic which is able to differentiate LCDM from ‘something else’ with as few priors as possible being set on other cosmological parameters.

In this paper we introduce a new diagnostic, Om , which is constructed from the Hubble parameter $H \equiv \dot{a}/a$ determined directly from observational data and provides a *null test* of the LCDM hypothesis. Here $a(t)$ is the scale factor of a Friedmann-Robertson-Walker (FRW) cosmology. We show that Om is able to distinguish dynamical DE from the cosmological constant in a robust manner *both with and without reference* to the value of the matter density, which can be a significant source of uncertainty for cosmological reconstruction. The Om diagnostic is in many respects the logical companion to the statefinder $r = \ddot{a} / aH^3$ [19] (otherwise dubbed jerk j , see e.g. [20, 21]

as well as the earlier paper [22]). We remind the reader that $r = 1$ for LCDM while $r \neq 1$ for evolving DE models. Hence $r(z_1) - r(z_2)$ provides a *null test* for the cosmological constant. Similarly, the unevolving nature of $Om(z)$ in LCDM furnishes $Om(z_1) - Om(z_2)$ as a *null test* for the cosmological constant. (For null tests based on gravitational clustering see [23, 24].) Like the statefinder, Om depends only upon the expansion history of our Universe. However, while the statefinder r involves the third derivative of the expansion factor $a(t)$, Om depends upon its *first* derivative only. Therefore, as we demonstrate in this paper, Om is much easier to reconstruct from observations [49].

It is clear that a determination of $H(z)$ from any single test may suffer from systematic uncertainties. That is why, in order to obtain Om with sufficient accuracy, it is necessary to combine information about $H(z)$ obtained from different independent tests, such as the supernovae luminosity distance, the scale of baryon acoustic oscillations (BAO) in the matter power spectrum as a function of z , the acoustic scale in the angular power spectrum of the cosmic microwave background (CMB) temperature fluctuations, etc.

The second diagnostic – *Acceleration probe* \bar{q} – is constructed out of the Hubble parameter and the lookback time. Like Om it does not depend upon the current value of the matter density. We apply \bar{q} to current data and show that it provides an independent test of the present acceleration of the Universe.

The plan of the rest of the paper is as follows. In Sec. 2 the Om diagnostic is introduced for a flat as well as a spatially curved FRW background, and determined using SNLS and Union supernovae data. In Sec. 3 the \bar{q} diagnostic is introduced. In Sec. 4 both these diagnostics are determined from a combination of existing data sets including type Ia supernovae, BAO and WMAP5 CMB data. Sec. 5 contains our conclusions.

2. THE OM DIAGNOSTIC – A NULL TEST OF LCDM

A. Influence of Ω_{0m} on properties of dark energy

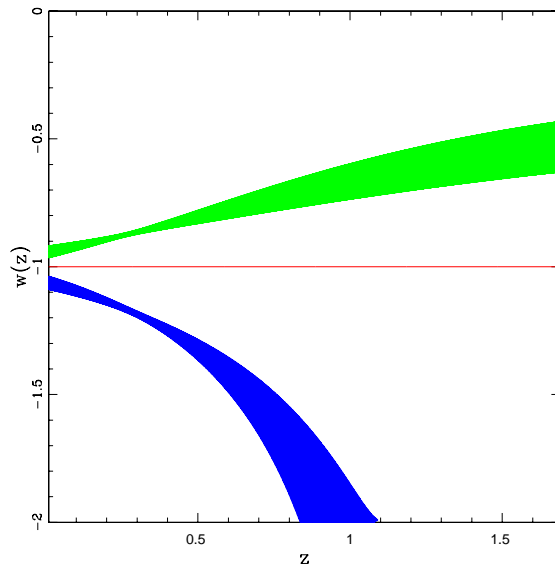


FIG. 1: The equation of state of a fiducial LCDM model ($w = -1, \Omega_{0m}^{true} = 0.27$) is reconstructed using an incorrect value of the matter density. For $\Omega_{0m}^{erroneous} = 0.22$ the resulting EOS shows quintessence-like behavior and its $1 - \sigma$ contour is shown in green. In the opposite case, when $\Omega_{0m}^{erroneous} = 0.32$, the EOS is phantom-like and its $1 - \sigma$ contour is shown in blue. Note that in both cases the true fiducial model (red) is excluded in the reconstruction. (The parametric reconstruction scheme suggested in [19] was applied to SNAP-quality data to construct this figure.)

Given many alternative models of dark energy it is useful to try and understand the properties of DE in a model independent manner. An important model independent quantity is the expansion history, $H(z)$, whose value can be reconstructed from observations of the luminosity distance, D_L , via a single differentiation [25, 26, 27, 28, 29]

$$H(z) = \left[\frac{d}{dz} \left(\frac{D_L(z)}{1+z} \right) \right]^{-1}. \quad (2.1)$$

The equation of state, $w(z)$, of DE is more cumbersome to reconstruct since it involves two derivatives of $D_L(z)$ and is therefore a noisier quantity than $H(z)$. An additional source of uncertainty relating to $w(z)$ is caused by the fact that the value of the matter density, Ω_{0m} enters into the determination of $w(z)$ explicitly, through the expression

$$w(x) = \frac{(2x/3) d \ln H / dx - 1}{1 - (H_0/H)^2 \Omega_{0m} x^3} . \quad (2.2)$$

Clearly an uncertainty in Ω_{0m} propagates into the EOS of dark energy even if $H(z)$ has been reconstructed quite accurately. This fact has been emphasized in several papers [14, 17, 18, 30, 31] and is illustrated in figure 1, which shows how an erroneous estimate of Ω_{0m} adversely affects the reconstructed EOS by making a LCDM model appear as if it were quintessence (if $\Omega_{0m}^{erroneous} < \Omega_{0m}^{true}$) or phantom (if $\Omega_{0m}^{erroneous} > \Omega_{0m}^{true}$).

The influence of dark matter on dark energy persists if a parametric ansatz such as CPL [32]

$$w(z) = w_0 + w_1 \frac{z}{1+z}, \quad (2.3)$$

is employed in the determination of

$$\begin{aligned} H^2(z) &= H_0^2 [\Omega_{0m}(1+z)^3 + \Omega_{DE}], \\ \Omega_{DE} &= (1 - \Omega_{0m}) \exp \left\{ 3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right\} . \end{aligned} \quad (2.4)$$

In this case, if Ω_{0m} is wrongly specified then, in a maximum likelihood approach, the DE parameters w_0, w_1 will adjust to make $H(z)$ as close to its real value as possible, leading once more to an erroneous reconstruction of the cosmic equation of state.

