AN EMPIRICAL APPROACH TO COSMOLOGY

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(Received 23 May, 1980)

Abstract. A two-component model of the universe is proposed, based on the observations of discrete extragalactic sources and the microwave background radiation. The large scale dynamics of the universe is determined by the radiation component and it leads to a characteristic size of the universe $\sim 6 \times 10^5$ Mpc and an age $\sim 10^{12}$ yr. The second component, that of matter, occurs in discrete sources which group together in super-superclusters of characteristic size $\sim 6 \times 10^3$ Mpc and age $\sim 10^{10}$ yr. It is suggested that our Galaxy belongs to one of these super-superclusters and that observations of discrete sources are confined to this unit. A reasonable agreement with the cosmological tests is obtained on the assumption that the geometry within a typical super-supercluster is Euclidean and that the redshifts of galaxies arise from Doppler effect due to motions originating in a local explosion which gave birth to the super-supercluster. Further observational checks on this model are proposed.

1. Introduction

The early success of Friedmann cosmology in predicting the phenomenon of the nebular redshift and the Hubble law went a long way towards establishing general relativity as *the* proper theory for describing the large-scale structure of the universe. The discovery of the microwave background radiation and the subsequent confirmation of its near-Planckian spectrum have given a further boost to the belief in the overall validity of relativistic cosmology. Indeed, in spite of its difficulties and imperfections, general relativity is the best working theory of gravity that we have today. Its successes in the experimental tests in the solar system and in the recent measurements of the binary pulsar PSR 1913+16 indicate the correctness of the theory at least in its weak field approximation. Nevertheless, on the cosmological front there are certain features which are disturbing in the context of the present-day observations of discrete sources.

First, the most striking feature of general relativity, that the distribution of matter in the universe determines the geometry of the large-scale structure of space-time, still remains to be demonstrated. It is this feature which was

* Operated by the Association of Universities for Research in Astronomy, Inc. under contract with the National Science Foundation

Astrophysics and Space Science 74 (1981) 111–129. 0004-640X/81/0741-0111$02.85.

emphasized by Einstein (1917) when, in his pioneering paper on relativistic cosmology, he proposed the model of a finite static universe. The spatial geometry of the Einstein universe is that of the surface of a hypersphere of radius R which is related to the mean matter density ρ by the simple relation

$$R = \frac{c}{\sqrt{4\pi G\rho}}. \tag{1}$$

In the dynamic Friedmann universe a corresponding critical role is played by the so-called closure density

$$\rho_c = \frac{3H_0^2}{8\pi G}, \tag{2}$$

where H_0 = Hubble's constant. For $\rho > \rho_c$ the geometry of space is closed with the same topology as in the Einstein universe, while for $\rho \leqslant \rho_c$ the space geometry is open. *For $H_0 = 50\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$, $\rho_c \cong 5\times 10^{-30}\ \mathrm{g\ cm^{-3}}$. A striking demonstration of Einstein's expectation that 'matter closes space' would have been the observation of a mean matter density $\rho \gtrsim \rho_c$. Yet, observations to date have failed to reveal the existence of matter density more than a few percent of ρ_c.

Secondly, all observational attempts to ascertain the non-Euclidean nature of the geometry of the universe have so far proved inconclusive. The early attempt of Hubble (1936) to look for non-Euclidean effects in the number counts of galaxies had to be abandoned, as it turned out to be impractical. The more recent tests using optical as well as radio astronomy, although claiming to have probed the universe much deeper than Hubble did, have not succeeded in solving this fundamental problem. In the redshift-magnitude relation the uncertain evolutionary corrections may completely mask the geometrical differences between the various cosmological models. A very curious situation exists with regard to the radio source counts. The observed $\log N$–$\log S$ relation except at the brighter end is very nearly what one would expect in a static Euclidean universe. To explain this relation in a typical non-Euclidean universe of relativistic cosmology it is necessary to postulate evolutionary effects which just about cancel the non-Euclidean effects (Kellermann, 1977). A similar contrived cancellation has to be invoked to explain the lack of a minimum in the angular size-redshift relation of radio sources as predicted by the Friedmann cosmologies. Here it is assumed that the radio sources were progressively smaller (in linear size) in the past.

Our third and final point concerns the sudden change of approach encountered in going from extragalactic astronomy to cosmology. Astronomical objects (with the esoteric exceptions of black holes) are usually studied adequately without recourse to general relativity: and these span a range of length scale right from

* Throughout this work we will use this value of Hubble's constant.

that of the solar system to that of the clusters of galaxies. The change over at the next (and largest) length scale to a homogeneous and isotropic universe is drastic. The fundamental assumptions of cosmology, viz. the Weyl postulate and the cosmological principle are extrapolations based on the observations of our local neighbourhood, and are necessary if one is to make headway with equations of general relativity. There is a danger, however, that this oversimplified continuum (smooth dust) description may be counterproductive to our understanding of the discrete objects in the universe. The lack of a satisfactory theory of galaxy formation in a Friedmann universe bears testimony to this fact. Until the discrete objects are properly recognized in cosmology, that subject will continue to stand apart from the rest of astronomy.

Can we understand the present data on cosmology without having to use a sophisticated theory like general relativity, without having to put in space-time curvature effects, without the need for elaborate evolutionary schemes? In presenting an empirical model of the Universe in this paper we are guided by the considerations of Occam's razor. These considerations suggest a two-pronged approach in which the background radiation is treated relativistically, while the discrete objects are treated in the weak field approximation, i.e., with a combination of Newtonian gravity and special relativity. The reasons for this will become clear as we go into the details of the two approaches. We will consider the discrete objects first.

2. Super-Superclusters

We first consider the dynamics of the SSC and then describe the kinematical region of linear size $\sim cH_0^{-1}$. We imagine this region in the form of a super-supercluster (SSC in brief) which came into existence in á local explosion. The dynamics of an SSC do not have to be considered within the framework of general relativity. Because the present density of the discrete objects is very much less than the closure density, Newtonian gravity can be used with confidence. It is also worth emphasizing that since the SSC is a finite object our discussion does not suffer from the ambiguities associated with the Newtonian cosmology of an infinite universe (Milne and McCrea, 1934; McCrea, 1954; Layzer, 1954).

We first consider the dynamics of the SSC and then describe the kinematical effects within it.

