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Finite-time response of inertial and uniformly accelerated Unruh–DeWitt detectors

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Abstract. We study the response of inertial and uniformly accelerated Unruh–DeWitt detectors in the Minkowski vacuum when they are coupled to the quantum field for a finite time interval. A finite-time detector will respond even on an inertial trajectory due to transient effects. Also, the response will depend on the manner in which the detector is switched on and off. We study the response for smooth as well as abrupt switching of the detector. The detectors are switched on and off with window functions whose width, T, determines the effective timescale for which the detector is coupled to the field. We work out in detail the response of inertial and uniformly accelerated detectors for Gaussian, exponential and rectangular window functions and also obtain a general formula for the response of these detectors when a window function is specified. The $T \to 0$ and $T \to \infty$ limits are discussed in detail and several subtleties in the limiting procedure are clarified.

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1. Introduction

In the canonical formulation of quantum field theory in flat spacetime, we identify the coefficients of the positive frequency component of the normal modes of the quantum field to be the annihilation operators and define the vacuum state to be the state that is annihilated by these operators. Such a vacuum state is invariant only under action of the Poincaré group. Concepts such as vacuum, particles, etc, defined by conventional field theoretic methods do not, in general, possess a covariant meaning, but rather seem to have an observer-dependent quality about them. It is well known, for example, that quantization in Minkowski and Rindler coordinates are not equivalent [1]. Similar problems are encountered when quantum fields are studied in curved spacetimes.

The original motivation behind the idea of detectors [2, 3] was to improve our understanding of the concept of a particle in a curved spacetime. The philosophy being that, 'particles are what the particle detectors detect' [4]. With this motivation, the response of different types of detectors that are coupled to the quantum field has been studied in the literature (see, e.g., [5–7]). The response of these detectors has always been studied for their entire history, i.e. from the infinite past to the infinite future in the detector's proper time. But, in any realistic situation, the detectors can be kept switched on only for a finite period of time and, due to this reason, the study of the response of a detector for a finite interval in proper time becomes important.

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There is also another motivation for studying the response of finite-time detectors. Consider a detector that is coupled to the field in such a way that it responds to the energy–momentum content of the quantum field [7]. We can utilize this detector to analyse the back-reaction problem as follows. This detector can be set in motion on a certain trajectory, in the spacetime of our interest, and switched on for a finite proper time interval during its motion. The response of this finite-time detector will then reflect the energy–momentum content of the quantum field in that localized region of spacetime. (The detector has to be constructed so that it has only a small extension in space.) We can then attempt to relate the response of this detector to the term that is responsible for the back-reaction of the quantum field on the background metric in that localized region of spacetime (see also [8], pp 780 and 781 in this context).

There have been a couple of examples in the recent literature where the response of a detector has been evaluated for a finite time interval [9, 10]. The authors in [9] study the response of a Unruh–DeWitt detector that is turned on and off abruptly with the aid of a rectangular window function. They encounter certain ultraviolet divergences and resort to a regularization procedure to remove these divergences. In [10] the authors point out that no divergences arise in the detector response function when the Unruh–DeWitt detector is switched on and off with a smooth window function. They also show that in the limit when their window function matches a rectangular window function the ultraviolet divergences reported in [9] do appear in the detector response function.

We re-analyse this problem in this paper. We begin by noting that a detector which is kept on only for a finite time interval T will be affected by the transients related to the process of switching. This has the consequence that even an inertial detector in the Minkowski vacuum will respond if it is switched on for a finite T. This effect, as we shall see, needs to be clearly identified before one studies the response of an accelerated detector for finite T. Furthermore, we expect the response to vanish when $T \to 0$ for *any* realistic detector on *any* trajectory. This is simply a physical requirement arising from the demand that '*a detector which was never switched on should not detect anything*'. While this demand sounds reasonable, its mathematical implementation turns out to be fairly subtle. We will see that spurious results can arise if one does not implement the limiting procedure with care.

The response of a detector, in general, depends on the following three elements: (a) the state of the quantum field; (b) the trajectory of the detector; and (c) the nature of coupling that exists between the field and the detector. The quantum field we consider here is a massless, minimally coupled, scalar field and it is assumed to be in the Minkowski vacuum state. The detector we consider here is the Unruh–DeWitt detector and we study the response of this detector when it is in motion on inertial and uniformly accelerated trajectories.

This paper is organized as follows. In section 2 we briefly discuss some of the essential results from standard Unruh–DeWitt detector response theory and also comment on certain aspects of finite-time detection. In section 3 we study the response of the detector which is operational only for a finite interval of time, the cases of smooth window functions as well as that of abrupt switching are considered. In section 4 we discuss the conclusions that can be drawn from our analysis.

2. Unruh–DeWitt detector response theory—essential results

In this section, we gather some of the essential results from Unruh–DeWitt detector response theory and also point out certain features of finite-time detection.

The interaction of the Unruh–DeWitt detector with the scalar field, Φ, is described by the interaction Lagrangian $cm(\tau)\Phi(x)$, where c is a small coupling constant and $m(\tau)$ is the detector's monopole operator. The time evolution of $m(\tau)$, where τ is the proper time in the frame of the detector, is assumed to be $m(\tau) = \mathrm{e}^{\mathrm{i}H_0\tau}\, m(0)\, \mathrm{e}^{-\mathrm{i}H_0\tau}$, where H_0 is the Hamiltonian of the detector. Consider an Unruh–DeWitt detector in motion on a trajectory $x(\tau)$. Its amplitude for transition from a lower energy state $|E_0\rangle$ to a higher energy state $|E\rangle$, with energy eigenvalues E_0 and E, respectively, up to the first order in perturbation theory, is given by [11]

$$\mathcal{A}(\Omega) = \mathcal{M} \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\Omega\tau}\, \langle \Psi|\Phi[x(\tau)]|0_M\rangle \tag{1}$$

where $\Omega = (E - E_0)$, $\mathcal{M} = \mathrm{i}c\,\langle E|m(0)|E_0\rangle$, $|0_M\rangle$ is the Minkowski vacuum state and $|\Psi\rangle$ is the state of the scalar field after its interaction with the detector. (We will hereafter drop the term $\mathcal{M}$ in the transition amplitude as it depends only on the internal structure of the detector and not on its motion.)

When the quantum field, $\Phi(x)$, is expressed in terms of the Minkowski normal modes, it is clear from (1) that the non-zero contribution to the transition amplitude arises only from the state $|\Psi\rangle = |1_{\boldsymbol{k}}\rangle$. For a detector on an inertial trajectory in $(1 + 1)$ dimensions, i.e. for $x(\tau) = x_0 + vt = x_0 + v\gamma\tau$ where $\gamma = (1 - v^2)^{-1/2}$, x_0 and v are constants and $|v| < 1$, the amplitude (1) turns out to be [11]

$$\mathcal{A}_{\mathrm{ine},\omega}(\Omega) = \frac{\mathrm{e}^{-\mathrm{i}kx_0}}{\sqrt{4\pi\omega}} \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\Omega\tau}\, \mathrm{e}^{\mathrm{i}\gamma\tau(\omega - kv)} = \sqrt{\frac{\pi}{\omega}}\, \mathrm{e}^{-\mathrm{i}kx_0}\, \delta(a) = 0, \tag{2}$$

where $\omega = |k|$ and $a = (\Omega + \gamma(\omega - kv))$. The last equality in the above equation follows from noting that, since $kv \leqslant |k||v| < \omega$ and $E > E_0$, the argument of the δ-function is always greater than zero, the transition in the detector being essentially forbidden on the grounds of energy conservation.

The following points should be stressed regarding the above—apparently trivial—calculation: the amplitude is being calculated for the system to make a transition from the state $|E_0\rangle$ in the *infinite past*, to the state $|E\rangle$ in the *infinite future*. To do so we need to know the trajectory $x(\tau)$ for all τ, i.e. for $-\infty < \tau < \infty$. No realistic detector can be kept switched on forever. Suppose the detector was kept switched on only during the time interval $-T \leqslant \tau \leqslant T$, *then the amplitude will be non-zero*:

$$\mathcal{A}_{\mathrm{ine},\omega}(\Omega, T) = \frac{\mathrm{e}^{-\mathrm{i}kx_0}}{\sqrt{4\pi\omega}} \int_{-T}^{T} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\Omega\tau}\, \mathrm{e}^{\mathrm{i}\gamma\tau(\omega - kv)} = \frac{\mathrm{e}^{-\mathrm{i}kx_0}}{\sqrt{4\pi\omega}} \left(\frac{2\sin(aT)}{a}\right), \tag{3}$$

and the probability for transtition for a fixed ω will be

$$\mathcal{P}_{\mathrm{ine},\omega}(\Omega, T) = |\mathcal{A}_{\mathrm{ine},\omega}(T)|^2 = \frac{1}{\pi\omega} \left(\frac{\sin(aT)}{a}\right)^2, \tag{4}$$

which is finite for all finite T. For small T, $\mathcal{P}_{\mathrm{ine},\omega} \propto T^2$ and hence vanishes for $T \to 0$; for large T, we use the relations

$$\begin{aligned}
\lim_{T\to\infty} \left\{\frac{\sin(aT)}{\pi a}\right\}^2 &= \lim_{T\to\infty} \left\{\left(\lim_{T\to\infty} \frac{\sin(aT)}{\pi a}\right)\left(\frac{\sin(aT)}{\pi a}\right)\right\} \\
&= \lim_{T\to\infty} \left\{\delta(a)\, \frac{\sin(aT)}{\pi a}\right\} \\
&= \lim_{T\to\infty} \left\{\frac{T}{\pi}\, \delta(a)\right\},
\end{aligned} \tag{5}$$

i.e.

$$\lim_{T\to\infty}\left\{\frac{\mathcal{P}_{\mathrm{ine},\omega}(T)}{T}\right\}=\frac{1}{\omega}\,\delta(a). \tag{6}$$

Clearly the rate of transitions $\mathcal{R}_{\mathrm{ine},\omega}(\Omega,T)=(\mathcal{P}_{\mathrm{ine},\omega}(\Omega,T)/T)$ has the following behaviour: $\mathcal{R}_{\mathrm{ine},\omega}\propto T$ for small T and $\mathcal{R}_{\mathrm{ine},\omega}\propto\delta(a)$ for large T. Hence $\mathcal{R}_{\mathrm{ine},\omega}$ vanishes in both the limits.

