IUCAA - 10/94
February 1994

GENERAL RELATIVITY, COLLAPSE OF ROTATING STAR AND EMISSION OF GRAVITY WAVE

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ABSTRACT : We have discussed the non-dimensional quantity q of massive stars ($M \gtrsim 10\ M_\odot$) as a case of collapse of rotating stars. For $q > 1$ it is found that almost all cases (except few) no apparent horizon appears i.e. all the matter will not be swallowed into a blackhole. The main aim of this paper is what are the possible consequences of it.

Subject headings: Relativity:General - Star: collapse - Star:neutron - Blackhole - Gravitational Wave

I. INTRODUCTION

The collapse of rotating stellar cores is one of the most interesting source of gravitational waves. Because

i) the gravitational waves from these processes should be measurable on the Earth

ii) from the measurement of typical frequencies and wave amplitudes, the final state of collapse could be identified (neutron star or blackhole or any ?)

iii) arrival time measurements of the gravity wave pulse would allow to set upper bounds onto the restmass of the gravitational field.

Presented in the XVII th IAGRG Conference held on 12-14 Feb 1994 at IUCAA, Pune, India.

Not only that, other importances are :

i) the collapse of rotating stellar cores is very likely the optically unseen trigger mechanism of type II supernova explosion

b) the stellar core collapse is accompanied by neutrino emission from the core

c) the stellar core collapse may create blackhole.

We know from the standard view point of gravitational collapse when a rotating star begins to collapse, the gravity becomes so strong that there appears a region from which even a photon cannot escape. After the distortion of space-time is radiated as gravitational waves, a Kerr black hole is formed finally. But the details of dynamics of matter and the propagation of gravitational radiation is not yet fully known.

In the physics of gravitational collapse, the non-dimensional quantity q is defined as :

$$q \equiv \vec{J} / \left(M \cdot \frac{GM}{c^2} \cdot c \right) \tag{1}$$

where J and M are the total angular momentum and the gravitational mass of the star, c the speed of light. As q corresponds to ' a / M ' in Kerr Metric, q should be smaller than unity if a Kerr black hole is formed finally (see fig 1). It is believed that q of a collapsing massive star ($M \gtrsim 10\ M_\odot$) can be greater than unity [de Felice and Yunquiang 1984]. If the collapsing star has q >1 then what happens ?

To know the answer many works [i.e. Smarr 1977, Eppley 1979, Miyama 1981, Bardeen 1982, Maeda etal 1981, Piran 1980, 1982, York 1982, Nakamura 1981, 1984, Nakamura etal 1980a, 1980b, 1982, Nakamura and Sato 1981a, b, 1982, Sasaki 1980] have been done considering rotation through 2-Dimensional numerical simulations. It has found that for $q > 1$

a) apparent horizon appears when $q \lesssim 1.05$ but no horizon for $q > 1.05$

b) the dynamics of matter strongly depend on the initial conditions and the equation of state (EOS)

c) all the matter will not be swallowed into a black hole

d) matter with large specific angular momentum is ejected as jets or an expanding disk or an expanding ring.

Based on the above , we have discussed, as a consequence, about the various open issues .

II. BASIC EQUATIONS AND NUMERICAL SIMULATIONS

The first relativistic computation of dynamical collapse of a rapidly rotating objects were done by Nakamura and his collaborators [Nakamura 1981, Nakamura etal 1982, Nakamura etal 1987]. In order to consider the general relativistic collapse of rotating star they consider the basic conditions in the [(2+1) + 1] formalism strongly resemble to those of a non-rotating system with a toroidal magnetic field B_φ, angular momentum density J_φ, as [for details cf. Nakamura 1984, York 1982] :

Hamiltonian constraint conditions

$$\rho_H \phi^6 = \begin{cases} \rho_3(r) & \text{for } r < r^* \\ 10^{-6}\rho_3(0) & \text{for } r > r^* \end{cases} \tag{2}$$

and angular momentum constraint

$$J_R / R = J_Z / Z = \begin{cases} \rho_H C_V \\ \rho_H C_V \exp[1-(r/r_0)^2] \quad \text{for } r > r_0 \end{cases} \tag{3}$$

$$J_\varphi = x \rho_H \Omega_0 \exp(-C_\varphi x / r_0^2) \tag{4}$$

where $$r^* \equiv \rho_3^{-1} (10^{-6} \rho_3(0)) \tag{5}$$

and $\rho_3(r)$ and r_0 be the density distribution of N = 3 polytrope and initial redius of the star, respectively. C_V, Ω_0, C_φ are the constants that determine the initial velocity, the initial central angular velocity and the rotation law , respectively. The asymptotic behaviour of ϕ for $r \longrightarrow \infty$ is as

$$\phi = 1 + C_1 / r , \tag{6}$$

R, Z, φ are the cylindrical coordinates.
They adopt cylindrical coordinates initially because the system has reflection symmetry about Z = 0 , the regularity conditions at R = 0. In their calculations they also take r_0 = 10.5 and Cv is chosen so that the infall velocity at $r = r_0$ becomes free fall velocity. Instead of Ω_0 and K, they use $\tilde{J}$ and U defined as $\tilde{J}$ = E rot / | E grav | and U = E int / |E grav | where E rot, E int and E grav are the rotational energy, the internal energy and the gravitational energy of a star, respectively.

As the rotation laws, they consider three cases :

a) Rotational Law A — Here C_φ = 2 i.e. almost rigidly rotating case in this model

b) Rotational Law B -- C_φ = 10 i.e. strongly differential rotation case

c) Case C corresponds to the collapse of a star of 10 $M_\odot$ [Nakamura 1981] with

$$J_R = J_z = 0,$$
$$\rho = 3 \times 10^{13} \exp\{ - (x + y) / 4.5 \} \ \mathrm{g/cm^3} \qquad (7)$$
$$\Omega = \Omega_0 \exp (- x / 4.5)$$
$$P = \begin{cases} \rho\epsilon / 3 & \text{for } \rho \leq \rho^* \equiv 3 \times 10^{14} \ \mathrm{g/cm^3} \\ (\rho - \rho^*)\epsilon + \rho^*\epsilon / 3 & \text{for } \rho > \rho^* \end{cases}$$

They use the equation of state in all models $p = \rho\epsilon / 3$ and initial distribution of the internal energy per gram ϵ is taken as $\epsilon = k (\rho)^{1/3}$

ROTATIONA LAW A

In this rotationa law, the centrifugal force has a maximum value at R = 0.35 r_0 where f_3 (R) is less than 0.1 f_3 (0)

They use typical models such as :

Model A50 : In this model, the rotation is very slow. At t = 0.287, the matter distribution is alsmost spherical and the velocity pattern (see fig.2a) shows a spherical collapse. This feature is kept through the entire time up to when the apparent horizon (dashed line) is formed (fig.2b). Although the matter distribution becomes slightly oblate by the effect of rotation, all the matter will be swallowed into the slowly rotating blackhole.