(A) THE DYNAMICS OF A SUPER-SUPERCLUSTER

We imagine the SSC to arise from a finite explosion (a mini-bang) and suppose that the distribution of discrete objects (e.g., galaxies) in it simulates a homogeneous dust-ball. Taking a typical galaxy at a distance R from the centre and assuming spherical symmetry, the equation of motion of the Galaxy is given by

$$\ddot{R} = -\frac{GM(R)}{R^2}, \tag{3}$$

where $M(R)$ is the mass of the dust ball within a distance R from the centre. Let $\rho(t)$ be the density at epoch t, so that

$$M(R) = \frac{4\pi}{3}\rho(t)R^3 . \tag{4}$$

It is then easy to see that (3) can be rewritten in the form

$$\ddot{S}(t) = -\frac{4\pi G}{3}\rho(t)S(t), \tag{5}$$

with

$$R = rS(t), \quad r = \text{constant} . \tag{6}$$

The r-coordinate labels a typical galaxy at all epochs. We also have, by conservation of matter

$$\rho(t)S^3(t) = \text{constant} = \rho(t_0)S^3(t_0) = \rho_0 S_0^3 ; \tag{7}$$

t_0 denoting the present epoch. Equation (5) can be written in the form

$$q(t)H^2(t) = \frac{4\pi G\rho_0 S_0^3}{3S^3}, \tag{5}$$

where

$$H(t) \equiv \frac{\dot{S}}{S}, \qquad -qH^2 \equiv \frac{\ddot{S}}{S} . \tag{8}$$

At the present epoch we know from observations that

$$\rho_0 \equiv q(t_0) \cdot \frac{3H_0^2}{4\pi G} \ll 1 . \tag{9}$$

Thus it is a good approximation to have

$$\ddot{S} \approx 0, S(t) \propto t, \quad H(t) \cong 1/t; \tag{10}$$

That is, the expansion is *free*; the self gravity of the SSC is negligible. This approximation is a good one, not only at the present epoch but also at the past epochs down to which our observations of discrete objects have been made. We will accordingly assume Equation (10) to hold, and proceed to calculate the kinematics within the SSC.

(B) KINEMATICS

Subject to the above approximation (10) we can visualize the expanding SSC in the form of a velocity distribution function given by

$$f(\mathbf{V}, \mathbf{R}) \propto \delta_3(\mathbf{R} - \mathbf{V}t)\theta(V_0 - |\mathbf{V}|) . \tag{11}$$

Equation (11) expresses the fact that at $t = 0$ the explosion generated a uniform velocity distribution with an upper limit V_0 on velocity magnitude. At a typical time t, an object at $\mathbf{R}$ will have a (radial) velocity V relative to the centre. It is well known (cf. Bondi, 1960) that in Newtonian kinematics, the same radial velocity distribution is seen by any of the typical objects partaking the expansion of the SSC. We show below that the same conclusion holds even when the effects of special relativity are taken into consideration.

To see this, consider a typical object A having a speed V^A relative to the centre O. Without loss of generality take the z-axis along the direction of motion of A. Take another typical object B in the SSC with velocity $\mathbf{V}^B$ in a direction making an angle θ with OZ. Take OX in the plane of AOB in the direction perpendicular to OZ and choose OY to make up a cartesian system of axes at O. Choose a parallel set of axes (AX^A, AY^A, AZ^A) at A, and denote the space coordinates measured by A as (x^A, y^A, z^A). Let t^A denote the time in the inertial frame moving with A. We then have the following relations connecting the coordinate systems at O and A:

$$\left.\begin{aligned} &z^A = \gamma_A(z - V^A t) \qquad t^A = \gamma_A\left(t - \frac{V^A z}{c^2}\right) \\ &x^A = x, \qquad y^A = y, \qquad \gamma_A = \left[1 - \frac{(V^A)^2}{c^2}\right]^{-1/2} \end{aligned}\right\} \tag{12}$$

and their inverse relations. What is the velocity of B as seen by A? In the Lorentz frame of O this velocity is ($V^B \sin\theta$, 0, $V^B\cos\theta$). In the Lorentz frame of A this transforms to

$$\mathbf{V}^{BA} = \left(\frac{V^B \sin\theta}{1 - (V^B V^A/c^2)\cos\theta}\,\gamma^{A^{-1}}, 0, \frac{V^B\cos\theta - V^A}{1 - (V^B V^A/c^2)\cos\theta}\right). \tag{13}$$

The radial coordinates of B relative to A are given by $\mathbf{R}^{BA} \equiv (x^{BA}, y^{BA}, z^{BA})$. At time t

$$x^{BA} = V^B \sin\theta \cdot t, \qquad y^{BA} = 0\,, \qquad z^{BA} = (V^B\cos\theta - V^A)\gamma^A t\,. \tag{14}$$

Notice first that as seen by A, the velocity of B is radial and given by the *same* velocity distance relation as that given in the rest frame of the centre:

$$\mathbf{V}^{BA} = \frac{1}{\gamma^A\left(1 - \dfrac{V^A V^B\cos\theta}{c^2}\right)t}\,\mathbf{R}^{BA} = \frac{1}{t^A}\,\mathbf{R}^{BA}\,. \tag{15}$$

There is also a simple relation connecting the γ-factors of A and B:

$$\gamma^A\gamma^B\left(1 - \frac{V^A V^B\cos\theta}{c^2}\right) = \gamma^{BA}\,. \tag{16}$$

Using (13) and (14) we can now transform the delta distribution function (11). We get

$$\begin{aligned} \delta(\mathbf{R} - \mathbf{V}^B t) &= \left[\gamma^A\left(1 - \frac{V^A V^B\cos\theta}{c^2}\right)\right]^{-1} \cdot \delta(\mathbf{R}^{BA} - \mathbf{V}^{BA} t^A) \\ &= \frac{\gamma^B}{\gamma^{BA}} \cdot \delta(\mathbf{R}^{BA} - \mathbf{V}^{BA} t^A)\,, \end{aligned}$$

i.e.,

$$(\gamma^{\mathrm{B}})^{-1}\cdot\delta(\mathbf{R}-\mathbf{V}^{\mathrm{B}}t)=(\gamma^{\mathrm{BA}})^{-1}\cdot\delta(\mathbf{R}^{\mathrm{BA}}-\mathbf{V}^{\mathrm{BA}}t^{\mathrm{A}})\,. \tag{17}$$

In this form the distribution function is manifestly symmetric between any pair of observers.

Equation (17) implies that an observer located off-centre also sees a radially expanding distribution of matter with a uniform density, just as an observer at the centre would. We will use these results to examine what would be the observable quantities as seen from our Galaxy, located not necessarily at the centre of the SSC. We will henceforth ignore the fact that we are located off-centre, except that we will not suppose ourselves to be located too near the edge. The 'edge effects' will be considered separately later.