The above analysis should teach us two lessons. Firstly, even an inertial detector will respond if it is switched on and off. This is merely a manifestation of the energy–time uncertainty principle; a detection process lasting for a time $(2T)$ cannot measure energy differences with an accuracy greater than $(1/2T)$. So for $(a\,2T)\lesssim 1$, the rate $\mathcal{R}$ will be significantly non-zero. Secondly, the rate $\mathcal{R}$ is a more reliable quantity to compute than $\mathcal{P}$, especially if one is considering the $T\to\infty$ limit. In particular, $\mathcal{P}$ is infinite if we take the $T\to\infty$ limit naively in (4). The limits also need to be handled with care to obtain sensible results. We shall say more about the limiting procedures later on.

For the case of a uniformly accelerated trajectory in $(1+1)$ dimensions, the transformations from the Minkowski to the accelerated frame are

$$x=g^{-1}\xi\cosh(g\tau)\,;\qquad t=g^{-1}\xi\sinh(g\tau), \tag{7}$$

where τ is the proper time of the accelerated observer at ξ. In what follows we shall set $\xi=1$ without any loss of generality. For a wave travelling to the right, i.e. when $k=\omega$, the transition amplitude for a detector on an accelerated trajectory turns out to be [12]

$$\begin{aligned}\mathcal{A}_{\mathrm{acc},\omega}(\Omega)&=\frac{1}{\sqrt{4\pi\omega}}\int_{-\infty}^{\infty}\mathrm{d}\tau\,\mathrm{e}^{\mathrm{i}\Omega\tau}\exp-\mathrm{i}g^{-1}\omega\,\mathrm{e}^{-g\tau}\\&=\frac{1}{\sqrt{4\pi\omega}}\,g^{-1}\left(\omega g^{-1}\right)^{\mathrm{i}\Omega g^{-1}}\Gamma(-\mathrm{i}\Omega g^{-1})\;\exp-(\pi\Omega/2g),\end{aligned} \tag{8}$$

which is clearly non-zero. The probability for the transition $\mathcal{P}_\omega=|\mathcal{A}_\omega|^2$ for a particular ω will then be

$$\mathcal{P}_{\mathrm{acc},\omega}(\Omega)=|\mathcal{A}_{\mathrm{acc},\omega}(\Omega)|^2=\frac{g^{-2}}{4\pi\omega}\;\mathrm{e}^{-\pi\Omega g^{-1}}\,|\Gamma(-\mathrm{i}\Omega g^{-1})|^2=\frac{1}{2\omega g}\left(\frac{1}{\Omega\,(\mathrm{e}^{2\pi\Omega g^{-1}}-1)}\right) \tag{9}$$

which has a Planckian form in Ω with a temperature $\beta^{-1}=(g/2\pi)$.

There is another feature that needs emphasis as regards both (9) and (4): *these are probabilities for transitions to fixed final states* $|1_k\rangle$ *characterized by a given momentum* k. Normally one would like to integrate over all k so as to find the net probability for the detector to have made a transition from $|E_0\rangle$ to $|E\rangle$. This will lead to an integral

$$I_{\mathrm{ine}}=\int_0^\infty\frac{\mathrm{d}\omega}{\omega}\left(\frac{\sin^2((\Omega+\omega)T)}{(\Omega+\omega)^2}\right) \tag{10}$$

in the case of (4) and to an integral

$$I_{\mathrm{acc}}=\int_0^\infty\frac{\mathrm{d}\omega}{\omega} \tag{11}$$

in the case of (9). Both these integrals are formally divergent. However, consider the limit

$$\lim_{T\to\infty}\left(\frac{I_{\mathrm{ine}}}{T}\right)=\int_0^\infty\frac{\mathrm{d}\omega}{\omega}\left\{\lim_{T\to\infty}\left(\frac{1}{T}\,\frac{\sin^2((\Omega+\omega)T)}{(\Omega+\omega)^2}\right)\right\}=\frac{1}{\pi}\int_0^\infty\frac{\mathrm{d}\omega}{\omega}\,\delta(\Omega+\omega). \tag{12}$$

If $\Omega>0$, $\omega>0$ the integrand identically vanishes and we may take this integral to be zero, thereby recovering the earlier result (see also [10] for a similar discussion). This result shows that $(I_{\rm ine}/T)$ is formally divergent for all finite T but can be interpreted to be zero as $T \to \infty$! Such a contradiction arises because of an illegitimate interchange of limits. We will elaborate on the limiting procedures later on.

The integral (10) is divergent in both the lower and upper limits of ω. The divergence for small ω (infrared divergence) is a feature of massless scalar fields in $(1+1)$ dimensions. For the $(3+1)$ dimensional case, we will, later in this paper, find that no infrared divergences arise and only logarithmic divergences for large ω (ultraviolet divergences) are encountered. These divergences have been reported earlier in [9, 10]. We shall see later that the divergences in (10) for a finite T can be attributed to the abrupt switching of the detector.

We conclude this section by collecting together some more results we need. The probability of transition to all possible $|\Psi\rangle$ from $|0_M\rangle$ can be expressed in a more formal and concise manner as [11]:

$$\mathcal{P}(\Omega) = \sum_{|\Psi\rangle} |\mathcal{A}(\Omega)|^2 = \mathcal{F}(\Omega)\,; \qquad \mathcal{F}(\Omega) = \int_{-\infty}^{\infty} \mathrm{d}\tau \int_{-\infty}^{\infty} \mathrm{d}\tau'\, \mathrm{e}^{-\mathrm{i}\Omega(\tau-\tau')}\, G^+(x(\tau), x(\tau')). \tag{13}$$

The detector response function $\mathcal{F}$, is independent of the details of the detector and is determined completely by the Wightman function $G^+(x(\tau), x(\tau')) = \langle 0_M|\Phi[x(\tau)]\Phi[x(\tau')]|0_M\rangle$. For trajectories in Minkowski space, which are integral curves of timelike Killing vector fields, for, e.g., the inertial and the accelerated trajectories, the Wightman function is invariant under time translations in the reference frame of the detector [13, 14]. Hence $G^+(x(\tau), x(\tau')) = G^+(\tau - \tau') \equiv G^+(\Delta\tau)$ and the double integration in (13) reduces to a Fourier transform of the two-point function multiplied by an infinite time interval. This is usually handled by interpreting the Fourier transform of the two-point function to be the transition probability per unit time, i.e. as the rate of transition probability, given by

$$\mathcal{R}(\Omega) = \int_{-\infty}^{\infty} \mathrm{d}\Delta\tau\, \mathrm{e}^{-\mathrm{i}\Omega\Delta\tau}\, G^+(\Delta\tau). \tag{14}$$

(It should, however, be stressed that the expression for probability *is* divergent and only the rate is finite. On the other hand, the probability of transition to any specific one-particle state $|1_{\boldsymbol{k}}\rangle$ is finite.) The Wightman function for a massless scalar field in $(3+1)$ dimensions is [11]

$$G^+(x, x') = \frac{-1}{4\pi^2\left((t - t' - \mathrm{i}\epsilon)^2 - |\boldsymbol{x} - \boldsymbol{x}'|^2\right)}, \tag{15}$$

(where $\epsilon \to 0^+$) and for the case of an inertial trajectory it reduces to [11]

$$G^+_{\rm ine}(\Delta\tau) = \frac{-1}{4\pi^2\,(\Delta\tau - \mathrm{i}\epsilon)^2}. \tag{16}$$

Since $E > E_0$, the integral (14) can be performed by closing the contour in an infinite semi-circle in the lower half of the complex $\Delta\tau$ plane. Since the pole of the two-point function (16) is at $\Delta\tau = \mathrm{i}\epsilon$, it does not contribute to the integral and the detector response is zero; in other words the inertial detector does not respond in the Minkowski vacuum.

