Nonparametric Analysis of The CMB Angular Power Spectrum WMAP 1-3-5-7 and beyond

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August 10, 2011

CMB Anisotropies and the Power Spectrum

- **Expansion in spherical harmonics**

$$\Delta T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{l,m} Y_{l,m}(\theta, \phi),$$

- $\Delta T(\theta, \phi)$ **is a Gaussian random field**

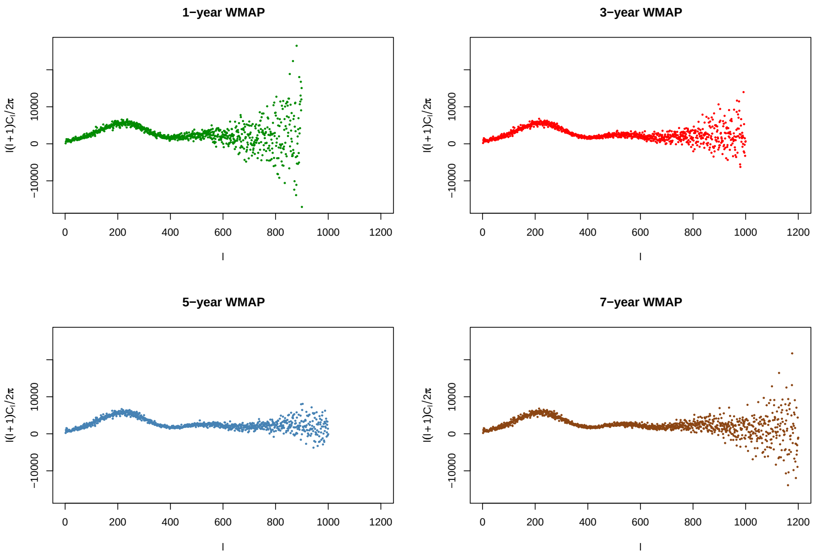
$\longrightarrow$ $a_{l,m}$ are mean-0 random variables with variance $C_l := E|a_{l,m}|^2$.

$$\tilde{C}_l := \frac{1}{2l+1} \sum_{m=-l}^{+l} |a_{l,m}|^2$$

- **Expect larger variability in the $\tilde{C}_l$ at low- and high-l ends**

  - low l: due to what is called *cosmic variance*
  - high l: due incomplete sky coverage + instrumental noises

WMAP 1/3/5/7: Power Spectrum Data



Regression: Generalities

- **Canonical form of the regression problem:** Given data $(X_1, Y_1), \ldots, (X_n, Y_n)$ where $Y_i = f(x_i) + \epsilon_i$ estimate relationship f between X and Y.
- **Formally:** $f(x) = E(Y|X = x)$.
- **Hitch:** Typically, there is only one realization of Y at each x.
- **Solution:** Make reasonable assumptions about the noise ϵ_i, try to make a reasonable guess $\widehat{f}(x)$.
- **Parametric/model-based regression:** Assume a specific functional form for $\widehat{f}(x)$ with *finite* number of adjustable parameters. Estimate adjustable parameters by maximizing the likelihood function.
- **Nonparametric/model-independent regression:** No functional form assumed; let the data drive the fit through optimal smoothing. Technically, infinite number of parameters.

Nonparametric Regression

$Y_i = f(x_i) + \epsilon_i$

- "Loss function" $\equiv$ a measure of closeness between estimator $\widehat{f}$ and the unknown truth f:

$$L(\widehat{f}, f) = \int \left(\widehat{f}(x) - f(x)\right)^2 dx.$$

- "Risk" or average error can be written as

$$R(\widehat{f}, f) = E\left[L(\widehat{f}, f)\right] = \int \text{Bias}^2(x)dx + \int \text{Var}(x)dx$$

