On the removal of divergences in electrodynamics: a global point of view

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This paper explores the implications of Dirac's seminal work on the concept of self-energy of a charged particle in classical electrodynamics. To avoid the notion of divergent acceleration through self-action, Dirac offered an alternative that involved the existence of preacceleration and an apparent departure from the inherent causality of special relativity. It is argued that Dirac's solution appears naturally in the electrodynamics described by action at a distance. In this framework the notion of self-action is replaced by that of the response of the universe on the large scale. Provided the universe has the correct large-scale structure, there are no divergent integrals either in the classical or the quantum version of electrodynamics. The price one has to pay involves replacing the purely local Lorentz invariant picture by a global cosmological one. On the other hand the price of standard renormalizable quantum electrodynamics is that of the theoretical mass of the electron is infinitely negative, a requirement that Dirac regarded as absurd, far worse than the loss of local invariance in favour of global invariance, the position adopted here.

1. Introduction

The three initials in Dirac's name could very well stand for attributes that made him the outstanding theoretical physicist that he was: he was a pragmatist, an aesthete and a mathematician. While guided by considerations of elegance and simplicity in basic physical laws, he would not hesitate to take unconventional steps if they achieved his expected end results and he would, if required, create mathematical tools to back those steps. The non-commutative algebra of the Poisson brackets, the delta function, the relativistic wave equation, the large numbers hypothesis, etc., are hallmarks of the P, A, M. Equally illustrative but not so well known was Dirac's attack on the problem of the self-energy of the electron, the problem that concerns us here.

The problem which Dirac (1938) was addressing was a purely classical one, and can be summarized as follows. Consider an electron a of charge e and mass m, moving in the flat space-time of special relativity. The Maxwell equations for the field of the point particle a are given by

$$F^{(a)ik}_{;k} = 4\pi e \int \delta_4(X, A) \frac{\mathrm{d}a^i}{\mathrm{d}s_a} \mathrm{d}s_a \equiv 4\pi j^{(a)i}, \tag{1}$$

where $X \equiv x(^i)$ is a general point with coordinates x^i and $A \equiv (a^i)$ is a typical point on the worldline of charge a. The element of proper time on the worldline of a is given by $\mathrm{d}s_a$ where

$$\mathrm{d}s_a^2 = \eta_{ik}\,\mathrm{d}a^i\,\mathrm{d}a^k. \tag{2}$$

The function $\delta_4(X, A)$ is the four-dimensional delta function which vanishes unless $X \equiv A$.

The normal tendency is to choose a solution for the field tensor $F^{(a)ik}$ that has compact support on the future light cone of the source. This is the commonly known retarded solution. Using this solution and a limiting process one can evaluate the force exerted by this field on its own source charge. Although $F^{(a)}$ diverges on the worldline of a, the divergent terms in the force expression can be shown to vanish by symmetry arguments. So the equation of motion becomes

$$m\frac{\mathrm{d}^2 a^i}{\mathrm{d}s_a^2} = e\frac{\mathrm{d}a^k}{\mathrm{d}s_a}\tilde{F}^{(a)i}_{\ \ k}, \tag{3}$$

where

$$\tilde{F}^{(a)ki} = \tfrac{2}{3}e\left\{\frac{\mathrm{d}^3 a^k}{\mathrm{d}s_a^3}\frac{\mathrm{d}a^i}{\mathrm{d}s_a} - \frac{\mathrm{d}^3 a^i}{\mathrm{d}s_a^3}\frac{\mathrm{d}a^k}{\mathrm{d}s_a}\right\}. \tag{4}$$

The field is none other than that of radiative reaction, i.e. the damping force experienced by the charge because of loss of energy through electromagnetic radiation.

In fact Dirac showed that (4) can be obtained by evaluating the following expression:

$$\tfrac{1}{2}\{F^{(a)ki}_{\mathrm{ret}} - F^{(a)ki}_{\mathrm{adv}}\} \equiv \tilde{F}^{(a)ki} \tag{5}$$

on the electric charge a. The two fields on the left-hand side are respectively the retarded and advanced fields of the charge a. Both fields individually diverge on the worldline of the charge but their difference remains finite, being given by the right-hand side of (4). This limiting procedure is more clearcut and elegant than the method using the retarded solution alone.

Dirac (op. cit.) showed, however, that this equation leads to an unphysical solution in which a free electron accelerates exponentially in time so as to attain a near light speed over a characteristic timescale of $\sim e^2/mc^3$. To avoid such solutions Dirac prescribed the following rule: Whenever an electron is subjected to an external force and then left alone, it must get into the state of uniform motion $\mathrm{d}a^i/\mathrm{d}s_a = \text{const}$. This makes the solution finite and bounded but it introduces an unusual aspect. Instead of specifying everything initially we are now specifying one condition (zero acceleration) finally and this has acausal connotations. When the external force is withdrawn the electron has to reach a state of zero acceleration, thus requiring it to adjust its earlier motion in anticipation of that final state. The anticipation period is

$$\tau \sim e^2/mc^3. \tag{6}$$

Dirac commented on this situation as follows:

> This is a fundamental departure from the ordinary ideas of relativity and is to be interpreted by saying that it is possible for a signal to be transmitted faster than light through the interior of an electron. The finite size of the electron now reappears in a new sense, the interior of the electron being a region of failure, not of the field equations of electromagnetic theory, but of some of the

elementary properties of space-time. In spite of this departure from ordinary relativistic ideas, our whole theory is Lorentz invariant.

The apparent inconsistency of the above scenario with the causality implicit in the light speed barrier of special relativity may, however, have a different interpretation to that given above by Dirac. The clue may lie not within the electron but outside it. In the following sections we will review the later attempts to come to grips with the single electron problem, not only classically but also quantum mechanically. The crucial ingredient turns out to be cosmology. Finally, we will show that the infinities of quantum electrodynamics can be avoided by dropping the purely local approach of field theory and that it is cosmology that produces the 'cut off' to divergent integrals in classical and quantum electrodynamics.