For the case of an accelerated trajectory, given by (7), the Wightman function reduces to

$$G^+_{\rm acc}(\Delta\tau) = -\left(16\pi^2 g^{-2}\,\sinh^2(g\Delta\tau/2 - \mathrm{i}\epsilon)\right)^{-1}, \tag{17}$$

which can be expressed as [11]

$$G^{+}_{\rm acc}(\Delta\tau) = -\frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty}\left(\Delta\tau - 2{\rm i}\epsilon + 2\pi{\rm i}ng^{-1}\right)^{-2}. \tag{18}$$

Substituting (18) into (14) and performing the contour integral in the lower half of the complex $\Delta\tau$ plane, we obtain the transition probability rate of the detector to be

$$\mathcal{R}_{\rm acc}(\Omega) = \frac{1}{2\pi}\,\frac{\Omega}{({\rm e}^{2\pi g^{-1}\Omega} - 1)} \tag{19}$$

which is the thermal spectrum well known in the literature.

Having thus discussed some of the essential results from Unruh–DeWitt detector response theory, we will now move on to the crux of this paper, namely finite-time detectors.

3. Detector response with window functions

To understand some of the subtleties mentioned earlier regarding the limiting procedures, we shall begin this section with the following discussion.

Consider a Unruh–DeWitt detector which is moving on a trajectory $x(\tau)$ and is switched on during the interval $\tau = -T$ to $\tau = T$. The response of such a detector is governed by the integral

$$\mathcal{F}(\Omega, T) = \int_{-T}^{T}{\rm d}\tau\int_{-T}^{T}{\rm d}\tau'\,{\rm e}^{-{\rm i}\Omega(\tau-\tau')}\,G^{+}(\tau,\tau'). \tag{20}$$

We shall further assume that the trajectory of the detector is along the integral curve of a timelike Killing vector field so that $G^{+}(\tau,\tau') = G^{+}(\tau-\tau')$. *It is clear from this expression that* $\mathcal{F} \to 0$ *as* $T \to 0$ *irrespective of any other details*. Also, we should recover the standard results when $T \to \infty$.

We shall now rewrite the integral (20) in different variables and then take the limits $T \to 0$ and $T \to \infty$. Changing the variables to

$$x = \tau - \tau'; \qquad y = \tau + \tau' \tag{21}$$

we obtain that

$$\int_{-T}^{T}{\rm d}\tau\int_{-T}^{T}{\rm d}\tau'\,{\rm e}^{-{\rm i}\Omega(\tau-\tau')}\,G^{+}(\tau-\tau') = \tfrac{1}{2}\int_{-2T}^{2T}{\rm d}x\int_{-2T+|x|}^{2T-|x|}{\rm d}y\,{\rm e}^{-{\rm i}\Omega x}G^{+}(x), \tag{22}$$

where the factor $\frac{1}{2}$ is the Jacobian of the transformation from the (τ,τ') coordinates to the (x, y) coordinates. After integrating with respect to y, we obtain

$$\mathcal{F}(\Omega, T) = \int_{-2T}^{2T}{\rm d}x\,{\rm e}^{-{\rm i}\Omega x}\,G^{+}(x)\,(2T - |x|). \tag{23}$$

Let us now consider the limits $T \to \infty$ and $T \to 0$ of this integral. When $T \to \infty$, we get

$$\mathcal{F}(\Omega) = \lim_{T\to\infty}\left\{(2T)\,\tilde{G}^{+}(\Omega) - \int_{-2T}^{2T}{\rm d}x\,{\rm e}^{-{\rm i}\Omega x}\,G^{+}(x)\,|x|\right\} \tag{24}$$

where $\tilde{G}^{+}(\Omega)$ is the Fourier transform of $G^{+}(x)$. Clearly,

$$\mathcal{R}(\Omega) = \lim_{T\to\infty}\left\{\frac{\mathcal{F}(\Omega,T)}{2T}\right\} = \lim_{T\to\infty}\left\{\tilde{G}^{+}(\Omega) - \frac{1}{2T}\int_{-\infty}^{\infty}{\rm d}x\,{\rm e}^{-{\rm i}\Omega x}\,G^{+}(x)\,|x|\right\} = \tilde{G}^{+}(\Omega), \tag{25}$$

provided the second integral is well defined. This expression is finite and represents a constant rate of transition; we have thus recovered the standard result in the $T \to \infty$ limit.

Let us next consider the $T \to 0$ limit which is rather tricky. We need to evaluate

$$\mathcal{F}(\Omega, T) = \lim_{T\to 0} \left\{ \int_{-2T}^{2T} \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}\Omega x}\, G^+(x)\, (2T - |x|) \right\}. \tag{26}$$

The integral over x is confined to a small range $(-2T, 2T)$ around the origin. This implies that we can expand the integrand in a Taylor series around the origin to obtain the required limit. We write

$$\mathrm{e}^{-\mathrm{i}\Omega x}\, G^+(x) \simeq \left(1 - \mathrm{i}\Omega x - \frac{\Omega^2 x^2}{2} + \cdots\right)\left(G^+(0) + G^{+'}(0)\, x + G^{+''}(0)\, \frac{x^2}{2} + \cdots\right). \tag{27}$$

Substituting the above expression into (26) and performing the integration, we obtain that

$$\begin{aligned}\mathcal{F}(\Omega, T) &\simeq 4T^2 G^+(0) + \tfrac{4}{3} T^4 \big(G^{+''}(0) - \Omega^2 G^+(0) - 2\mathrm{i}\Omega G^{+'}(0)\big) + \mathrm{O}(\Omega^4 T^4) \\ &\simeq 4T^2 G^+(0),\end{aligned} \tag{28}$$

to the lowest order. All derivatives of $G^+(x)$ in $(3+1)$ dimensions behave as ϵ^{-n} at the origin and, in particular, $G^+(0) = (1/4\pi^2\epsilon^2)$, giving

$$\mathcal{F}(\Omega, T) \simeq \frac{T^2}{\pi^2\epsilon^2}. \tag{29}$$

The above expression shows that care should be exercised when the limits $T \to 0$ and $\epsilon \to 0$ are taken. It is clear from the fundamental definition of the integral in (20) that we must have $\mathcal{F}(\Omega, 0) = 0$ for all regular integrands. If the integrand has a pole on the real axis (requiring an $\mathrm{i}\epsilon$ prescription to give meaning to the integral) then we should *arrange* the limiting procedure in such a way that $\mathcal{F}(\Omega, 0) = 0$. This can be achieved by using the rule that the $\epsilon \to 0$ limit should be taken right at the end, after the limit $T \to 0$ has been taken. Since

$$\lim_{\epsilon\to 0} \left\{ \lim_{T\to 0} \frac{T^2}{\epsilon^2} \right\} = 0\,; \qquad \lim_{T\to 0} \left\{ \lim_{\epsilon\to 0} \frac{T^2}{\epsilon^2} \right\} = \infty, \tag{30}$$

only the former ordering will provide physically reasonable results. This prescription is also necessary to ensure that $G^+(0), G^{+'}(0), \ldots$, exist in the Taylor expansion for $G^+(x)$. For $\epsilon = 0$, this expansion ceases to exist.

In $(1+1)$ dimensions $G^+(x)$ has a logarithmic dependence in x; hence the limit will be modified to the form

$$\mathcal{F}(\Omega, T) \propto T^2 \ln(\epsilon^2). \tag{31}$$

Taking the $T \to 0$ limit first will give the sensible result $\mathcal{F}(\Omega, 0) = 0$, while if the $\epsilon \to 0$ limit is taken first we will obtain a logarithmic divergence. This logarithmic divergence has been mentioned earlier in the discussion following equations (10) and (11). We shall see explicit examples of such ambiguities (and their resolution) in what follows.

We shall now calculate the response of inertial and uniformly accelerated Unruh–DeWitt detectors that are switched on and off with the help of three different window functions. Hence, instead of working with (20), we will consider the integral of the form

$$\mathcal{F}(\Omega, T) = \int_{-\infty}^{\infty} \mathrm{d}\tau \int_{-\infty}^{\infty} \mathrm{d}\tau'\, \mathrm{e}^{-\mathrm{i}\Omega(\tau-\tau')}\, W(\tau, T)\, W(\tau', T)\, G^+(x(\tau), x(\tau'))$$

where $W(\tau, T)$ is a window function with the properties $W(\tau, T) \approx 1$ for $|\tau| \ll T$ and $W(\tau, T) \approx 0$ for $|\tau| \gg T$. The abrupt switching corresponds to $W(\tau, T) =$

$\big(\Theta(T-\tau)+\Theta(T+\tau)\big)$. More gradual switching on and off can be achieved with the window functions

$$W_1(\tau,T)=\exp-\left(\frac{\tau^2}{2T^2}\right); \qquad W_2(\tau,T)=\exp-\left(\frac{|\tau|}{T}\right). \tag{32}$$

The motivation to study the detector response with smooth window functions W_1 and W_2 is twofold. One is to carefully identify any divergence that may arise when a finite-time detection is performed. The other is to check whether a certain lack of smoothness in the window function is responsible for the appearance of divergences in the detector response, as has been reported in [9, 10].

In the rest of this section, we will evaluate the response of inertial and uniformly accelerated finite-time detectors that are switched on and off by the window functions W_1, W_2 and W.