Model A 105 : In this model, the star is rotating rather rapidly. The obtained results are shown in fig.3a & 3b. In fig.3a we see that at $t = 11.5$ the matter falls vertically for $R \lesssim 2$. The collapse in the equatorial plane is considerably suppressed by the effect of rotation. In the range $2.4 \lesssim R \lesssim 7.7$, the outflow velocity reaches upto 0.3c. Finally the oblate shaped core is formed in the central region and an apparent horizon is formed outside this core (see fig3b). The outer envelope expands alongwith the lateral direction with relativistic velocity. It is expected that some part of this envelope will return to the central blackhole and the other part will expand to infinity.

Model A 146 : From fig.4a, it is seen that at $t = 5.76$ the matter in the central part falls almost vertically. For $2 \lesssim R \lesssim 6.5$, a strong outflow with the velocity upto 0.5 is seen. At $t = 18.8$ the central core bounces and a shock wave is formed. Near the equatorial plane the outflow extends upto $R = 9$. The outer thin envelope falls vertically to this outflow and the shock front is formed (see fig 4b At $t = 23.2$ a strong jet exists along the rotational axis. It is noticeable that the central core has almost stopped moving and the rather dense envelope expands both in the lateral direction and in the Z-direction (fig.4c) and finally a strong jet reaches $Z = 9$ (fig.4d). As the core has a rather small q value (i.e. ≈ 0.1), it may recollapse eventually and a blackhole may be formed after all.

ROTATIONAL LAW B

In this case, the centrifual force is more effective for small R i.e. if the mass shedding occurs, it will occur from the central region. Typical models they used are :

Model B 51 : The density contours and the flow patter at t= 21.6 are shown in fig.5 . Although A 50 and B 51 have almost the same angular momentum, the difference is that the central core is deformed rather strongly in B51. This is due to a rapidly rotating core because of the differential rotation of the model.

Model B 92 : In this model, the centrifugal force near the centre is greater than the gravitational force at t=0. This causes the outflow of the matter from a small R region. As the rotation is very slow for large R, the matter in the outerpart falls almost spherically (see fig.6a). From fig.6a we see that at t = 11.5 the inflow in the Z-direction form a disk in the central region. While in fig. 6c we see that at t = 17.3 the outgoing velocity of the disk is considerably decelerated. The outer envelope falls into this disk vertically and forms the almost steady shock. For large R, a very thin envelope expands in the lateral direction. However, no apparent horizon appears.

Model B 143 : In this model we can see the outflow for small R even at a rather early time (see fig.7a). Fig.7b and fig.7c show an expanding disk clearly formed at t = 11.6 and the expansion velocity of the disk decelerated in the central part at t = 17.5 which is the fastest at the edge of the disk, respectively. At t= 20.8 the central part of the disk is almost stopped. The outer thin envelope falls into the disk continuously and forms the almost steady shock front. As the expansion velocity is very large in this model, all the matter except the central part will go away from the system. No jet is formed (see fig.7d) in this model in contrast to the model A 146.

In the above we have described the special features of the important models although Nakamura considered many other models (a summary is given in Table I).

From the above numerical simulations it can be concluded that

1) the apparent horizons are formed for $q \lesssim 1.05$ in case A , $q \lesssim 0.92$ in case B and $q \lesssim 0.86$ in case C, although the models are different from each other in their equation of state, initial density distribution and rotation law ,
2) for $q > 1$, the dynamics of matter strongly depend on the initial conditions and the equation of state. Jet appears in case A while an expanding disk and an expanding ring appear in case B and case C, respectively.
3) The important results for $q > 1$ are that all the matter will not be swallowed into a blackhole and the matter with large angular momentum is ejected as jets, an expanding disk .

4) Matter with $q < 1$ may recollapse to a blackhole.

For $q < 1$ the appearence of event horizon is possible because we have considered the end point of a rotating collapse core is Kerr black hole. So singularity of the Kerr black hole may be hidden by the event horizon. But the appearence of apparent horizon, return of some part of the envelope of collapse star to the central blackhole and expansion of the other part of it to infinity give a new area of thought in astrophysics. Questions naturally arise : what are the consequences of the appeatence of event horizon for $q > 1$? What informations we will get from the expansion part of the envelope ? What is the significance of recollapse , etc ?

III. CONSEQUENCES AND SOME OPEN ISSUES

A) Do all the rotating star collapse to Black hole for $q < 1$?

Let us see fig. 10 - 12 obtained by Nakamura [1984 for final stages of models A 97 ($q = 0.97$), B 97 & C 93 for $q < 1$. No apparent horizon appears i.e. they will not collapse to blackhole. Where they will go ? They can collapse to Neutron Stars (except few) because of our known stellar configuration . Well, then what will be the exact value of q that determines whether the collapse of the rotating core goes to neutron star or blackhole ? It is still unknown.

B) Examination of Cosmic Censorship Hypothesis

As q correspondence to ' a / M ' in Kerr Metric, q should be smaller than unity if a Kerr blackhole is finally formed i.e. any equilibrium rotating blackhole has its external field represented by the Kerr metric [Robinson 1975] and the Kerr metric only represent a blackhole solution only when

$$a / M < 1$$

What happens if $a > M$? In our previous discussion we can see that the apparent event horizon is formed outside the core only for model A 105 ($q \lesssim 1.05$) . Although for axi-symmetry, it seems that collapse of a realistic rotating object with $a > M$ will be stopped by centrifugal support before blackhole dimensions are reached (Miller & de Felice 1985) , the question of whether a collapse could ever give rise to a singularity not surrounded by an event horizon is still an open one. Does model A 105 help to solve this problem ?