(C) OBSERVABLE COSMOLOGICAL PARAMETERS

Having established that we can view the problem as if we were located at the centre of expansion, we can use the treatment given by Strittmatter (cf. Burbidge and Burbidge, 1967) in connection with the local hypothesis of QSOs coming out of an explosion in our Galaxy (Terrell, 1964).

Let V be the radial velocity of the object which is viewed at the present epoch t_0 by us. The Doppler redshift of the object is given by

$$1+z=\sqrt{\frac{1+(V/c)}{1-(V/c)}}\,. \tag{18}$$

The light received by us at time $t=t_0$ must have left the object at time $t=t_e$ where

$$t_e=t_0\left(1+\frac{V}{c}\right)^{-1}. \tag{19}$$

The distance of the object at time $t=t_e$ (from us) was

$$D=Vt_e\,. \tag{20}$$

What does the emitter see in its frame of reference? Let t'_e be the time of emission by its watch (which was synchronized to register zero time at the mini-bang) and let t'_r be the time of reception of the signal by us in the rest frame of the emitter. Then in the emitter's rest frame light has travelled the distance

$$D'=c(t'_r-t'_e)\,. \tag{21}$$

To calculate t'_r and t'_e we note that the Lorentz transformation connecting our rest frame to the rest frame of the emitter (assumed to move in the z-direction) is given by

$$\left.\begin{aligned}&z'=\gamma(z-Vt)\,,\quad t'=\gamma\left(t-\frac{Vz}{c^2}\right),\quad x'=x\,,\quad y'=y\,,\\&\gamma=\left(1-\frac{V^2}{c^2}\right)^{-1/2}.\end{aligned}\right\} \tag{22}$$

At emission ($t = t_e$), the emitter had the z-coordinate $= Vt_e$. At reception the receiver (i.e., ourselves) had coordinates $t = t_0$, $z = 0$. Using (18)–(22) we get

$$D' = D(1+z)\,. \tag{23}$$

Suppose the emitter has the bolometric luminosity L and an intensity distribution function normalized to unity as $I(\nu)$. Then the amount of energy emitted in the frequency range ν, $\nu + d\nu$ is given, in the rest frame of the emitter, by

$$LI(\nu)\,\mathrm{d}\nu\,. \tag{24}$$

At $t = t_0$ this energy is redshifted and distributed over a sphere of surface area $4\pi D'^2$. Hence the apparent luminosity in the frequency range (ν, $\nu + \mathrm{d}\nu$) is given by $l(\nu)\,\mathrm{d}\nu$ at our receiving end, where

$$l(\nu) = \frac{LI(\nu\cdot\overline{1+z})}{4\pi D'^2(1+z)} = \frac{LI(\nu\cdot\overline{1+z})}{4\pi D^2(1+z)^3}\,. \tag{25}$$

However, using (18), (19) and (20) we get

$$D = \frac{z(1+(z/2))}{(1+z)^2}\,ct_0\,. \tag{26}$$

Hence, we get

$$l(\nu) = \frac{LI(\nu\cdot\overline{1+z})(1+z)}{4\pi c^2 t_0^2 z^2(1+(z/2))^2}\,. \tag{27}$$

This formula is the *same* as predicted by a Friedmann cosmology with $q_0 = 0$. The corresponding expression for the bolometric luminosity is of the form

$$l_{\mathrm{bol}} = \frac{L}{4\pi c^2 t_0^2 z^2(1+(z/2))^2}\,. \tag{28}$$

For a typical radio source Equation (28) can be translated into a power (per steradian)-flux density relation of the form

$$S = \frac{P(1+z)^{1-\alpha}}{c^2 t_0^2 z^2(1+(z/2))^2}\,, \tag{29}$$

where α is the spectral index in the relevant frequency of observation.

This similarity with the $q_0 = 0$ Friedmann model is hardly surprising when one notes that this Friedmann space–time is actually flat and can be transformed to the Minkowski form. We emphasize, however, that unlike the Friedmann model, this model of the SST relates to a compact system for which $q_0 = 0$ is only an approximation to $q_0 \ll 1$. The above treatment based on Newtonian gravity + special relativity will give a good approximation for $q_0 \ll 1$, although the corresponding Friedmann model ($q_0 \neq 0$) cannot be described by a flat space–time. We now apply these results to cosmological observations.

3. Observations of Discrete Sources

In the comparison of the predictions of this theory with the available cosmological data, two main points of difference will be made, from a similar comparison between observations and standard cosmology. The first point is that there are no curvature effects in this simple theory; whatever effects are noticed arise from the combination of Euclidean geometry with Doppler effect. Secondly, we will try to argue that the present data for discrete sources do not show any evolution. This is not to say that the theory precludes evolution: evolution can be accommodated if necessary. We are simply invoking Occam's razor to minimize the number of parameters in the theory.

(A) THE REDSHIFT-MAGNITUDE RELATION

The redshift magnitude relation corresponding to (28) is given by

$$m = 5 \log z + 5 \log(1 + (z/2)) + \text{constant}\,. \tag{30}$$

The effective Hubble constant is given by

$$H_0 = t_0^{-1}\,. \tag{31}$$

The relation (30) has to be corrected for the K-correction for the specific wavelength band used for observation. This is given by the general formula (27).

In Figure 1 we show the theoretical curve plotted against the data given by Kristian, *et al.* (1978). It is usual to invoke luminosity evolution and to argue that galaxies at larger redshifts were systematically brighter. In this way an argument is made to fit a $q_0 \approx 0$ model to a data plot which is best fitted to a linear Hubble law:

$$m = 5 \log z + \text{constant}\,. \tag{32}$$

In our present model it would be possible to argue in a similar way, since luminosity evolution is not ruled out. However, we question whether it is absolutely necessary. For example, the magnitude difference at $z = 0.5$ between (30) and (32) is ~ 0.5. The uncertainties of the K-correction and of the absolute magnitude of the standard candle are surely larger than this! Gunn and Oke (1975) have already pointed out the considerable uncertainty which surrounds a precise measurement of q_0.

The large scatter in the Hubble diagram of the QSOs is considerably larger than the magnitude difference between (30) and (32) even at redshift of $z \sim 3$. At $z \sim 3$, the magnitude difference is ~ 2 whereas the scatter in the magnitudes of the QSOs is $\gtrsim 5$ at typical redshifts. Clearly our simple model is not inconsistent with the Hubble diagrams of galaxies or QSOs. It is of course likely that the large scatter in the m–z diagram of QSOs arises because the redshifts are not of Doppler origin in this model.