3.1. Gaussian window function

The detector response integral with the window function W_1 is

$$\mathcal{F}(\Omega,T)=\int_{-\infty}^{\infty}\mathrm{d}\tau\int_{-\infty}^{\infty}\mathrm{d}\tau'\,\mathrm{e}^{-\mathrm{i}\Omega(\tau-\tau')}\,G^{+}(\tau,\tau')\,\exp-\left(\frac{\tau}{T}\right)^2\,\exp-\left(\frac{\tau'}{T}\right)^2 \tag{33}$$

which can be rewritten as

$$\begin{aligned}\mathcal{F}(\Omega,T)=\int_{-\infty}^{\infty}\mathrm{d}\tau\int_{-\infty}^{\infty}\mathrm{d}\tau'\,\mathrm{e}^{-\mathrm{i}\Omega(\tau-\tau')}\,G^{+}(x(\tau),x(\tau'))\\ \times\exp-\left\{\frac{1}{2T^2}\left((\tau+\tau')^2\;+\;(\tau-\tau')^2\right)\right\}.\end{aligned} \tag{34}$$

Let us first consider the response of a detector on an inertial trajectory. Substituting the Wightman function (16) for the inertial trajectory in the above integral and performing the transformations (21), the integral for the detector response function simplifies to

$$\mathcal{F}_{\mathrm{ine}}(\Omega,T)=-\frac{1}{8\pi^2}\int_{-\infty}^{\infty}\mathrm{d}y\,\exp-\left(\frac{y^2}{2T^2}\right)\int_{-\infty}^{\infty}\mathrm{d}x\,\frac{\mathrm{e}^{-\mathrm{i}\Omega x}\,\exp-(x^2/2T^2)}{(x-\mathrm{i}\epsilon)^2}=-\frac{T}{\sqrt{32\pi^3}}\,I \tag{35}$$

where

$$I=\int_{-\infty}^{\infty}\mathrm{d}x\,\frac{\mathrm{e}^{-\mathrm{i}\Omega x}}{(x-\mathrm{i}\epsilon)^2}\,\exp-\left(\frac{x^2}{2T^2}\right). \tag{36}$$

Writing the Gaussian function in the above integral as a Fourier transform

$$\exp-\left(\frac{x^2}{2T^2}\right)=\frac{T}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{d}k\,\mathrm{e}^{\mathrm{i}kx}\,\exp-\left(\frac{k^2T^2}{2}\right) \tag{37}$$

and interchanging the order of integration, we obtain

$$I=\frac{T}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{d}k\,\exp-\left(\frac{k^2T^2}{2}\right)\int_{-\infty}^{\infty}\mathrm{d}x\,\frac{\mathrm{e}^{\mathrm{i}(k-\Omega)x}}{(x-\mathrm{i}\epsilon)^2}. \tag{38}$$

When $k>\Omega$, the x integral can be performed as a contour integral by closing the contour in the upper half of the complex x plane and the second-order pole at $x=\mathrm{i}\epsilon$ gives a non-trivial contribution to the integral. When $k<\Omega$ the contour has to be closed in the lower half of the complex x plane and, since the integrand is analytic in this half, the integral vanishes.

Hence the lower limit of the k integral is Ω. After some manipulation and substituting this result in (35), we obtain

$$\mathcal{F}_{\rm ine}(\Omega, T) = \frac{{\rm e}^{\Omega\epsilon}}{2\pi}\ \exp(\epsilon^2/2T^2)\int_r^\infty {\rm d}p\,{\rm e}^{-p^2}(p-r) \tag{39}$$

where

$$p = \frac{1}{\sqrt{2}}\left(kT + \frac{\epsilon}{T}\right); \qquad r = \frac{1}{\sqrt{2}}\left(\Omega T + \frac{\epsilon}{T}\right). \tag{40}$$

Before proceeding further, let us check whether (39) gives sensible results for the limits $T \to 0$ and $T \to \infty$. Since this is an inertial detector we must have $\mathcal{F}(\Omega) = 0$; also for a detector on any trajectory we demand that $\mathcal{F}(\Omega, 0) = 0$. These two limits can be obtained from the above result. When $T \to \infty$, the lower and the upper limits of the p integral in (39) coincide, thereby giving a null result as expected for the inertial detector. (Note that for large r, the expression

$$r\int_r^\infty {\rm d}p\,{\rm e}^{-p^2} \simeq \tfrac{1}{2}{\rm e}^{-r^2}\left\{1 + {\rm O}\left(\frac{1}{r^2}\right)\right\} \tag{41}$$

vanishes exponentially.) Hence, there is no ambiguity in this result.

Studying the limit $T \to 0$ of (39), when the window function is sharply peaked at the origin, has to be done more carefully. In this case, it matters crucially whether the limit $T \to 0$ is taken first and the condition $\epsilon \to 0$ is incorporated later or *vice versa*. The first alternative will be adopted (as has been mentioned earlier) for the reason that ϵ helps us to identify the poles in the contour integrals; hence unless and until all the other limits in the problem have already been taken care of, the limit on ϵ should not be taken. Then, as $T \to 0$, $r \to (\epsilon/\sqrt{2}T)$ and $\mathcal{F}_{\rm ine}(\Omega, T)$ can be rewritten as

$$\begin{aligned}\mathcal{F}_{\rm ine}(\Omega, T) = \frac{{\rm e}^{\Omega\epsilon}}{2\pi}\ \exp(\epsilon^2/2T^2)&\left\{\int_{(\epsilon/\sqrt{2}T)}^\infty {\rm d}p\,{\rm e}^{-p^2}p\right.\\ &\left.-\frac{\epsilon}{\sqrt{2}\,T}\left(\int_0^\infty {\rm d}p\,{\rm e}^{-p^2} - \int_0^{(\epsilon/\sqrt{2}T)} {\rm d}p\,{\rm e}^{-p^2}\right)\right\}.\end{aligned} \tag{42}$$

The last term in the above expression is the error function and its asymptotic form for large arguments is as follows:

$$\frac{2}{\sqrt{\pi}}\int_0^x {\rm d}v\,{\rm e}^{-v^2} = 1 - \frac{{\rm e}^{-x^2}}{\sqrt{\pi}}\left(\frac{1}{x} - \frac{1}{2x^3} + \frac{3}{4x^5}\cdots\right). \tag{43}$$

Substituting the above expression in (42), we obtain the detector response when $T \to 0$ to be

$$\mathcal{F}_{\rm ine}(\Omega, 0) = \frac{{\rm e}^{\Omega\,\epsilon}\,T^2}{4\pi\epsilon^2} \to 0 \tag{44}$$

for finite ϵ. This expression has the same form as (29) and clearly illustrates the need to keep $\epsilon \neq 0$ until the end. Note that the detector response function as well the rate of transition $\mathcal{R}_{\rm ine}(\Omega, T) = (\mathcal{F}_{\rm ine}(\Omega, T)/T)$ vanish when $T \to 0$. The non-commutativity of the limiting procedure as regards $T \to 0$, $\epsilon \to 0$ in the detector response functions is evidently due to the presence of factors like (ϵ/T).

If the condition $\epsilon \to 0$ is incorporated first in (39), the expression factorizes to

$$\mathcal{F'}_{\rm ine}(\Omega, T) = \frac{1}{2\pi}\int_{(\Omega\,T/\sqrt{2})}^\infty {\rm d}p\,{\rm e}^{-p^2}\left(p - \frac{\Omega T}{\sqrt{2}}\right). \tag{45}$$

If we now take the limit $T \to 0$, we obtain that

$$\mathcal{F}'_{\rm ine}(\Omega, 0) = \frac{1}{2\pi}\int_0^\infty {\rm d}p\, {\rm e}^{-p^2}\, p = \frac{1}{4\pi}. \tag{46}$$

On the other hand, $\mathcal{F}_{\rm ine}(\Omega, T)$ vanishes if we take the limit $T \to 0$ before we set $\epsilon = 0$. We stress again that the procedure of setting ϵ to zero only after the $T \to 0$ limit has been taken is the proper one.

If we are only interested in finite, non-zero values of T then we can set $\epsilon = 0$ in the integral (39). The response of the inertial detector for a finite T can then be written in closed form as

$$\mathcal{F}_{\rm ine}(\Omega, T) = \frac{1}{4\pi}\left\{\exp -\left(\frac{\Omega^2 T^2}{2}\right) - \left(\frac{\Omega T}{\sqrt{2}}\right)\Gamma\left(\frac{1}{2}, \frac{\Omega^2 T^2}{2}\right)\right\}. \tag{47}$$

For $\Omega T \gg 1$, this expression has the asymptotic form

$$\mathcal{F}_{\rm ine}(\Omega, T) \simeq \frac{\exp -(\Omega^2 T^2/2)}{4\pi\,\Omega^2 T^2}. \tag{48}$$

This shows that an inertial detector, switched on for a finite period of time, does give a non-zero response which goes to zero as $T \to \infty$.