C) Accreting Neutron Star and Maximum mass of rotating Neutron Star

In the previous section it was concluded that for $q > 1$ all the matter will not be swallowed into a blackhole but what will be for a non-axisymmetric case for $q <1$? Let us consider the situation --- infall of an unejected envelope of a massive star of Mass > $10\ M_{\odot}$ in type II supernova explosion problem. According to Hillebrandt (see Nakamura 1984) — a star of ($8\ M_{\odot} \lesssim M \lesssim 10\ M_{\odot}$) may result in supernova explosion but the outer envelope of a star of $M \gtrsim 10\ M_{\odot}$ will not be ejected. Fig.8 and 9 show the situation of accreting neutron star with rotation where three kinds of rotation laws A, B, C are included in calculation. The situation is as follows :

First consider a non-rotating neutron star with $\rho_c = 10^{15}$ g/cm^3, then add an envelope to this neutron star, which is a model of an infalling envelope and later considered rotational situation. Fig.9a shows the almost initial state for model A 07 ($q = 0.07$). As the total mass of the system is $1.9\ M_{\odot}$ which is greater than the maximium mass of a stable neutron star $1.4\ M_{\odot}$, a blackhole is formed finally (see fig.9b). If we consider the non-axially symmetric perturbation, the similar situation will arise. The maximum mass of rotating neutron star will need to decide the point from where a blackhole forms finally.

Table II (Dutta and Alpar 1993) gives the calculated values of (a) the maximum gravitational mass of stable neutron stars, (b) the maximum mass that is constraint with $\alpha \geq 3.4 \times 10^{-2}$ and (c) the value of α for 1.4 $M_\odot$ neutron star for different equation of state models [α = the ratio of the moment of inertia of the pinned superfluid crust w.r.t. the total moment of inertia of the star]. We know that all measured neutron star masses are consistent with a value of 1.4 $M_\odot$ [van Paradijs 1991] . Assuming that the mass of the Vela Pulsar is also 1.4 $M_\odot$, we see from various equation of state models that a 1.4 $M_\odot$ neutron star is able to support crust with $\alpha = 3.4 \times 10^{-2}$ If we use the PSR 1913 + 16 mass constraint, the maximum gravitational mass for vela pulsar implied by the limit on α is in the range (1.48 - 2.223) $M_\odot$ [from Table II]. But what is the exact Maximum Mass of a rotating Neutron Star ?

4) Mechanism which is likely to be responsible for the most powerful source of Gravitational Radiation

We know that equilibrium objects for which rotation supplies a significant proportion of the force required to balance gravity are susceptible to non-axisymmetric instabilities (Lindblom 1986) . A possible picture is that a collapsing stellar core could have its collapse halted by a combination of internal pressure and centrifugal support. If the rotation speed is sufficient, a non-axisymmetric mode could then start to grow on a secular time scale giving rise to gravitational radiation which would carry away both energy and angular momentum. As a result of this, together with a probable increase in the core mass coming from continuing infall of surrounding matter, the core might then be destabilized again and commence a second phase of collapse ending in blackhole formation. Such a picture could alter the predictions for Gravitational Radiation Emission (Miller 1993).

CONCLUSION: Rotation plays an important active role on the collapse of a star. Gravitational Waves would provide the way to get observational information about relativistic gravitational collapse and could also give important insights into the nature of compact objects. We have to wait for detection of gravitational radiation from astrophysical sources to go into deeper understanding the details of relativistic gravitational collapse physics.

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TABLE 1. Initial Parameters and Main Results of the Calculated Models[a] (Nakamura 1984)

Model	Case	q	U	J	Time Slice	Horizon?[b]	Remarks
A146	A	1.46	0.94	0.77	max	no	jet
A122	A	1.22	0.86	0.51	max	no	jet
A105	A	1.05	0.84	0.37	max	yes	
A93	A	0.93	0.82	0.29	max	yes	
A75	A	0.75	0.82	0.19	max	yes	
A50	A	0.50	0.81	0.08	max	yes	
B143	B	1.43	1.01	1.20	max	no	expanding disk
B121	B	1.21	0.88	0.76	max	no	expanding disk
B104	B	1.04	0.84	0.54	max	no	expanding disk
B92	B	0.92	0.82	0.42	max/hyper	yes	
B74	B	0.74	0.81	0.27	max/hyper	yes	
B51	B	0.51	0.81	0.12	max	yes	
C137	C	1.37	0.25	1.56	hyper	no	expanding ring
C109	C	1.09	0.23	0.79	hyper	no	expanding ring
C95	C	0.95	0.22	0.55	hyper	no	expanding ring
C86	C	0.86	0.21	0.45	hyper	yes	ring singularity
C80	C	0.80	0.21	0.36	hyper	yes	ring singularity
C64	C	0.64	0.20	0.22	hyper	yes	ring singularity
C56	C	0.56	0.20	0.17	hyper	yes	ring singularity
C48	C	0.48	0.20	0.12	hyper/max	yes	ring singularity
C32	C	0.32	0.20	0.05	hyper	yes	

[a] In the sixth column, max and hyper mean the maximal slicing and the hypergeometric slicing, respectively. In the eighth column, the important characteristic of each model is written.

[b] A method for identifying an apparent horizon is shown in Reference Sasaki etal 1980

TABLE II: Equation of state (EOS), the maximum mass (gravitational) of stable neutron stars. (Dutta and Alpar 1993)

Equation of state *	$M_{max}/M_{\odot}$	$M_{max}/M_{\odot}$ consistent with $\alpha \geq 3.4\,10^{-2}$	α for neutron star of $1.4 M_{\odot}$
A	1.658	1.572	$5.518\,10^{-2}$
B	1.415	1.214	$1.394\,10^{-2}$
C	1.751	1.481	$4.060\,10^{-2}$
D	1.758	1.742	$7.103\,10^{-2}$
E	1.653	1.645	$6.797\,10^{-2}$
F	1.367	1.242	—
G	2.236	2.230	$1.221\,10^{-1}$
H	1.038	—	—
I	1.812	—	$3.161\,10^{-1}$
J	1.787	—	$3.691\,10^{-1}$
K	1.973	—	$3.725\,10^{-1}$
L	1.978	—	$3.200\,10^{-1}$
M	1.510	—	$1.360\,10^{-1}$
N	1.770	—	$1.938\,10^{-1}$
O	1.431	1.400	$4.916\,10^{-2}$
P	1.705	1.700	$9.203\,10^{-2}$
Q	2.127	1.532	$4.181\,10^{-2}$
R	2.182	1.786	$6.329\,10^{-2}$
S	1.839	1.671	$5.902\,10^{-2}$

* The EOS used were taken from the references:

A – Pandharipande (1971a), neutron matter
B – — (1971b), hyperonic matter
C – Moszkowski 1974
D – Bethe – Johnson model 1974, neutron matter
E – — 1974, hyperonic matter
F – Canuto & Chitre 1974
G – Walecka 1974
H – Brown & Weise 1976,
I→L – Glendenning Models I – IV
M→P – Prakash etal 1988 Models PAL 1, 2, 3, 4.
Q→S – Wiringa etal 1989 Models AV 14 + UVII, UV 14 + UVII, & UV 14 + TNI.

