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The Luminosity–Volume Test and the Local Hypothesis of Quasars

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Abstract. It is shown that the luminosity-volume test for optically selected objects has an in-built bias towards increasing the average value of V/V_m above the Euclidean value of 1/2. A more satisfactory bias-free statistic is suggested in the form of In (V/V_m), The result of applying the test to a sample from the Bright Quasar Survey (BQS) shows that the local hypothesis of quasars is consistent with the data.

Key words: quasars, statistics– quasars, local hypothesis

1. Introduction

In spite of the discovery of nearly 3000 quasars the question of their distance remains a controversial one. The majority of astronomers believe in the cosmological hypothesis that the quasar redshifts are due solely to the expansion of the universe. This hypothesis implies that quasars of large redshifts are considerably farther than those with small redshifts. There are, however, several examples wherein quasars of very different redshifts are found too close to one another, and sometimes in very special linear configurations. There are also cases of quasars of high redshifts near galaxies of very small redshifts. For a review of such examples see Narlikar (1983). Statistics of quasar distributions on the sky suggest that the probability of obtaining such configurations by chance is extremely small (Burbidge, Narlikar & Hewitt 1985; Narlikar & Subramanian 1985). Perhaps the most striking case of a quasar-galaxy association with highly discrepant redshifts is that reported recently by Huchra *et al.* (1985). Even with gravitational lensing thrown in, it is hard to reconcile this case with a chance juxtaposition.

These examples make one wonder whether the cosmological hypothesis is correct. Certainly one cannot rule out the possibility that quasars are considerably nearer than implied by their redshifts. To test whether a group of quasars is in fact local rather than distant, other methods may therefore be used. For example, if quasars are distant then probes of their large-scale distributions should show some effects of the non-Euclidean geometry that describes the expanding universe.

The luminosity-volume test applied to a complete magnitude limited sample is one such probe. If the quasars are very nearby (say within 30–100 Mpc), then their distribution should follow Euclidean geometry. It is then easy to verify that a quasar of magnitude B observed in a survey of limiting magnitude B_m has the V/V_m ratio given by

$$\frac{V}{V_m} = \text{dex}\{0.6(B - B_m)\}. \tag{1.1}$$

Here V = observed volume of space up to the quasar and V_m = the maximum volume of space within which the quasar could be found within the prescribed magnitude limit. In a uniform distribution the average value of V/V_m is 1/2 when the geometry is Euclidean.

This test had earlier been applied to radio quasars by Wills & Lynds (1978) who concluded that the results were consistent with the local hypothesis. If, however, the test is applied to verify the cosmological hypothesis on the same sample, evolutionary effects have to be invoked.

This circumstance, that evolution (in number density or luminosity or both) needs to be invoked, has so far prevented any clear-cut demonstration of the hypothesis that quasars are at cosmological distances. The effects of non-Euclidean geometry implied by Hubble's law are masked by the evolutionary effects invoked to fit the data.

Schmidt & Green (1983), for example, have applied the V/V_m test to the quasars from the Bright Quasar Survey (BQS), a sample of quasars brighter than an average magnitude B = 16.16. On the basis of a comparison of their sample with other fainter quasar samples, these authors have to invoke strong evolutionary effects in a typical Friedmann model. This conclusion has, however, been challenged by Wampler & Ponz (1985, 1986) who find no evidence for evolution between the BQS quasars (mean redshift $z \simeq$ 1.8) and the CTIO sample of quasars (mean redshift $z \simeq 2.8$) of Osmer & Smith (1979).

By contrast, in a local hypothesis the evolutionary parameters cannot be invoked and it is consequently more vulnerable to observational disproof. Indeed, Schmidt & Green (1983) have already argued that the application of the V/V_m test to the BQS sample disproves the local hypothesis.

In this paper we suggest a variant of the standard V/V_m test as applied to optical samples. We do so because the standard test has an inbuilt theoretical bias that overestimates the average value of V/V_m We then apply the modified test to the BQS sample to test the validity of the local hypothesis.

2. The Modified V/V_m Test for Optical Quasars

Consider the theoretical expression for V/V_m given by Equation (1.1) in the following form

$$\frac{V}{V_m} = \exp\{0.6(\ln 10).(B - B_m)\} \simeq \exp\{1.38(B - B_m)\}. \tag{2.1}$$

Suppose that owing to errors in magnitude determinations the true magnitude in fact lies in the range (B —Δ B, B+ΔB). Although the average value of the magnitude in this range is B, the corresponding errors in V/V_m do not average out to zero. In fact the average of V/V_m for (B -Δ B) and (B + Δ B) is

$$\left(\frac{\overline{V}}{V_m}\right) = \cosh(1.38\Delta B)\cdot\exp\{1.38(B - B_m)\}. \tag{2.2}$$

In other words, on an average the observer tends to overestimate V/V_m by a factor cosh (1.38 ΔB) that exceeds unity. A similar effect arises from errors in the estimation of plate limits B_m.