Let us now carry out the same analysis for the accelerated detector. For this case, the Wightman function, given by (18), is substituted into (34) and the transformations (21) are performed; the result is

$$\mathcal{F}_{\rm acc}(\Omega, T) = -\frac{1}{8\pi^2}\int_{-\infty}^{\infty} {\rm d}y\, \exp -\left(\frac{y^2}{2T^2}\right)\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} {\rm d}x\, \frac{{\rm e}^{-{\rm i}\Omega x}\, \exp -(x^2/2T^2)}{(x - {\rm i}b_n)^2} \tag{49}$$

where $b_n = \epsilon - 2\pi g^{-1} n$. With the aid of (37), the above integral can then be simplified to the form

$$\begin{aligned}\mathcal{F}_{\rm acc}(\Omega, T) &= -\frac{T}{\sqrt{32\pi^3}}\sum_{n=-\infty}^{\infty} I_n;\\ I_n &= \frac{T}{\sqrt{2\pi}}\int_{-\infty}^{\infty} {\rm d}k\, \exp -\left(\frac{k^2 T^2}{2}\right)\int_{-\infty}^{\infty} {\rm d}x\, \frac{{\rm e}^{{\rm i}(k-\Omega)x}}{(x - {\rm i}b_n)^2}.\end{aligned} \tag{50}$$

When $k > \Omega$ the x integration can be performed by closing the contour in the upper half of the complex x plane and the poles corresponding to the values of n between $-\infty$ and zero contribute non-trivially to $\mathcal{F}_{\rm acc}(\Omega, T)$, giving

$$\mathcal{F}_{\rm acc1}(\Omega, T) = \frac{1}{2\pi}\sum_{n=-\infty}^{0} {\rm e}^{\Omega b_n}\, \exp({b_n}^2/2T^2)\int_{r'}^{\infty} {\rm d}p'\, {\rm e}^{-p'^2}\,(p' - r') \tag{51}$$

where

$$p' = \frac{1}{\sqrt{2}}\left(kT + \frac{b_n}{T}\right); \qquad r' = \frac{1}{\sqrt{2}}\left(\Omega T + \frac{b_n}{T}\right). \tag{52}$$

When $k < \Omega$ the contour can be closed in the lower half of the complex x plane and the poles corresponding to the values of n between one and infinity contribute non-trivially, with the result

$$\mathcal{F}_{\rm acc2}(\Omega, T) = \frac{1}{2\pi}\sum_{n=1}^{\infty} {\rm e}^{\Omega b_n}\, \exp(b_n^2/2T^2)\int_{-r'}^{\infty} {\rm d}p'\, {\rm e}^{-p'^2}(p' + r'). \tag{53}$$

The complete result is $\mathcal{F}_{\rm acc}(\Omega, T) = (\mathcal{F}_{\rm acc1}(\Omega, T) + \mathcal{F}_{\rm acc2}(\Omega, T))$, i.e.

$$\mathcal{F}_{\rm acc}(\Omega, T) = \frac{1}{2\pi} \sum_{n=-\infty}^{0} {\rm e}^{\Omega b_n} \exp({b_n}^2/2T^2) \int_{r'}^{\infty} {\rm d}p' \, {\rm e}^{-p'^2} (p' - r') + \frac{1}{2\pi} \sum_{n=1}^{\infty} {\rm e}^{\Omega b_n} \exp({b_n}^2/2T^2) \int_{-r'}^{\infty} {\rm d}p' \, {\rm e}^{-p'^2} (p' + r'). \tag{54}$$

Let us again check the two relevant limits. In the limit $T \to \infty$ the lower limits of the above integrals reduce to ∞ and $-\infty$, respectively, so that only $\mathcal{F}_{\rm acc2}(\Omega, T)$ contributes to the detector response. Evaluating this and imposing the condition $\epsilon \to 0$, we get the standard result:

$$\lim_{T\to\infty} \left\{ \frac{\mathcal{F}_{\rm acc}(\Omega, T)}{T} \right\} = \frac{1}{\sqrt{8\pi}} \left(\frac{\Omega}{{\rm e}^{2\pi g^{-1}\Omega} - 1} \right). \tag{55}$$

In this case the ratio $\mathcal{R}_{\rm acc}(\Omega, T) = (\mathcal{F}_{\rm acc}(\Omega, T)/T)$ should be interpreted as the transition probability rate.

When $T \to 0$, we can perform the analysis as was carried out for the inertial detector and, since only the $n = 0$ term in the series (54) contributes non-trivially, we obtain the result

$$\mathcal{F}_{\rm acc}(\Omega, 0) = \frac{{\rm e}^{\Omega\epsilon} \, T^2}{2\pi\epsilon^2}. \tag{56}$$

This is identical to the inertial detector result and shows that the transition probability (as well as the rate) will go to zero as $T \to 0$.

The fact that both accelerated and inertial detectors give identical results for the $T \to 0$ limit is to be expected on physical grounds. The curvature of the trajectory cannot make its presence felt for infinitesimal intervals and the response of the detector cannot depend on parameters like g which characterize the detector trajectory.

It is possible to state some of these results in a more general form for this window function. Note that for a detector moving along any trajectory for which $G^+(x, x') = G^+(\tau - \tau')$, the response function is

$$\begin{aligned} \mathcal{F}(\Omega, T) &= \int_{-\infty}^{\infty} {\rm d}\tau \int_{-\infty}^{\infty} {\rm d}\tau' \, \exp - \left\{ \frac{1}{2T^2} (\tau^2 + \tau'^2) \right\} {\rm e}^{-{\rm i}\Omega(\tau-\tau')} G^+(\tau - \tau') \\ &= \frac{1}{2} \int_{-\infty}^{\infty} {\rm d}y \, \exp - \left(\frac{y^2}{2T^2} \right) \int_{-\infty}^{\infty} {\rm d}x \, \exp - \left(\frac{x^2}{2T^2} \right) {\rm e}^{-{\rm i}\Omega x} G^+(x) \\ &= \sqrt{\frac{\pi}{2}} \, T \int_{-\infty}^{\infty} {\rm d}x \, \exp - \left(\frac{x^2}{2T^2} \right) {\rm e}^{-{\rm i}\Omega x} G^+(x). \end{aligned} \tag{57}$$

We can write

$$f(x) \big[{\rm e}^{-{\rm i}\Omega x} G^+(x) \big] = f \left({\rm i} \frac{\partial}{\partial\Omega} \right) \big[{\rm e}^{-{\rm i}\Omega x} G^+(x) \big] \tag{58}$$

for any function $f(x)$ which has a power series expansion around $x = 0$. Hence we have

$$\mathcal{F}(\Omega, T) = \sqrt{\frac{\pi}{2}} \, T \int_{-\infty}^{\infty} {\rm d}x \, \exp \left(\frac{1}{T} \frac{\partial^2}{\partial\Omega^2} \right) \big[{\rm e}^{-{\rm i}\Omega x} G^+(x) \big] = \exp \left(\frac{1}{2T^2} \frac{\partial^2}{\partial\Omega^2} \right) [\mathcal{F}(\Omega)]. \tag{59}$$

The expression in the square brackets, $\mathcal{F}(\Omega)$, is the result for the infinite-time detector. The corresponding rates are

$$\mathcal{R}(\Omega, T) = \exp \left(\frac{1}{2T^2} \frac{\partial^2}{\partial\Omega^2} \right) [\mathcal{R}(\Omega)]. \tag{60}$$

This formula allows us to systematically calculate finite-time corrections as a series in $(1/T)$. To the lowest order, the correction is

$$\mathcal{R}(\Omega, T) = \mathcal{R}(\Omega) + \frac{1}{2T^2}\,\frac{\partial^2\mathcal{R}(\Omega)}{\partial\Omega^2} + \mathrm{O}\left(\frac{1}{T^4}\right). \tag{61}$$

In the case of a uniformly accelerated detector up to the lowest order, we obtain that

$$\mathcal{R}(\Omega, T) \simeq \mathcal{R}(\Omega)\left\{1 - \frac{2\pi}{g\Omega T^2}\left(\frac{\mathrm{e}^{2\pi\Omega g^{-1}}}{(\mathrm{e}^{2\pi\Omega g^{-1}} - 1)^2}\right)\left\{\mathrm{e}^{2\pi\Omega g^{-1}}\,(1 - \pi\Omega g^{-1}) - 1 - \pi\Omega g^{-1}\right\}\right\}. \tag{62}$$

3.2. Window function with an exponential cut-off

Having studied the detector response with a Gaussian window function, we now study it with the window function W_2. In this case the response function turns out to be

$$\mathcal{F}(\Omega, T) = \int_{-\infty}^{\infty}\mathrm{d}\tau\int_{-\infty}^{\infty}\mathrm{d}\tau'\,\mathrm{e}^{-\mathrm{i}\Omega(\tau-\tau')}\exp-\left\{\frac{1}{T}\,(|\tau| + |\tau'|)\right\}G^{+}(x(\tau), x(\tau')). \tag{63}$$

Introducing the window functions as Fourier transforms, i.e.

