

Cylindrical universes with heat and null radiation flow

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Abstract

In paper-I (Patel and Dadhich, 1992) we have discussed cylindrically symmetric viscous fluid models with the Kasnerian time evolution. In this paper we incorporate heat flow and null radiation flow with the perfect fluid. Here again, in the case of heat flow the Kasner spacetime is the matter-free limit of the model. We establish a general result that a static perfect fluid distribution will on Kasnerisation (introduce $t^{4/3}$, $t^{4/3}$ and $t^{-2/3}$ in the coefficients of dr^2 , $d\phi^2$ and dz^2) yield a time dependent distribution with the same equation of state and with or without heat flow.

Subject Headings : General relativity, cylindrical universes, fluid with heat and null radiation flow

PACS No. : 04.20, 98.80

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1. Introduction

In very early stages of evolution of the universe near the big-bang singularity, the only thing we can say about the nature of matter is that it would be in highly dense state having very exotic and unusual behaviour. The distribution will be inhomogeneous and anisotropic and the geometry of spacetime will be near Kasnerian.

It is evident that spherical symmetry will have to be traded for a lesser order of symmetry. So we come to cylindrical symmetry which is less restrictive than spherical and can accommodate the Kasnerian behaviour as well as provide enough avenue for inhomogeneity and anisotropy (Patel and Dadhich 1992, hereafter to be referred as paper-I). As the form of matter will be exotic, hence we should keep an open mind to consider the attributes of viscosity, heat flow and null radiation flux etc. In paper-I we have considered cylindrical viscous-fluid models exhibiting the Kasnerian evolutionary behaviour (which turns out to be a general feature) and in some cases the matter-free limit is the non-flat empty Kasner metric.

Following the philosophy of the paper-I, we shall, in this paper, consider heat flow and null radiation flux alongwith perfect fluid distribution. As the matter is not expected to attain thermal equilibrium in the early stages, it is understandable that there would be heat flow in the universe. Though the dominant component of matter is supposed to be the incoherent radiation with the equation of state $\rho = 3p$, but there could as well be some pure null radiation flowing along a particular direction. The effect of heat flux in evolution of the cosmological models has been investigated by several authors (Deng 1989; Mukherjee 1986; Novello and Reboucas 1978; Ray

1980; Reboucas and de Limma 1981, 1982; Reboucas 1982; Bradley and Sviestins 1984; and Sviestins 1985) while null radiation flux has also been considered (Vaidya and Patel 1986, 1989; Patel and Yadav 1987; and Patel and Koppar 1988).

We shall consider the general cylindrically symmetric metric and shall assume that the metric coefficients are separable functions of r at t . In §2 we set up the general equations. The universe with heat flow and null flowing radiation will be considered in §3 and §4. Finally we conclude with discussion in §5.

For inclusion of heat flow there is a general procedure. We establish the following general result : Take a given cylindrically symmetric spacetime describing static perfect fluid distribution and Kasnerise it to get a time dependent distribution with the same equation of state and with or without heat flow.

2. The general equations

We begin with cylindrically symmetric metric in the general form :

$$ds^2 = D^2 dt^2 - A^2 dr^2 - B^2 dz^2 - C^2 d\phi^2 \quad (2.1)$$

with D, A, B, C being functions of r and t . We introduce the tetrad :

$$\theta^1 = A dr, \theta^2 = B dz, \theta^3 = C d\phi, \theta^4 = D dt$$

so that the metric (2.1) takes the form

$$ds^2 = (\theta^4)^2 - (\theta^1)^2 - (\theta^2)^2 - (\theta^3)^2 = g_{(ab)} \theta^a \theta^b.$$