2. The response of the universe

History tells us that there have been two approaches to electrodynamics. The theory started in the action at a distance format with Coulomb's laws for electric charges and magnetic poles along lines similar to Newton's inverse square law for gravity. Subsequently, experiments with rapidly moving charges showed the inadequacy of these laws to explain all the observed facts. Suspecting that the fault lay with the concept of instantaneous action at a distance, Gauss wrote to Weber on March 19, 1845:

> I would doubtless have published my researches long since were it not that at the time I gave them up I had failed to find what I regarded as the keystone, *Nil actum reputans si quid superesset agendum*: namely, the derivation of the additional forces – to be added to the interaction of electric charges at rest, when they are both in motion – from an action which is propagated not instantaneously but in time as is the case with light.

Unfortunately for progress on this problem, other issues had greater demands on Gauss's intellect and he did not follow up on his conjecture. Nevertheless several decades later theoreticians like Schwarzschild (1903), Tetrode (1922) and Fokker (1929) found ways of embodying Gauss's concept into an action integral. In modern notation we may write it as

$$J = -\sum_{a} \int m_a \,\mathrm{d}s_a - \sum_{a<b}\sum e_a\, e_b \iint \delta(s^2_{AB})\, \eta_{ik}\, \mathrm{d}a^i\, \mathrm{d}b^k, \tag{7}$$

where the charged particles are labelled $a, b, c, \ldots, e_a, m_a$ being the charge and mass of particle a. The argument of the delta function is the invariant square of the distance between points A, B lying respectively on the worldlines of charges a and b:

$$s^2_{AB} = \eta_{ik}(a^i - b^i)\,(a^k - b^k). \tag{8}$$

Thus the electromagnetic part of the action is non-zero only when typical worldpoints are connected by a light ray. In other words, the action is not instantaneous but propagates with the speed of light, as foreseen by Gauss. Further, the action (7) does not have $a = b$ in the double integral, thus excluding self action.

This formulation looks quite different from Maxwell's field theory, the other (more popular) approach to electrodynamics. A contact with field theory can, however, be made in (7) by defining the so called 'direct particle potentials'

$$A_i^{(a)}(X) = e_a \int \delta(s_{AX}^2)\, \eta_{ik}\, \mathrm{d}a^k. \tag{9}$$

These potentials identically satisfy the gauge condition and the wave equation

$$\Box A_i^{(a)}(X) = 4\pi j_i^{(a)}(X), \tag{10}$$

where $j_i^{(a)}$ is defined by (1).

Unlike the maxwellian fields and potentials, which have degrees of freedom independent of the source charges, the potentials (9) and the corresponding direct particle fields

$$F_{ik}^{(a)} = A_{k,i}^{(a)} - A_{i,k}^{(a)} \tag{11}$$

are uniquely determined by their sources. As such they have no independent degrees of freedom. In fact the definition (9) tells us that the direct particle field $F_{ik}^{(a)}$ is made up as the time-symmetric solution of (10):

$$F^{(a)} = \tfrac{1}{2}\{F_{\mathrm{ret}}^{(a)} + F_{\mathrm{adv}}^{(a)}\}. \tag{12}$$

This expression illustrates the practical difficulty faced by the action at a distance theory, namely its equal commitment to advanced and retarded solutions. Causality demands that we choose the full retarded solution $F_{\mathrm{ret}}^{(a)}$ as the solution of (10) whereas the theory forces us to the combination (12). Even the starting point (7) should have alerted us to this situation. For $\delta(s_{AB}^2)$ is non-zero only when A lies on the past or future light cone of B, thus making A interact with B both in the past as well as future. This 'unphysical' aspect had been the main stumbling block of the action at a distance until it was surmounted by Wheeler & Feynman (1945, 1949).

Wheeler & Feynman demonstrated the global rather than local nature of the problem. They pointed out that no electric charge is isolated. Rather, it is in continuous interaction with all other charges in the universe. Since in the action at a distance picture (see their fig. 1) the points A and B on the worldlines a and b are connected by a light ray, action and reaction between A and B both propagate along the segment joining them. So if B is to the future of A, reaction from B must travel backwards in time to arrive at A, no matter how far (spatially) B is from A. In practical terms it implies that any motion of the charge a at A must take into account the responses from typical particles b all over the universe on its future light cone. Using the Minkowski spacetime with a uniform distribution of electric charges Wheeler & Feynman (1945) showed that, provided each electric charge a ultimately produces the full retarded field $F_{\mathrm{ret}}^{(a)}$, the response of the universe is given by

$$\tilde{F}^{(a)} = \tfrac{1}{2}\{F_{\mathrm{ret}}^{(a)} - F_{\mathrm{adv}}^{(a)}\}. \tag{13}$$

This is none other than (5) prescribed by Dirac. The difference is that (13) is not an *ad hoc* prescription but turns out to have a deeper significance as the response of the universe. Notice also that if we subtract (13) from the assumed full retarded field, we get back (12). The force on charge a is then given by the total effective field

$$F_{\mathrm{total}}^{(a)} = \sum_{b \neq a} F_{\mathrm{ret}}^{(b)} + \tilde{F}^{(a)}. \tag{14}$$

The scenario is thus self consistent. Even though each charge taken by itself does not distinguish between the advanced and retarded parts, the universe as a whole does. Its response (13) is time-asymmetric (changing sign as $t \to -t$). This response, added to the individual field of the charge gives us the retarded field (14) for every charge, the field assumed for computing the response.

In the Maxwell field theory the choice of the retarded field is dictated by considerations of causality. From a general solution of the form

$$\alpha F_{\mathrm{ret}} + (1-\alpha) F_{\mathrm{adv}}$$

the selection of $\alpha = 1$ is made to be consistent with causality. In the action at a distance theory there is no freedom to choose: but it is the universe that sets $\alpha = 1$.

What aspect of the universe determines this response? Wheeler & Feynman (1945) could generalize their statement of the problem to demonstrate that if the universe is a perfect absorber in the future, i.e. if it has sufficient asymptotic density of electric charges on the future light cone of the typical charge a, then the response produced is (13). They also found that if the perfect absorption condition were satisfied in the past light cone the response would be the opposite of (13) and only advanced fields ($\alpha = 0$) would prevail.