(B) THE RADIO-SOURCE COUNT

In (29) we obtained the formula for the flux density of a radio source of spectral

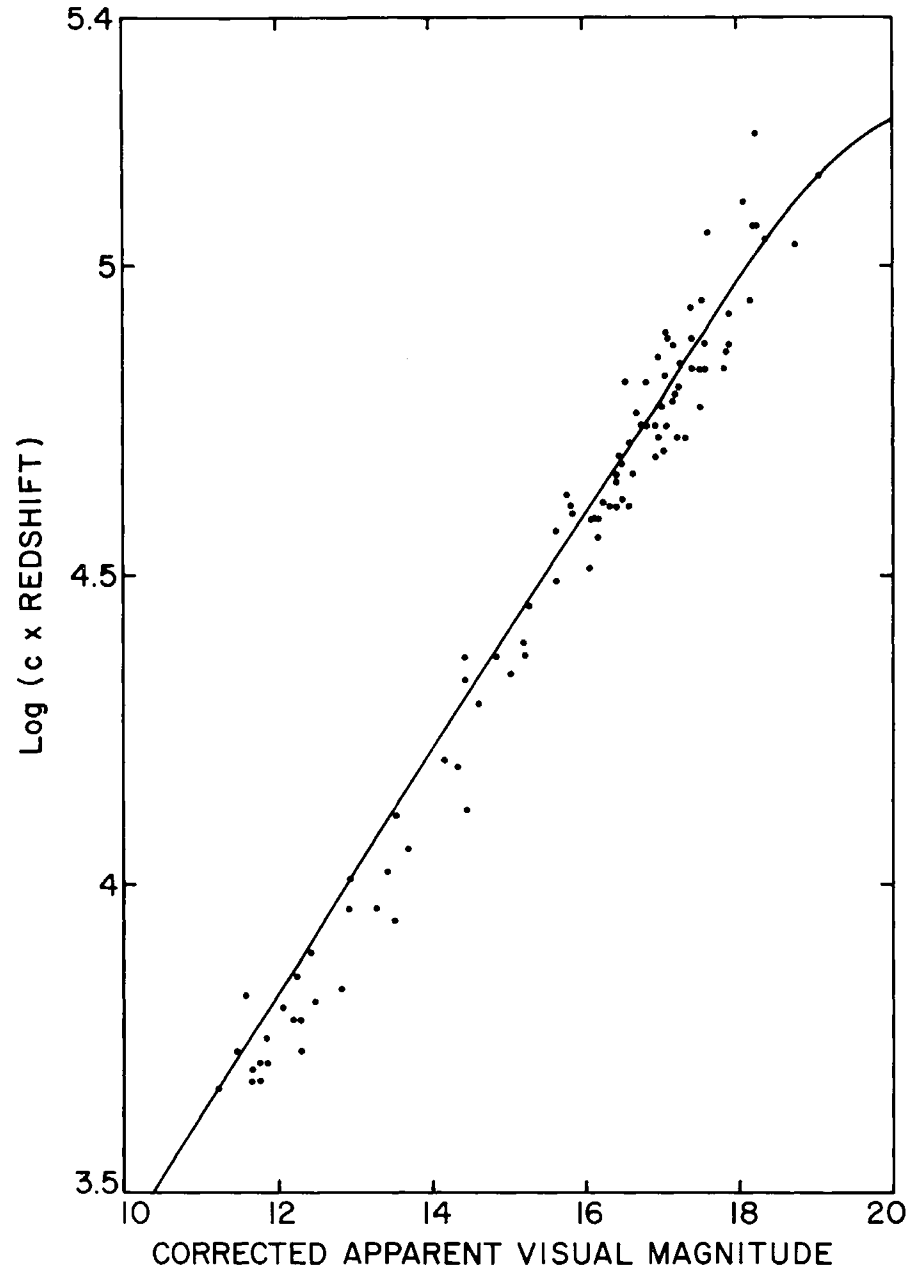

Fig. 1. The theoretical curve given by (30) plotted on the m–z diagram of Kristian *et al.* (1978), Tables 4 and 5. The constant in (30) has been determined by minimizing $\Sigma_i \{m_i - 5 \log z_i(1 + (z_i/2))\}^2$ for the data points.

index α. The radio-luminosity function (RLF) is inevitably brought into the calculation of the number N of radio sources down to a flux level S. The calculation in the present theory will go as follows.

With the uniform velocity distribution function (11), the number of objects with velocities between V and $V + \mathrm{d}\,V$ is given by $n(V)\,\mathrm{d}V$, where

$$n(V) \propto V^2 . \tag{33}$$

Let $f(P)\,\mathrm{d}P$ denote the fraction of radio sources with power in the range

$(P, P + \mathrm{d}P)$. We then get the required number N as

$$N(>S) \propto \int\int V^2 f(P)\,\mathrm{d}V\,\mathrm{d}P\,, \tag{34}$$

where the range of integration is covered by $P_{\min} \leqslant P \leqslant P_{\max}$, the range over which $f(P)$ is significant and by the values of V between O and that implied by the flux limit

$$\frac{P(1+z)^{1-\alpha}}{c^2 t_0^2 z^2 (1 + (z/2))^2} \geqslant S\,, \tag{35}$$

with z related to V by (18).

For steep spectrum sources we may approximate (35) by using $\alpha \approx 1$. We then get

$$N(>S) \propto \int_{P_{\min}}^{P_{\max}} f(P)(1 + ct_0\sqrt{S/P})^{-3}\,\mathrm{d}P\,. \tag{36}$$

The corresponding differential count goes as

$$\frac{\mathrm{d}N}{\mathrm{d}S} \propto S^{-1/2} \int_{P_{\min}}^{P_{\max}} \frac{1}{\sqrt{P}} f(P)(1 + ct_0\sqrt{S/P})^{-4}\,\mathrm{d}P\,. \tag{37}$$

At large flux densities ($S \gg P_{\max}/c^2 t_0^2$), we approximate to Euclidean counts $N \simeq N_0$: i.e.,

$$N_0(>S) \propto S^{-3/2}\,, \qquad \frac{\mathrm{d}N}{\mathrm{d}S} \propto S^{-5/2}. \tag{38}$$

Over limited ranges of power the RLF can be approximated by the power law

$$f(P) \propto P^{-\gamma}\,. \tag{39}$$

The larger the index γ the less effective are the more powerful sources ($P \approx P_{\max}$) in controlling N. The effective $P_{\max}$ is thus reduced and the Euclidean result gives a good approximation down to lower values of S. Nevertheless, subsequently N/N_0 will fall to values significantly below 1.