Here and in what follows the bracketed indices indicate tetrad components. Following the standard calculations, we obtain the following non-zero components of the Ricci tensor in the tetrad form :

$$R_{(14)} = \frac{1}{AD} \left[\frac{\dot{B}'}{B} + \frac{\dot{C}'}{C} - \frac{\dot{A}}{A} \left(\frac{B'}{B} + \frac{C'}{C} \right) - \frac{D'}{D} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] \quad (2.2)$$

$$R_{(44)} = \frac{1}{D^2} \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{D}}{D} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] - \frac{1}{A^2} \left[\frac{D''}{D} - \frac{D''}{D} \left(\frac{A'}{A} - \frac{B'}{B} - \frac{C'}{C} \right) \right] \quad (2.3)$$

$$R_{(11)} = \frac{1}{A^2} \left[\frac{B''}{B} + \frac{C''}{C} + \frac{D''}{D} - \frac{A'}{A} \left(\frac{B'}{B} + \frac{C'}{C} + \frac{D'}{D} \right) \right] - \frac{1}{D^2} \left[\frac{\ddot{A}}{A} + \frac{\dot{A}}{A} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{D}}{D} \right) \right] \quad (2.4)$$

$$R_{(22)} = \frac{1}{A^2} \left[\frac{B''}{B} + \frac{B'}{B} \left(\frac{C'}{C} + \frac{D'}{D} - \frac{A'}{A} \right) \right] - \frac{1}{D^2} \left[\frac{\ddot{B}}{B} + \frac{\dot{B}}{B} \left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A} - \frac{\dot{D}}{D} \right) \right] \quad (2.5)$$

$$R_{(33)} = \frac{1}{A^2} \left[\frac{C''}{C} + \frac{C'}{C} \left(\frac{B'}{B} + \frac{D'}{D} - \frac{A'}{A} \right) \right] - \frac{1}{D^2} \left[\frac{\ddot{C}}{C} + \frac{\dot{C}}{C} \left(\frac{\dot{B}}{B} + \frac{\dot{A}}{A} - \frac{\dot{D}}{D} \right) \right]. \quad (2.6)$$

A prime and a dot indicate differentiation with respect to r and t respectively. We shall name the coordinates as $x^4 = t$, $x^1 = r$, $x^2 = z$, $x^3 = \phi$.

We write the Einstein field equations for non-empty spacetime as

$$R_{ik} = -8\pi(T_{ik} - \frac{1}{2}Tg_{ik}) \quad (2.7)$$

where the energy-momentum tensor T_{ik} for perfect fluid with heat flow is given by

$$T_{ik} = (p + \rho)\nu_i\nu_k - pg_{ik} + q_i\nu_k + q_k\nu_i \quad (2.8)$$

and that with null radiation by

$$T_{ik} = (p + \rho)\nu_i\nu_k - pg_{ik} + \sigma\omega_i\omega_k. \quad (2.9)$$

We note that $\nu_i\nu^i = 1$, $q_i\nu^i = 0$, $\omega_i\omega^i = 0$ and the normalization condition $\nu^i\omega_i = 1$. q_i and σ represent the heat flow vector and the null radiation density respectively.

3. Fluid with heat flow

We consider a radial heat flow and use co-moving coordinates. Therefore we have

$$\nu_{(a)} = (0, 0, 0, 1), \quad q_{(a)} = (q, 0, 0, 0) \quad (3.1)$$

where $\nu_{(a)}$ and $q_{(a)}$ are tetrad components of ν_i and q_i respectively. From equations (2.7), (2.8) and (3.1) we obtain

$$R_{(11)} = R_{(22)} = R_{(33)},$$

$$8\pi p = \frac{1}{2}[R_{(11)} - R_{(44)}],$$

$$8\pi\rho = -\frac{1}{2}[3R_{(11)} + R_{(44)}],$$

$$8\pi q = -R_{(14)}. \quad (3.2)$$

We assume the separability of metric potentials as $A = t^\alpha a(r)$, $B = t^\beta b(r)$, $C = t^\gamma c(r)$, $D = D(r)$ (the t -dependence of D is absorbed in the redefinition of the time).