These two factors: the larger errors caused by the double differentiation of a noisy quantity (D_L) and the strong dependence of $w(z)$ on an uncertain quantity (Ω_{0m}) adversely impact the cosmological reconstruction of the EOS making it difficult to differentiate a cosmological constant from evolving DE from an analysis of $w(z)$ alone.

B. The Om diagnostic introduced

In this paper we suggest an alternative route which enables us to distinguish LCDM from other DE models without directly involving the cosmic EOS. Our starting point is the Hubble parameter which is used to determine the Om diagnostic

$$Om(x) \equiv \frac{h^2(x) - 1}{x^3 - 1}, \quad x = 1 + z, \quad h(x) = H(x)/H_0 . \quad (2.5)$$

For dark energy with a constant equation of state $w = const$,

$$h^2(x) = \Omega_{0m} x^3 + (1 - \Omega_{0m}) x^\alpha, \quad \alpha = 3(1+w) \quad (2.6)$$

(we assume that the universe is spatially flat for simplicity). Consequently,

$$Om(x) = \Omega_{0m} + (1 - \Omega_{0m}) \frac{x^\alpha - 1}{x^3 - 1}, \quad (2.7)$$

from where we find

$$Om(x) = \Omega_{0m} \quad (2.8)$$

in LCDM, whereas $Om(x) > \Omega_{0m}$ in quintessence ($\alpha > 0$) while $Om(x) < \Omega_{0m}$ in phantom ($\alpha < 0$). We therefore conclude that: $Om(x) - \Omega_{0m} = 0$ iff DE is a cosmological constant [50]. In other words, the Om diagnostic provides us with a *null test* of the cosmological constant. This is a simple consequence of the fact that $h^2(x)$ plotted against x^3 results in a straight line for LCDM, whose slope is given by Ω_{0m} , as shown in figure 2. For other DE models the line describing $Om(x)$ is curved, since the equality

$$\frac{dh^2}{dx^3} = constant, \quad (2.9)$$

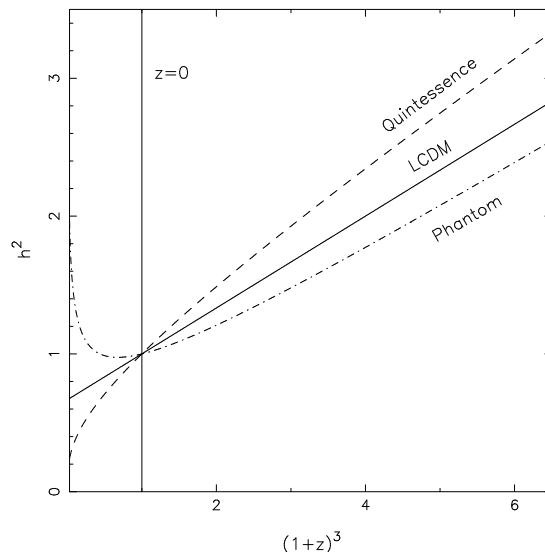


FIG. 2: The Hubble parameter squared is plotted against the cube of $1+z$ for Quintessence ($w = -0.7$, dashed), LCDM ($w = -1$, solid) and Phantom ($w = -1.3$, dot-dash). The universe is assumed to be spatially flat and $\Omega_{\text{DE}} = 2/3$ in all models. For LCDM the plot h^2 vs $(1+z)^3$ is a straight line whereas for P and Q this line is curved in the interval $-1 < z \lesssim 1$. This forms the basis for the observation that $\overline{Om(x_1, x_2)} \equiv Om(x_1) - Om(x_2) = 0$ in LCDM, while $Om(x_1, x_2) > 0$ in Quintessence and $Om(x_1, x_2) < 0$ in Phantom, if $x_1 < x_2$. Thus $Om(x_1, x_2)$ furnishes us with a *null test* for the cosmological constant. (At $z < 0$ the Hubble parameter for Phantom diverges at the ‘Big Rip’ future singularity, while for Quintessence $h(z) \rightarrow 0$ as $z \rightarrow -1$. LCDM approaches the de Sitter space-time at late times.)

(which always holds for LCDM) is satisfied in quintessence/phantom type models only at redshifts significantly greater than unity, when the effects of DE on the expansion rate can safely be ignored. As a result the efficiency of the Om diagnostic improves at low $z \lesssim 2$ precisely where there is likely to be an abundance of cosmological data in the coming years !

In practice, the construction of Om requires a knowledge of the Hubble parameter, $h(z)$, which can be determined using a number of model independent approaches [19, 30, 33]. In figure 3 we show the Om diagnostic reconstructed from SNAP-quality data using the non-parametric prescription of [30]. One clearly sees that for quintessence as well as phantom the line describing $Om(x)$ is curved, which helps distinguish these models from LCDM even if the value of the matter density is not accurately known.

Clearly, a comparison of Om at two different redshifts can lead to insights about the nature of DE *even if the value of Ω_{0m} is not accurately known*. Thus, the two-point difference diagnostic

$$Om(x_1, x_2) \equiv Om(x_1) - Om(x_2) = (1 - \Omega_{0m}) \left[\frac{x_1^\alpha - 1}{x_1^3 - 1} - \frac{x_2^\alpha - 1}{x_2^3 - 1} \right], \quad (2.10)$$

can serve as a *null test* of the cosmological constant hypothesis, since

$$Om(x_1) = Om(x_2) \quad (\Lambda - \text{term}). \quad (2.11)$$

In other words, $Om(x_1, x_2) = 0$ iff DE is a cosmological constant; $Om(x_1, x_2) > 0$ for quintessence while $Om(x_1, x_2) < 0$ for phantom ($x_1 < x_2$). Thus, the value of Om determined at two redshifts can help distinguish between DE models *without reference either to the matter density or H_0* !

We have reconstructed the Om diagnostic and the cosmic EOS for two SNe data sets: SNLS [6] and Union [9].