In Figure 2 we show the differential source counts at 2700 MHz and at 5000 MHz against (37) for the luminosity function determined by Meier *et al.* (1979) with $\gamma \simeq 2.1$ over the range $24.6 < \log 4\pi P < 27$ at 408 MHz. The above qualitative remarks are well illustrated. The steeper luminosity functions ($\gamma =$ 2.5, 3) give a better fit than the flatter one ($\gamma = 2$).

With regard to the cosmological significance of the radio source count, the following points can be made (some of these have been made earlier in different contexts):

(i) In order to test the source count against any cosmology the RLF must be determined independently of the cosmological models. This has not been possible so far (Fanti and Perola, 1977). At the high-P end (powerful sources) it is usually necessary to invoke a cosmological theory and γ is

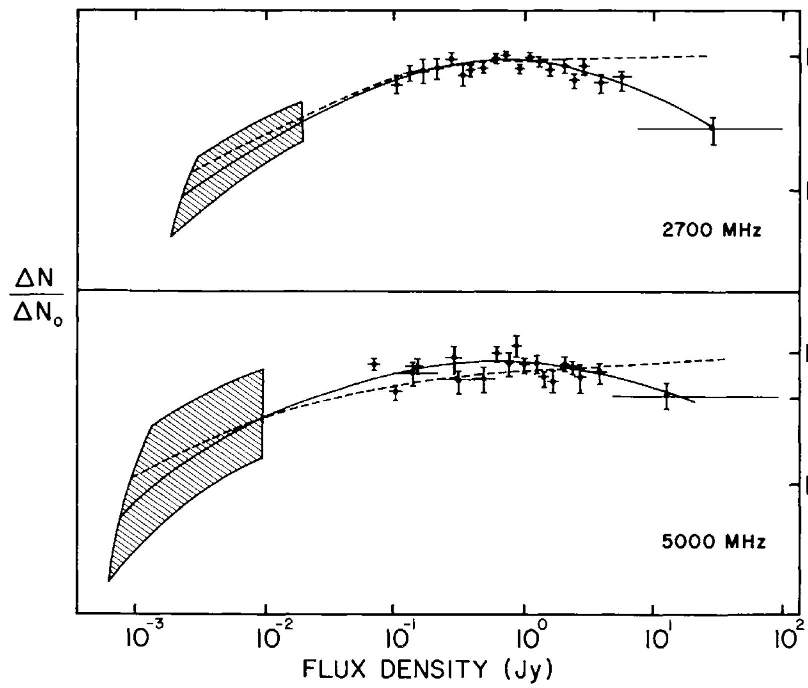

Fig. 2. The differential source counts at 2700 MHz and 5000 MHz. The data points and the continuous curves through them are as given by Wall and Cooke (1975). The dotted curves are theoretical ones drawn on the basis of the formulae given in the text.

determined with some evolutionary model in mind. This procedure is a circular one and robs the test of its main thrust.

(ii) It can be rightly argued that (36) and (37) can never give super-Euclidean counts which are found at high flux ends, especially in low frequency surveys. Taken at a face value these surveys require either some kind of evolution or a local hole. Although either possibility is not ruled out in our model the present data cannot distinguish between the two.

(iii) If the data at high flux end, e.g. in the 3CR survey, are further examined (Burbidge and Narlikar, 1976) it turns out that there are three types of sources. Of these the radio galaxies do show a sub-Euclidean slope. The slope for quasars is near-Euclidean, although its cosmological significance is in doubt so long as the interpretation of QSO redshift is a subject of controversy. The third class of optically unidentified objects (in the 3CR catalogue) shows a very steep slope ($d \log N/d \log S \sim -2.5$). The nature and significance of these objects is unclear in the usual context. If these objects are very powerful and distant, they could belong to another SSC which is our nearest neighbour. We shall return to this point towards the end of this paper.

(C) NUMBER COUNT OF GALAXIES

This test of the non-Euclidean geometry of the universe was attempted by Hubble in 1936 and later abandoned as impractical. Recently a number of deep galaxy samples have become available going down to $M > 22$ (Kron, 1979; Tyson and Jarvis, 1979) which enable one to devise a meaningful test of the number count in the optical part of the spectrum. As in the radio source count the optical luminosity function (OLF) plays an important role here. Felten (1977) has reviewed the recent investigations of OLF. Although a few differences exist in the works of different groups, for our purpose the form suggested by Schechter (1976) is sufficient: namely,

$$f(L) = \phi\left(\frac{L^*}{L}\right)^{5/4} \exp\left(-\frac{L}{L^*}\right) L^{*-1}, \tag{40}$$

where $f(L)\,\mathrm{d}L$ is the number of galaxies in the luminosity range $(L, L + \mathrm{d}L)$ per unit volume, expressed in Mpc^3. The constant luminosity L^* corresponds to an absolute magnitude -20.6 and $\phi = 0.005$. As in the case of radio sources, we have corresponding to (36) the formula

$$N(<m) \propto \int_0^\infty \frac{x^{1/4}\,\mathrm{e}^{-x}\,\mathrm{d}x}{[x^{1/2} + \mathrm{dex}\,(4.658 - (m/5))]^3}, \tag{41}$$

in which N stands for the number of galaxies with apparent magnitude less than m. For small m, we recover the Euclidean result

$$\log N(<m) = 0.6m + \text{constant}. \tag{42}$$

Tyson and Jarvis have claimed that outside the local supercluster, over a magnitude range 17–24 the $N(m)$ relation satisfies the linear law

$$\log N = 0.41m + \text{constant}. \tag{43}$$

In Figure 3 we show the $N(m)$ curve given by (41) with the data points of Tyson and Jarvis. The continuous curve shows the uncorrected curve as given by (21) while the dotted curves incorporate the range of possible K-corrections. In spite of the recent work on ultraviolet spectroscopy of galaxies (Code and Welch, 1979) the nature of K-correction is still highly uncertain, especially at redshifts in the range $z \geqslant 0.5$ which may dominate the $N(m)$ curve. Although the agreement between the prediction of our model and the observed curve is reasonably good, the uncertain nature of the K-corrections should not be ignored.

Kron's recent survey (op. cit.) suggests that there is divergence between different galaxy counts at faint magnitudes. Within the observed scatter of observed counts it is possible to accommodate non-evolutionary as well as the evolutionary models. But there is certainly no need to invoke an evolutionary model at present.