From (2.3) - (2.6) it turns out that the Kasnerian values, $\beta = -\frac{1}{3}$ and $\alpha = \gamma = \frac{2}{3}$ solve out the terms involving time derivatives and hence the time variation of the density ρ and the pressure p will only come from $\frac{1}{A^2}$ (i.e. $t^{-4/3}$). From (2.2) we get

$$8\pi q = \frac{X}{3tAD}, \quad X = 3\frac{b'}{b} + \frac{D'}{D}. \quad (3.3)$$

From (3.2) for the equation of state $\rho = 3p$, we obtain, after substantial algebra, the condition

$$X' + X \left[\frac{D'}{D} + \frac{b'}{b} + \frac{c'}{c} - \frac{a'}{a} \right] = 0. \quad (3.4)$$

The trivial solution $X = 0$ implies $q = 0$. That means if we start with a static perfect fluid distribution satisfying $X = 0$ implying $\rho = 3p$ and introduce the Kasnerian time dependence, then we shall have a non-stationary perfect fluid distribution with the same equation of state. In particular $\rho = \rho_s t^{-4/3}$ where ρ_s is the density in the static case. Davidson's (1991) solution is one such example.

The non-trivial solution of (3.4) will give rise to non-static perfect fluid with $\rho = 3p$ and $q \neq 0$. That is, given a static perfect fluid solution of Einstein equations with $\rho = 3p$ but $X \neq 0$, by introducing the Kasnerian time dependence we obtain a non-static perfect fluid distribution with the heat flow. The values of ρ and p in the

non-static case are obtained as before by multiplying their static values by $t^{-4/3}$.

We construct an example of this case.

The equation (3.4) can be easily integrated to give

$$X = aK|bcD \quad (3.5)$$

where K is a constant of integration.

The equations $R_{11} = R_{22} = R_{33}$ and (3.5) admit a solution

$$D = r^d, \quad a = r^m, \quad b = r^n, \quad c = r \quad (3.6)$$

where

$$6d = 3(K + 2) - 3\Delta, \quad 6m = 2(K + 2) - 2\Delta,$$

$$6n = K - 2 + \Delta, \quad \Delta = (K^2 - 16K + 4)^{1/2}. \quad (3.7)$$

The constant K has to satisfy the inequality $0 \leq K \leq 8 - 2\sqrt{15}$. The pressure p and the heat flux parameter q are given by

$$8\pi p = 8\pi(\rho/3) = 2Kt^{-4/3}r^{-2(m+1)}$$

$$8\pi q = Kt^{-5/3}r^{-(m+d+1)} \quad (3.8)$$

Clearly p and q tend to zero as $r \rightarrow \infty$ or $t \rightarrow \infty$.

The metric for the above solution is written as

$$ds^2 = r^{2d} dt^2 - t^{4/3} (r^{2m} dr^2 + r^2 d\phi^2) - t^{-2/3} r^{2n} dz^2 \quad (3.9)$$

where d, m and n are given by (3.7). When $K = 0$, pressure p and the heat flux parameter q vanish and we get the non-flat vacuum Kasner spacetime. Note that $K = 0$ implies $d = m = n = 0$.

In addition to $t = 0$ singularity, here $r = 0$ is also a singularity which is an undesirable feature.

The above discussion can be summarised in the form of a theorem as follows :

Theorem : Let the metric (2.1) represent a static perfect fluid distribution (ρ_s, p_s) .

To this we introduce the Kasnerian time dependence then the following situations arise :

- (a) the Kasnerised metric will always represent the fluid distribution with the same equation of state and $\rho = t^{-4/3} \rho_s, p = t^{-4/3} p_s,$
- (b) when $\rho_s \neq 3p_s,$ there will always be heat flow in the fluid,
- (c) when $\rho_s = 3p_s,$ the fluid will be with or without heat flow depending upon the equation (3.4) has a non-trivial ($X \neq 0$) or trivial ($X = 0$) solution.

The above theorem also holds good for the plane-symmetric distributions, for which there will always be heat flow. Details are given in the Appendix.

We now give an example to illustrate the case (b).

Kramer (1988) has discussed a simple metric describing a static cylindrically symmetric perfect-fluid solution. It is given by

$$ds^2 = e^{k^2 r^2} (dt^2 - dr^2) - r^2 d\phi^2 - dz^2 \quad (3.10)$$

where k is an arbitrary constant and

$$8\pi\rho_s = 8\pi p_s = k^2 e^{-k^2 r^2}. \quad (3.11)$$

Note that $\rho_s \neq 3p_s$. Using the above theorem, we have the metric

$$ds^2 = e^{k^2 r^2} dt^2 - t^{4/3} (e^{k^2 r^2} dr^2 + r^2 d\phi^2) - t^{-2/3} dz^2 \quad (3.12)$$

that satisfies the Einstein field equations for a perfect fluid with radial heat flow.