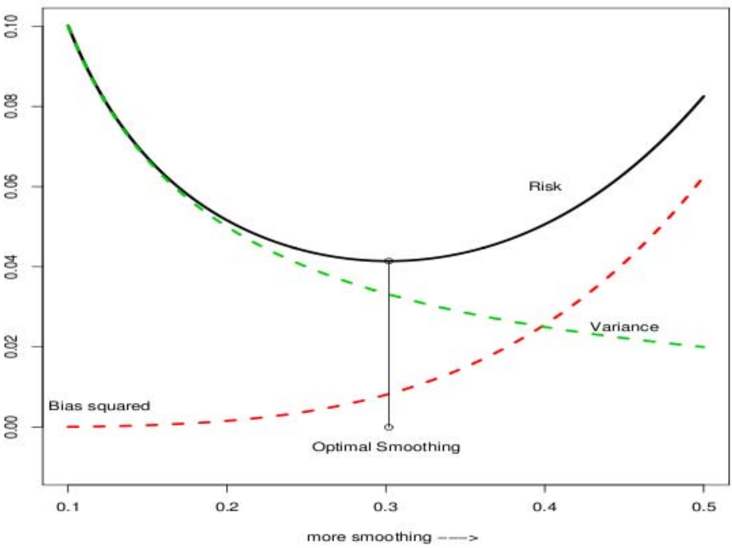
where

$\text{Bias}(x) = E\left[\ \widehat{f}(x)\ \right] - f(x)$

$\text{Var}(x) = E\left[\ \left(\ \widehat{f}(x) - E[\ \widehat{f}(x)\ ]\ \right)^2\ \right]$

- Minimize risk R to find the best $\widehat{f}$.

Bias-Variance Tradeoff and Optimal Smoothing



REACT

Genovese et al. (2004), Beran (2000), Beran and Dümbgen (1998), Larray Wasserman books (2004, 2006)

- $Y_i = f(x_i) + \epsilon_i$, with $\epsilon_i \sim N(0, \sigma^2)$ IID, σ^2 known.
- Assume $f \in L_2(a, b)$ and a complete orthonormal basis $\{\phi_j(x)\}$.

$$f(x) = \sum_{j=0}^{\infty} \beta_j \phi_j(x),\ \beta_j = \int_a^b f(x)\phi_j(x)dx$$

- Regression estimator $\widehat{f}(x)$:

$$f(x) = \sum_{j=0}^{n-1} \widehat{\beta}_j \phi_j(x) + \text{(some truncation bias)}$$

$\widehat{\beta}_j := \lambda_j Z_j$ with $1 \geq \lambda_0 \geq \ldots \geq \lambda_{n-1} \geq 0.$ and $Z_j = \sum_{i=1}^{n} Y_i \phi_j(x_i)$

REACT Confidence Set

Beran and Dümbgen (1998)

- The REACT formalism also gives a powerful inferential entity: the REACT confidence set around the fit $\widehat{f}$.
- $$B = \left\{\beta \in R^n : ||\beta - \widehat{\beta}||^2 \leq \rho^2\right\}$$

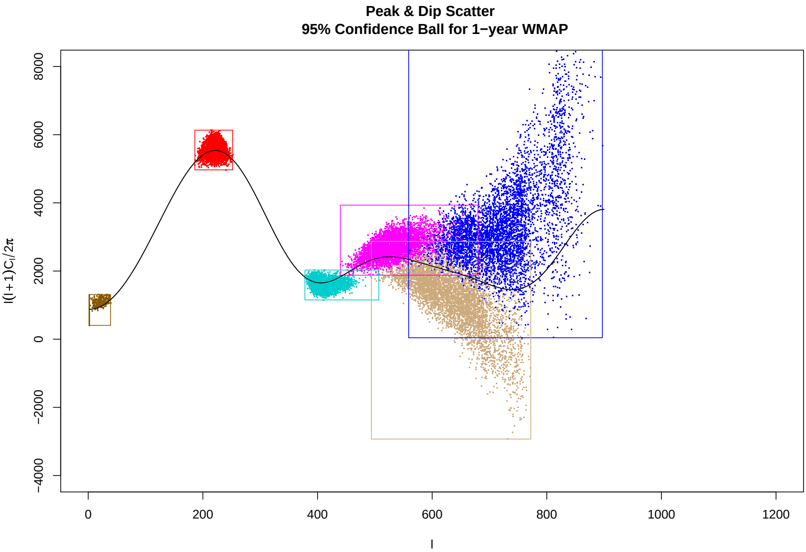
  $$\rho^2 = R(\lambda_*) + \widehat{\tau} z_\alpha / \sqrt{n}$$

  $$\widehat{\tau}^2 = 2\sigma^2 \left(\sigma^2 + 2\sum_J^{n-1} \left(Z_j^2 - \frac{\sigma^2}{n}\right)\right)$$
- This is a spherical object centered around the fit $\widehat{f}$ in the L_2 space such that