In retrospect this conclusion appears reasonable. The behaviour of the universe in the asymptotic future or the asymptotic past seems to determine the retarded/advanced character of electrodynamics. But what happens (as Wheeler & Feynman found for the static Minkowski universe) if both the past and the future absorbers are perfect? In that case one is forced to look for another source for breaking the symmetry between past and future. Wheeler & Feynman resorted to thermodynamic time asymmetry to justify the retarded as opposed to the advanced solution.

Subsequently, Hogarth (1962) argued that the particular case considered by Wheeler & Feynman happened to be a singular one in that both the $\alpha = 0$, $\alpha = 1$ cases were consistent. If one takes the expanding universe into consideration it provides a time-symmetry-breaking mechanism and it follows that the responses of the past and future absorbers would necessarily be different. The nature of the response depends on the specific model of the universe chosen for calculating it. Hogarth demonstrated that the future absorber of a steady state model is perfect but not its past absorber while the reverse is true of the ever-expanding Friedman models. In other words the retarded solutions ($\alpha = 1$) and the Dirac formula for radiative reaction would hold in the steady state model whereas advanced solutions ($\alpha = 0$) and the Dirac formula with the wrong sign would hold in the above Friedman models.

Hogarth's arguments were basically sound although his computation of the absorption properties of the past and future absorbers had a technical error which was criticised by Feynman. Later Hoyle & Narlikar (1963) redid the calculations with the error eliminated and still confirmed Hogarth's conclusions. The classical picture thus far developed had therefore the following attractive features when compared to the field theory.

1. With far fewer degrees of freedom than in field theory the action at a distance picture was able to account for the observed electrodynamic phenomena at the classical level.

2. The choice of retarded solutions is not *ad hoc* (as in field theory) but turns out to have deeper cosmological significance.

3. The Dirac formula of radiative reaction is shown to arise naturally as the response of the universe rather than being prescribed intuitively.

4. With the explicit presence of advanced solutions in the theory, the phenomenon of pre-acceleration noticed by Dirac receives a logical interpretation.

Nevertheless the action at distance picture failed to attract support for two reasons. From the theoretical physicists' point of view, it had not been quantized and hence had not come up with the successes claimed by the quantized field theory, such as the Lamb shift, the anomalous magnetic moment of the electron, the various two-particle scattering phenomena like the Compton effect, etc. The cosmologist, on the other hand having put his eggs in the Friedman basket, was not happy to look at a theory which was manifestly inconsistent with Friedman cosmology.

The situations on both these fronts have changed in the last three decades. Although the strict steady state model is still under a cloud so far as extragalactic observations are concerned, these are now shown to fit naturally with the quasi-steady-state model of Hoyle *et al.* (1993) whereas the Friedman cosmologies face a number of awkward difficulties (Arp *et al.* 1990). Since in both the asymptotic future and past the quasi-steady-state model behaves like the steady state model, the classical Wheeler–Feynman theory will work there too.

So far as the quantization of the action at a distance electrodynamics is concerned this has been achieved (Hoyle & Narlikar 1969, 1971) and it is known to reproduce the familiar results of quantum field theory. However, as we shall show in the next section, it is cosmology that brings about convergence of the crucial integrals which are normally considered divergent but renormalizable in quantum field theory.

3. The quantum interaction with the universe

In our earlier work (Hoyle & Narlikar 1969) we had missed a subtle point which turns out to have a bearing on the divergence problem of quantum electrodynamics. The work concerned the explanation of the formula of spontaneous transition of the atomic electron in the Wheeler–Feynman framework. We had selected this problem as the starting point for our attack on the quantization of the above framework, for the following reason.

The interaction of an electron with a given electromagnetic disturbance is described in the usual notion by the Dirac equation ($c = 1$, $\hbar = 1$)

$$(\nabla\!\!\!/ + \mathrm{i}e A\!\!\!/)\,\psi + \mathrm{i}m\psi = 0, \tag{15}$$

where e and m are the charge and mass of the electron and ψ its wavefunction. In the non-relativistic limit applicable to the atomic electron, this equation reduces to the Schrödinger equation. Now if A_i is the classical electromagnetic 4-potential, it leads to the explanation of induced transitions of the electron. Such transitions can be downward or upward with equal probability provided the energy difference between the concerned levels matches the energy corresponding to the angular frequency of the electromagnetic disturbance ($\Delta E = \hbar\omega$). However, in addition to these the electron also has a finite probability for downward transition only which does not seem to require the ambient field.

To explain this spontaneous transition, it is apparently necessary to quantize the electromagnetic field. Since the quantum field has a zero point energy of fluctuations (these have no classical counterpart) it can account for the spontaneous transition even when the classical external field is absent. However, in the Wheeler–Feynman version there are no independent degrees of freedom vested in the direct particle fields and hence the above explanation cannot be invoked. Can we therefore really do away with fields as independent entities?

This challenge to the action at a distance picture must be met if the picture is to survive. Fortunately the clue to the solution is not difficult to grasp: it lies in the response of the universe. One has to argue in a self-consistent loop, in the following way.

(a) *The path integral approach*

In a quantum mechanical system the evolution from an initial state to a final state is not deterministic, as it is for its classical counterpart. To give an example, if a particle a moves from an initial space-time point A_{I} to a final one A_{F} its motion in classical mechanics is determined by the principle of stationary action. If the action is given by J, a functional of the path Γ joining A_{I} to A_{F}, then the chosen classical path Γ_{c} satisfies the relation

$$J[\Gamma_{\mathrm{c}}+\delta\Gamma] \cong J[\Gamma_{\mathrm{c}}] \tag{16}$$

for first-order deviations $\delta\Gamma$ from Γ_{c}. Normally, given the initial and final conditions at A_{I} and A_{F}, Γ_{c} is uniquely determined.

In the corresponding quantum mechanical problem one considers a 'probability amplitude' $K[A_{\mathrm{F}};A_{\mathrm{I}}]$ of arriving at A_{F} given the starting point A_{I}. This quantity K is a complex number, being expressed as shown by Feynman (1948) by the sum over paths:

$$K[A_{\mathrm{F}};A_{\mathrm{I}}] = \sum_{\Gamma} \exp\{\mathrm{i}J/\hbar\}. \tag{17}$$

The sum in (17) is of probability amplitudes over all geometrically possible paths from A_{I} to A_{F}. In quantum mechanics all paths have equal probability. It is only in the classical limit $|J| \gg \hbar$ that a unique path given by (16) emerges. Again, it was a statement by Dirac (1935, pp. 123–124) that had inspired this approach by Feynman. Dirac had essentially suggested (17) as a starting point for quantum mechanics, with the argument that as $\hbar \to 0$ this reduces to the classical limit of stationary action.