$$\exp-\left(\frac{|\tau|}{T}\right) = \int_{-\infty}^{\infty}\mathrm{d}k\, f(k)\mathrm{e}^{\mathrm{i}k\tau}\,; \qquad f(k) = \frac{T}{\pi}\left(\frac{1}{1 + k^2T^2}\right) \tag{64}$$

and performing the transformations (21) we obtain the response function for an inertial detector to be

$$\begin{aligned}\mathcal{F}_{\mathrm{ine}}(\Omega, T) = -\frac{1}{8\pi^2}\int_{-\infty}^{\infty}\mathrm{d}k\, f(k)\int_{-\infty}^{\infty}\mathrm{d}q\, f(q)\int_{-\infty}^{\infty}\mathrm{d}y\,\mathrm{e}^{\mathrm{i}\{y(k+q)/2\}}\\ \times\int_{-\infty}^{\infty}\mathrm{d}x\left\{\frac{\mathrm{e}^{\mathrm{i}x\{(k-q)/2-\Omega\}}}{(x - \mathrm{i}\epsilon)^2}\right\}.\end{aligned} \tag{65}$$

When the y and q integrals in the above expression are performed in that order, the result is

$$\mathcal{F}_{\mathrm{ine}}(\Omega, T) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}k\, f(k)\, f(-k)\int_{-\infty}^{\infty}\mathrm{d}x\,\frac{\mathrm{e}^{\mathrm{i}(k-\Omega)x}}{(x - \mathrm{i}\epsilon)^2}. \tag{66}$$

Performing the contour integral after substituting for $f(k)$, the detector response function reduces to

$$\mathcal{F}_{\mathrm{ine}}(\Omega, T) = \frac{\mathrm{e}^{\Omega\epsilon}}{\pi^2}\,T^2\int_{\Omega}^{\infty}\mathrm{d}k\,\frac{\mathrm{e}^{-\epsilon k}}{(1 + k^2T^2)^2}\,(k - \Omega). \tag{67}$$

If ϵ is kept non-zero, the above expression, up to the lowest order in T, clearly decays as T^2 as $T \to 0$. We can rewrite the above integral as

$$\mathcal{F}_{\mathrm{ine}}(\Omega, T) = \frac{\mathrm{e}^{\Omega\epsilon}}{\pi^2}\int_{\Omega T}^{\infty}\mathrm{d}p\,\frac{\exp-(p\epsilon/T)}{(1 + p^2)^2}\,(p - \Omega T), \tag{68}$$

where $p = kT$. When $T \to \infty$ the limits of the above integral coincide, giving a null result as expected.

We again note the crucial role played by the ϵ factor. The limits $\epsilon \to 0$, $T \to 0$ do not (again!) commute in the function $\exp-(p\epsilon/T)$:

$$\lim_{T\to 0}\left\{\lim_{\epsilon\to 0}\exp-(p\epsilon/T)\right\} = 1\,; \qquad \lim_{\epsilon\to 0}\left\{\lim_{T\to 0}\exp-(p\epsilon/T)\right\} = 0. \tag{69}$$

If ϵ is set to zero in the integral (68), we obtain that

$$\mathcal{F}'_{\rm ine}(\Omega, T) = \frac{1}{\pi^2}\int_{\Omega T}^{\infty}\frac{{\rm d}p}{(1+p^2)^2}(p - \Omega T). \tag{70}$$

When the limit $T \to 0$ is taken in the above integral, it reduces to

$$\mathcal{F}'_{\rm ine}(\Omega, T) = \frac{1}{\pi^2}\int_{0}^{\infty}\frac{{\rm d}p\, p}{(1+p^2)^2} = \frac{1}{2\pi^2}, \tag{71}$$

i.e. the detector response is non-zero, even as $T \to 0$. As we have emphasized several times by now, a physically sensible result (that the response of the detector is zero when it is not switched on at all) can be obtained only if ϵ is kept non-zero until all the other limits have been taken.

If we are interested only in the $T \neq 0$ case, then we can set $\epsilon = 0$ in (68) and this integral with $\epsilon = 0$ can be expressed in a closed form as follows:

$$\mathcal{F}_{\rm ine}(\Omega, T) = \frac{1}{2\pi^2}\left(\frac{1}{(1+\Omega^2 T^2)} - \frac{\Omega T}{2}\left\{\pi - 2\tan^{-1}(\Omega T) - \sin 2(\tan^{-1}(\Omega T))\right\}\right). \tag{72}$$

For $\Omega T \gg 1$, this function behaves as

$$\mathcal{F}_{\rm ine}(\Omega, T) \simeq \frac{1}{6\pi^2\Omega^2 T^2}. \tag{73}$$

We once again see that the inertial detector responds in the Minkowski vacuum if it is switched on only for a finite T. As $T \to \infty$, this response decays as $(1/T^2)$.

Let us next consider the case of the accelerated detector. The response function of the accelerated detector is given by the integral

$$\mathcal{F}_{\rm acc}(\Omega, T) = -\frac{1}{8\pi^2}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}{\rm d}k\, f(k)\int_{-\infty}^{\infty}{\rm d}q\, f(q)\int_{-\infty}^{\infty}{\rm d}y\, {\rm e}^{{\rm i}\{y(k+q)/2\}} \times \int_{-\infty}^{\infty}{\rm d}x\left\{\frac{{\rm e}^{{\rm i}\{(k-q)/2-\Omega\}x}}{(x-{\rm i}b_n)^2}\right\}, \tag{74}$$

where $b_n = \epsilon - 2\pi g^{-1} n$. When the y and q integrals are carried out, in that order, the detector response function reduces to

$$\mathcal{F}_{\rm acc}(\Omega, T) = -\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}{\rm d}k\, f(k) f(-k)\int_{-\infty}^{\infty}{\rm d}x\,\frac{{\rm e}^{{\rm i}(k-\Omega)}}{(x-{\rm i}b_n)^2}. \tag{75}$$

The above contour integral can be performed in the same fashion as in the previous subsection, to give the following result:

$$\mathcal{F}_{\rm acc}(\Omega, T) = \frac{1}{\pi^2}\sum_{n=-\infty}^{0}{\rm e}^{\Omega b_n}\int_{\Omega T}^{\infty}{\rm d}p\,\frac{\exp-(p b_n/T)}{(1+p^2)^2}(p - \Omega T) + \frac{1}{\pi^2}\sum_{n=1}^{\infty}{\rm e}^{\Omega b_n}\int_{-\Omega T}^{\infty}{\rm d}p\,\frac{\exp(p b_n/T)}{(1+p^2)^2}(p + \Omega T), \tag{76}$$

where $p = kT$. When $T \to \infty$, the $\exp-(pb_n/T)$ factors in the integrand reduce to unity and the lower limit of the integrals are ∞ and $-\infty$, respectively. As the limits coincide the first integral vanishes. In the second integral, only the second term contributes, the first term being an odd function it reduces to zero on integration under symmetric limits. Thus, in the $T \to \infty$ limit, we recover the thermal spectrum after ϵ is set to zero:

$$\lim_{T\to\infty}\left\{\frac{\mathcal{F}_{\rm acc}(\Omega, T)}{T}\right\} = \frac{\Omega}{2\pi}\sum_{n=1}^{\infty}{\rm e}^{-2\pi g^{-1}\Omega n} = \frac{1}{2\pi}\left(\frac{\Omega}{{\rm e}^{2\pi\Omega g^{-1}} - 1}\right). \tag{77}$$

As mentioned before, $(\mathcal{F}_{\rm acc}(\Omega, T)/T)$ is to be interpreted as the rate of transition probability of the detector. When the $T \to 0$ limit is considered keeping $\epsilon \neq 0$, all the integrands in (76) decay exponentially thereby giving a null result.

Before concluding this section we shall provide an asymptotic formula for the detector response with any smooth window function of the form $W(\tau/T)$. This is a direct generalization of the results in (57)–(61). For such a window function we can write

$$\mathcal{F}(\Omega, T) = \int_{-\infty}^{\infty} {\rm d}\tau \int_{-\infty}^{\infty} {\rm d}\tau'\, W(\tau, T) W(\tau', T) {\rm e}^{-{\rm i}\Omega(\tau-\tau')} G^{+}(\tau - \tau')$$
$$= W\left({\rm i}\frac{\partial}{\partial\Omega}, T\right) W\left(-{\rm i}\frac{\partial}{\partial\Omega}, T\right) \mathcal{F}(\Omega). \tag{78}$$

Expanding $W(\tau, T) = W(\tau/T)$ as a Taylor series around $\tau = 0$ and assuming that $W(0) = 1$, $W'(0) = 0$, i.e.

$$W\left(\frac{\tau}{T}\right) \simeq W(0) + W'(0)\left(\frac{\tau}{T}\right) + \tfrac{1}{2} W''(0)\left(\frac{\tau}{T}\right)^2 \simeq 1 + \tfrac{1}{2} W''(0)\left(\frac{\tau}{T}\right)^2, \tag{79}$$

we obtain that

$$\mathcal{F}(\Omega, T) \simeq \left(1 - \frac{W''(0)}{2T^2}\, \frac{\partial^2}{\partial\Omega^2}\right)^2 \mathcal{F}(\Omega) \simeq \mathcal{F}(\Omega) - \frac{W''(0)}{T^2}\, \frac{\partial^2[\mathcal{F}(\Omega)]}{\partial\Omega^2}. \tag{80}$$

This gives the rate to be

$$\mathcal{R}(\Omega, T) = \mathcal{R}(\Omega) - \frac{W''(0)}{T^2}\, \frac{\partial^2[\mathcal{R}(\Omega)]}{\partial\Omega^2} + {\rm O}\left(\frac{1}{T^4}\right), \tag{81}$$

for any window function and trajectory. Note that the response at finite T depends on the derivatives of the window function, e.g. $W''(0)$. Hence, if the detector is switched on abruptly, these derivatives will diverge, thereby leading to divergent responses. We shall discuss this case explicitly in the following section.

