Astronomy and Astrophysics (in press)

On the evolution of magnetic moment of pulsars

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The running head: U. S. Pandey & S. S. Prasad. On the evolution of magnetic moment of pulsars

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Proposed choice of journal: Main Journal/Letter

Proposed section: 6. Formation, structure and evolution of stars

Thesaurus code numbers: 08.16.6 - 08.13.1 - 08.05.3

XC Send proofs to: Dr. U. S. Pandey, Institut für Astrophysik der Universität Bonn, Auf dem Hügel 71, D-53121 Bonn, Germany

Abstract. We have derived an analytic equation for time evolution of inclination angle ξ between the magnetic moment and the spin axis of radio pulsars under the assumption of magnetic dipole braking model. We find that ξ is sensitive to the characteristic time t_c and the initial inclination angle ξ_0 for a magnetic dipole torque. We note the maximum effect of alignment torque for pulsars born as almost orthogonal rotator. Implications of the alignment scenario are discussed.

Keywords: 08.16.6 - 08.13.1 - 08.05.3

1. Introduction

Radio pulsars are the astronomical objects emitting primarily radio waves. Soon after their discovery, they were identified as rotating neutron stars having very high magnetic fields ($B \sim 10^{12} G$) . The stable radiation pulses are the result of stability of rotation of neutron star. It is under stood that coherent emission and directivity of observed pulsar radio emission is due mainly to the strong magnetic field. An important fact is that period P of almost all pulsars increase gradually. However, its rate of increment, dP/dt, ranges from 10^{-14} to 10^{-21} (Michel 1991 ; Beskin et al. 1993). Rate of increment for PSR 2127+11, found in globular cluster M15 , is observed to be negative although fairly small , i.e., $dP/dt = -(20 \pm 1) \times 10^{-18}$ (Wolszczan et al. 1989). The interpretation is that the centre of mass of neutron star is accelerated in the gravitational field of the cluster and above value of dP/dt results due to Doppler shift of this acceleration rather than the rotation acceleration of the pulsar itself.

For pure magnetic dipole radiation, one finds (Michel 1991) a value of $n_b = 3$ for braking index. Departures of braking index from the theoretical value may be interpreted to arise due to short term components of variations. Groth (1975) and Lyne et al. (1988) obtain values of $n_b = 2.515 \pm 0.005$ and $n_b = 2.509 \pm 0.001$ respectively for the Crab pulsar. It is believed that glitches , multipole electromagnetic radiation, pulsar wind, magnetic field decay, gravitational quadrupole radiation, neutrino emission and plate tectonics (see e.g. Alpar & Ögelman 1990; Ruderman 1991) cast doubts on slowing down of pulsars. Rate of change of slowing down $\ddot{P}$ is hard to determine because it may be mislead by those components of variation which are of the order of or larger than any secular component. Evolution of braking index with time has been noticed by Demianski & Proszynski(1983) for Crab pulsar (see also Lyne et al. 1988

). These studies lead to the analysis of the possibility of secular variation of magnetic field (Blandford & Romani 1988) of pulsars.

The purpose of the present communication is to discuss time evolution of inclination angle ξ between magnetic moment and spin axis alongwith its immediate consequences on other pulsar parameters. The alignment of magnetic moment with spin axis is an important question that needs to be answered. We assume magnetic dipole braking model. We find that there is considerable scatter in the angle of alignment of pulsars with initial inclination angles $0 < \xi_0 \leq 90^0$. We note the maximum effect of alignment torque for pulsars born as almost orthogonal rotator. We derive an analytic equation for time evolution of ξ in Sect. 2. The characteristic time t_c and inclination angle ξ are calculated for a number of plausible pulsar periods and initial inclination angles ξ_0 (see e.g. Table 1). Discussions regarding the implications of this analysis are given in the last section.

2. Variation of inclination angle

It has been established that magnetic torque due to an oblique rotator acts to slow down the rotation and also it tries to align magnetic moment with spin axis (Mestel 1968 ; Michel & Goldwire 1970 ; Davis & Goldstein 1970). However, Goldreich (1970) favours otherwise view regarding alignment. Good & Ng (1985) suggest two types of torques : the alignment torque and the counteralignment torque. Beskin et al. (1993) show that out of the two surface currents, Pedersen current $J_s^{(1)}$ along electric field E_s and Hall current $J_s^{(2)}$ orthogonal to it, only the former causes slowing down of pulsar rotation.