The Feynman 'sum over path' formula (17) not only gives a quantitative background to Dirac's arguments, it also gives us the Schrödinger equation in the full quantum case when $|J| \sim \hbar$. In our present problem it points the way to the solution of the spontaneous transition problem. Because the details having been given in our earlier work (Hoyle & Narlikar 1969, hereafter referred to as Paper I) we will be brief here with cross references to some crucial formulae derived there. We will, however, go beyond that work to highlight a critical new point missed earlier.

To fix ideas consider an electron a in one of the intermediate energy states of an atom, say with energy E_2. There are levels above it with energies $\ldots > E_4 > E_3 > E_2$ while there is at least one state of energy level E_1 below it. By induced transitions caused by external disturbances (given by the first part of (14))

$$F_{\mathrm{ext}} = \sum_{b\neq a} F^{(b)}_{\mathrm{ret}}$$

the electron may go to either E_1 or to E_3, E_4, etc. However, there is also another force acting on it, that due to the response of the universe. Even when the $F_{\mathrm{ext}} = 0$, the response would still be

$$\tilde{F}^{(a)} = \tfrac{1}{2}\{F^{(a)}_{\mathrm{ret}} - F^{(a)}_{\mathrm{adv}}\}, \tag{18}$$

provided, as we assume here, the universe is a perfect future absorber. What role does it play in such situations?

We could argue that with $F_{\text{ext}} = 0$, the electron stays put at level E_2. It is therefore unaccelerated and hence $\tilde{F}^{(a)} = 0$. Thus the response appears to have no role to play. But this is the classical view of the situation. Quantum mechanically we must allow other paths $\Gamma \neq \Gamma_{\text{c}}$ for the electron, leaving it to the action formula and the Feynman prescription (17) to determine the overall K. Thus, we can, in principle, calculate the transition probability for the electron to go from the state of energy E_2 to the state of energy E_1 as well as to states of energy $E_3, E_4, \ldots$. The way $\tilde{F}^{(a)}$ enters the calculation will determine whether the electron spontaneously jumps up or down, or neither. Paper I had carried out this calculation in the steady state universe which is a perfect future absorber. We summarize the crucial parts of that argument first and then show how it can be further improved.

(*b*) *The spontaneous transition formula*

Consider the electron as particle a in our notation and denote by $0 \leqslant t \leqslant T$ the time interval relevant to the above situation. Let $\boldsymbol{a}(t)$ denote the displacement of a at time t. The typical absorber particle b has its worldline intersected respectively in segments Δ_- and Δ_+ by the past and future light cones from the above section of the worldline of a. In a universe with perfect future absorber and imperfect past absorber the induced transitions come from sections like Δ_- and the spontaneous ones from Δ_+. We will consider here the latter only.

The transition of absorber particle b by the full retarded effect of a is governed by the following term in the action:

$$-e_b \int_{\Delta_+} \boldsymbol{A}^{(a)}_{\text{ret}}(\boldsymbol{b}) \cdot \dot{\boldsymbol{b}}\, \mathrm{d}t. \tag{19}$$

The motion of b so generated produces a reaction on a via the half advanced component. Since the absorber particles all act independently the combined influence of all of them is described by an influence functional of the form

$$F[\boldsymbol{a}(t), \boldsymbol{a}'(t)] = \prod_{b \neq a} F^{(b)}[\boldsymbol{a}(t), \boldsymbol{a}'(t)], \tag{20}$$

where

$$\begin{aligned} F^{(b)}[\boldsymbol{a}, \boldsymbol{a}'] = 1 + \sum_{f \neq i} & \iiiint\!\!\iint \psi_f^*(\boldsymbol{b}_f)\, \psi_f(\boldsymbol{b}_f')\, \psi_i^*(\boldsymbol{b}_i')\, \psi_i(\boldsymbol{b}_i) \\ & \times \frac{1}{\hbar^2} \left[e_b^2 \int_{\Delta_+} \boldsymbol{A}^{(a)}_{\text{ret}}(\boldsymbol{b}) \cdot \dot{\boldsymbol{b}}\, \mathrm{d}t \int_{\Delta_+'} \boldsymbol{A}^{(a)}_{\text{ret}}(\boldsymbol{b}') \cdot \dot{\boldsymbol{b}}'\, \mathrm{d}t' \right] \\ & \times \exp{(\mathrm{i}/\hbar)} \{ J_E[\boldsymbol{b}] - J_E[\boldsymbol{b}'] \} \mathscr{D}\boldsymbol{b}\, \mathscr{D}\boldsymbol{b}'\, \mathrm{d}^3 \boldsymbol{b}_i\, \mathrm{d}^3 \boldsymbol{b}_i'\, \mathrm{d}^3 \boldsymbol{b}_f\, \mathrm{d}^3 \boldsymbol{b}_f' \\ & + \text{terms in} \int_{\Delta_+} \cdot \int_{\Delta_+}, \int_{\Delta_+'} \cdot \int_{\Delta_+'}, \text{etc.} \end{aligned} \tag{21}$$

Here J_E denotes the part of action of b relating to its own degrees of freedom. If E_f and E_i are the energies of the states ψ_f and ψ_i respectively, then the contribution to the above expression from the absorber transition $\psi_i \rightarrow \psi_f$ is given to first order by

$$F^{(b)}_{fi} = \frac{e_b^2}{\hbar^2} \frac{(E_f - E_i)^2}{\hbar^2} M[\boldsymbol{a}(t)] M^*[\boldsymbol{a}'(t)], \tag{22}$$

where

$$M[\boldsymbol{a}] = \int_{\Delta_+} \exp\left[\frac{\mathrm{i}(E_f - E_i)\,t}{\hbar}\right] \mathrm{d}t \int \psi_f^*(\boldsymbol{b})\, \boldsymbol{A}_{\mathrm{ret}}^{(a)}(\boldsymbol{b}) \cdot \boldsymbol{b}\psi_i(\boldsymbol{b})\, \mathrm{d}^3\boldsymbol{b}. \tag{23}$$