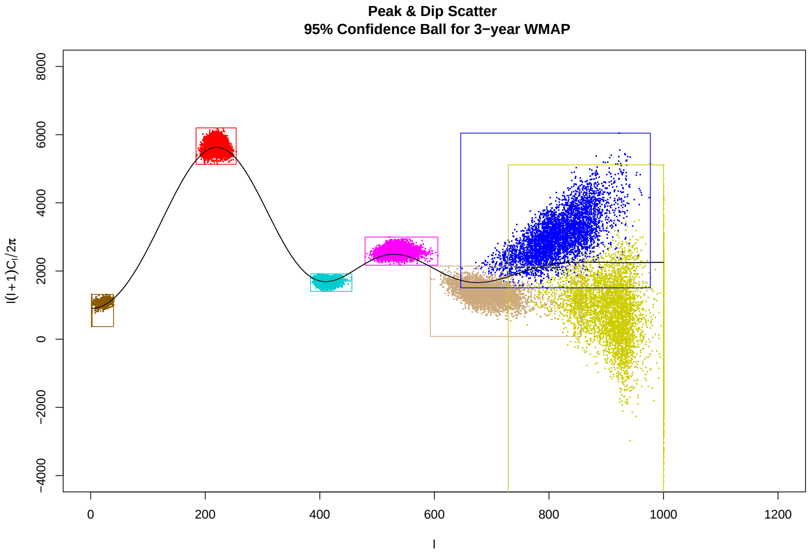
  $$P(\beta \in B) \longrightarrow 1 - \alpha$$

  (asymptotically)

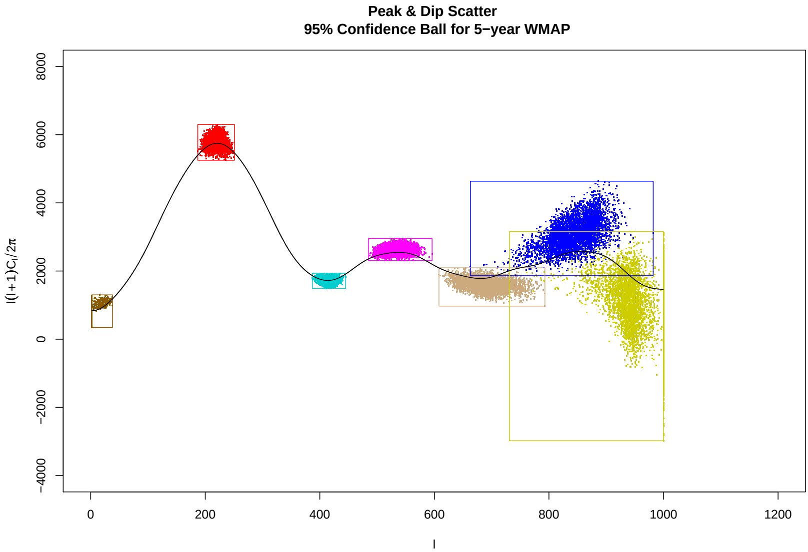
WMAP 1-year: Confidence Ball Scatter of Peaks/Dips



