

Phantom damping of matter perturbations

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Cosmological scaling solutions are particularly important in solving the coincidence problem of dark energy. We derive the equations of sub-Hubble linear matter perturbations for a general scalar-field Lagrangian—including quintessence, tachyon, dilatonic ghost condensate and k-essence—and solve them analytically for scaling solutions. We find that matter perturbations are always damped if a phantom field is coupled to dark matter and identify the cases in which the gravitational potential is constant. This provides an interesting possibility to place stringent observational constraints on scaling dark energy models.

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I. INTRODUCTION

In the past years much efforts have been made to understand the nature and the origin of dark energy—a major puzzle of modern cosmology. The accumulating observational data continues to confirm that about 70% of the total energy density in the present universe corresponds to unknown energy with a negative pressure [1]. A wide variety of dark energy models have been proposed to address this problem [2] (see Ref. [3] for review). For a viable cosmological evolution the field energy density should remain sub-dominant during the radiation/matter dominant era and become important only at late times. Though the dynamically evolving scalar-field models have an edge over the cosmological constant scenario, they too, in general, are plagued with fine-tuning problems of initial conditions and parameters of the models. Cosmological scaling solutions, in which matter and dark energy follow the same background evolution, can alleviate the so-called coincidence problem by providing a dynamical attractor [4]. For a minimally coupled scalar field, however, the energy density of the field decreases proportionally to that of the background fluid for scaling solutions and hence the acceleration of universe cannot be realized. Ordinary dark energy models are then supplemented by additional features tuned to allow the exit from the scaling regime at late times [5].

It has, however, been shown that if the coupling between dark energy and dark matter is taken into account, one can achieve an accelerated expansion in the scaling regime, thus alleviating the coincidence problem [6, 7]. It is therefore important to be able to distinguish between scaling and non-scaling solutions observationally. It is already known that scaling solutions with accelerated expansion give an acceptable fit to the supernovae (SN) Ia [8]; therefore the background behaviour seems to be insufficient in distinguishing between a scaling universe and a non-scaling one. What is still lacking is the investigation of the evolution of density perturbations in such models, except for specific cases [9, 10]. The goal of this paper is to find the perturbation equation in the

sub-Hubble regime for a very general Lagrangian and to solve it along scaling solutions. Our analysis is applied to a wide variety of coupled dark energy models including quintessence, tachyon, dilatonic ghost condensate and k-essence. We will show that along scaling solutions the equation of matter perturbations can be solved *analytically* in terms of observable background quantities (equation of state and density parameter). In particular, the growth rate is found to be unbounded, both from below and from above. Finally, we shall see that when the field behaves as phantom (equation of state $w_\phi < -1$) then linear matter perturbations are always damped. We call this phenomenon *phantom damping*.

The prototype of these stationary solutions is the standard coupled scalar field with an exponential potential [9]. Here, however, the perturbations grow too fast due to the extra attraction induced by the coupling and drive an unacceptable Integrated Sachs-Wolfe (ISW) effect [11] on the Cosmic Microwave Background (CMB). This problem is generally expected in accelerated scaling regimes, both because of the stronger interaction and because the onset of acceleration may occur quite earlier than usual. An intriguing result in our paper is that the ISW effect vanishes for some parameter values. Although these parameters are not observationally acceptable due to current supernovae constraints, this result shows that the problem of an unacceptable ISW effect can be alleviated by allowing the phantom field.

II. DENSITY PERTURBATIONS AND SCALING SOLUTIONS

We start with the following general Lagrangian [12]

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{2} + p(X, \phi) \right] + \mathcal{S}_m[\phi, g_{\mu\nu}], \quad (1)$$

where X is the kinematic term of a scalar field ϕ , i.e., $X \equiv -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2$. Here p is a scalar field Lagrangian which is the function of X and ϕ , and \mathcal{S}_m is the action for matter fields which are generally dependent on ϕ .