We consider a simple model resulting in dipole torques. Further complexities regarding magnetic field and surface currents might be added, though qualitative conclusions are expected to remain true in general. The kind of geometry we have in mind is as follows : the spin axis is along z, and x is perpendicular to z lying in the plane containing spin vector and the magnetic moment. The y axis is perpendicular to both. The resultant torque has a component opposite to the angular momentum vector that causes slowing down of the pulsar. There is also a component of torque that causes alignment of magnetic moment. Following Michel (1991), we write the components of stresses as

$$T_x = T\sin\xi\cos\xi \quad (1)$$

$$T_y = 0 \quad (2)$$

$$T_z = -T\sin^2\xi \quad (3)$$

where $T = 2\pi a^6 B^2 \Omega^3 / 3\mu_0 c^3$ with a = radius of neutron star, B = surface magnetic field, Ω = spin angular velocity, c = speed of light. There are two equations of motion (Michel & Goldwire 1970) expressed as

$$I(d\Omega/dt) = T_z \quad (4)$$

$$I\Omega(d\xi/dt = -T_x. \quad (5)$$

Differentiate Eq.(5) with respect to time and make use of Eq.(4) to get

$$d^2\xi/dt^2 = (T^2/I^2\Omega^2)\sin\xi\cos\xi. \quad (6)$$

A solution of Eq.(6) will provide the required variation of ξ. We integrate Eq.(6) with the initial condition, at $\xi = \xi_0$, $d\xi_0/dt = 0$ and get

$$(d\xi/dt)^2 = (T^2/2I^2\Omega^2)(\cos 2\xi_0 - \cos 2\xi) \quad (7)$$

In the derivation of Eq.(7), we have made use of the following integral of motion

$$\Omega^2 = \Omega_0^2 cos^2\xi_0/(1 - sin^2\xi_0 \exp(-\tau)) \quad (8)$$

for arbitrary ξ_0 and $\tau \equiv t/t_c$. The characteristic time t_c can be obtained through Eq.(4) as

$$t_c = \left((\Omega/2\dot{\Omega})\tan^2\xi\right)_{\xi=\xi_0} = \left(3\mu_0 I c^3/(4\pi a^6 B^2 \Omega^2 cos^2\xi)\right)_{\xi=\xi_0} \tag{9}$$

keeping in mind that at $\xi = \xi_0$, $\Omega = \Omega_0$. We further tabulate t_c (see Table 1) for 1 ms , 10 ms and 10^3 ms pulsar periods for a range of initial inclination angles ξ_0. In view of Eq.(8), further integration of Eq.(7) yields

$$\int d\xi/(cos2\xi_0 - cos2\xi)^{1/2} = -(T/\sqrt{2}I\Omega_0 cos\xi_0)\int(1 - sin^2\xi_0\exp(-\tau))^{1/2}dt \tag{10}$$

We simplify Eq.(10) using standard integrals and neglecting higher order terms. We get the following

$$\sin^2 2\xi(\cos 2\xi_0 - \cos 2\xi) = 2t_c^2 T^2 sin^4 2\xi_0/I^2\Omega^2 \tag{11}$$

which may be rewritten as

$$x^2 + 2(A-1)x - C = 0 \tag{12}$$

where $x \equiv \sin^2 2\xi, A \equiv cos2\xi_0$ and $C \equiv 4t_c^2 T^2 sin^4 2\xi_0/I^2\Omega^2$. Finally, the time derivative of ξ is expressed as

$$sin2\xi = \left(\left((cos2\xi_0 - 1)^2 + sin^4 2\xi_0/(1 - sin^2\xi_0 exp(-\tau))\right)^{1/2} - |cos2\xi_0 - 1|\right)^{1/2}. \tag{13}$$

Table 1 gives inclination angle ξ after a lapse of time t_c. It may be noted that the maximum effect of alignment torque is observed for pulsars born as almost orthogonal rotator (e.g., for $\xi_0 = 85°$, after time t_c, $\xi \sim 0.7°$, i.e. almost alignment). We have thus obtained a simple analytic equation which depicts the evolution of inclination angle and/or the magnetic moment of a pulsar. The inclusion of other torque compo nents (viz. magnetospheric torque, gravitational torque) might affect the analysis presented here. These effects should be taken into account as pointed out in Sect. 1 .

