ELIMINATION OF THE STANDARD BIG BANG SINGULARITY AND PARTICLE HORIZON THROUGH QUANTUM CONFORMAL FLUCTUATIONS

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Received 13 April 1983

It is shown that within the framework of quantum conformal fluctuations it is extremely unlikely that the universe originated from a state of singularity and small particle horizon.

The standard big bang cosmology is known to suffer from two defects: (1) the space–time singularity at the beginning of the universe [1] and (2) the existence of a particle horizon which severely limits the communication between the different parts of the universe [2]. The standard models of cosmology are, however, based on the classical equations of general relativity. Earlier work [3–5] has shown that quantum fluctuations of the conformal degree of freedom diverge at the classical singularity, thus rendering the classical solution of doubtful validity. Here we show that within this framework the quantum-mechanical probability that the universe had the above two defects turns out to be vanishingly small.

To demonstrate the above result we proceed in two steps. First we show that if a conformal fluctuation eliminates the first defect it also eliminates the second. Next we compute the probability of obtaining singular models under the quantum conformal fluctuations (QCF), and show that it tends to zero at the classical singular epoch. We choose units such that $c = 1, \hbar = 1$, $G = 1$.

The geometry of the standard models is given by the Robertson–Walker (RW) line element

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{1}$$

with $k = 0$, 1 or -1. The space–time singularity arises because $S \to 0$ at some past epoch which we take as $t = 0$. The horizon problem arises because the integral

$$I(t) = \int_0^t dt/S(t) \tag{2}$$

is convergent, and tends to zero as $t \to 0$. We will denote by $\bar{S}(t)$ the classical solution for $S(t)$ obtained from Einstein's equations. Let $d\bar{s}^2$ denote the corresponding classical RW line element.

By conformal fluctuations we mean the class of metrics obtained by the conformal transformation

$$ds = \Omega(t)\, d\bar{s}\,, \tag{3}$$

where $\Omega(t)$ is an arbitrary C^2 function of t. Obviously (3) can be transformed back to the RW line element by the time transformation

$$\tau = \int \Omega(t) dt\,, \quad S = \Omega\bar{S}\,. \tag{4}$$

The conformal transformation (3) therefore preserves the homogeneity and isotropy of space–time.

Denote by $\mathcal{M}$ and $\bar{\mathcal{M}}$ the space–times with metrics ds and $d\bar{s}$ related through (3). For $\mathcal{M}$ to be non-singular at $t = 0$ we need

$$\Omega\bar{S} \to a\,, \tag{5}$$

where a is finite (i.e., $a \neq 0$). The criterion for the absence of horizon is somewhat different. For, suppose in (4), $t \to 0$ leads to $\tau \to -\infty$. Then the integral

$$\int_{-\infty}^{\tau_0} \mathrm{d}\tau/S = \int_0^{t_0} \mathrm{d}t/\bar{S} \tag{6}$$

is still finite, and it would appear that a particle horizon cannot be eliminated by a conformal transformation. However, there is a loophole in this argument. If as $t \to 0$, $\tau \to$ a finite value, which may be taken as $\tau = 0$, then the past null cones can be extended beyond the hypersurface $\tau = 0$, and with a suitable continuation of the metric in $\tau < 0$ the horizon can be eliminated. A simple example of this case is $\Omega = (\bar{S})^{-1}$ for $k = 0$ which gives us the horizon-free Minkowski space–time.

There is one trivial and isolated instance where $\mathcal{M}$ is nonsingular at $t = 0$ even though $S \to 0$ as $t \to 0$. This is the well-known example of the flat space–time for $S \propto t$, $k = -1$. However, as in the general case, this space–time is free from particle horizons. Our main concern here will be with the general class of nonsingular models which obey condition (5).

Since close to $t = 0$, condition (5) gives $\Omega \sim a(\bar{S})^{-1}$, the above criterion requires that

$$\tau \sim \int a\,\mathrm{d}t/\bar{S} \sim \text{finite} \quad \text{as} \quad t \to 0\,. \tag{7}$$

We now show that (7) is satisfied by $\bar{S}$. The Einstein field equations give for energy density ϵ and pressure p,

$$3(\dot{\bar{S}}^2 + k)/\bar{S}^2 = 8\pi\epsilon\,, \quad \mathrm{d}(\bar{S}^3\epsilon)/\mathrm{d}\bar{S} + 3p\bar{S}^2 = 0\,. \tag{8}$$

For $p \geqslant 0, \bar{S}^3\epsilon$ is a nonincreasing function of $\bar{S}$. Hence for any $\bar{S} < S_0, \bar{S}_0^3\epsilon_0 \leqslant \bar{S}^3\epsilon$. From the first equation of (8)

$$\dot{\bar{S}}^2\bar{S}^2 = -k\bar{S}^2 + \tfrac{8}{3}\pi\epsilon\bar{S}^4 \geqslant -k\bar{S}^2 + \tfrac{8}{3}\pi\epsilon_0 S_0^3\bar{S}\,.$$