The perturbation equations have been discussed in Refs. [9, 10] for the system of a coupled scalar field. Here we shall study the evolution of matter perturbations for scaling solutions with a general Lagrangian (1). Let us consider metric perturbations Ψ and Φ in the longitudinal gauge about the flat Friedmann-Robertson-Walker (FRW) background:

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Phi)dx_i dx^i, \quad (2)$$

where $a(t)$ is a scale factor. One has $\Phi = \Psi$ in the absence of anisotropic stress. We shall study a cosmological scenario in which the Universe is filled by the field ϕ with an energy density ρ and by only one type of matter fluid with an energy density ρ_m . We also assume that the field is coupled to matter fluid with a coupling Q defined by $Q \equiv -1/(\rho_m \sqrt{-g})\delta\mathcal{S}_m/\delta\phi$. In the general case one should insert also radiation and baryons and leave them uncoupled in order not to violate the equivalence principle and other observations. Defining the matter density contrast $\delta_m \equiv \delta\rho_m/\rho_m$ and the dimensionless matter velocity divergence $\theta_m \equiv \nabla_i v_{(m)i}/H$, where H is the Hubble rate, we obtain the following perturbation equations in Fourier space with wavelength $\lambda_k = Ha/k$ [10]:

$$\delta'_m = -\theta_m + 3\Phi' + \sqrt{6}Q\varphi', \quad (3)$$

$$\theta'_m = -\left(2 + \frac{H'}{H} + \sqrt{6}Qx\right)\theta_m + \frac{1}{\lambda_k^2}(\Phi + \sqrt{6}Q\varphi), \quad (4)$$

where $\varphi \equiv \delta\phi/\sqrt{6}$ and $x \equiv \dot{\phi}/\sqrt{6}H$. Here a prime denotes the derivative with respect to $N \equiv \log(a)$. In Newtonian limit the perturbation equation for the field ϕ takes the following form

$$\varphi'' + F(\phi)\varphi' + m(\phi)_{\text{eff}}^2\varphi + \frac{c_s^2}{\lambda_k^2}\varphi = -\frac{\sqrt{6}c_s^2 Q\Omega_m\delta_m}{2p_X}, \quad (5)$$

where $p_X \equiv \partial p/\partial X$, $c_s^2 \equiv p_X/\rho_X$ and $\Omega_m \equiv \rho_m/(3H^2)$. $F(\phi)$ and $m(\phi)_{\text{eff}}$ are functions of $p(X, \phi)$.

The effective mass $m(\phi)_{\text{eff}}$ is expected to be negligible if the field ϕ is responsible for dark energy. Then on sub-Hubble scales the $(c_s^2/\lambda_k^2)\varphi$ term is important, and its amplitude is forced to balance with the r.h.s. of Eq. (5). Then we find

$$\varphi \simeq -\frac{\sqrt{6}\lambda_k^2 Q\Omega_m\delta_m}{2p_X}. \quad (6)$$

The gravitational potential is expressed as [10]

$$\Phi = -\frac{3}{2}\lambda_k^2 [(\delta_m + 3\lambda_k^2\theta_m)\Omega_m + \delta_\phi\Omega_\phi + 6x\varphi p_X]. \quad (7)$$

Since φ is proportional to λ_k^2 on sub-Hubble scales, Φ is approximately given by $\Phi \simeq -(3\lambda_k^2/2)\Omega_m\delta_m$. Then by using Eqs. (3), (4) and (6), we find finally

$$\delta''_m + \left(2 + \frac{H'}{H} + \sqrt{6}Qx\right)\delta'_m - \frac{3}{2}\Omega_m \left(1 + 2\frac{Q^2}{p_X}\right)\delta_m = 0. \quad (8)$$

This is a very general equation which holds for any coupled scalar field with Lagrangian $p(X, \phi)$, even when the coupling Q depends on the field ϕ . In the general case it can be integrated numerically. In the following we show that it may be integrated analytically for a scaling cosmology in which the equation of state $w_\phi \equiv p/\rho$ and the energy ratio $\Omega_\phi \equiv \rho/(3H^2)$ are constant.