The values of p, ρ and the radial component q_1 of q_i are given by

$$8\pi p = 8\pi\rho = k^2 t^{-4/3} e^{-k^2 r^2}, \quad 8\pi q_1 = \frac{k^2}{3} r t^{-1} e^{-k^2 r^2/2} \quad (3.13)$$

It is clear that $t = 0$ is the singularity but the spacetime is well-behaved for

$t > 0$ and the physical parameters go to zero as $t \rightarrow \infty$.

The fluid velocity is

$$\nu_i = (0, 0, 0, e^{k^2 r^2/2}) \quad (3.14)$$

and hence the acceleration vector is

$$f_i = (-k^2 r, 0, 0, 0). \quad (3.15)$$

Thus the stream lines of the fluid are not geodesic.

The expansion scalar θ and the shear σ have the expressions

$$\sigma = \sqrt{\frac{2}{3}}\theta, \quad \theta = t^{-1}e^{-k^2 r^2/2}. \quad (3.16)$$

We have verified that the fluid motion is irrotational. Clearly θ and σ tend to zero as $t \rightarrow \infty$ or $r \rightarrow \infty$.

Now, the phenomenological expression for the heat conduction is given by

$$q_i = \psi(T_{,i} + T f_i)(\delta_i^j - \nu^j \nu_i) \quad (3.17)$$

where a comma denotes partial derivative. Here ψ is the thermal conductivity and T is the temperature. Since in our case only the radial component of heat flux is retained, from the above equation we obtain

$$\frac{\partial T}{\partial r} - k^2 r T = (k^2/24\pi t \psi) r e^{-k^2 r^2/2}. \quad (3.18)$$

If we assume ψ to be a function of t alone, equation (3.18) can be easily integrated to give

$$T = l_0(t) e^{k^2 r^2/2} - (1/12\pi t \psi) e^{-k^2 r^2/2} \quad (3.19)$$

where l_0 is an arbitrary function of t .

In general ψ should be a function of both r and t . But we have only one equation connecting two unknowns ψ and T and hence we have to put one additional restriction on the behaviour of T and ψ .

If we switch off the heat flux ($k = 0$), then pressure and density also vanish and we go over to the Kasner empty spacetime

$$ds^2 = dt^2 - t^{4/3}(dr^2 + r^2 d\phi^2) - t^{-2/3} dz^2 \quad (3.20)$$

As discussed in paper-I, the property, matter-free limit being the Kasner spacetime, is a novel feature of this class of cylindrical universes.

4. Fluid with null radiation

It is easy to see that the field equations (2.7) alongwith (2.9) can be expressed in the tetrad form as

$$R_{(ab)} = -8\pi[(p + \rho)\nu_{(a)}\nu_{(b)} - \frac{1}{2}(\rho - p)g_{(ab)}] - 8\pi\sigma\omega_{(a)}\omega_{(b)} \quad (4.1)$$

where $\nu_{(a)}$ and $\omega_{(a)}$ are the tetrad components of the fluid velocity ν_i and the null radiation velocity ω_i respectively. We take the tetrad components $\nu_{(a)}$ and $\omega_{(a)}$ as

$$\nu_{(a)} = (0, \sinh\lambda, 0, \cosh\lambda), \quad \omega_{(a)} = (0, e^\lambda, 0, e^\lambda) \quad (4.2)$$

where λ is a function of co-ordinates to be determined from the field equations.

Obviously ν_i and ω_i satisfy $\nu^i\nu_i = 1$, $\omega^i\omega_i = 0$ and $\nu^i\omega_i = 1$.