(D) ANGULAR SIZES

Another startling effect of the non-Euclidean geometry, which was looked for and not found, is the expected upturn in the angular size redshift relation of radio sources. In most Friedmann cosmologies the departure from the Euclidean

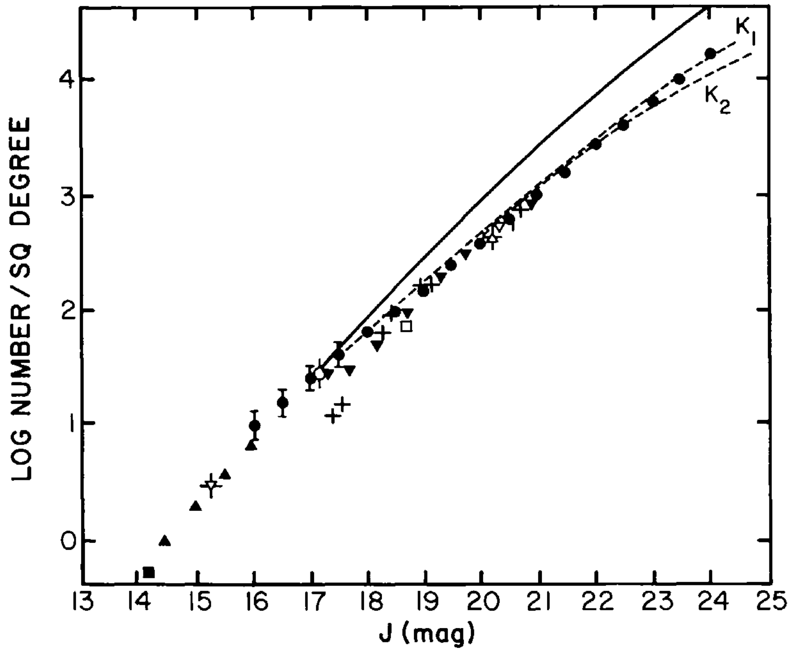

Fig. 3. The number magnitude relation. The data points are taken from the compilation of Tyson and Jarvis (1979). The uncorrected theoretical N–m relation is shown by the continuous curve. The dotted curves arise from the application of the K-correction. The two dotted curves K_1, K_2 reflect the range and uncertainty of the K-correction, as shown by Code and Welch (1979), for M31 and NGC 4486. The conversion of the Code and Welch K-correction to J-magnitude has been done on the basis of the linear extrapolation given by Kron (1978).

result becomes significant around the redshift $z \sim 1$. The data of Wardle and Miley (1974) giving angular sizes and redshifts of radio sources show an enormous scatter. The scatter in the data is due to the scatter in the linear sizes of the sources and to projection effects. The present model predicts a relation of the type

$$\alpha = \frac{dH_0(1+z)^2}{cz(1+(z/2))} = \alpha_0 \frac{(1+z)^2}{1+(z/2)}, \tag{44}$$

where d = linear size and α = angular size. α_0 is the size predicted by a Euclidean geometry (without redshift corrections):

$$\alpha_0 = \frac{dH_0}{cz}. \tag{45}$$

Note that (44) does not show an upturn; $\alpha \to 2dH_0/c$ as $z \to \infty$. It may be possible to reduce the scatter in the θ–z diagram by plotting a median value of the angular size against the redshift. Such a curve has been drawn by Katgert-Merkelijn *et al.* (1979) and its agreement with (44) is reasonably good as shown in Figure 4. However, completeness of the sample must be guaranteed before the median value can be trusted.

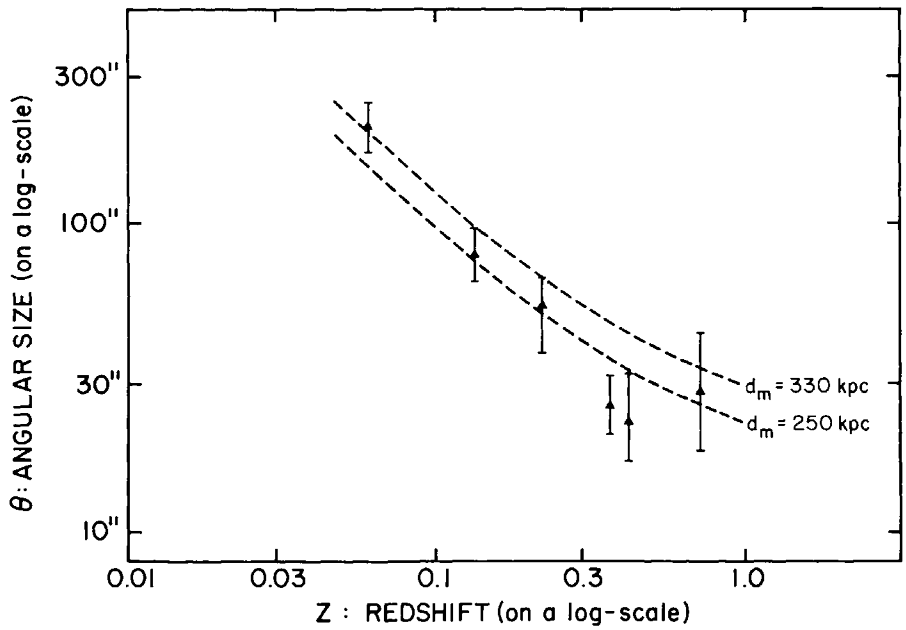

Fig. 4. The angular size-redshift relation for median angular sizes. The data points are taken over from Figure 9 of Katgert-Merkelijn *et al.* (1979). We have not included the last point of their plot as it refers to unidentified radio sources (empty fields). The largest redshift point in the remaining plot is from Laing *et al.* (1978). The dotted curves are for our model with characteristic (unevolving) median sizes of $d_m = 330$ Kpc and 250 Kpc. While comparing this figure with Figure 9 of Katgert-Merkelijn *et al.* (op. cit.) it must be noted that their value of H_0 is twice ours.

Finally we refer to the work of Dodd *et al.* (1975) who have plotted a number–angular size relation for 3000 galaxies up to $B \equiv 23^m$. The expected redshifts do not exceed 0.5 and so the test is not able to make a strong distinction between the evolutionary and non-evolutionary theories. The present model is, therefore, consistent with this data.

4. Relation to the Cosmic Microwave Background

The above discussion of discrete objects in the universe suggests that so far as the present data are concerned a finite non-evolving, expanding blob of discrete objects having a Euclidean geometry together with Doppler redshift can adequately represent the universe. There is, however, another important observation which is still to be taken note of, viz. the microwave background.