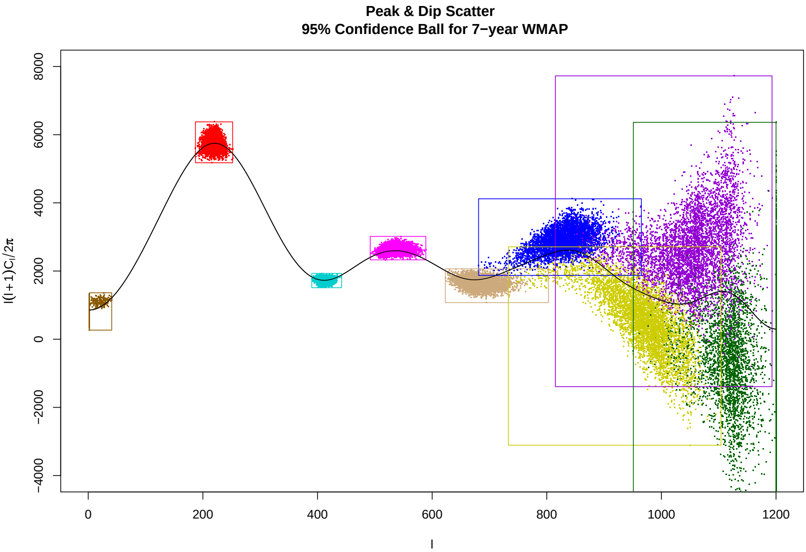
WMAP 3-year: Confidence Ball Scatter of Peaks/Dips



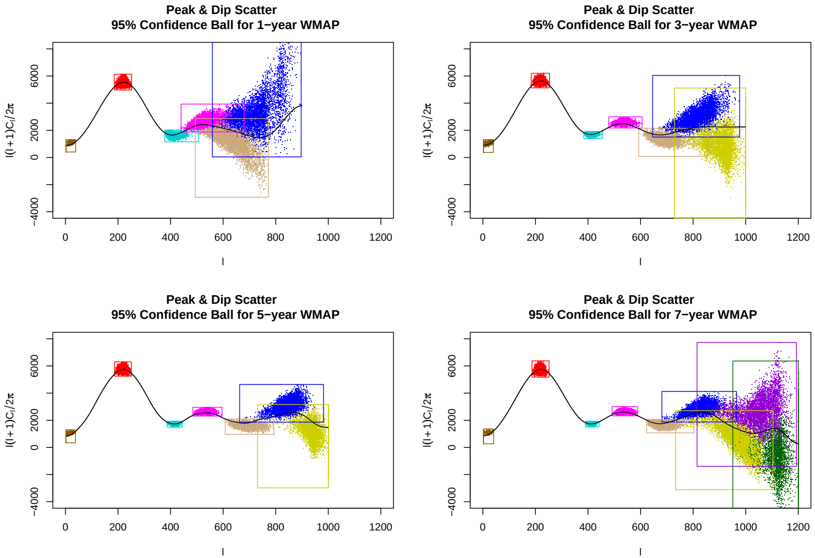
WMAP 5-year: Confidence Ball Scatter of Peaks/Dips



WMAP 7-year: Confidence Ball Scatter of Peaks/Dips

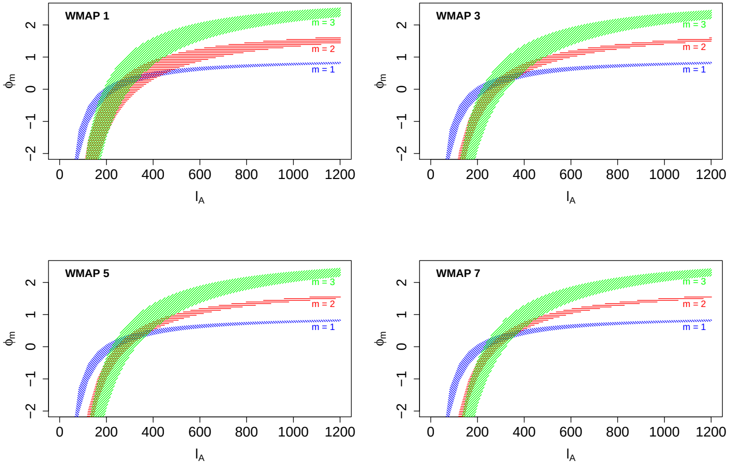


WMAP 1/3/5/7: Confidence Ball Scatter of Peaks/Dips



Confidence bands for Acoustic Scale

$$l_m = l_A(m - \phi_m)$$ (Hu, et al. 2000)