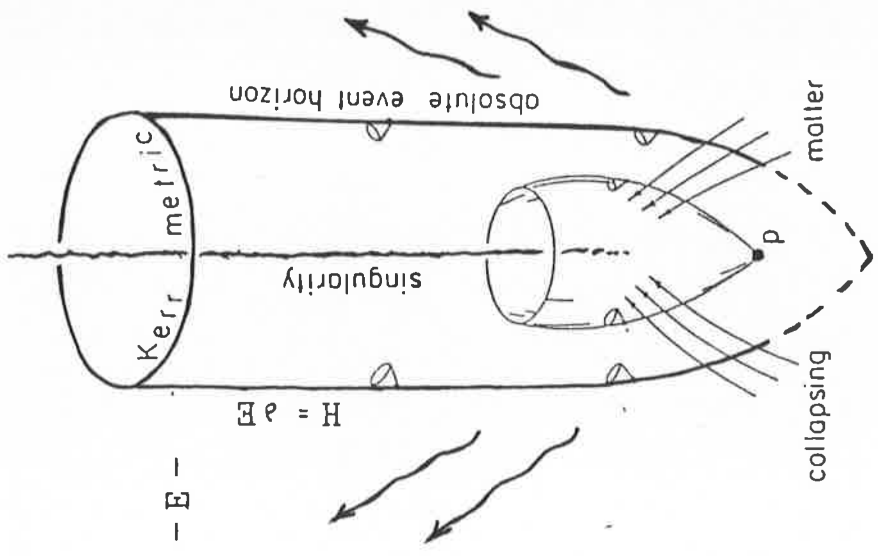

absolute event horizon
singularity
Kerr metric
collapsing matter
p
H = ∂E
– E –

Fig – 1

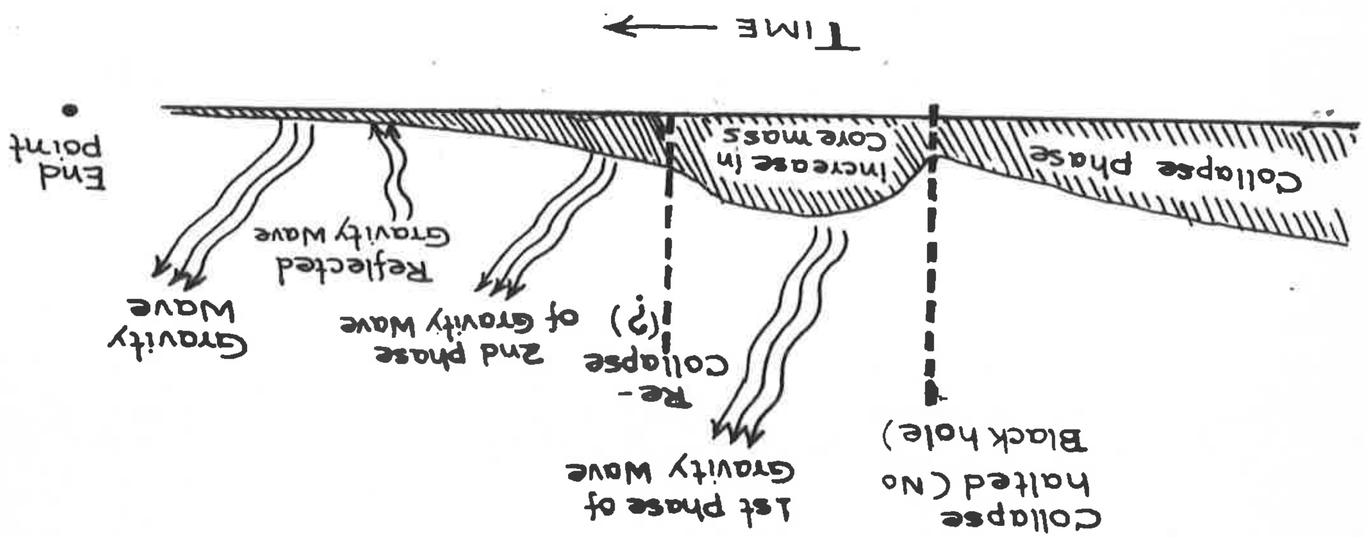
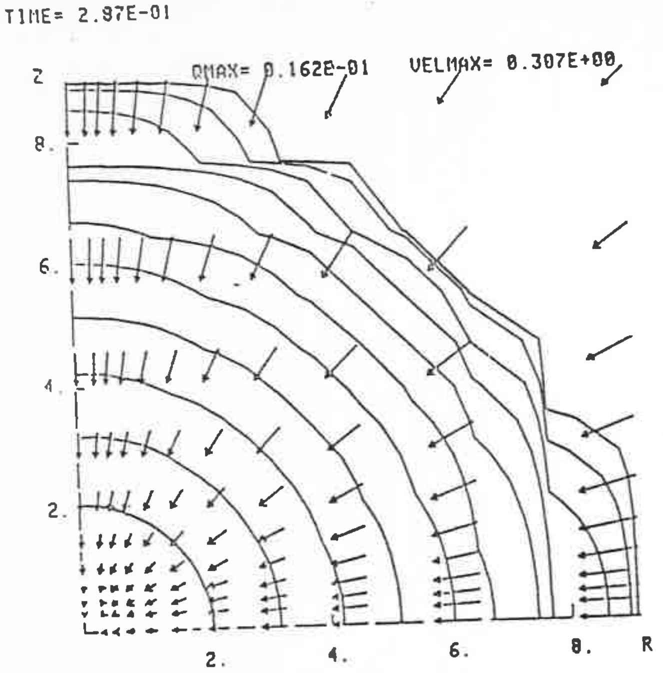
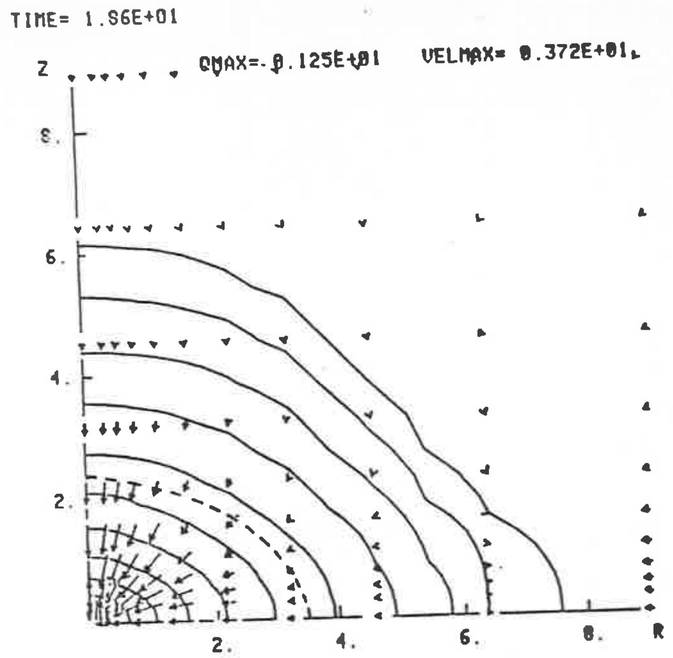