In the flat FRW background one obtains the following conservation equations

$$\dot{\rho} + 3H(1 + w_\phi)\rho = -Q\rho_m\dot{\phi}, \quad (9)$$

$$\dot{\rho}_m + 3H(1 + w_m)\rho_m = Q\rho_m\dot{\phi}, \quad (10)$$

where $w_m \equiv p_m/\rho_m$. The equation for H is given by

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1 + w_s), \quad (11)$$

where the effective equation of state is defined as $w_s \equiv (w_\phi\rho + w_m\rho_m)/(\rho + \rho_m)$.

Scaling solutions satisfy the condition $\rho \propto \rho_m$, i.e., $d\log\rho/dt = d\log\rho_m/dt$. Assuming that the coupling Q is a constant in the scaling regime, it was shown in Refs. [12, 13] that the existence of scaling solutions restricts the form of the scalar field Lagrangian to be

$$p(X, \phi) = Xg(Xe^{\lambda\phi}), \quad (12)$$

where g is any function in terms of $Y \equiv Xe^{\lambda\phi}$ and λ is given by

$$\lambda \equiv Q\frac{1 + w_m - \Omega_\phi(w_m - w_\phi)}{\Omega_\phi(w_m - w_\phi)}. \quad (13)$$

For the Lagrangian (12) we find that [14]

$$\Omega_\phi = x^2(g + 2g_1), \quad w_\phi\Omega_\phi = x^2g, \quad (14)$$

where $g_n \equiv Y^n g^{(n)}$.

In what follows we shall specialize the formula to the most relevant case $w_m = 0$, i.e., that of cold dark matter. Then by using Eqs. (13) and (14), the effective equation of state is given by

$$w_s = \Omega_\phi w_\phi = -\frac{Q}{Q + \lambda}. \quad (15)$$

We note that this property holds irrespective of the form of the function $g(Y)$. One has $w_s = 0$ for $Q = 0$ and $w_s \rightarrow -1$ in the limit $Q \gg \lambda > 0$.

To discuss the fixed points of our system, it is convenient to introduce two dimensionless quantities x and y , defined by $x \equiv \dot{\phi}/(\sqrt{6}H)$ and $y \equiv e^{-\lambda\phi/2}/(\sqrt{3}H)$. Then the evolution equations (9), (10) and (11) for the Lagrangian (12) can be casted in the following autonomous dynamical system:

$$x' = \frac{3}{2}x[1 + x^2g - 2A(g + g_1)] + \frac{\sqrt{6}}{2}[A(Q + \lambda)(g + 2g_1)x^2 - \lambda x^2 - QA], \quad (16)$$

$$y' = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y(x^2g + 1), \quad (17)$$

where $A \equiv (g + 5g_1 + 2g_2)^{-1}$. We note that the equation of state for the field ϕ reads

$$w_\phi = -1 + \frac{2x^2 p_X}{\Omega_\phi}. \quad (18)$$

Therefore the field behaves as a phantom ($w_\phi < -1$) for $p_X < 0$.

The ordinary (phantom) scalar field with an exponential potential corresponds to the choice $g(Y) = \epsilon - c/Y$ (negative ϵ is a phantom). The dilatonic ghost condensate [12] and the tachyon [15] also has scaling solutions, since the Lagrangians in these models are written in the form (12) by the choice $g(Y) = -1 + cY$ and $g(Y) = -c\sqrt{1 - 2Y}/Y$, respectively. The critical points can be found by setting $x' = y' = 0$ in Eqs. (16) and (17). In fact the property of fixed points for coupled systems was discussed in Ref. [14] for three classes of dark energy models mentioned above.