The SNLS (Supernova Legacy Survey) dataset contains 115 Type Ia supernovae (SNe Ia) in the range $0.1 < z < 1.0$. The Union dataset [9] is a new compilation of SNe Ia and consists of 307 SNe after selection cuts, includes the recent samples from the SNLS [6] and ESSENCE Surveys [8], older datasets, as well as the recently extended dataset of distant supernovae observed with HST [7].

Results for the SNLS dataset, shown in figure 4, indicate that while the EOS is quite sensitive to the value of the matter density, the Om diagnostic is not. Note that the three distinct models of dark energy in figure 4 result in virtually the same luminosity distance since: (i) $\chi^2 = 110.93$ for $\Omega_{0m} = 0.32$ (Phantom), (ii) $\chi^2 = 110.99$ for $\Omega_{0m} = 0.28$ (LCDM), (iii) $\chi^2 = 111.02$ for $\Omega_{0m} = 0.22$ (Quintessence). This shows that different values of Ω_{0m} and

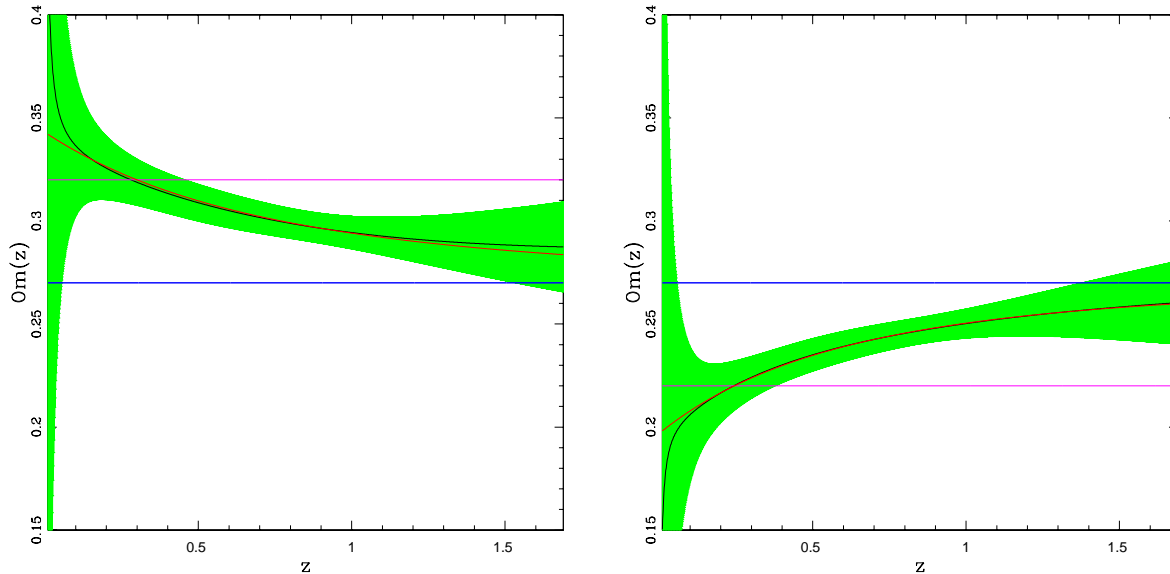


FIG. 3: The *left panel* shows the $Om(z)$ diagnostic reconstructed for a fiducial *quintessence* model with $w = -0.9$ and $\Omega_{0m} = 0.27$ (black line, green shaded region shows 1σ CL, the red line is the exact analytical result for Om). The horizontal blue line shows the value of Om for a Λ CDM model with the same value of Ω_{0m} as quintessence. Note that any horizontal line in this figure represents Λ CDM with a *different value of Ω_{0m}* . For instance Λ CDM with $\Omega_{0m} = 0.32$ is shown by the horizontal magenta line. As this figure shows, the negative curvature of quintessence allows us to distinguish this model from (zero-curvature) Λ CDM independently of the current value of the matter density. The *right panel* shows the $Om(z)$ diagnostic reconstructed for a fiducial *phantom* model with $w = -1.1$ and $\Omega_{0m} = 0.27$ (black line, green shaded region shows 1σ CL). The positive curvature of phantom allows us to distinguish this model from (zero-curvature) Λ CDM independently of the current value of the matter density. For instance, phantom can easily be distinguished from Λ CDM both with the correct $\Omega_{0m} = 0.27$ (horizontal blue) as well as incorrect $\Omega_{0m} = 0.22$ (horizontal magenta). (The non-parametric reconstruction scheme suggested in [30] has been employed on SNAP quality data for this reconstruction.)

$w(z)$ can provide an excellent fit to the same set of data, as originally pointed out by [18]. From the upper panel of figure 4 we find that the Om diagnostic is virtually independent of the input value of Ω_{0m} and its flat form is suggestive of LCDM [51]. Note that the degeneracy between evolving dark energy and the cosmological constant can be broken by studying the behavior of $Om(z)$ at higher redshifts. For most models the effect of DE on expansion becomes negligible for $z > \text{few}$ leading to $Om(z) \rightarrow \Omega_{0m}$ at moderately high redshifts. Consequently the differential diagnostic $Om(z_1, z_2)$ evaluated at $|z_1 - z_2| \gg 1$ can help break the degeneracy caused by uncertainties in the value of Ω_{0m} . For the parameters in figure 4, $Om(z_1, z_2) = 0$ for the cosmological constant, while $|Om(z_1, z_2)| = 0.05$ for evolving DE; see section 2 C for a related discussion.

Figure 5 shows results for the more recent Union dataset. Again we see that the behaviour of the EOS can range from being quintessence-like (for $\Omega_{0m} = 0.22$) to being phantom-like (for $\Omega_{0m} = 0.32$). The behaviour of Om is less sensitive to the value of the matter density and leads us to conclude that while a cosmological constant appears to be strongly preferred by SNLS, constraints from the Union dataset allow evolving DE as well as Λ .

One can improve the efficiency of the Om diagnostic (2.10) by determining it selectively in regions where there is better quality data. The error in the reconstructed value of the Hubble parameter is [34]

$$\frac{\delta H}{H}(z) \propto \frac{\sigma}{N(z)^{1/2}}, \quad (2.12)$$

where $N(z)$ is the number of supernovae in a given redshift interval and σ is the noise of the data. Since $N(z)$ is never likely to be a perfectly uniform distribution, there will always be regions where $N(z)$ is larger and $H(z)$ better reconstructed. Consequently, by determining $Om(z_1, z_2)$ selectively in such regions, one can improve the efficiency of this diagnostic by ‘tuning it’ to the data.