TIME
Collapse phase
Increase in Core mass
Collapse halted (No Black hole)
1st phase of Gravity Wave
Re-Collapse (?)
2nd phase of Gravity Wave
Reflected Gravity Wave
Gravity Wave
End point

Fig – 13

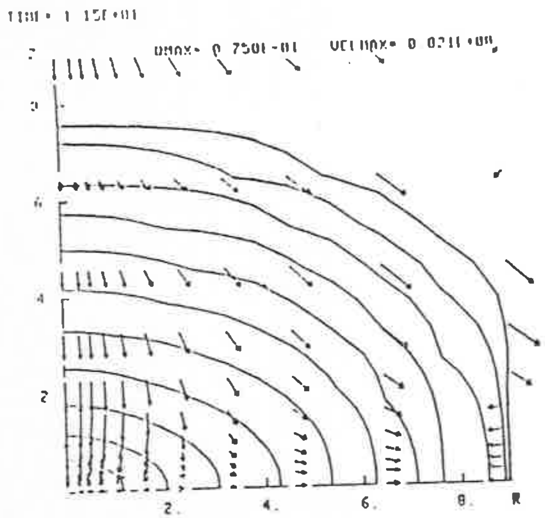
Fig(2a) →

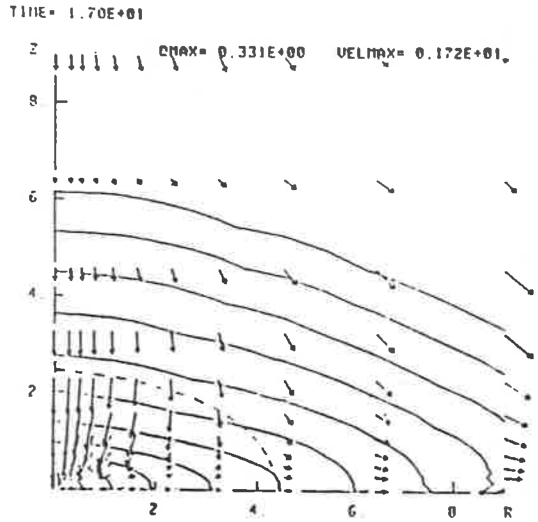




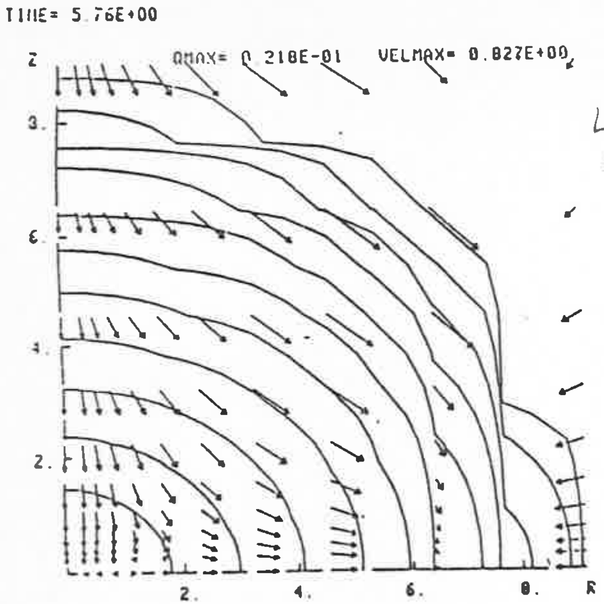
←(2b)

3a →

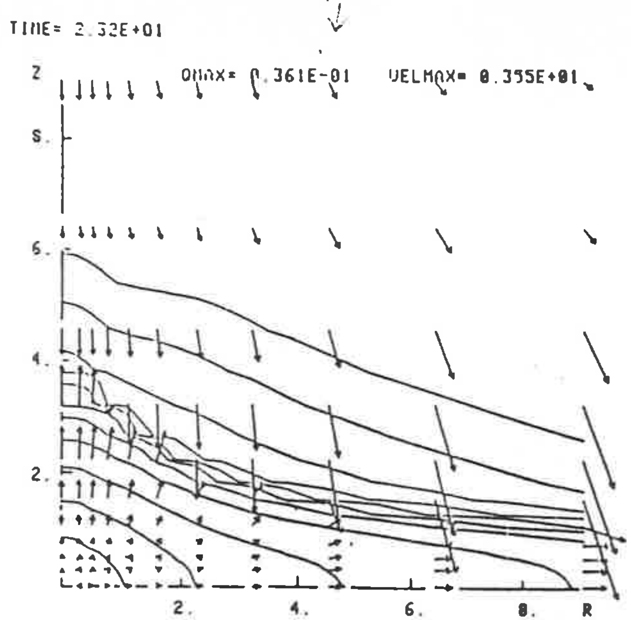




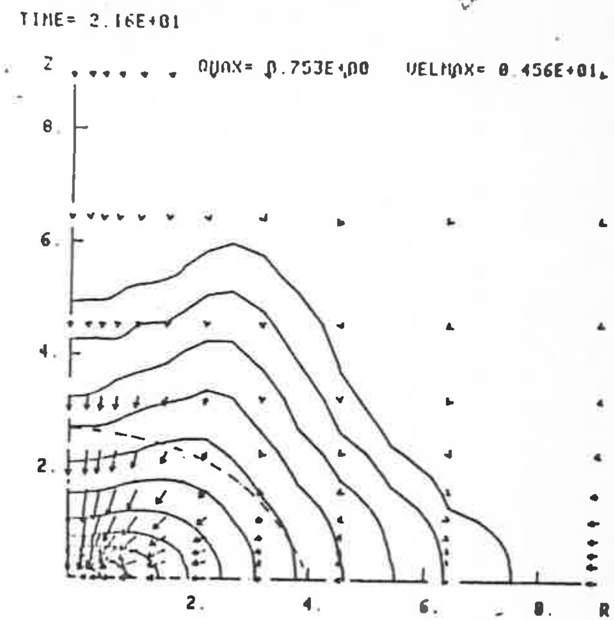
←(3b)