The observation of the microwave background radiation is different from the observation of discrete sources in that so far the best interpretation of its existence can be given in terms of the big bang. This is the only observation which (for lack of any other satisfactory explanations) forces us to consider the state of the universe radically different from what it is today. None of the observations of discrete sources (which go up to at most $z \sim 3.5$ even if QSO redshifts are cosmological) lead us to think of a universe which was radically different from today—as we saw in the previous section. The conventional big bang interpretation of the microwave background takes us back to past epochs of redshifts as high as ~ 1000. A considerable gap exists therefore between the bits of information about the past history of the universe from these two components, and this leads us to view the two entities – the background radiation and the discrete objects – separately. We feel that the former can be used (because of its direct connection with the big bang) to study the dynamics of the universe; whereas the latter have a relatively minor role in this connection. We now show how to construct a self-consistent picture out of such a hypothesis.

Using general relativity to study the dynamics of a radiation-dominated universe we immediately conclude that the expansion factor in the $k = 0$ Robertson-Walker case is given by

$$S(t) = (t/t^*)^{1/2}, \tag{46}$$

where S has been normalized to unity to the present epoch t^*. (This 'present' epoch t^* is not the same as t_0 which we considered in the context of the exploding SSC.) The radiation temperature in this universe is given by

$$\theta(t) = \left(\frac{3}{32\pi G a}\right)^{1/4} t^{-1/2} \cong \frac{1.5 \times 10^{10}}{t_{\mathrm{sec}}^{1/2}}\,\mathrm{K}, \tag{47}$$

where a is the radiation constant. The inclusion of neutrinos pair creations etc. (and other zero rest mass particles) in the early stages will modify the coefficient but we will ignore this small effect.

Setting $\theta = 2.7$ K then determines t_{sec} as

$$t_{\mathrm{sec}} = \left(\frac{1.5 \times 10^{10}}{2.7}\right)^2 \cong 3 \times 10^{19}, \tag{48}$$

corresponding to a Hubble constant of $1.6 \times 10^{-20}\,\mathrm{s}^{-1}$ and a distance scale of 2×10^{30} cm.

Note that this Hubble constant is not the one measured by the redshift-magnitude test. That test refers to discrete objects in the SSC which expands as a subunit of this larger scale (infinite) universe. Suppose there are N such SSCs in the distance scale of 2×10^{30} cm. If the smoothed out density of discrete objects in a SSC is $\sim 3 \times 10^{-31}\,\mathrm{g\,cm^{-3}}$, then the overall average matter density in the Universe is

$$\rho_m = \frac{N \times 3 \times 10^{-31}}{(100)^3} = N \times 3 \times 10^{-37}\,\mathrm{g\,cm^{-3}}. \tag{49}$$

Our assumption of a radiation dominated universe is valid provided ρ_m is less than the microwave background energy density

$$\rho_r \cong 10^{-34}\,\mathrm{g\,cm^{-3}}. \tag{50}$$

Thus our assumption is valid for as many as 100 SSCs in the characteristic size of the Universe.

Although the universe is radiation dominated right up to the present epoch, it was optically thick in the past, up to an epoch when its expansion factor was $\sim 10^6$ times smaller than at present. (In standard Friedmann cosmology this occurred up to a shrinkage factor of $\sim 10^3$).

The apparent discrepancy in the anisotropy of the microwave background and the anisotropy of the local Hubble flow (Rubin-Ford effect) can be interpreted to mean that the SSC is moving against the microwave background at a speed of 817 km s^{-1} in the direction $b = 30°$, $l = 299°$.

A very serious problem, which has been pointed out to us by Dr S. O'Dell and also by other critics, is associated with the fact that the motion of our Galaxy with respect to the radiation background is so *small*. The motions in the SSC must range up to a significant fraction of c provided that the large redshifts seen in galaxies are due to expansion. Thus, most members of the SSC would see motion with respect to the radiation field which is very large. Only a very small fraction of the galaxies $\sim (V/c)^3$ would see the smaller motion measured in our case. This may pose to some readers a serious objection to the model.

5. Background Radiation from Discrete Sources

We consider here a possible consequence of a much enlarged universe on the radiation background from the discrete sources. Instead of a single observable universe of linear dimension $\sim cH_0^{-1}$ we now have several SSCs each of this size. Would discrete radiators in other SSCs besides ours also produce a large background of radiation in our neighbourhood? In particular we can address this question within the context of starlight, radio background and the X-ray (as well as γ-ray) background, all of which arise wholly or substantially from discrete sources. A simple argument given below will show that, so far as our local neighbourhood is concerned, the other SSCs produce a negligible effect.

To show this we consider the larger scale universe of size $\sim 2 \times 10^{30}$ cm. We may approximate each SSC as a point source of luminosity L in the specified wavelength region (i.e., optical, radio etc.) These SSCs are distributed in a universe whose space-time line element is given by

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - (t/t^*)[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)]\,. \tag{51}$$

The number of SSCs with radial coordinates between r and $r + \mathrm{d}r$ is given by

$$\mathrm{d}N = 4\pi r^2\,\mathrm{d}r \times n\,, \tag{52}$$

where n is the present proper number density of SSCs. The radiation leaving an SSC at coordinate r at time t will reach our SSC at $r = 0$ at time t_* provided

$$r = \int_t^{t_*} c\sqrt{\frac{t_*}{t}}\,\mathrm{d}t = 2ct_*\left(1 - \sqrt{\frac{t}{t_*}}\right) = \frac{2ct_* z}{1+z}, \tag{53}$$

where z is the redshift of the radiation as a result of passage through the space-time given by (51). Because of this redshift, the amount of radiation reaching us from the SSC is also reduced in intensity. The flux of radiation is given by

$$l=\frac{L}{4\pi r^2(1+z)^2}=\frac{L}{16\pi c^2 t_*^2 z^2}. \tag{54}$$

To find the total radiation-flux from all sources in a given solid angle $d\Omega$ around any direction we use (52), (53) and (54). The answer is $F\,d\Omega$ where

$$F=\int_0^\infty r^2\,dr nl=\int_0^\infty \frac{nL}{16\pi c^2 t_*^2}(2ct_*)^3\frac{dz}{(1+z)^4}=\frac{nL}{6\pi}ct_*. \tag{55}$$

A similar calculation for background generated by sources *within our own SSC* will give the answer as $F_0\,d\Omega$, where

$$F_0\sim\frac{3L}{4\pi}\left(\frac{c}{H_0}\right)^{-2}. \tag{56}$$

The ratio F/F_0 gives the relative importance of the background generated by all other SSCs (except ours), compared to that arising from sources in our SSC. We get

$$\frac{F}{F_0}=\tfrac{2}{9}n(ct_*)\left(\frac{c}{H_0}\right)^2=\tfrac{2}{9}n(ct_*)^3\left(\frac{1}{H_0t_*}\right)^2\sim N(H_0t_*)^{-2}; \tag{57}$$

where N is the total number of SSCs in a characteristic size ct_* of the Universe, a number which we estimated to be $\sim10^2$. The ratio $(H_0t_*)^{-2}$ is, however, 10^{-4}. Hence,

$$\frac{F}{F_0}\sim10^{-2}\ll1. \tag{58}$$

Thus, provided the other SSCs are statistically no different from ours, their contribution to the background (generated by discrete sources) in our locality will be negligible.