Let us now look at the cosmological framework before proceeding further. Although the ideas to be described work within the quasi-steady-state model equally well, we shall work within the simpler framework of the steady-state cosmology which also generates the correct response of the universe at the classical level. The line element for this model is given by

$$\mathrm{d}s^2 = c^2\,\mathrm{d}\tau^2 - \mathrm{e}^{2H\tau}(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2 \sin^2\theta\,\mathrm{d}\phi^2). \tag{24}$$

This line element can be expressed in a manifestly conformally invariant form by defining a time coordinate

$$t = H^{-1}(1 - \mathrm{e}^{-H\tau}). \tag{25}$$

We have chosen an arbitrary constant at our disposal to ensure that $\tau = 0$ corresponds to $t = 0$. Notice, however, that while $\tau \to -\infty$ gives $t \to -\infty$, the corresponding limit in the future, $\tau \to \infty$ gives a finite value $t = 1/H$ for t. This circumstance will play an important role in this calculation, which we will highlight at an appropriate stage. The line element (25) now becomes

$$\mathrm{d}s^2 = (1 - Ht)^{-2}\,[\mathrm{d}t^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)]. \tag{26}$$

Since the action at a distance electrodynamics used here is conformally invariant we can use many of the flat space-time expressions without any modification.

Thus, if the origin of b has a relative radius vector $\boldsymbol{R}$ with respect to the origin of a, we can write the radial coordinate displacement from a to b as

$$\boldsymbol{r} = \boldsymbol{R} + \boldsymbol{b} - \boldsymbol{a}, \quad \text{and} \quad |\boldsymbol{r}| \equiv r \cong R + (\boldsymbol{R}/R)\cdot(\boldsymbol{b} - \boldsymbol{a}). \tag{27}$$

Correspondingly, when t runs in the range $0 \leqslant t \leqslant T$ at a, the time at b will be given by

$$t' = t + R + \boldsymbol{R}\cdot(\boldsymbol{b} - \boldsymbol{a})/R; \quad 0 \leqslant t \leqslant T. \tag{28}$$

Thus t' will have a corresponding range T'.

Returning now to electrodynamics, we express $\boldsymbol{A}_{\mathrm{ret}}^{(a)}(\boldsymbol{b})$ as a Fourier series at b,

$$\boldsymbol{A}_{\mathrm{ret}}^{(a)}(\boldsymbol{b}) = \sum_{l=-\infty}^{\infty} \sum_{J=1,2} A_l^J\, \boldsymbol{e}_J \exp\left(-\frac{2\pi\,\mathrm{i}lt'}{T'}\right). \tag{29}$$

Here $\boldsymbol{e}_J$ are the two independent spin states in the Coulomb gauge and A_l^J are the Fourier coefficients. Following the analysis of Paper I we finally arrive at the following expression for (22)

$$F_{fi}^{(b)} = \frac{e_a^2\, e_b^2\, k^2}{3R^2\hbar^2}\,\mathrm{e}^{-\tau(k)}\,|\boldsymbol{b}_{if}(k)|^2$$

$$\times \sum_{J=1,2} \int_0^T \boldsymbol{e}_J\cdot\boldsymbol{a}\exp\,(+\mathrm{i}kt - \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a})\,\mathrm{d}t \int_0^T \boldsymbol{e}_J\cdot\dot{\boldsymbol{a}}'\exp\,(-\mathrm{i}kt + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}')\,\mathrm{d}t, \tag{30}$$

where $k = (E_f - E_i)/\hbar$, and $\boldsymbol{b}_{fi}$ is the matrix element of the absorber displacement with respect to states ψ_i and ψ_f.

Considering next the distribution of such absorbers all along the future light cone of a we recover the familiar expression for F that gives the usual spontaneous transition formula:

$$F[\boldsymbol{a}, \boldsymbol{a}'] = \exp\left\{\frac{e_a^2}{4\pi^2\hbar}\int \mathrm{d}\Omega \int_0^\infty k\,\mathrm{d}k Q(\boldsymbol{k})\right\}, \tag{31}$$

$$\begin{aligned} Q(\boldsymbol{k}) = \sum_{J=1,2}\Bigg[&\int_0^T (\boldsymbol{e}_{kJ}\cdot\dot{\boldsymbol{a}})\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}+\mathrm{i}kt)\,\mathrm{d}t\int_0^T(\boldsymbol{e}_{kJ}\cdot\dot{\boldsymbol{a}}')\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}'-\mathrm{i}kt')\,\mathrm{d}t' \\ &-\int_0^T(\boldsymbol{e}_{kJ}\cdot\dot{\boldsymbol{a}})\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}-\mathrm{i}kt)\,\mathrm{d}t\int_0^t(\boldsymbol{e}_{kJ}\cdot\dot{\boldsymbol{a}})\exp(+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}+\mathrm{i}k\tilde{t})\,\mathrm{d}\tilde{t} \\ &-\int_0^T(\boldsymbol{e}_{kJ}\cdot\dot{\boldsymbol{a}}')\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}'+\mathrm{i}kt)\,\mathrm{d}t\int_0^t(\boldsymbol{e}_{kJ}\cdot\dot{\boldsymbol{a}}')\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{a}-\mathrm{i}k\tilde{t})\,\mathrm{d}\tilde{t}\Bigg]. \end{aligned} \tag{32}$$

Notice that we have introduced the suffix $\boldsymbol{k}$ for $\boldsymbol{e}$ to remind ourselves that it is perpendicular to $\boldsymbol{k}$.

Since the quantum field theory of maxwellian electrodynamics leads to the same influence functional, we have demonstrated that the action at a distance formulation together with the right cosmological boundary conditions is at least as successful as field theory at the quantum level. We will next consider an additional aspect of the universal response that we have missed so far, and which in our opinion gives an important measure of superiority to the present theory.