4a

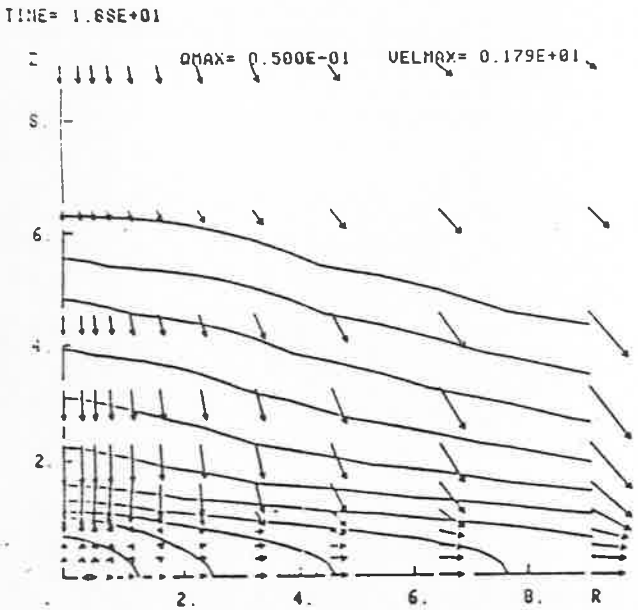


4c

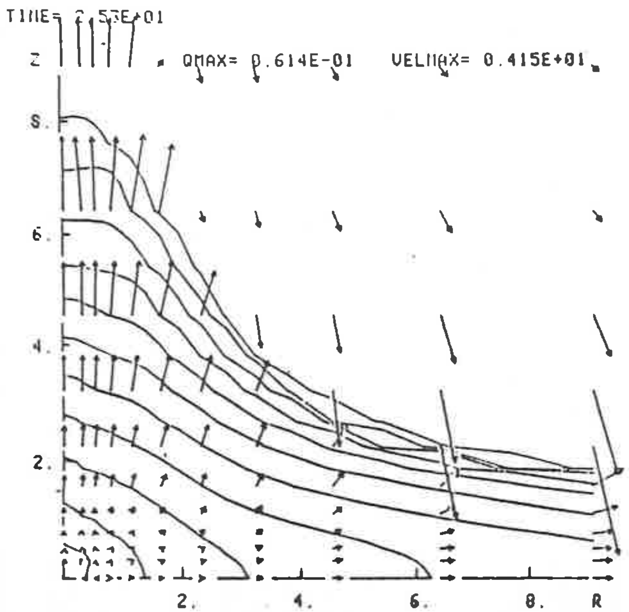


5

Fig - 4

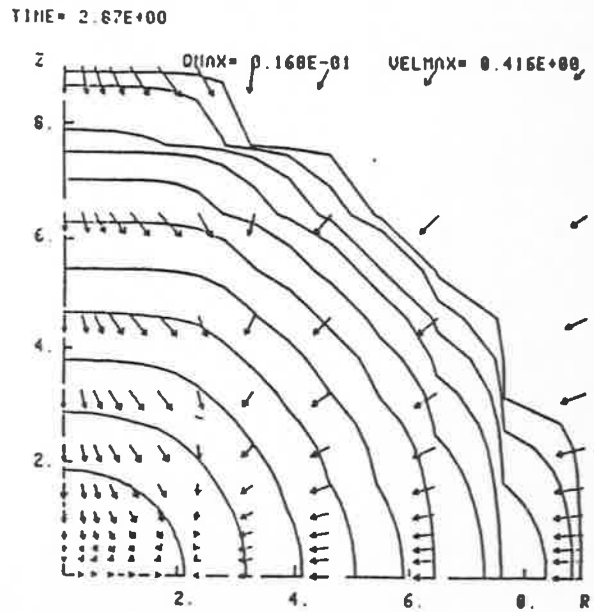


4b



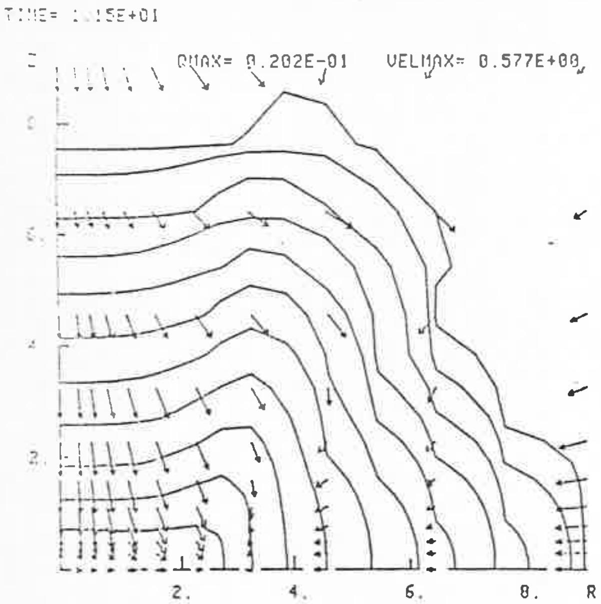
4d

6

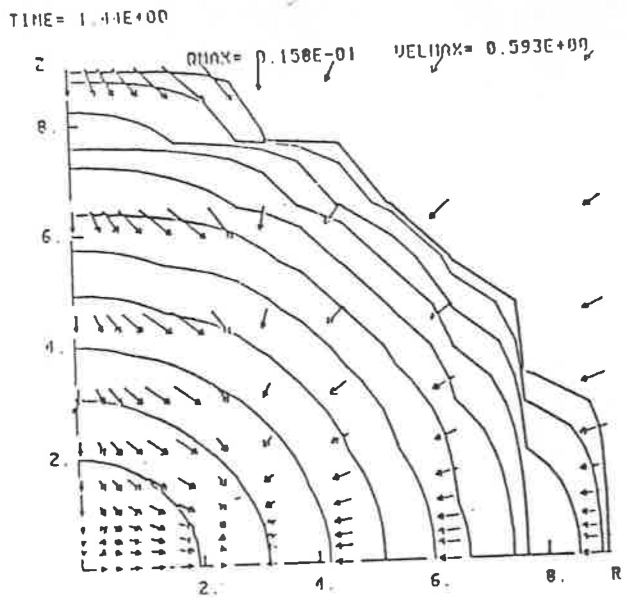


6a

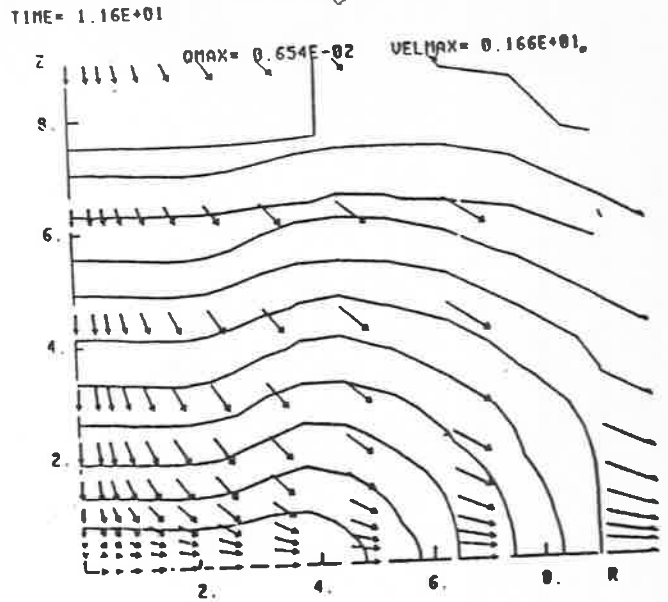
6b

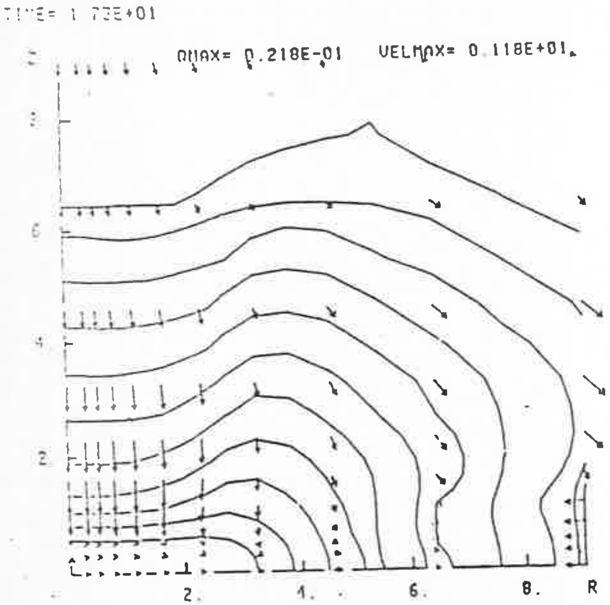


7a