Bearing in mind that both ΔB and ΔB_m are larger at fainter magnitudes where more sources are expected to lie, the above bias could lead to an average $\langle V/V_m \rangle$ significantly higher than the Euclidean value when in fact the true value may well be Euclidean. For example, an uncertainty of ΔB = 0.27 increases the value of $\langle V/V_m \rangle$ for a Euclidean distribution from 0.5 to 0.535 while ΔB = 0.5 raises it to 0.62.

So far as an optically selected sample is concerned we therefore suggest that instead of using

$$x \equiv \frac{V}{V_m} \tag{2.3}$$

as the variable we should choose

$$y = \ln x = 1.38(B - B_m). \tag{2.4}$$

It is evident that the errors in y due to errors $\pm \Delta B$ in B or $\pm \Delta B_m$ in B_m cancel out in a random distribution.

Since x has a uniform distribution in the range [0,1],

$$\langle y \rangle = \int_0^1 \ln x \, dx = -1, \tag{2.5}$$

while

$$\sigma_y^2 = \int_0^1 (\ln x)^2 \, dx - \langle y \rangle^2 = 1. \tag{2.6}$$

Therefore, the mean value $\langle y \rangle$ computed for a sample of N members has the Standard error $1/\sqrt{N}$. For a Euclidean distribution the mean value of y is therefore expected to lie in the range $[-1-3/\sqrt{N}, -1+3/\sqrt{N}]$ with a probability of 99.7 per cent.

3. The BQS sample and local quasars

To illustrate the above test we choose the BQS sample since it is a single homogeneous optically selected sample containing a large number of quasars. Schmidt & Green (1983) quote that their magnitudes are accurate to ΔB = 0.27. However, Wampler & Ponz (1985) have discussed the various observational biases in the BQS data and have given more reliable magnitudes for the samples studied by Schmidt & Green. A comparison of the Wampler–Ponz magnitudes with the Schmidt–Green magnitudes turns up many cases of magnitude differences considerably in excess of 0.27. For example, in 15 out of the 67 entries in Table 3 of Wampler & Ponz the difference exceeds 0.50 mag. To justify the greater reliability of their magnitudes, Wampler & Ponz (1986) have shown that they are in agreement with another independent estimate.

We therefore apply the above test to the BQS quasars with the revised B magnitudes as given by Wampler & Ponz (1985). There is, however, one difficulty in this procedure. In many cases (24 out of 67) the magnitudes listed by Wampler & Ponz in Table 3 of their paper exceed the corresponding limiting magnitudes B_{lim} of the BQS survey. It is therefore not possible to set $B_m = B_{lim}$ as originally given by Schmidt & Green.

Wampler & Ponz do not give the revised values of B_{lim}, for the various plates, but for overall consistency it is obviously necessary to suppose that the revised values of B_{lim} must be fainter than those quoted by Schmidt & Green.

In their Table 2, Wampler & Ponz have taken the average magnitude to be 16.5 for the BQS survey. For the 67 cases of their Table 3 the average excess of their magnitudes over the Schmidt—Green magnitudes is about 0.21. Thus it is not unreasonable to set an overall average limiting magnitude for the Wampler–Ponz data at ~ 16.4 instead of the average value ~ 16.16 given by Schmidt & Green. We will therefore consider the two cases of B_m = 16.5 and 16.4.

Accordingly, we first take B_m= 16.5 in Equation (2.4). In Table 3 of Wampler & Ponz there are 58 quasars brighter than this magnitude. For this sample we find

$$\langle y \rangle = -0.92 \tag{3.1}$$

Since $1/\sqrt{58} = 0.13$, it is clear that the observed value of $\langle y \rangle$ does not differ significantly from the Euclidean value –1, the departure being ~ 0.6 σ_y.

A similar analysis for B_m = 16.4 gives $\langle y \rangle$ differing from the Euclidean value of –1 by $\lesssim 1.3\ \sigma_y$. This difference also cannot be considered statistically significant. It can be easily verified that if instead of the statistic y we had used for the same data the statistic $x = V/V_m$, the σ -departures of $\langle x \rangle$ from the Euclidean value of 0.5 would have been systematically higher, thus illustrating the in-built bias of x.

4. Conclusion

From our application of the modified bias-free luminosity volume test it is clear that the BQS sample studied here is consistent with the local hypothesis of quasars. Taken in conjunction with the earlier studies of V/V_m for radio quasars we therefore find that quasar counts do not rule out the local hypothesis. This result also bears out an earlier conclusion by one of us (Hoyle 1984) that with suitable correction for the bias in favour of objects chosen too near the plate limit, $\langle V/V_m \rangle$ for quasars is not significantly higher than 1/2.

Since the same tests applied on the basis of the cosmological hypothesis force the supporter of this hypothesis to invent various evolutionary scenarios to ensure a good fit, we feel that on the grounds of Occam's razor the local hypothesis comes out better.

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