(c) *The horizon effect*

A modification to the above at very high frequencies is needed, relating to redshift and the existence of the event horizon in the future absorber. Consider a light signal emitted from $r = 0$ at $t = 0$. Its path along the future light cone is given by $r = t$ (with $c = 1$). The t-coordinate is not, however, the proper time of an observer. The line element (26) tells us that for an observer with $r = \text{const.}$, $\theta, \phi = \text{const.}$ the proper time differential is

$$|\mathrm{d}s| = |\mathrm{d}t|/(1-Ht). \tag{33}$$

Thus an absorber particle b at r sees the radiation from a at $r = 0$, $t = 0$ redshifted by

$$1+z = 1/(1-Hr). \tag{34}$$

As $r \rightarrow H^{-1}$ we get to the event horizon with $z \rightarrow \infty$. Let us now see how the redshift enters the above argument.

Consider a typical Fourier component of (32) with frequency k at $r = 0$. This is reduced by redshift to $k(1+z)^{-1}$ in the absorber, where it gets absorbed. As shown by Hoyle & Narlikar (1963) this absorption occurs in the limit at the plasma frequency

$$\omega_\mathrm{p} = (4\pi N e^2/m)^{\frac{1}{2}}, \tag{35}$$

where N is the particle number density and m = mass of the electron. Taking $N \sim 3.10^{-9}\ \mathrm{cm}^{-3}$ (we can legitimately take the present intergalactic estimate of N for the asymptotic future in a steady state universe) we get $\omega_\mathrm{p} \sim 3\ \mathrm{s}^{-1}$. We will, however, keep to the general expression for the present. For our argument it is sufficient to assume the existence of a limiting frequency. Thus for absorption of frequency k we need the redshift

$$1+z = k/\omega_\mathrm{p}. \tag{36}$$

Consider the light ray leaving $r = 0$ at some time t in the range $0 \leqslant t \leqslant T$. This ray will reach the absorber particle b at time $t+r$. For this instant to be within the event horizon we need to satisfy the inequality

$$t+r < 1/H.$$

Here for our calculation above to be valid for the entire duration $0 \leqslant t \leqslant T$, we need

$$T+r < 1/H. \tag{37}$$

Using (34) we get

$$(1+z)\,T < 1/H. \tag{38}$$

Combining (37) with (35) we have

$$k < \omega_{\mathrm{p}}/HT. \tag{39}$$

In short, the response calculation works only for frequencies limited in the above fashion. That the cut off given by the right-hand side of (39) is finite is seen from the following argument.

The spontaneous transition calculation given above makes sense only if T is large enough for the transition probability to be established. That is T should be large compared to

$$t_{\mathrm{atomic}} = \hbar/\Delta E \tag{40}$$

where ΔE is the energy difference between the energy levels of transition. Typically $t_{\mathrm{atomic}} \sim 10^{-14}$–$10^{-13}$ s. Thus we find that cosmology provides a cut-off at high frequency. Although its existence does not have any significant impact on the spontaneous transition formula, it turns out to be highly relevant to the renormalization programme as we shall show next.

4. Relation to renormalization

The cut-off on k at the high frequency and given by (39) works out to *ca.* 10^{31} s^{-1} for the atomic and cosmological parameters described above. This cut off may vary from one microscopic process to another; it also is linked with the properties of the cosmic absorber. However, the reasoning given above tells us that for every microscopic process in electrodynamics a cut-off exists.

The purely local approach to QED demands Lorentz invariance in every operation that may be performed. Our method, on the other hand, picks out a specific local reference frame, namely the so-called cosmological rest frame, to define the response of the universe. Thus Lorentz invariance is manifestly not present, although one can use the Lorentz transformation to describe any process of QED in a frame different from the cosmological rest frame. We will briefly review the earlier work of Hoyle & Narlikar (1971, hereafter referred to as Paper II) in this context. This was a sequel to Paper I and the present idea of a high frequency cut off will be shown to be relevant there.

Paper II began with the extension of path integral concept to relativistic Dirac particles and then brought in the concept of the influence functional arising from the response of the universe that modifies the path of a 'free' particle. This led to issues of self-energy and vacuum polarization, etc. where renormalization techniques are normally invoked. We will consider these topics briefly in that order before arriving at the notion of a high frequency cut-off.

(a) Path integral for a Dirac particle

By a Dirac particle we mean a spin $\frac{1}{2}$ particle of mass m satisfying the free-particle Dirac equation

$$(\nabla\!\!\!/ + \mathrm{i}m)\psi = 0 \tag{41}$$

in the absence of any electromagnetic interaction. For the time being we consider the spacetime to be given by the Minkowski line element.

Define the propagators $K_0^{\pm}(2;1)$ by the sums

$$\left.\begin{aligned} K_0^{+}(2;1) &= \theta(t_2-t_1)\sum_n u_n(2)\,\bar{u}_n(1), \\ K_0^{-}(2;1) &= -\theta(t_1-t_2)\sum_n u_n(2)\,\bar{u}_n(1), \end{aligned}\right\} \tag{42}$$

where 1 and 2 are two spacetime points with time coordinates t_1, t_2 respectively and the $\{u_n\}$ are the complete set of stationary solutions of (41) with different energy levels $\{E_n\}$. Thus K_0^+ denotes propagation into the future ($t_2 > t_1$) while K_0^- denotes propagation into the past ($t_2 < t_1$). Any general path from point 1 to 2, denoted by Γ_{21}, can be made up of a large number of small zig-zag segments of time-like nature but each segment may be forward going or backward going. If the interior points are P_i, so that a typical segment is $P_i P_{i+1}$, then, with $P_0 \equiv 1$ and $P_N \equiv 2$ we may define the probability amplitude for Γ_{21} by the expression

$$P(\Gamma_{21}) = \prod_{i=1}^{N} B_i^{-1} K_0^{\pm}(i;i-1), B_i = \text{const.}, \tag{43}$$

where we choose the appropriate propagator depending on whether the segment $[i-1, i]$ is forward or backward directed in time.