There exists the following scaling solution for *any* form of the function $g(Y)$:

$$x = \frac{\sqrt{6}}{2(Q + \lambda)}, \quad (19)$$

as was shown in Eq. (37) in Ref. [12]. We note that $|Q + \lambda| > \sqrt{6}/2$ and that $g = -2Q(Q + \lambda)/3$ along the scaling solution. One can easily check that this solution actually satisfies Eqs. (16) and (17). We recall that Ω_ϕ (and g_1) remains undetermined and depends on the specific Lagrangian.

For the scaling solution (19) we have $Qx = -\sqrt{6}w_s/2$ and $p_X = g + g_1 = (\Omega_\phi + w_s)/(2x^2)$ by Eqs. (14) and (15). Then one can write the perturbation equation (8) in terms of w_s and Ω_ϕ :

$$\delta_m'' + \xi_1 \delta_m' + \xi_2 \delta_m = 0, \quad (20)$$

where

$$\xi_1 \equiv \frac{1}{2} - \frac{9}{2}w_s, \quad (21)$$

$$\xi_2 \equiv -\frac{3}{2}(1 - \Omega_\phi) \left(1 + \frac{6w_s^2}{\Omega_\phi + w_s} \right). \quad (22)$$

Observationally, we expect Ω_ϕ to be in the range 0.6-0.8 (from the complementary matter fraction in clustered objects) and w_s in the range $(-0.45, -0.75)$ from comparison with SNIa [8].

Since w_s and Ω_ϕ are constants in the scaling regime, we obtain the following solution

$$\delta_m = c_+ a^{n_+} + c_- a^{n_-}, \quad (23)$$

where c_\pm are integration constants and

$$n_\pm = \frac{1}{2} \left[-\xi_1 \pm \sqrt{\xi_1^2 - 4\xi_2} \right]. \quad (24)$$

One has $\Omega_\phi + w_s \equiv \bar{\Omega}_\phi(1 + w_\phi) > 0$ for a non-phantom scalar field, thus giving $n_+ > 0$ and $n_- < 0$. Therefore δ_m (and Φ) grows in the scaling regime ($\delta_m \propto a^{n_+}$).

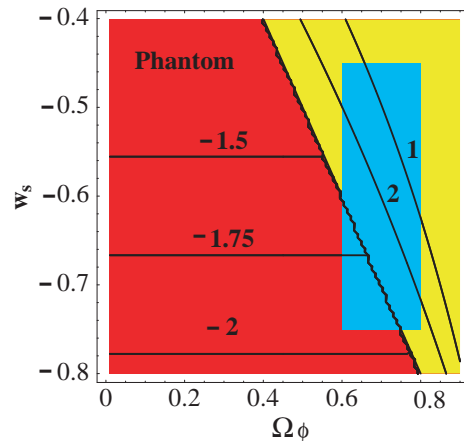


FIG. 1: Contour plot of the index n_+ in terms of the functions of Ω_ϕ and w_s . The numbers which we show in the figure correspond to the values n_+ . In the non-phantom region characterized by $\Omega_\phi + w_s > 0$, n_+ are always positive. Meanwhile in the phantom region ($\Omega_\phi + w_s < 0$) with $w_s > -1$, n_+ take complex values with negative real parts. We plot the real parts of n_+ in the phantom region. The box (blue in the color version) represents schematically the observational constraints on w_s, Ω_ϕ coming from the SNIa data.