Near small enough $\bar{S}$, the second term on the right-hand side will dominate so that for a suitable constant λ,

$$\int \mathrm{d}t/\bar{S} = \int \mathrm{d}\bar{S}/\bar{S}\dot{\bar{S}} < \int \mathrm{d}\bar{S}/\lambda(\bar{S})^{1/2} \sim \text{finite}$$

$$\text{as} \quad t \to 0\,.$$

Hence for all classical $\bar{S}$, condition (5) for avoidance of singularity by a conformal transformation guarantees that the particle horizon is also eliminated.

Note that because of the above result it becomes possible to extend the nonsingular conformal space–time suitably to avoid the particle horizon. In this case the support of the conformal function can be suitably extended to $t < 0$ as was done in the example of the Minkowski space–time given earlier. It is important to remember that such extensions are not arbitrary and pathological but are subject to the dynamics of quantum cosmology as outlined below.

How probable is the realization of condition (5)? To answer this question we treat Ω as a quantum variable and compute its propagator by the path-integral method. To this end we write

$$\Omega = 1 + \Phi \tag{9}$$

and identify Φ as the variable denoting QCF from the classical solution. The propagator $K[\Phi_2, t_2; \Phi_1, t_1]$ then denotes the probability amplitude for $\Phi = \Phi_2$ at $t = t_2$ given that $\Phi = \Phi_1$ at $t = t_1$.

The method of computation of K has been described in earlier papers [3,4]. The quantum state of the universe may be described by a wavefunction $\psi(\Phi, t)$ and the time development of ψ can be studied with the help of K. Since we are interested in calculating the probability that the universe has attained a given state at $t = t_1$ from a group of states in the past, we will assume that $t_2 < t_1$. It is convenient to describe the quantum-mechanical state of the universe at time t_1 by a wavepacket ψ of spread Δ_1:

$$\psi(\Phi, t_1) = (2\pi\Delta_1^2)^{-1/4}\exp(-\Phi^2/4\Delta_1^2)\,. \tag{10}$$

Δ_1 represents the uncertainty in specifying Φ precisely at its classical value. It was shown in ref. [3] that for the classical dust models ($p = 0$) the corresponding wavefunction at t_2 also had a wavepacket form with a spread Δ_2 that grows indefinitely as the classical singular epoch is approached. The growth in uncertainty is given by

$$\Delta_2 \propto t_2^{-2/3}, \quad t_2 < t_1\,. \tag{11}$$

This result can be generalized for matter with pressure [6] and it can be shown that for QCF around a classical Friedman universe $\bar{\mathcal{M}}$ with the expansion factor $\bar{S}(t)$, the uncertainty in Φ grows as we go further back in time as

$$\Delta = b(\bar{S})^{-1}\,, \quad b = \text{constant}\,. \tag{12}$$

Let us now return to condition (5) and ask for the condition that $\mathcal{M}$ is singular at $t = 0$. This condition is

$$\eta \equiv \Omega \bar{S} \to 0 \,, \quad \text{as} \quad t \to 0 \,. \tag{13}$$

From (12), write $\Delta = b\bar{S}$ and express (13) in the form

$$|\Phi/\Delta| \equiv b^{-1} |\eta - \bar{S}| \to 0 \quad \text{as} \quad t \to 0 \,. \tag{14}$$

For a wavepacket in Φ centred on $\Phi = 0$, the probability that $\mathcal{M}$ is singular therefore tends to zero as $t \to 0$.

The extreme unlikelihood of the classical singularity and the consequent removal of horizon makes it possible to extend the manifold to the past of $t = 0$. Clearly such extensions are not conformal to the original classical manifold $\bar{\mathcal{M}}$. However, they can be considered in the following manner. For any S nonzero at $t = 0$ we can continue the solution to $t < 0$. If S tends to zero at some past epoch $t_p < 0$, the above calculation will again show that such solutions have vanishing probability. The important point is that in the regime of QCF not all extensions have equal status: the nonsingular solutions dominate over singular ones.

The physical significance of this conclusion is as follows. Given the expansion function $\bar{S}$ at an epoch t_1, classical gravity uniquely fixes the past history of the universe and it tells us that the universe had a singular origin as well as a particle horizon. The quantum conformal fluctuations around this classical solution however admit other histories of the universe for $t < t_1$. Some of these histories are with singularity and horizon while the rest are not. As we probe backwards to $t = 0$, the former set becomes less and less probable and the latter set more and more so. Hence, given that the universe is at present in a given Friedmann state, it is extremely unlikely that it would have got there from a state with singularity and a particle horizon.

I thank T. Padmanabhan for discussions.

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