When the particle has charge e and it moves in an electromagnetic field of potential A_i, the expression (43) gets modified to

$$P^A(\Gamma_{21}) = \prod_{i=1}^{N} B_i^{-1} K_0^{\pm}(i;i-1)\exp\left\{\int_{P_{i-1}}^{P_i} -\mathrm{i}eA_l\,\mathrm{d}x^l\right\}. \tag{44}$$

Since the A_l are not the gradient of a scalar, the evaluation of (44) is non-trivial and needs the perturbation expansion to carry it out. The net result, as discussed in detail in Paper II is to lead to a propagator $K_+^A(2,1)$ for propagating a charged particle wavefunction from $\psi(1)$ to $\psi(2)$:

$$\begin{aligned} K_+^A(2,1) = K_+(2,1) &- \mathrm{i}e\int K_+(2,3)\,A\!\!\!/(3)\,K_+(3,1)\,\mathrm{d}\tau_3 \\ &+ (-\mathrm{i}e)^2 \iint K_+(2,4)\,A\!\!\!/(4)\,K_+(4,3)\,A\!\!\!/(3)\,K_+(3,1)\,\mathrm{d}\tau_3\,\mathrm{d}\tau_4 + \dots, \end{aligned} \tag{45}$$

where the $K_+(2,1)$ is the propagator defined by:

$$K_+(2,1) = \begin{cases} \sum\limits_{E_n>0} u_n(2)\,\bar{u}_n(1) & t_2 > t_1, \\ -\sum\limits_{E_n<0} u_n(2)\,\bar{u}_n(1) & t_2 < t_1. \end{cases} \tag{46}$$

The use of the $K_+(2,1)$ propagator allows us to bypass the 'hole' theory and to interpret (in the Feynman sense) the backward going negative energy paths of particles as forward going positive energy paths of anti-particles.

(b) *The response of the universe*

The condition of 'perfect absorber in the future' allows us, in the Wheeler–Feynman theory to use the classical formula (14). What happens in the quantum analogue of perfect future absorption? The clue is provided by the non-relativistic formula (31) for the influence functional. Still working in the locally Minkowskian spacetime we write the integrals for advanced and retarded potentials as:

$$A^i(X)_{\mathrm{ret}\pm} = \sum_b e_b \int \frac{\delta_\pm(t_X - t_B - |\boldsymbol{x}-\boldsymbol{x}_B|)}{2\,|\boldsymbol{x}-\boldsymbol{x}_B|}\,\mathrm{d}b^i, \tag{47}$$

$$A^i(X)_{\mathrm{adv}\pm} = \sum_a e_a \int \frac{\delta_\pm(t_X - t_B + |\boldsymbol{x}-\boldsymbol{x}_B|)}{2\,|\boldsymbol{x}-\boldsymbol{x}_B|}\,\mathrm{d}b^i, \tag{48}$$

The notation is self-explanatory, the $\delta_\pm$ functions being given by

$$\delta_\pm(x) = \frac{1}{\pi}\int_0^\infty \mathrm{e}^{\mp\mathrm{i}wx}\,\mathrm{d}\omega. \tag{49}$$

Since the future absorber is essentially in a 'cold environment', a typical absorber particle b makes an upward transition on receiving a disturbance from the source particle. Hence only the positive frequencies in the potential A_{ret} are responded to. The calculation of the influence functional for a pair of particles a, b in Paper II led to the result

$$\begin{aligned} F[\boldsymbol{a},\boldsymbol{b};\boldsymbol{a}',\boldsymbol{b}'] = \exp\Big\{\mathrm{i}e_a\,e_b\Big[\int_{t'_A > t_B}\!\!\int \delta_+(s^2_{A'B})\,\mathrm{d}a'_i\,\mathrm{d}b^i \\ - \int_{t_B > t'_A}\!\!\int \delta_-(s^2_{BA'})\,\mathrm{d}a'_i\,\mathrm{d}b^i \\ + \int_{t'_B > t_A}\!\!\int \delta_+(s^2_{B'A})\,\mathrm{d}a^i\,\mathrm{d}b'_i - \int_{t_A > t'_B}\!\!\int \delta_-(s^2_{AB'})\,\mathrm{d}a^i\,\mathrm{d}b'_i \\ - \iint \delta_+(s^2_{AB})\,\mathrm{d}a^i\,\mathrm{d}b_i + \iint \delta_-(s^2_{A'B'})\,\mathrm{d}a'_i\,\mathrm{d}b'_i\Big]\Big\}. \end{aligned} \tag{50}$$

This result shows how (i) the time symmetric $\delta(s^2)$ interaction gets modified due to the response of the universe (ii) the paths a, b and conjugate paths a', b' together are needed to describe the outcome of the interaction. This influence functional leads to all the known results of QED normally derived by quantum field theory.

In particular, it reduces to (32) that gives the spontaneous transition formula of Paper I. In Paper II we also showed how the closed loops of vacuum polarization and the self-energy diagrams for charged particles in the present theory lead to the usual results like the Lamb Shift, anomalous magnetic moments, etc.

(c) *The elimination of divergences*

The direct particle QED so obtained also suffers from the problems of divergence that have bedevilled the quantum field theory; although in the former case the reason for infinities is more easy to grasp. It is as follows.

Once we admit the possibility of charged particle paths going backward as well as forward in time the classical fiat of 'no self-action' gets modified. For, in a purely classical theory wherein worldlines are confined to the interior of light cones $\delta(s^2_{PQ}) = 0$ if P and Q belong to the same worldline. In the full quantum theory, the zig-zagging of paths forward and backward in time makes it possible for two such points to be connectible by a null ray. Thus completeness demands that we allow such interactions to exist. There is experimental evidence in the form of positronium annihilation to demonstrate the reality of this concept. However, we can still preserve the spirit of the classical 'no self-action' requirement by demanding that $P \neq Q$. It is because P is allowed to coincide with Q in the calculation of $\delta_{\pm}(s^2_{PQ})$ that we run into the divergence problem.

Can P be prevented from approaching Q? In Paper II we had conjectured that a gravitational length scale such as the Schwarzschild radius of the electron may provide the required cut-off. Later Padmanabhan (1985) showed that Planck length $(G\hbar/c^3)^{\frac{1}{2}}$ provides a natural cut-off when quantum gravitational fluctuations are taken into account.

The idea presented at the end of the preceding section now becomes relevant. Referring to equation (121) of Paper II, we can impose a restriction on how close P can come to Q in $\delta_+(s^2_{PQ})$, etc. by the following prescription.