When $Q = 0$ the solution of Eq. (8) for constant w_s and Ω_m is given by Eq. (23) with an index

$$n_\pm = \frac{1}{2} \left[\frac{3}{2}w_s - \frac{1}{2} \pm \sqrt{\left(\frac{3}{2}w_s - \frac{1}{2}\right)^2 + 6\Omega_m} \right]. \quad (25)$$

Then we obtain $n_+ = 1$ and $n_- = -3/2$ in the matter dominant era with $w_s \simeq 0$ and $\Omega_m \simeq 1$. From Eq. (15) one has $w_s = 0$ in the uncoupled case ($Q = 0$). Since $0 \leq \Omega_m \leq 1$ in the scaling regime, the index n_+ satisfies $n_+ \leq 1$ for uncoupled scaling solutions. Meanwhile the coupling Q can lead to the index n_+ larger than 1. In Fig. 1 we show the contour plot of n_+ as the functions of Ω_ϕ and w_s . The growth of the perturbations gets unboundedly larger as we approach the border $\Omega_\phi + w_s = 0$. The large index n_+ obviously gives rise to a strong ISW effect on CMB, which is not acceptable. However we caution that a precise bound on n_+ will depend on the specific choice of Lagrangian.

In the phantom region corresponding to $\Omega_\phi + w_s < 0$, we find that n_+ are complex with negative real parts for $w_s > -1$. Therefore the perturbations exhibit damped oscillations in this case. If $w_s < -1$, n_+ are either negative real values or complex values with negative real parts. In any case the perturbations always decay when the field ϕ corresponds to a phantom: we call this phenomenon *phantom damping*. In other words, the repulsive effect of the phantom coupling dissipates the perturbations (at least in the linear regime).

Along this scaling solution the gravitational potential $\Phi \simeq -(3H^2 a^2 / 2k^2) \Omega_m \delta_m$ evolves as $a^{n_+ - 1 - 3w_s}$. It can

be seen therefore that the potential is constant for $n_+ = 3w_s + 1$, which corresponds to $w_s^\pm = [-2 \pm \sqrt{4 - 3\Omega_\phi}]/3$. Since $0 \leq \Omega_\phi \leq 1$ we find $-1/3 \leq w_s^+ \leq 0$ and $-4/3 \leq w_s^- \leq -1$ (for instance $w_s^+ = -0.207$ and $w_s^- = -1.126$ for $\Omega_\phi = 0.7$). Although these values of w_s are currently excluded by SN observations, it is interesting to observe that there exist scaling solutions for which the gravitational potential is exactly constant. This shows that, generally speaking, the absence of the ISW effect does not imply the absence of dark energy. We should also mention that values of w_s smaller than -1 are allowed if part of dark matter itself is not coupled, see Ref. [8]. Clearly, the full investigations of such cases require the numerical integration of the Boltzmann equations and it is beyond the scope of this paper.

III. CONCLUSIONS

In this paper we have studied the evolution of sub-Hubble linear perturbations in the universe filled with a general scalar field coupled to dark matter. We analytically derived the solutions for matter perturbations when the background is described by the scaling solution given by Eq. (19). This analysis can be applied to any dark energy models which possess scaling solutions.

The power-law index n_\pm for perturbations is expressed by the functions of Ω_ϕ and w_s only. The evolution of perturbations is neatly divided by the border $\Omega_\phi + w_s = 0$

between the ordinary field and the phantom field. While the perturbations grow for $w_\phi > -1$, they are always suppressed in the phantom case. In Fig. 1 we plot the index n_+ as the functions of Ω_ϕ and w_s together with the constraint coming from the SNIa datasets. In the non-phantom region the growth of the perturbations gets larger as the parameters approach the border $\Omega_\phi + w_s = 0$, which would give rise to an unacceptable ISW effect. Therefore it is likely that large part of the parameters space in Fig. 1 is excluded from the CMB constraints. The phantom region allowed by the SNIa constraint corresponds to the strong suppression with $\text{Re}(n_+) \lesssim -1.6$.

It is certainly of interest to place constraints on Ω_ϕ and w_s using the latest CMB datasets. This requires a full detailed analysis of the evolution of perturbations for each Fourier mode without using the short wavelength approximation, which we leave to future work. This will provide a powerful way to distinguish between coupled dark energy models and other alternatives.

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