In view of the equations (4.1) and (4.2), for the metric (2.1) we have

$$R_{(14)} = 0, R_{(11)} = R_{(33)} \quad (4.3)$$

$$8\pi p = -(1/2)[R_{(44)} - R_{(22)}] \quad (4.4)$$

$$8\pi\rho = -2R_{(11)} - (1/2)[R_{(44)} - R_{(22)}] \quad (4.5)$$

$$e^{2\lambda} = [2R_{(11)} + R_{(44)} - R_{(22)}]/[R_{(44)} + R_{(22)}] \quad (4.6)$$

$$8\pi\sigma e^{2\lambda} = (1/4)(e^{2\lambda} - e^{-2\lambda})[2R_{(11)} + R_{(44)} - R_{(22)}] \quad (4.7)$$

where $R_{(ab)}$ are given by (2.2) - (2.6).

We are interested in the solutions of the above equations which are separable. Therefore following paper-I and Davidson (1991) we shall assume the following form of the metric potentials :

$$A = t^\alpha(1+r^2)^a, \quad B = t^\beta(1+r^2)^b, \quad C = t^\gamma(1+r^2)^c, \quad D = (1+r^2)^d \quad (4.8)$$

where $a, b, c, d, \alpha, \beta$ and γ are real constants. Now equations (4.3) determine

$$\gamma = \alpha, \quad b(\beta - \alpha) = d(\beta + \alpha), \quad d^2 + b^2 = (d + b)(a + c + 1). \quad (4.9)$$

The remaining four equations (4.4) - (4.7) give us the physical parameters $\rho, p, e^{2\lambda}$ and σ .

From (2.2) - (2.6) it turns out that the Kasnerian values, $\alpha = \gamma = 2/3$ and $\beta = -1/3$ solve out the terms containing time derivatives and hence time variation of ρ, p and σ will only come from $1/A^2$ (i.e. $t^{-4/3}$). The physical requirement of ρ, p, σ being positive and finite for $t > 0$ and (4.9) determine a, b, c and d as follows :

$$a = (2 + 7\epsilon)/10, \quad b = -(1 + \epsilon)/5, \quad c = (-2 + 3\epsilon)/10, \quad d = 3(1 + \epsilon)/5 \quad (4.10)$$

where ϵ is an arbitrary constant.

Then we have

$$8\pi\rho = \frac{4}{5}(3 - 2\epsilon)t^{-4/3}(1 + r^2)^{-(12+7\epsilon)/5},$$

$$p = \frac{(1 + \epsilon)}{(3 - 2\epsilon)}\rho, \quad \sigma = \frac{5\epsilon(8 + 3\epsilon)}{4(3 - 2\epsilon)(4 - \epsilon)}\rho, \quad e^{2\lambda} = \frac{(4 - \epsilon)}{4(1 + \epsilon)}. \quad (4.11)$$

Clearly $\rho \geq 0, \sigma \geq 0$ restrict ϵ to the range $0 \leq \epsilon < \frac{2}{3}$. It is interesting to note that

$\lambda, \frac{p}{\rho}, \frac{\sigma}{\rho}$ turn out to be constants.

The spacetime metric is given by

$$ds^2 = (1 + r^2)^{6(1+\epsilon)/5} dt^2 - t^{4/3}(1 + r^2)^{(2+7\epsilon)/5} dr^2 \\ - r^2 t^{4/3}(1 + r^2)^{(-2+3\epsilon)/5} d\phi^2 - t^{-2/3}(1 + r^2)^{-2(1+\epsilon)/5} dz^2. \quad (4.12)$$

The interesting point about the above metric is that for $-\frac{6}{11} \leq \epsilon \leq 0$, it was shown in paper-I that it represents a viscous fluid model. That is, it can describe a viscous fluid or a fluid with null flowing radiation depending upon the permissible range for ϵ . This however gives rise to an important question : When can in general a fluid with null radiation flux and a viscous fluid be described by the same metric, through different range of values for parameters? This deserves further consideration.

When $\epsilon = 0$, the radiation density σ and the parameter λ vanish and we recover the Davidson's (1991) universe filled with incoherent radiation.

5. Discussion

As argued in paper-I, cylindrical universes with heat and null radiation flow will be relevant only in the very early stages of evolution of the universe. The overall qualitative features of these models are the same as those of the viscous universes of paper-I. That is, time evolution is Kasnerian and matter-free limit of fluid with heat flow turns out to be the empty Kasner metric.