3. Discussions

We have expressed time evolution of spin frequency Ω through Eq.(8). We find that t_c and ξ_0 emerge as two fundamental parameters which determine time evolution of ξ. Figure 1 shows the variation of t_c with ξ_0 for three representative pulsar periods and magnetic fields. Old pulsars with less magnetic field ($B \sim 10^9 G$) thus harbour millisecond regime.

We notice another constant of motion, $\Omega \cos\xi = \Omega_0 \cos\xi_0$, which shows that evolution of pulsars from millisecond to second regime takes place provided they started their evolution as almost orthogonal rotator. Time elapsed since the Crab was observed is small compared to its charact eristic time. This gives $\xi \simeq \xi_0$. Using the relation (Davis & Goldstein 1970), $\sin\xi = exp(-\tau)$, we calculate t_c for the Crab. As t = 940yr(distance∼2 kpc), $\xi \sim 86°$, we obtain $t_c = 3.1 \times 10^6$yr. This is in agreement with the value of t_c derived from Eq.(9) with P=33 ms, $B = 3.7 \times 10^{13} G, a = 10^6$cm, and I = 10^{45}gm cm^2.

Alignment of magnetic dipole moment with the spin axis of pulsars has been considered by Jones (1976). It is found that after characteristic time t_c since the birth of a pulsar, alignment of magnetic moment by the electromagnetic torque may be identified as the decay of magnetic moment iniatially suggested by Lyne et al. (1975). Kundt (1981) has concluded that alignment of magnetic moment with spin axis of pulsars occurs until they reach the non-pulsing phase. An analysis by Lyne & Manchester(1988) suggests alignment of magnetic moment with increasing age of pulsars and they conclude that timescale for alignment is

$\sim 10^7$ yr similar to the field decay time determined by Lyne et al. (1985). Accordingly, magnetic field both aligns and decays on the above timescale at least for non-millisecond pulsars. More so, the beaming factor (especially for old pulsars) is significantly affected by magnetic alignment. Our analysis is expected to provide some important clues to the time evolution of magnetic moment and its consequent effect on other physical parameters , e.g. magnetic field, spin down torque, spin period and its derivative, pulsar age, beam structure and its evolution, beaming factor and luminosity of radio pulsars.

Acknowledgements. We are thankful to Professor Jayant V. Narlikar, Director, IUCAA (India) for awarding us the Associateship. Local hospitality and library facility utilised at IUCAA (Pune), India where this work was carried out are thankfully acknowledged. Our thanks are due to Dr. Oleg Doroshenko (MPIfR, Bonn) and Dr. Alexander Muslimov (IfA ,Univ.Bonn)for a critical reading of the manuscript and also many helpful suggestions.