7b





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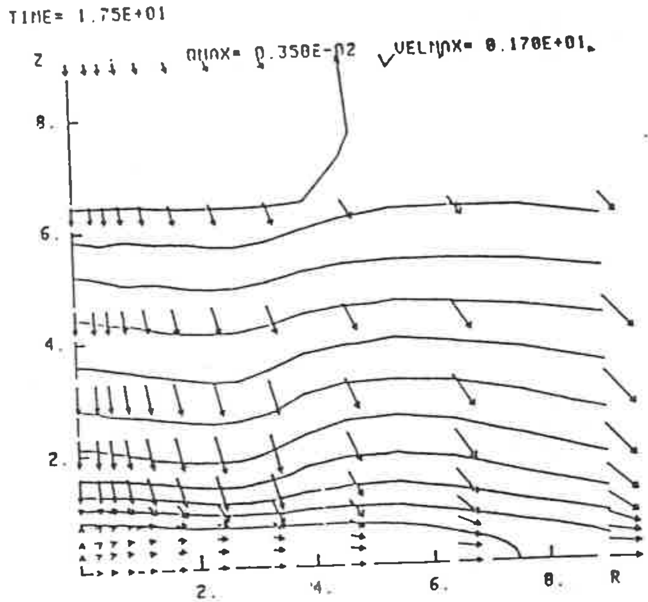

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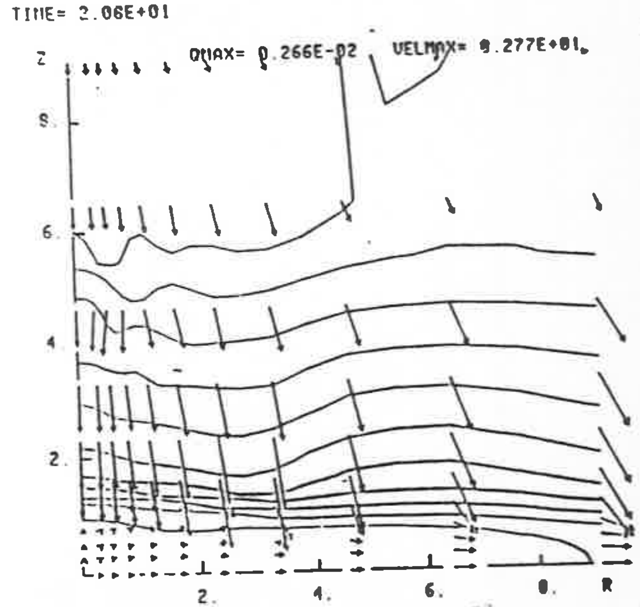
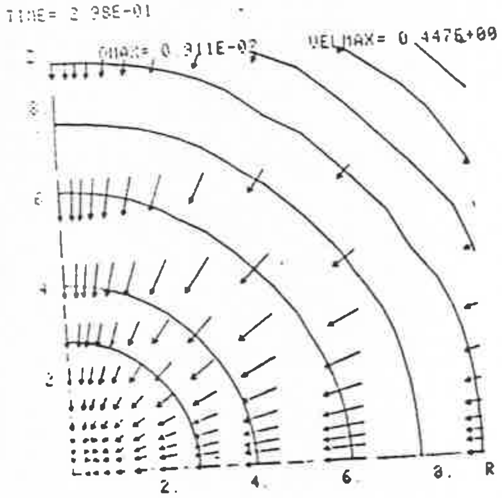


7d

9a →



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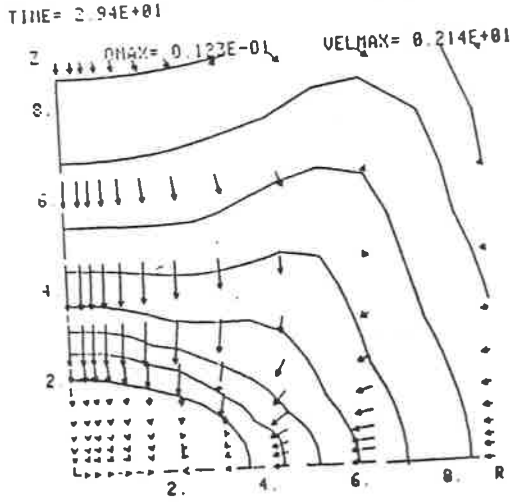