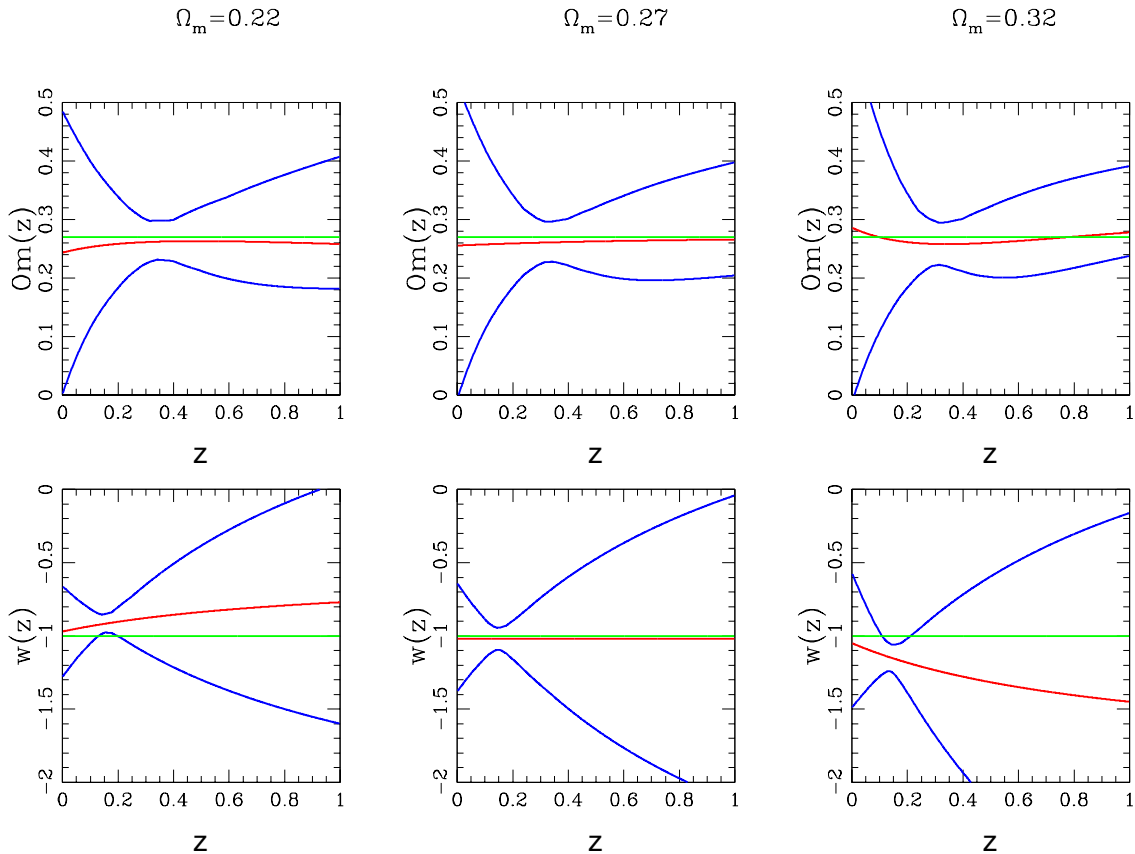


FIG. 4: Reconstructed $Om(z)$ and $w(z)$ from SNLS supernovae data using the CPL ansatz (2.3) and assuming three different values $\Omega_m = 0.22, 0.27, 0.32$ for the matter density. Notice that while the best fit value of $Om(z)$ is virtually independent of the redshift (top panel, red curve) and is therefore consistent with LCDM (with $\Omega_m = 0.27$: green line), the reconstructed EOS strongly depends upon the value of the matter density. Thus, for the same data set, the best fit value of $w(z)$ is suggestive of quintessence for $\Omega_m = 0.22$, LCDM for $\Omega_m = 0.27$ and phantom for $\Omega_m = 0.32$, while $Om(z)$ favours LCDM throughout. Note that the small variations in $Om(z)$ in the three upper panels are a consequence of the CPL ansatz which requires, as input, the value of the matter density Ω_m . A non-parametric ansatz such as [30], or the parametric ansatz [28], would have led to a uniquely reconstructed $Om(z)$ with no dependence on Ω_m . Blue lines show 1σ error bars.

C. Dark Energy Metamorphosis

An important example of quintessence is provided by tracker DE models, which give rise to cosmic acceleration at late times while earlier, during the radiation and matter dominated epochs, the density in the tracker remains proportional to the background matter density [35, 36]. This last property leads to $\rho_{\text{track}}/\rho_B \simeq \text{constant} \ll 1$ at $z > z_t$, where ρ_B is the background density of matter or radiation and z_t is the redshift when tracking ends. As an example consider the double exponential model [35] $V(\phi) = M^4[\exp(-\alpha\phi) + \exp(-\beta\phi)]$ with $\alpha \gg \beta$, $\beta \ll 1$, which has the attractor solution $\Omega_\phi = 3(1+w_B)/\alpha^2$, $w_\phi = w_B$, at high redshift $z > z_t$, while $w_\phi \simeq -1 + \alpha^2/3$ at the present time. Since these models behave like quintessence at late times, their behavior is similar to that shown in figure 1 for a typical quintessence model. Consequently these models may be distinguished from LCDM by applying the Om diagnostic shown in figure 1. An interesting limiting case corresponds to *metamorphosis* models which have $w_0 \simeq -1$ today and $w \rightarrow 0$ at earlier times [12, 37]. For such models $Om(x) = \Omega_{0m}$ for $x < x_t$ and $Om(x) = \tilde{\Omega}_{0m}$, for $x \gg x_t$ where

$$\tilde{\Omega}_{0m} \simeq \Omega_{0m} + \frac{1 - \Omega_{0m}}{(1 + z_t)^3}. \quad (2.13)$$