We have a general prescription for generating a non-stationary fluid distribution with heat flow from a static distribution : Take a static perfect fluid solution which on Kasnerisation will in general give a non-stationary fluid distribution with heat flow. In the case of $\rho = 3p$, the presence of heat flow will be determined by whether the equation (3.4) admits a non-trivial or trivial solution. On Kasnerisation the equation of state remains unaltered and $(\rho, p) = t^{-4/3}(\rho_s, p_s)$.

It is interesting that the same metric for different range of values for the parameter ϵ can represent a fluid with null flowing radiation and a viscous fluid, e.g. the metric (4.12) represents the former for $0 \leq \epsilon < 2/3$ while the latter for $-\frac{6}{11} \leq \epsilon \leq 0$ (paper-I). The similarity in energy-momentum structure of fluid with null radiation flux and viscous fluid deserves to be investigated further. In paper-I we have seen that geodesic viscous fluid model could as well be interpreted as string dust distribution.

The same spacetime metric can describe more than one kind of matter dis-

tribution is a very interesting feature. For very close to the big-bang singularity matter will be in highly dense and exotic form that may include viscosity (that may even dominate over hydrodynamical pressure), heat flow, null radiation flow as well as cosmic strings etc. It is therefore very important that we have a spacetime metric which is capable of describing almost all these attributes for suitable values for certain parameters. The change in the parameter values may be accomplished by phase transitions.

Finally what we have is : Very close to the "big-bang" a cylindrical universe endowed with the Kasner time evolution and matter may occur in various forms; viscous fluid, string dust, fluid with heat and null radiation flux and so on. We have models to describe each of these situations separately, what we would like to find is one grand model which synthesises all these aspects.

Acknowledgements

One of us (LKP) wishes to thank Indian National Science Academy for a visiting fellowship and Inter-University Centre for Astronomy and Astrophysics for hospitality.

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Appendix

Let us suppose that a static plane-symmetric metric

$$ds^2 = D^2(r)dt^2 - a^2(r)dr^2 - b^2(r)(dz^2 + d\phi^2) \quad (A.1)$$

represents a perfect fluid distribution with pressure p_s and density ρ_s .

On Kasnerisation we obtain the metric

$$ds^2 = D^2 dt^2 - t^{-2/3} a^2 dr^2 - t^{4/3} b^2 (dz^2 + d\phi^2) \quad (A.2)$$

which gives rise to a non-static fluid distribution with

$$\rho = t^{2/3} \rho_s, \quad p = t^{2/3} p_s$$

$$8\pi q = \frac{2}{3t^{1/3} a D} \left[2 \frac{D'}{D} - 3 \frac{b'}{b} \right] \quad (A.3)$$

where

$$8\pi p_s = \frac{1}{a^2} \frac{b'}{b} \left(\frac{b'}{b} + 2 \frac{D'}{D} \right),$$

$$8\pi \rho_s = \frac{1}{a^2} \left[\frac{a'}{a} \left(3 \frac{b'}{b} + \frac{D'}{D} \right) + \frac{b'}{b} \frac{D'}{D} - \frac{D''}{D} - 3 \frac{b''}{b} \right] \quad (A.4)$$

and the pressure isotropy equation is

$$\frac{b''}{b} + \frac{D''}{D} - \frac{a'}{a} \left(\frac{b'}{b} + \frac{D'}{D} \right) - \frac{b'}{b} \left(\frac{b'}{b} + \frac{D'}{D} \right) = 0. \quad (A.5)$$

From the above equations it can be seen that the heat flux cannot vanish for $\rho \geq 0$. As an example, the Davidson's (1987) plane-symmetric perfect fluid solution can be Kasnerised.

$$(1.1) \quad \dots = \dots + \dots = \dots$$

The Kasnerisation is obtained by...

$$(1.2) \quad \dots = \dots + \dots = \dots$$

...

$$\dots = \dots$$

$$\dots = \dots$$

$$\dots = \dots$$

$$(1.3) \quad \dots = \dots + \dots + \dots = \dots$$

and the remaining equations are

$$(1.4) \quad \dots = \dots + \dots + \dots = \dots$$