3.3. *A rectangular window function (sum of two step-functions)*

In this section we study the response of a detector that has been switched on and off abruptly. The detector response integral for this case is given by (20) and when the transformations (21) are carried out it reduces to (23), i.e.

$$\mathcal{F}(\Omega, T) = \int_{-2T}^{2T} {\rm d}x\, {\rm e}^{-{\rm i}\Omega x} G^{+}(x)(2T - |x|). \tag{82}$$

For the response of an inertial detector that is turned on and off abruptly, the integrals to be evaluated are

$$\mathcal{F}_{\rm ine1}(\Omega, T) = -\frac{T}{2\pi^2} \int_{-2T}^{2T} {\rm d}x\, \frac{{\rm e}^{-{\rm i}\Omega x}}{(x - {\rm i}\epsilon)^2}\, ; \qquad \mathcal{F}_{\rm ine2}(\Omega, T) = \frac{1}{4\pi^2} \int_{-2T}^{2T} {\rm d}x\, \frac{{\rm e}^{-{\rm i}\Omega x}|x|}{(x - {\rm i}\epsilon)^2} \tag{83}$$

so that $\mathcal{F}_{\rm ine}(\Omega, T) = \big(\mathcal{F}_{\rm ine1}(\Omega, T) + \mathcal{F}_{\rm ine2}(\Omega, T)\big)$. The integral $\mathcal{F}_{\rm ine1}$ can be evaluated on a rectangular contour in the lower half of the complex x plane with its vertices at $(-2T, 0)$, $(2T, 0)$, $(2T, -{\rm i}\infty)$ and $(-2T, -{\rm i}\infty)$. $\mathcal{F}_{\rm ine2}$ has a $|x|$ term in the integrand and hence it can be rewritten as a sum of two integrals over the limits $(-2T, 0)$ and $(0, 2T)$. The integrals that have to be evaluated are

$$\mathcal{F}_{\rm ine2}(\Omega, T) = \frac{1}{4\pi^2} \left\{ \int_{0}^{2T} {\rm d}x\, \frac{{\rm e}^{{\rm i}\Omega x}\, x}{(x + {\rm i}\epsilon)^2} + \int_{0}^{2T} {\rm d}x\, \frac{{\rm e}^{-{\rm i}\Omega x}\, x}{(x - {\rm i}\epsilon)^2} \right\}. \tag{84}$$

The first of these integrals can be evaluated on a rectangular contour in the upper half of the complex x plane with vertices at $(0, 0)$, $(2T, 0)$, $(2T, \mathrm{i}\infty)$ and $(0, \mathrm{i}\infty)$ and the second integral can be evaluated on another rectangular contour, this time in the lower half of the complex x plane with its vertices at $(0, 0)$, $(2T, 0)$, $(2T, -\mathrm{i}\infty)$ and $(0, -\mathrm{i}\infty)$. After the integrations along the contours mentioned above and some simple algebraic manipulations, we obtain the response of an inertial detector that has been switched on abruptly for a finite time interval T to be

$$\mathcal{F}_{\mathrm{ine}}(\Omega, T) = \frac{1}{4\pi^2}\left\{2\int_0^\infty \mathrm{d}v\, \frac{\mathrm{e}^{-\Omega v}\, v}{(v+\epsilon)^2} - \mathrm{e}^{2\mathrm{i}\Omega T}\int_0^\infty \mathrm{d}v\, \frac{\mathrm{e}^{-\Omega v}\, v}{(v+\epsilon-2\mathrm{i}T)^2} - \mathrm{e}^{-2\mathrm{i}\Omega T}\int_0^\infty \mathrm{d}v\, \frac{\mathrm{e}^{-\Omega v}\, v}{(v+\epsilon+2\mathrm{i}T)^2}\right\}. \tag{85}$$

For a finite T, if we take the limit $\epsilon \to 0$, the second and the third integrals in the above result remain finite; but the first integral diverges logarithmically. Hence $\mathcal{F}_{\mathrm{ine}}$ is divergent for all finite T. The appearance of this ultraviolet divergence has been reported in [9].

It was shown towards the end of the previous subsection that the response of a finite-time detector and its rate involve the derivatives of the window function. The rectangular window function we consider here is continuous but has derivatives which diverge at $\tau = -T$ and $\tau = T$. The origin of the logarithmic divergences in $\mathcal{F}_{\mathrm{ine}}$ and $\mathcal{R}_{\mathrm{ine}}$ when ϵ is set to zero for a finite T can be attributed to these divergent derivatives.

The two relevant limits, $T \to 0$ and $T \to \infty$, however, give sensible results. When $T \to 0$, the second and the third integrals exactly cancel the first thus giving $\mathcal{F}_{\mathrm{ine}} = 0$, provided we keep $\epsilon \neq 0$. For large T, i.e. when $T \to \infty$, the rate $\mathcal{R}_{\mathrm{ine}} = (\mathcal{F}_{\mathrm{ine}}/T)$ vanishes because $\mathcal{F}_{\mathrm{ine}}$ is bounded (when $\epsilon \neq 0$) and well defined.

For a small T and a finite ϵ, such that $T < \epsilon$ the integrands in (85) can be Taylor expanded in T and the result up to $\mathrm{O}(T^2)$ is

$$\mathcal{F}_{\mathrm{ine}}(\Omega, T) \simeq \frac{1}{\pi^2}\left\{2\Omega^2 T^2\, I_1 + \frac{2\Omega T^2}{\epsilon}\, I_2 + \frac{6T^2}{\epsilon^2}\, I_3\right\} \tag{86}$$

where

$$I_1 = \int_0^\infty \mathrm{d}y\, \frac{\mathrm{e}^{-\Omega\epsilon y}\, y}{(y+1)^2}; \qquad I_2 = \int_0^\infty \mathrm{d}y\, \frac{\mathrm{e}^{-\Omega\epsilon y}\, y}{(y+1)^3}; \qquad I_3 = \int_0^\infty \mathrm{d}y\, \frac{\mathrm{e}^{-\Omega\epsilon y}\, y}{(y+1)^4}. \tag{87}$$

i.e. the detector response decays as T^2 for $T \to 0$.

For the finite-time response of an accelerated detector, the integrals to be evaluated are similar to those of the inertial case. The response function is given by

$$\mathcal{F}_{\mathrm{acc}}(\Omega, T) = -\frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty}\int_{-T}^{T}\mathrm{d}\tau' \int_{-T}^{T}\mathrm{d}\tau\, \frac{\mathrm{e}^{-\mathrm{i}\Omega(\tau-\tau')}}{(\tau-\tau'-\mathrm{i}b_n)^2} \tag{88}$$

where $b_n = \epsilon - 2\pi g^{-1} n$. After carrying out the transformations (21) we obtain the response function

$$\mathcal{F}_{\mathrm{acc}}(\Omega, T) = \sum_{n=-\infty}^{\infty}\mathcal{F}_{\mathrm{acc}1n}(\Omega, T) + \mathcal{F}_{\mathrm{acc}2n}(\Omega, T) \tag{89}$$

where

$$\mathcal{F}_{\mathrm{acc}1n}(\Omega, T) = -\frac{T}{2\pi^2}\int_{-2T}^{2T}\mathrm{d}x\, \frac{\mathrm{e}^{-\mathrm{i}\Omega x}}{(x-\mathrm{i}b_n)^2}; \qquad \mathcal{F}_{\mathrm{acc}2n}(\Omega, T) = \frac{1}{4\pi^2}\int_{-2T}^{2T}\mathrm{d}x\, \frac{\mathrm{e}^{-\mathrm{i}\Omega x}\,|x|}{(x-\mathrm{i}b_n)^2}. \tag{90}$$

$\mathcal{F}_{\mathrm{acc1}n}(\Omega, T)$ can be evaluated on a rectangular contour with its vertices at $(-2T, 0)$, $(2T, 0)$, $(2T, -\mathrm{i}\infty)$ and $(-2T, -\mathrm{i}\infty)$. This contour encloses the poles corresponding to the values of n between one and infinity. $\mathcal{F}_{\mathrm{acc2}n}(\Omega, T)$, after it has been split into two integrals with the limits $(-2T, 0)$ and $(0, 2T)$, is given by

$$\mathcal{F}_{\mathrm{acc2}n}(\Omega, T) = \frac{1}{4\pi^2}\left\{\int_0^{2T} \mathrm{d}x\, \frac{\mathrm{e}^{\mathrm{i}\Omega x}\, x}{(x+\mathrm{i}b_n)^2} + \int_0^{2T} \mathrm{d}x\, \frac{\mathrm{e}^{-\mathrm{i}\Omega x}\, x}{(x-\mathrm{i}b_n)^2}\right\}. \tag{91}$$