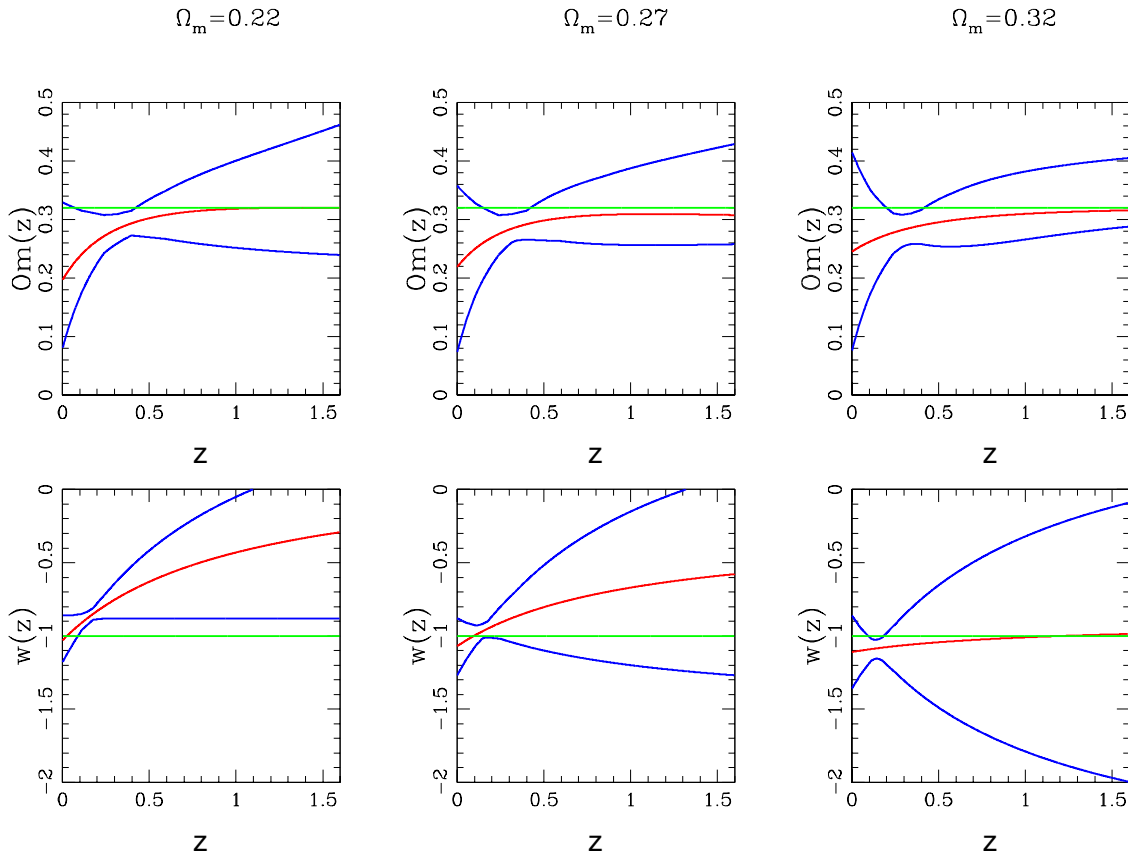


FIG. 5: Reconstructed $Om(z)$ and $w(z)$ from recent Union supernovae data using the CPL ansatz (2.3) and assuming three different values $\Omega_m = 0.22, 0.27, 0.32$ for the matter density. $Om(z)$ appears to be much more robust against variation in Ω_m in comparison with $w(z)$. The horizontal green line in the top panel indicates value of $Om(\equiv \Omega_m) = 0.32$ for LCDM model. The blue lines show 1σ error bars. Though LCDM is still consistent with the Union data, this consistency is not quite as strong as it was for the SNLS data shown in the previous figure. The top panel clearly indicates that evolving DE is also perfectly consistent with Union data.

Consequently, the Om diagnostic applied to data at low and high redshift, may help distinguish between tracker DE and LCDM as shown in figure 6. Tracker behavior can also arise in modified gravity theories such as Braneworld models [38] and scalar-tensor cosmology [39]. The growth of density perturbations provides a complementary means of distinguishing these models from LCDM.

An important property of the Om diagnostic is that the value of the cosmological density parameter Ω_{0m} does not enter into its definition (2.5) explicitly. As a result the diagnostic relation $Om(x_1, x_2) = 0$ (LCDM) does not require an a-priori knowledge of the matter density and therefore provide a means of differentiating the cosmological constant from evolving DE models even if uncertainties exist in the value of Ω_{0m} . (Current observations suggest an uncertainty of at least 25% in the value of Ω_{0m} [11].) For a constant EOS the dependence of Om on the value of the matter density can be altogether eliminated by constructing the ratio

$$\mathcal{R} = \frac{Om(x_1, x_2)}{Om(x_3, x_4)} \equiv \frac{\left[\frac{x_1^\alpha - 1}{x_1^3 - 1} - \frac{x_2^\alpha - 1}{x_2^3 - 1} \right]}{\left[\frac{x_3^\alpha - 1}{x_3^3 - 1} - \frac{x_4^\alpha - 1}{x_4^3 - 1} \right]}, \quad (2.14)$$

from where we see that the EOS encoded in the parameter $\alpha = 3(1 + w)$ can be determined from \mathcal{R} without any reference whatsoever to the value of Ω_{0m} !

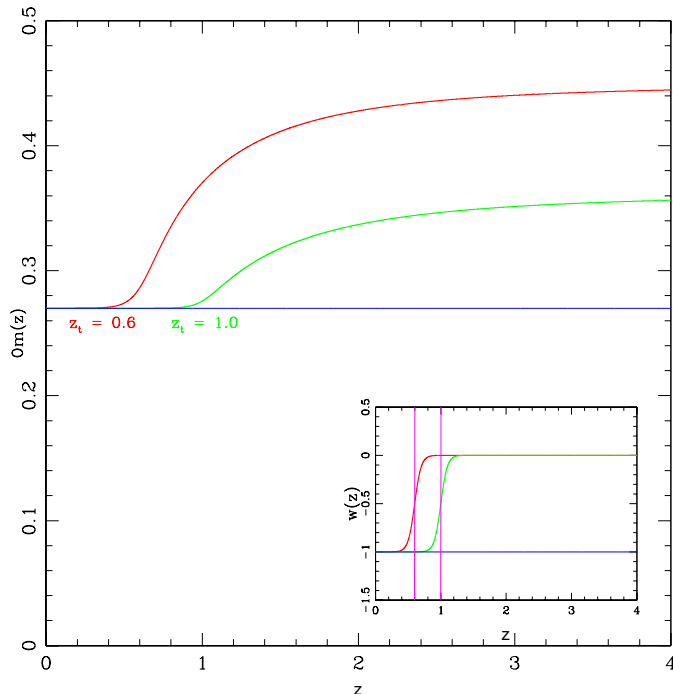


FIG. 6: The Om diagnostic is shown for two tracker models which mimic LCDM at low redshift ($z < z_t$) and dark matter at high redshift ($z > z_t$). The horizontal blue line shows LCDM. The inset shows the EOS for the tracker's as a function of redshift.