It is interesting to note that our choice of $N\leqslant10^2$ above was dictated by the overall consistency of the model. If N were considerably higher than this, our assumption of a radiation dominated universe with the line element given by (51) would not be valid. Thus, the inequality (58) follows from the requirement of overall consistency of the model.

6. Conclusion

To summarize, we have presented here a picture of the Universe whose dynamics is determined by the microwave background radiation. It is a universe $\sim10^{12}$ yr old since the big bang and has a characteristic linear dimension of 2×10^{30} cm. In it are super-superclusters of discrete objects like galaxies, radio sources etc. Our SSC is expanding as the result of an explosion $\sim10^{10}$ yr ago. A typical mini bang giving rise to an SSC describes matter creation in discrete form. The expansion of a typical SSC is free since the self gravity is negligible. All our tests of discrete objects are confined to one blob whose linear dimension is $\sim 10^{28}$ cm. These tests do not present any *prima facie* case for evolution and/or space-curvature. A Euclidean non-evolving expansion seems sufficient to explain the present observations within a blob. There may be as many as ~ 100 SSCs in the characteristic size of 2×10^{30} cm associated with the universe. The SSCs as a whole may move against the cosmic substratum as determined by the microwave background. The magnitude and direction of the Sun's velocity against the microwave background and against the local Hubble flow enables us to measure the velocity of our SSC against the microwave background.

We end by discussing possible future tests of this theory, especially of the finiteness of the SSC. For example, deep surveys, which will be possible with the space telescope, may show edge effects, such as a discontinuity in the distribution of discrete objects, as we look beyond our SSC into a neighbouring one. If the redshifts of QSOs are indeed due to the Doppler effect on the large scale, then we might consider the drop off in the density of QSOs beyond $z \sim 3$ as an indication of such a discontinuity. The unidentified radio sources could also belong to a neighbouring SSC, although for this hypothesis to be right they will have to be very powerful ($P > 10^{28}$ W Hz^{-1} Sr^{-1}). Galaxy counts going down to 28^m may also reveal anisotropies which would tell us how close we are to the edge of our SSC.

Acknowledgement

One of us (J. V. N.) gratefully acknowledges support from the Kitt Peak National Observatory which made this work possible.

References

Bondi, H.: 1960, *Cosmology*, University Press, Cambridge.
Brown, G. S.: 1974, Ph.D. Thesis, University of Texas at Austin.
Burbidge, G. R. and Burbidge, E. M.: 1967, *Quasi-Stellar Objects*, Freeman, San Francisco and London, p. 164.
Burbidge, G. R. and Narlikar, J. V.: 1976, *Astrophys. J.* **205**, 329.
Code, A. D. and Welch, G. A.: 1979, *Astrophys. J.* **228**, 95.
Dodd, R. J., Morgan, D. M., Nandy, K., Reddish, V. C. and Seddon, H.: 1975, *Monthly Notices Roy. Astron. Soc.* **171**, 329.
Einstein, A.: 1917 S. B., *Preuss. Akad. Wiss.*, 142.
Fanti, R. and Perola, G. C.: 1977, in D. L. Jauncey (ed.), 'Radio Astronomy and Cosmology', *IAU Symp.* **74**, D. Reidel Publ. Co., Dordrecht, p. 171.
Felten, J. E.: 1977, *Astrophys. J.* **82**, 861.
Gunn, J. E. and Oke J. B.: 1975, *Astrophys. J.* **195**, 255.
Hubble, E. P.: 1934, *Astrophys. J.* **79**, 8.
Hubble, E. P.: 1936, *Astrophys. J.* **84**, 517.
Katgert-Merkelijn, J., Lari, C. and Padrielli, L.: 1979, Preprint, Netherlands Foundation for Radio-Astronomy.
Kellermann, K.: 1977, in D. L. Jauncey (ed.), 'Radio Astronomy and Cosmology', *IAU Symp.* **74**, D. Reidel Publ. Co., Dordrecht, 80.

Kirshner, R. P., Oemler, A. and Schechter, P. L.: 1978, *Astrophys. J.* **83**, 1549.
Kristian, J., Sandage, A. and Westphal, J. A.: 1978, *Astrophys. J.* **221**, 383.
Kron, R. G.: 1978, 'Photometry of a Complete Sample of Faint Galaxies', Ph.D. Thesis, University of California, Berkeley.
Kron, R. G.: 1979, Preprint, Yerkes Observatory.
Laing, R. A., Longair, M. S., Riley, J. M., Kibblewhite, E. J. and Gunn, J. E.: 1978, *Monthly Notices Roy. Astron. Soc.* **183**, 547.
Layzer, D.: 1954, *Astrophys. J.* **59**, 268.
McCrea, W. H.: 1954, *Astrophys. J.* **60**, 271.
Meier, D. L., Ulrich, M. -H., Fanti, R., Gioia, I. and Lari, C.: 1979, *Astrophys. J.* **229**, 25.
Milne, E. A. and McCrea, W. H.: 1934, *Quart. J. Math. Oxford Ser.* **5**, 73.
Rainey, G. W.: 1977, Ph.D. Thesis, University of California at Los Angeles.
Reiz, A.: 1941, *Ann. Obs. Lund* **9**, 68.
Schechter, P.: 1976, *Astrophys. J.* **203**, 297.
Shane, C. D. and Wirtanen, C. A.: 1967, Lick Observatory Publication No. 22, Part 1.
Shapley, H.: 1957, *The Inner Metagalaxy*, Yale University Press, New Haven.
Terrell, J.: 1964, *Science* **145**, 918.
Tyson, J. A. and Jarvis, J. F.: 1979, *Astrophys. J.* **230**, L153.
Wall, J. V. and Cooke, D. J.: 1975, *Monthly Notices Roy. Astron. Soc.* **171**, 9.
Wardle, J. F. C. and Miley, G. K.: 1974, *Astron. Astrophys.* **30**, 305.
Zwicky, F.: 1961, *Catalogue of Galaxies and Clusters of Galaxies*, CIT Press, Pasadena.