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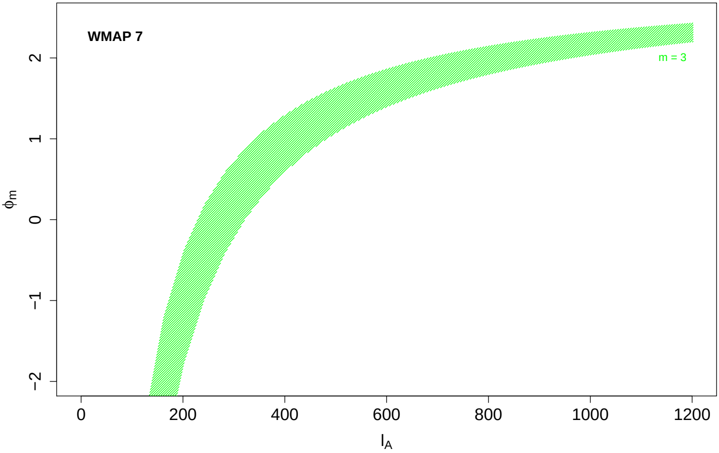

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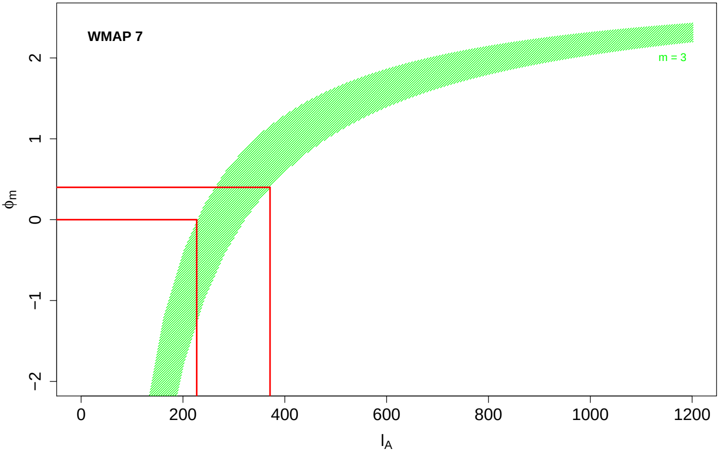

Checking Alternative Fits Against a REACT Fit

Parametric ΛCDM & HΛCDM Fits Relative to REACT Fit

Data	ΛCDM rejection probability	HΛCDM (with neutrino) rejection probability
1-year	16.70%	15.87%
3-year	14.52%	29.03%
5-year	19.66%	94.82%
7-year	9.08%	94.98%

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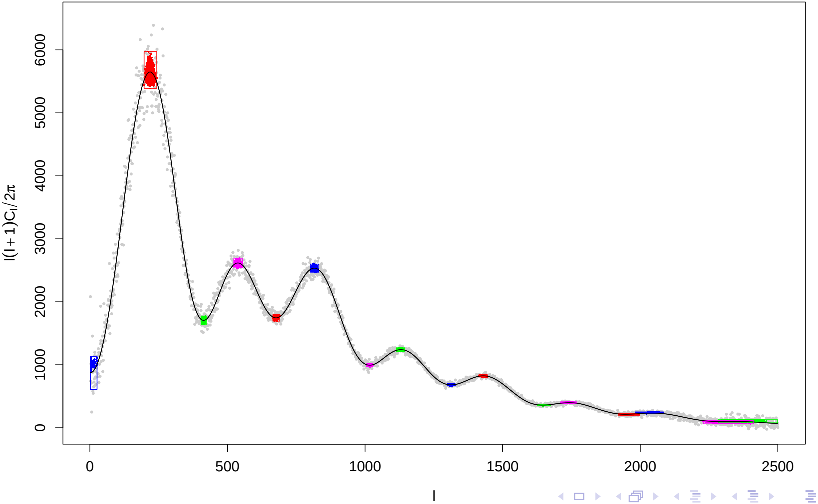
Simulated Planck data

REACT fit on Simulated Planck data (FutureCMB)



Credits

Collaborators

- **Mihir Arjunwadkar** (Guide)

  Centre for Modeling and Simulation, University of Pune

  National Centre for Radio Astrophysics

- **Tarun Souradeep** (Co-Guide)

  Inter-University Centre for Astronomy and Astrophysics