D. Influence of spatial curvature on Om

The preceding analysis, which showed how the Om diagnostic could distinguish between alternative models of DE, was based on the assumption that the universe was spatially flat. While this may well be true with sufficient accuracy, especially within the framework of the inflationary scenario which predicts $|\Omega_k - 1| \sim 10^{-5}$ today (due to the $l = 0$ mode of primordial scalar fluctuations), let us now consider the case of small, but non-zero spatial curvature. Then $Om(x)$ acquires the correction

$$\delta Om = \Omega_k \frac{x+1}{x^2+x+1}, \quad x = 1+z, \quad (2.15)$$

effectively, which has a fixed functional dependence on z . So, $|\delta Om| \leq 2|\Omega_k|/3$ and decreases with the growth of z . For the interval $-0.0175 < \Omega_k < 0.0085$ at the 95% CL admitted by the WMAP5 results [11], this correction does not exceed 0.01 (or $\sim 4\%$ of the value of the Om itself).

In this case (2.10) is modified to

$$Om(x_1, x_2) = \Omega_{DE} \left[\frac{x_1^\alpha - 1}{x_1^3 - 1} - \frac{x_2^\alpha - 1}{x_2^3 - 1} \right] + \Omega_k \left[\frac{x_1^2 - 1}{x_1^3 - 1} - \frac{x_2^2 - 1}{x_2^3 - 1} \right], \quad (2.16)$$

and we assume $x_2 > x_1$ as in figure 1.

The influence of the curvature term can be estimated by a simple ‘back of the envelope’ calculation which we carry out for quintessence (Q) and phantom (P). We assume $\Omega_{DE} = 0.7$, $\Omega_k = -0.0175$, $z_1 = 0.1$, $z_2 = 1$ and $w = -0.9$ for Q while $w = -1.1$ for P. In the case of Quintessence we find $Om(x_1, x_2) = 0.042$ when the curvature term is included in (2.16) and $Om(x_1, x_2) = 0.038$ when it is excluded. Thus the inequality $Om(x_1, x_2) > 0$, which generically holds for quintessence models, appears to be quite robust, since the contribution (read ‘contamination’) from the curvature term is only a fraction (9%) of the ‘signal’ from DE. Similar results are obtained for Phantom: $Om(x_1, x_2) = -0.037$ when the curvature term is included and $Om(x_1, x_2) = -0.041$ when it is not. The presence of curvature leads, once more, to a 9% change in our estimation of Om leading us to conclude that the phantom inequality $Om(x_1, x_2) < 0$ is

robust. (Of course, as $w_{\text{DE}} \rightarrow -1$, the relative influence of curvature in (2.16) becomes significant and can dominate the ‘signal’ from DE models which are very close to LCDM; see also [40].)

3. THE ACCELERATION PROBE

In the previous section we showed how the difference between the value of the Hubble parameter at nearby redshifts could be used to construct a null diagnostic for the LCDM model. In this section we construct another dimensionless quantity which could prove useful for determining the onset of cosmic acceleration in DE models which has also been the focus of other recent studies [43, 44].

Our diagnostic, *acceleration probe*, is the mean value of the deceleration parameter

$$\bar{q} = \frac{1}{t_1 - t_2} \int_{t_2}^{t_1} q(t) dt . \quad (3.1)$$

Since

$$q(t) = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 , \quad (3.2)$$

it follows that *acceleration probe* can be written in the following simple form

$$1 + \bar{q} = \frac{1}{\Delta t} \left(\frac{1}{H_1} - \frac{1}{H_2} \right) \quad (3.3)$$

where $\Delta t = t_1 - t_2 \equiv (t_0 - t_2) - (t_0 - t_1)$, and

$$t_0 - t(z) = \int_0^z \frac{dz}{(1+z)H(z)} \quad (3.4)$$

is the cosmic look-back time (also see [44, 45]).

Equation (3.3) expresses the mean deceleration parameter in terms of the look-back time and the value of the Hubble parameter at two distinct redshifts. From expressions (3.3) and (3.4) we find that, like the Om diagnostic, the acceleration probe \bar{q} does not depend upon the value of Ω_m and is, therefore, robust to uncertainties in the value of the matter density.

In figure 7 we show \bar{q} obtained using Union supernovae and the CPL ansatz. The behaviour of \bar{q} suggests $0.4 \leq z_a \leq 0.8$ for the redshift at which the universe began to accelerate. This result is independent of the value of the matter density. Close to the acceleration redshift, $\bar{q} \simeq 0$, and one obtains a very simple relationship linking the look-back time with the Hubble parameter

$$\Delta t = \frac{1}{H_1} - \frac{1}{H_2} , \quad (3.5)$$

where H_1 and H_2 lie on ‘either side’ of the acceleration redshift z_a when $q(z_a) = 0$. Since both the look-back time and the Hubble parameter can be reconstructed quite accurately (see for instance [17, 45]), it follows that one might be able to obtain the redshift of the acceleration epoch in a model independent manner using (3.5).

It is worth noting that the value of \bar{q} can also be obtained from an accurate determination of galactic ages. In this case we do not require a continuous form of $H(z)$ to compute the look-back time. We simply subtract the galactic ages at two distinct redshifts bins to determine Δt . The same information can also be used to derive $H(z)$ [45] since

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt} , \quad (3.6)$$

which is then used to determine \bar{q} . However at present errorbars in observed galactic ages are large and the number of data is small, so it is unlikely that this method will be useful for determining \bar{q} at this stage. In the nearby future, with better quality and quantity of data, \bar{q} can be used as a model independent probe of the acceleration of the universe using distinct and uncorrelated cosmological data.