Choose the cosmological rest frame in which, by the arguments of the preceding section all Fourier integrals in the computations of the influence functional have a high frequency cut-off at $k_{\max}$, say. With $c = 1$, $\hbar = 1$, this cut-off corresponds to a restriction in time coordinate

$$|t_P - t_Q| \geqslant k^{-1}_{\max}. \tag{51}$$

We will shortly specify $k_{\max}$; for the time being $k^{-1}_{\max}$ remains a small quantity akin to ϵ used in Paper II.

Using the derivation of Paper II and the formula (146) therein we find that in the cosmological rest frame the observed mass m_{obs} is related to the theoretical mass m_{th} of the electron by the relation

$$m_{\text{obs}} = m_{\text{th}}\left\{1 + \frac{3e^2}{2\pi}\ln\left(\frac{k_{\max}}{m_{\text{th}}}\right)\right\}. \tag{52}$$

Notice that a similar formula comes from quantum field theory but there the cut-off is a purely abstract quantity and so no numerical significance is attached to the mass difference

$$\Delta m = m_{\text{obs}} - m_{\text{th}}. \tag{53}$$

In the present theory $k_{\max}$ is related to physical parameters and as a result it is possible to estimate it and Δm, which we proceed to do now.

The upper limit on k is given by (39) in which we have $\omega_p \sim 3\ \text{s}^{-1}$, $H \sim 3\times10^{-17}\ \text{s}^{-1}$ and T a timescale large compared to the characteristic time for the process; in this case the free motion of the electron. We may take $T \sim \hbar/mc^2 \sim 10^{-21}$ s. Thus we have

$$k_{\max} \sim 10^{38}\ \text{s}^{-1}. \tag{54}$$

Using these values, (52) and (53) give in dimensionless form

$$\frac{\Delta m}{m} \sim \frac{3e^2}{2\pi\hbar c}\ln(10^{18}) \sim 0.14. \tag{55}$$

In fact, with $T \sim \hbar/mc^2$, (55) expressed in symbols is

$$\frac{\Delta m}{m} \sim \frac{3\alpha}{2\pi}\ln\left(\frac{\omega_p}{H}\right), \tag{56}$$

where α is the fine structure constant.

Two comments are needed to elaborate the above conclusion. First, (56) shows clearly the cosmological input to the correction term which no purely local attempt at resolving the divergence problem will arrive at. Second, the correction has been obtained in the cosmological rest frame: and so the statement is not strictly Lorentz invariant. This in our view is an unavoidable conclusion echoing the first comment that only a global theory can lead to the resolution of the divergence problem. In this context we again notice Dirac's intuitive perception when he wrote:

> With a cut-off we eliminate at once all difficulties about divergent integrals which have been plaguing theoretical physics for decades. These difficulties arise only because people want to have strict Lorentz invariance in an imperfect theory. In doing so they are aiming for something which may very well be impossible. (Dirac 1969)

We would agree with the above sentiment with one modification: replace the adjective 'imperfect' by 'incomplete' to underscore the one crucial element missed out in field theory, namely the response of the universe.

5. Conclusion

The so-called 'charge renormalization' was discussed in Paper II. In the present framework, the vacuum current j_k is related to the theoretical current J_k by

$$j_k = \frac{2e^2}{3\pi\hbar c} \ln\left(\frac{\omega_p}{H}\right) J_k. \tag{57}$$

The result is that closed loops effectively lower the theoretical (or 'base') value of the electron charge by some 0.04 fraction of its original value.

The renormalization programme in the quantum field theory of charged particles has the merit that it gives an unambiguous way of handling infinite integrals which are only logarithmically divergent. It has been felt that the actual values of these integrals do not contribute to observable quantities and as such the success of the programme is judged by how the residuals left after removing the infinite integrals pass the observational tests. However, as Dirac observed:

> ...this so called good theory (QED)...involves neglecting infinities, neglecting them in an arbitrary way. This is not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small – not neglecting it just because it is infinitely great and you do not want it. (Dirac 1978)

The proposed remedy in this paper solves this outstanding difficulty – at a price that the theoretical physicist trained at viewing the problem in a purely local way will find it difficult to appreciate. Yet, the merit of the solution presented here should induce him to take into account the missing link, namely the response of the universe. It is this link that forces us to consider cosmological boundary conditions for seemingly local problems.

References

Arp, H. C., Burbidge, G., Hoyle, F., Narlikar, J. V. & Wickramasinghe, N. C. 1990 *Nature, Lond.* **346**, 807.

Dirac, P. A. M. 1935 *The principles of quantum mechanics*, 2nd edn. Oxford: Clarendon Press.

Dirac, P. A. M. 1938 *Proc. R. Soc. Lond.* A **167**, 148.

Dirac, P. A. M. 1978 In *Directions in physics*, 36. New York: Wiley.

Dirac, P. A. M. 1969 In *Fundamental intersections at high energy* (ed. T. Gudehus, G. Kaiser & A. Perlmutter). New York: Gordon and Breach.

Feynman, R. P. 1948 *Rev. mod. Phys.* **20**, 367.

Fokker, A. D. 1929 *Z. Phys.* **58**, 386.

Hogarth, J. E. 1962 *Proc. R. Soc.* A **267**, 365.

Hoyle, F. & Narlikar, J. V. 1963 *Proc. R. Soc. Lond.* A **277**, 1.

Hoyle, F. & Narlikar, J. V. 1969 *Ann. Phys.* (*N.Y.*) **54**, 207 (Paper I).

Hoyle, F. & Narlikar, J. V. 1971 *Ann. Phys.* (*N.Y.*) **62**, 44 (Paper II).

Hoyle, F., Burbidge, G. & Narlikar, J. V. 1993 *Astrophys. J.* (Submitted.)

Padmanabhan, T. 1985 *G.R.G.* **17**, 215.

Schwarzschild, K. 1903 *Gott. Nach.* **128**, 132.

Tetrode, H. 1922 *Z. Phys.* **10**, 317.

Wheeler, J. A. & Feynman, R. P. 1945 *Rev. mod. Phys.* **17**, 157.

Wheeler, J. A. & Feynman, R. P. 1949 *Rev. mod. Phys.* **21**, 425.

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