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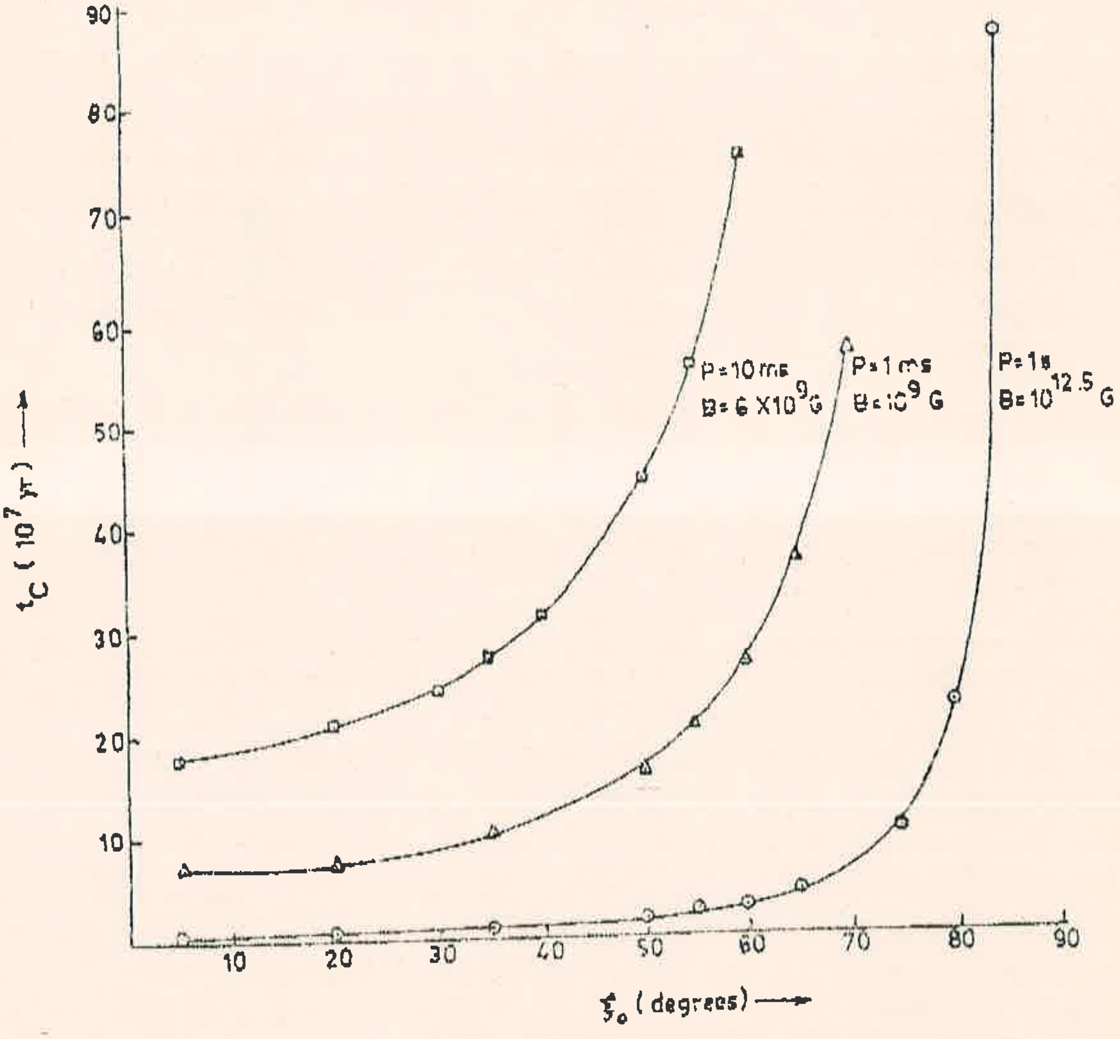
Figure caption

Fig. 1. Variation of characteristic time t_c with initial inclination angle ξ_0 for pulsars having different periods P and magnetic field $\mathbf{B}$

Table 1. Characteristic time t_c and inclination angle ξ for various ξ_0

		t_c (10^7 yrs) for various P(ms) and B(G)			
S. No.	ξ_0 (degrees)	P = 1 B = 10^9	P = 10 B = 6×10^9	P = 10^3 B = $10^{12.5}$	ξ (degrees)
1	05	07	18	0.7	3.9
2	10	07	19	0.7	7.7
3	15	07	19	0.7	11.4
4	20	07	21	0.7	14.7
5	25	08	22	0.8	17.5
6	30	09	24	0.9	21.5
7	35	10	27	1.0	23.7
8	40	11	31	1.1	24.9
9	45	13	36	1.3	24.9
10	50	16	44	1.6	23.4
11	55	20	55	2.0	20.8
12	60	26	72	2.6	17.3
13	65	36	101	3.6	13.4
14	70	56	154	5.6	9.4
15	75	97	269	9.7	5.7
16	80	216	600	21.6	2.7
17	85	855	2375	85.5	0.7


90
80
70
60
50
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30
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10
t_C (10^7 yr) ⟶
P=10 ms
B=6 X10^9 G
P=1 ms
B=10^9 G
P=1 s
B=$10^{12.5}$ G
10
20
30
40
50
60
70
80
90
ξ_o (degrees) ⟶