Note that the acceleration epoch is quite sensitive to the underlying DE model. For DE with a constant equation of state

$$1 + z_a = \left(\frac{|1 + 3w|\Omega_{0DE}}{\Omega_{0m}} \right)^{\frac{1}{|3w|}} , \quad (3.7)$$

and so an accurate determination of z_a using (3.5) could provide useful insights into the nature of DE.

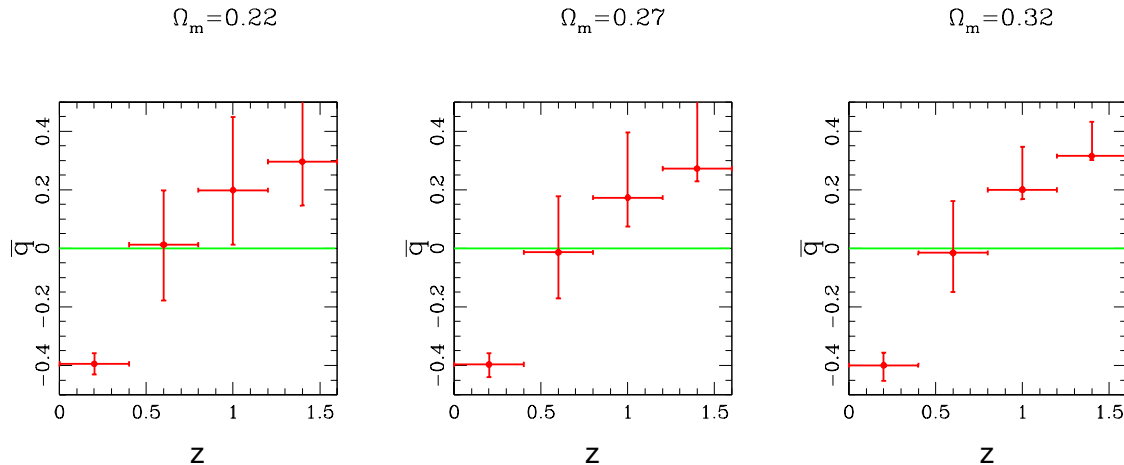


FIG. 7: The diagnostic \bar{q} is plotted in 4 bins using the recent Union supernovae data. The CPL ansatz has been used for three different values of the matter density. Error-bars in y-axis show $1 - \sigma$ CL. Note that the value of the acceleration redshift $0.4 \leq z_a \leq 0.8$ appears to be robust.

4. DETERMINING Om AND THE ACCELERATION PROBE FROM SNE, BAO AND CMB

In this section we determine our two new diagnostics, Om and \bar{q} , from a combination of: (i) the Union supernovae data set [9], (ii) data from baryon acoustic oscillations (BAO) [46], (iii) WMAP5 CMB data [47].

Acoustic oscillations in the photon-baryon plasma prior to recombination give rise to a peak in the correlation function of galaxies. This effect has recently been measured in a sample of luminous red galaxies observed by the Sloan Digital Sky Survey and leads to the value [46]

$$A = \frac{\sqrt{\Omega_{0m}}}{h(z_1)^{1/3}} \left[\frac{1}{z_1} \int_0^{z_1} \frac{dz}{h(z)} \right]^{2/3} = 0.469 \left(\frac{n}{0.98} \right)^{-0.35} \pm 0.017, \quad (4.1)$$

where $h(z) = H(z)/H_0$ and $z_1 = 0.35$ is the redshift at which the acoustic scale has been measured. The 5 year Wilkinson Microwave Anisotropy Probe (WMAP5) results, when combined with the results from BAO yield $n = 0.961$ for the spectral index of the primordial power spectrum [47, 48].

We also use the following value for the CMB ‘shift parameter’ (the reduced distance to the last scattering surface) deduced from WMAP5

$$R = \sqrt{\Omega_{0m}} \int_0^{z_{1s}} \frac{dz}{h(z)} = 1.715 \pm 0.021, \quad (4.2)$$

where $z_{1s} = 1089$. We use the two constraints, A , R together with the Union SNe data set to determine $Om(z)$ and \bar{q} . The CPL ansatz (2.3) has been used to parametrize the expansion history, and all three parameters in this ansatz: Ω_{0m} , w_0 and w_1 , are treated as being free in our maximum likelihood routine. Our results are summarized in figure 8. We find that LCDM is in excellent agreement with the data but other DE models fit the data too. These include quintessence and phantom models, some of which are shown in figure 9.

5. CONCLUSIONS

In this paper we propose two new diagnostics for determining the properties of dark energy. The first of these, $Om(z)$, is constructed from the Hubble parameter and results in the identity $Om(z) = \Omega_{0m}$ for LCDM. For other DE models $Om(z)$ is a function of the redshift. This allows one to construct a simple *null test* to distinguish the cosmological constant from evolving DE. We find that Om is a robust diagnostic whose value can be determined