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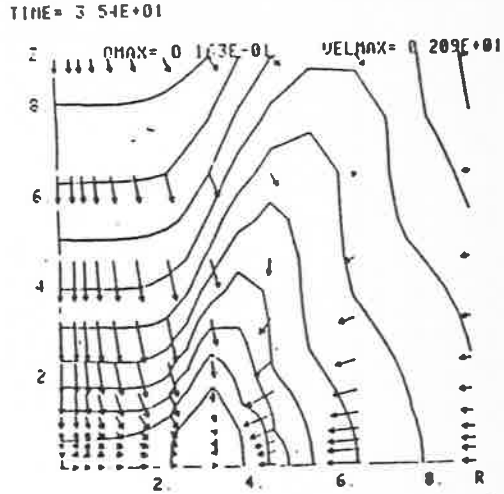
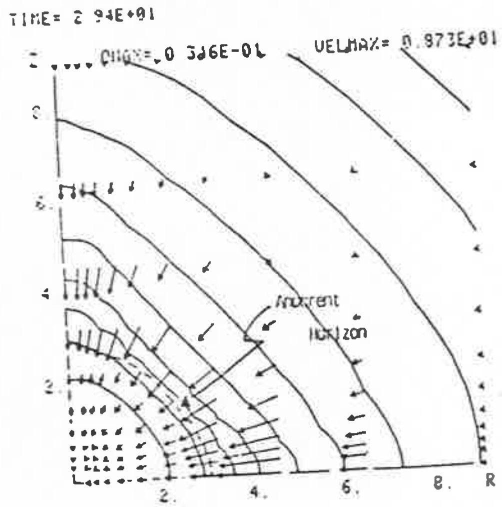
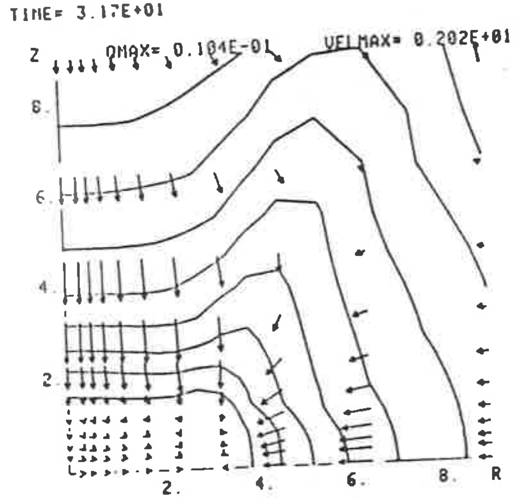




9b ↑



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8 ↙

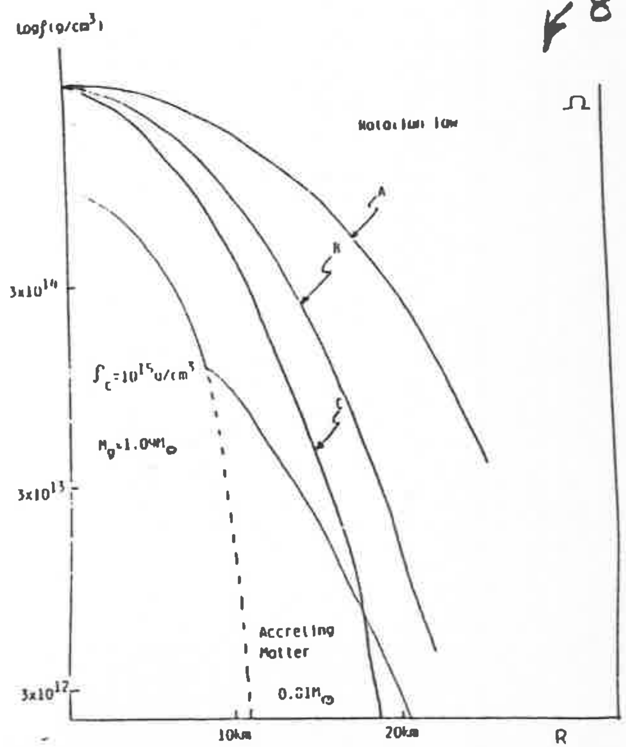

FIGURE CAPTIONS :

Fig.1 : Standard viewpoint of gravitational collapse of a rotating star to a blackhole (adopted from ref. Penrose, r., 1973, IAU Symp. 64, 82).

Fig.2 : Contour lines of Qb at t = 0.287 for A50 model. Each line corresponds to Qb = Qmax $10^{-n/2}$ for n = 1, 2, 3, ...11 (for details see ref. Nakamura 1984). The dashed line shows the apparent horizon.

Fig.2b: The contour lines of Qb for A50 at t = 18.6. The notations are the same as fig 2a. The dashed line shows the apparent horizon (see Nakamura 1984).

Fig.3a: The contour lines of Qb for A105 at t = 11.5. The The notations are the same as fig.2a

Fig.3b: The contour lines of Qb for A105 at t = 17.0. Notations are the same as fig.2b. (see ref. Nakamura 1984)

Fig.4a - 4d : The contour lines of Qb for A146 model at t = 5.76, 18.8, 23.2 and 25.3, respectively (ref. Nakamura 1984).

Fig.5 : The contour lines of Qb for B51 model. The notations are same as fig.2b (ref.Nakamura 1984)

Fig.6a - 6c : The contour lines of Qb for B92 model at various time t = 2.87, 11.5, 17.3, respectively. (see ref. Nakamura 1984)

Fig.7a - 7d : The contour lines of Qb for model B 143 at t = 1.44, 11.6, 17.5, 20.8, respectively (ref. Nakamura 1984)

Fig.8 : The density distribution and the rotation law of accreting neutron stars with rotation. The dashed line shows a neutron star model for $\rho_c = 10^{15}$ g/cm^3 .

Accreting matter is added to this neutron star. Solid lines with letters A, B, and C show the adopted rotation law (ref. Nakamura 1984).

Fig.9a: The contour lines of Qb for A07,

9b: The dashed line shows an apparent horizon (see Nakamura 1984)

Fig.10. The contour lines of Qb for A97 (Nakamura 1984)

Fig.11 : The contour lines of Qb for B97 (see Nakamura 1984)

Fig.12 :The contour lines of Qb for C93 (ref. Nakamura 1984)

Fig.13 : Imagined turning points for emission of Gravity Wave from the collapse of a rotating core.