The first of these integrals can be evaluated on a rectangular contour in the upper half of the complex x plane with vertices at $(0, 0)$, $(2T, 0)$, $(2T, \mathrm{i}\infty)$ and $(0, \mathrm{i}\infty)$. For evaluating the second integral in (91), a contour with vertices at $(0, 0)$, $(2T, 0)$, $(2T, -\mathrm{i}\infty)$ and $(0, -\mathrm{i}\infty)$ can be chosen and the poles that lie on the contour for the values of n between one and infinity can be avoided with an indentation of the contour so that they are left outside. The indentations on the contours contribute a residue corresponding to the infinitesimal semi-circle around the pole. After carrying out the contour integrations and some algebraic manipulation, we obtain the complete accelerated detector response to be

$$\begin{aligned}\mathcal{F}_{\mathrm{acc}}(\Omega, T) = \frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty}\Bigg\{&4\pi\Omega T\,\Theta(n)\,\mathrm{e}^{\Omega b_n} + 2\int_0^{\infty}\mathrm{d}v\,\frac{\mathrm{e}^{-\Omega v}\,v}{(v+b_n)^2}\\ &- \mathrm{e}^{2\mathrm{i}\Omega T}\int_0^{\infty}\mathrm{d}v\,\frac{\mathrm{e}^{-\Omega v}\,v}{(v+b_n-2\mathrm{i}T)^2} - \mathrm{e}^{-2\mathrm{i}\Omega T}\int_0^{\infty}\mathrm{d}v\,\frac{\mathrm{e}^{-\Omega v}\,v}{(v+b_n+2\mathrm{i}T)^2}\Bigg\},\end{aligned} \tag{92}$$

where $\Theta(n) = 1$ for $n > 0$ and zero otherwise. (When the pole happens to settle right on the axis of integration in any of the integrals in the above expression, the result of the integral over that axis is assumed to be given by the principal value of the integral.)

The nature of the divergence in (92) is the same as that of (85). This is because for a finite T when ϵ is set to zero it is the second integral in the above expression that diverges logarithmically (when $n = 0$ in the sum), which is exactly the term that exhibits a divergence for the case of the inertial detector.

In the limit $T \to 0$, for a non-zero ϵ, (92) reduces to zero, the first term vanishing identically being proportional to T, the second term being cancelled by the third and the fourth; whereas in the infinite-time limit, concentrating on the transition probability rate, we obtain that

$$\mathcal{R}_{\mathrm{acc}}(\Omega) = \lim_{T\to\infty}\left\{\frac{\mathcal{F}_{\mathrm{acc}}(\Omega, T)}{2T}\right\} = \frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty} 2\pi\Omega\,\Theta(n)\,\mathrm{e}^{\Omega b_n} = \frac{1}{2\pi}\left(\frac{\Omega}{\mathrm{e}^{2\pi\Omega g^{-1}} - 1}\right), \tag{93}$$

the thermal spectrum, the other terms in (92) vanishing when divided by the infinite time interval.

4. Conclusions

To clearly illustrate the conclusions we wish to draw from the analysis we have carried out in this paper, we tabulate here the response of an inertial detector for $T \to 0$, finite T and $T \to \infty$, when the limits on ϵ and T are taken in different orders. (Note that $\mathcal{F}_{\mathrm{ine}}$ and $\mathcal{R}_{\mathrm{ine}}$ are functions of ϵ before it is set to zero.)

	Gaussian	Exponential	Rectangular
$\lim_{T\to 0}\lim_{\epsilon\to 0}\mathcal{F}_{\rm ine}(\Omega,\epsilon,T)$	$(1/4\pi)$	$(1/2\pi^2)$	$(\ln(\epsilon)-\ln(T))$ (Divergence)
$\lim_{\epsilon\to 0}\lim_{T\to 0}\mathcal{F}_{\rm ine}(\Omega,\epsilon,T)$	0	0	0
$\lim_{T\to \infty}\lim_{\epsilon\to 0}\mathcal{R}_{\rm ine}(\Omega,\epsilon,T)$	0	0	$\ln(\epsilon)$ (Divergence)
$\lim_{\epsilon\to 0}\lim_{T\to \infty}\mathcal{R}_{\rm ine}(\Omega,\epsilon,T)$	0	0	0
$\lim_{\epsilon\to 0}\ \ T\neq 0\ \ \mathcal{F}_{\rm ine}(\Omega,\epsilon,T)$	Finite	Finite	$\ln(\epsilon)$ (Divergence)
$\lim_{T\to 0}\ \ \epsilon\neq 0\ \ \mathcal{F}_{\rm ine}(\Omega,\epsilon,T)$	0	0	0
$\lim_{T\to \infty}\ \ \epsilon\neq 0\ \ \mathcal{R}_{\rm ine}(\Omega,\epsilon,T)$	0	0	0

(In the last column of the above table whenever divergences arise we have just quoted the divergent terms, dropping the finite expressions.) The second and the fourth rows of the above table imply that when ϵ is kept non-zero the response of an inertial detector and its rate go to zero as $T\to 0$ and $T\to\infty$, respectively, for all window functions. This is just reiterated in the last two rows. When the $\epsilon\to 0$ limit is taken first, and the T is set to zero after that, as the first row of the above table shows, the detector response does not go to zero and, in fact, for the case of the rectangular window function, logarithmic divergences are encountered. When the $T\to\infty$ limit is considered after having set $\epsilon=0$ (third row) and when the detector has been switched on with the rectangular window function, logarithmic divergences appear in the detector response rate. Finally, for a finite T when ϵ has been set to zero (fifth row) logarithmic divergences arise again in the detector response for the case of the rectangular window function. The divergences that are listed in the first and the fifth rows of the above table for the case of the rectangular window function have been reported earlier in the literature (see [9]).

The role played by ϵ in producing the finite result for the different limits is by now obvious. In fact, by keeping ϵ finite until the end we are effectively introducing an ultraviolet cut-off. This can be seen by expressing the Wightman function (15) as

$$G^+(x,x')=\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3\,2\omega}\,\mathrm{e}^{-\mathrm{i}\omega(t-t'-\mathrm{i}\epsilon)+\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}=\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3\,2\omega}\,\mathrm{e}^{-\mathrm{i}\omega(t-t')+\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}\,\mathrm{e}^{-\epsilon\omega},\tag{94}$$

where $\omega=|\boldsymbol{k}|$. The results for the limits $T\to 0$ and $T\to\infty$ remain sensible even after the cut-off is removed, provided it is done right at the end. It may be argued that our procedure of taking $T\to 0$ before ϵ is set to zero is mathematically dubious on the grounds that the formulae are only reliable in the region $T\gg\epsilon$. Even then, we may justify the procedure on *physical* grounds as follows: it is likely that the standard quantum field theory breaks down for trans-Planckian frequencies, because of which it would be natural to introduce some ultraviolet cut-off parameter ϵ; hence it would be appropriate to take the limit $T\to 0$ with a finite ϵ. (We thank the referee for discussions on this point.)

The logarithmic divergences that appear in the response of a detector (for a finite T) when it is switched on abruptly can be attributed to the discontinuities that arise in the derivatives of the window function. (We would like to mention here that the authors of [10] had reached a similar conclusion earlier.) These divergences are certainly not the infinities that are inherent to quantum field theory, for had they been so, the response functions of the detectors would have diverged irrespective of the manner in which the detectors are switched on and off. To stress again, we state here the most important conclusion of our analysis: *the divergences appearing in the response of detectors that have been turned on and off abruptly are due to the discontinuities in the window function rather than the divergences of the underlying field theory.*

In what follows, we shall touch upon the relevance of the present work in a somewhat broader context.

In bringing together the principles of quantum theory and general relativity one notices a major issue of conflict: general relativity is inherently local in its description while the conventional formulation of field theory uses global structures to define even the most primitive concepts like the vacuum state. This point has been repeatedly made in the literature related to quantum gravity. However, it should also be noted that there is another, operational angle to the quantum theory as well. Quantum mechanics emphasizes the role of the operational definition of physical quantities including that of the quantum state. As a matter of principle, the same philosophy should be applicable to the field theory as well. In other words, one would like to define concepts, like the vacuum state, etc, in field theory using purely operational procedures similar to the ones used, for example, in defining the spin of an electron by using a magnetic field selector.

It is, however, well known that such procedures are exceedingly difficult to formulate in the case of a relativistic field. The role of detectors assumes special importance in this context. The work by Unruh and DeWitt comes closest to the operational definition of quantum states in field theory. In a simplified sense this detector model captures the essence of the actual particle detection which takes place in the laboratory. There is, however, one difficulty in the original Unruh–DeWitt model. This model uses the definition for detection which is based on asymptotic states. The calculations are done to estimate the transition probability from past infinity to future infinity. In any laboratory context, any detection is local in both space and time.

The analysis in this paper makes a first attempt in investigating the possibility of an inherently local definition of particle detection both in space and time. We have resolved the difficulties which arise in such a definition and we have provided general formulae to calculate the response of detectors which have been coupled to the field only for a finite interval of time. In a future publication we plan to investigate how these detectors respond in curved spacetimes in $(3+1)$ dimensions while on geodesic and non-geodesic trajectories. Since these toy-models mimic the physical situation as regards locality in space and time, we expect the results to shed some light on the operational definition of quantum processes in curved spacetime.

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