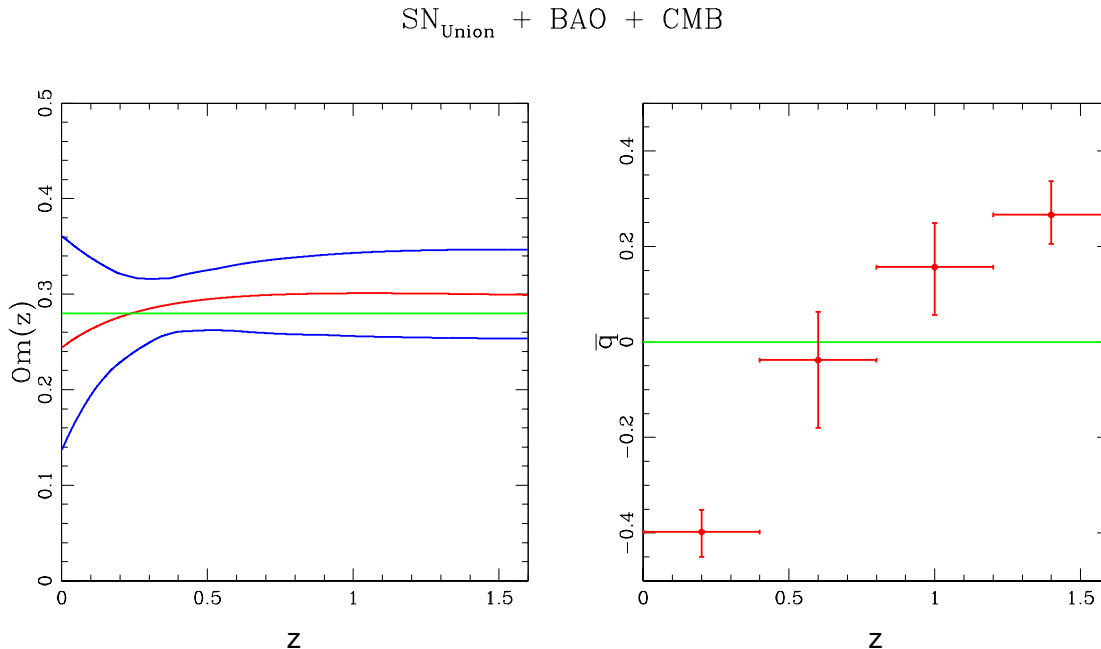


FIG. 8: Two new diagnostics, $Om(z)$ (left panel) and \bar{q} (right panel) are plotted using a combination of supernovae, BAO and CMB data. The CPL ansatz has been used assuming the matter density also to be a free parameter. Blue lines in left panel and red crosses in right panel show 1σ errorbars.

reasonably well even with current data. Unlike the equation of state w , Om relies only on a knowledge of the Hubble parameter and depends neither on $H'(z)$ nor on quantities like the growth factor $\delta(z)$. Errors in the reconstruction of Om are therefore bound to be smaller than those appearing in the EOS. To diminish systematic uncertainties, different tests which allow for the possibility of reconstructing $H(z)$ from observational data need to be used. Om can be determined using parametric as well as non-parametric reconstruction methods. It can shed light on the nature of dark energy even if the redshift distribution of supernovae is not uniform and the value of the dark matter density is not accurately known. The second diagnostic, *acceleration probe* \bar{q} , is the mean value of the deceleration parameter over a small redshift interval. The *acceleration probe* depends upon the value of the Hubble parameter and the look-back time and can be used to determine the epoch when the universe began to accelerate.

We use the current type Ia SNe data in conjunction with BAO and CMB data to estimate the values of $Om(z)$ and \bar{q} . Our results are consistent with LCDM but do not exclude evolving DE models including phantom and quintessence.

Note added in proof: At $z \ll 1$,

$$\frac{Om - \Omega_{0m}}{1 - \Omega_{0m}} \simeq 1 + w_0 ,$$

which suggests that the Om diagnostic can be used to probe w_0 from data available at low z , if Ω_{0m} is accurately known.

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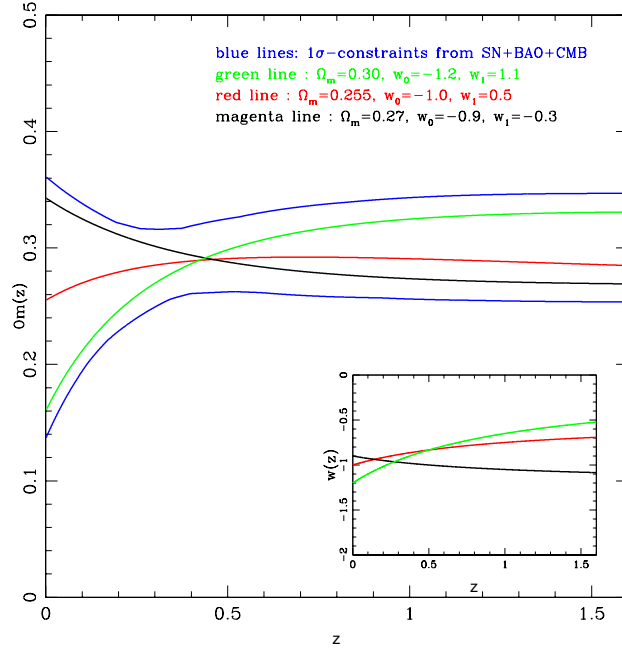


FIG. 9: The blue lines in the main figure show 1σ constraints on $Om(z)$ from a combination of SNe, BAO and CMB data. Also shown are values for $Om(z)$ from three DE models all of which are consistent with the data at the 1σ level. The green line is DE with $\Omega_m = 0.3, w_0 = -1.2, w_1 = 1.1$, this model crosses the phantom divide at $w = -1$. The red line shows a metamorphosis model with $\Omega_m = 0.255, w_0 = -1.0, w_1 = 0.5$, while magenta shows the model with $\Omega_m = 0.27, w_0 = -0.9, w_1 = -0.3$. In all cases the CPL ansatz (2.3) has been used and the bottom-right corner of the figure shows the EOS for these diverse DE models. Note that in both the main figure as well as the inset, Λ CDM corresponds to a horizontal straight line (not shown).

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- [49] Note that the Om diagnostic does not use any information about the evolution of inhomogeneities in an FRW background, usually represented by the growth factor $\delta(z) \equiv (\delta\rho/\rho)_m$. Therefore our proposal for a null test of LCDM is totally unrelated to earlier null tests [23, 24] (based on the analytical expression for Ω_m in terms of $\delta(z)$ obtained in [25]), and to the test of physical nature of dark energy (equivalently, that G_{eff} is a constant) suggested in [17].
- [50] Note that since $\Omega_\Lambda + \Omega_{0m} \simeq 1$ in LCDM, this model contains SCDM ($\Omega_{0m} = 1, \Omega_\Lambda = 0$) as an important limiting case. Consequently the Om diagnostic cannot distinguish between large and small values of the cosmological constant unless the value of the matter density is independently known. An identical degeneracy exists for the statefinder r as discussed in [19].
- [51] Note that although $Om(z) \simeq constant$, its value in the upper left and right panels of figure 4 is not equal to Ω_{0m} . Therefore from (2.8) it follows that the best fit DE model in these two cases is not LCDM. We conclude that although the behavior of $Om(z)$ appears tantalizingly similar to what one would expect for the cosmological constant, the severe degeneracies between Ω_{0m} and w prevent us from drawing a firm conclusion about the nature of dark energy on the basis of SNLS data alone.