

Inflation With Oscillations

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In this paper we investigate the general features of "Oscillatory Inflation". In adiabatic approximation, we derive a general formula for the number of e-foldings \tilde{N} which reduces to the standard expression in case of the slow roll approximation and leads to the Damour-Mukhanov type expression for the slowly varying adiabatic index. We apply our result to the logarithmic potential and arrive at a simple and elegant formula for the number of e-foldings. We evolve the field equations numerically and observe a remarkable agreement with the analytical result.

I. INTRODUCTION

The inflationary universe scenario has become an integral part of the standard model of universe. It does resolve the outstanding problems of the standard model like the flatness problem, the horizon problem, the homogeneity and isotropy problem etc. It also provides an important clue for the origin of structure formation in the universe [1,2, 3]. The underlying idea of inflation is that there was an epoch when the universe was slowly rolling down the flat wings of the scalar potential such that vacuum energy was dominant leading to the exponential growth of the scale factor. At the end of slow roll, the universe falls into the core of the potential and fast oscillates near the minimum leading to the re-heating ultimately. Damour and Mukhanov have discussed the possibility of inflation during the period of oscillations [4]. Further discussion on the same theme can be found in reference [5]. In fact, such a possibility was first pointed out by Turner[6]. Inflation with oscillation can be realized by a non-convex potential $V(\phi)$ ($V_{,\phi\phi} < 0$) with small convex core near the minimum of the potential and with large flat wings such that universe spends most of the time away from the core and inflates. In this paper we study general aspects of inflation during oscillations. We derive general expression for the number of e-folds which reduce to the standard formula in the slow roll regime and leads to the Damour-Mukhanov type expression for slowly varying adiabatic index. We apply our result to the logarithmic potential and show that the model exhibits simple analytical solution which has remarkable agreement with the numerical simulation.

II. EVOLUTION EQUATIONS AND CRITERIA OF INFLATION WITH OSCILLATIONS:

The field evolution equations in the Freedman cosmology have the form

$$\frac{d}{dt} \left(\frac{1}{2} \dot{\phi}^2 + V \right) = -3H\dot{\phi}^2 \quad (1)$$

$$H^2 = \frac{8\pi}{3M^2} \rho \quad (2)$$

$$\dot{\rho} = -3H(\rho + p) = -3H\dot{\phi}^2 \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M^2}(\rho + 3p) \quad (4)$$

where

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (5)$$

$$p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad (6)$$

Equations (3) and (4) are not independent of (1) and (2) but will be useful in view of forthcoming discussion. Equation (3) reveals that the energy density in scalar field ϕ decreases because of the red shifting away of the kinetic part $\frac{\dot{\phi}^2}{2}$.

In what follows , we shall assume that the potential is even and has minimum at $\phi = 0$. When the field initially being displaced from the minimum of the potential, rolls below its slow roll value, the coherence oscillation regime $\omega \gg H$ commences. The evolution equation can then be approximately solved by separating the two times scales namely the fast oscillation time scale and the longer expansion time scale. On the first time scale, the Hubble expansion can be neglected, and one obtain ϕ as a function of time;

$$t - t_0 = \pm \int \frac{1}{\sqrt{2(V_m - V(\phi))}} d\phi, \quad (7)$$

where $\rho \equiv V_m \equiv V(\phi_m)$; V_m being the maximum current value of the potential energy and ϕ_m being the field amplitude . On the longer time scale ρ and ϕ_m slowly decrease because Hubble damping term in equation (1) .The average adiabatic index γ is defined as ,

$$\gamma = \left\langle \frac{\rho + p}{\rho} \right\rangle = \left\langle \frac{\dot{\phi}^2}{\rho} \right\rangle \quad (8)$$

where $\langle . \rangle$ denotes the time average over one oscillation . Equation (4) then tells that expansion during oscillations would continue ($\ddot{a} > 0$) if $\gamma < \frac{2}{3}$. The adiabatic evolution of $a(t)$ and ρ is given by ,

$$a(t) \propto t^{\frac{2}{3\gamma}} \quad (9)$$

$$\rho \equiv V(\phi_m) \propto t^{-2} \quad (10)$$

As $\ddot{\phi} = -\frac{dV}{d\phi}$, the condition $\gamma < \frac{2}{3}$ can equivalently be written [6] ,

$$\begin{aligned} \gamma &= \left\langle \frac{\dot{\phi}^2}{\rho} \right\rangle = \frac{\langle \phi V_{,\phi} \rangle}{V_m} = 2\left(1 - \frac{\langle V \rangle}{V_m}\right) \\ &= 2 \frac{\int_0^{\phi_m} (1 - V(\phi)/V_m)^{\frac{1}{2}} d\phi}{\int_0^{\phi_m} (1 - V(\phi)/V_m)^{-\frac{1}{2}} d\phi} < \frac{2}{3} \end{aligned} \quad (11)$$

It is suggestive to write (11) in an equivalent form,

$$\langle U(\phi) \rangle \equiv \langle V - \phi V_{,\phi} \rangle > 0 \quad (12)$$

The expression (12) has a simple geometric meaning; the average over one oscillation of the intercept $U(\phi)$ at ϕ should be positive to ensure accelerated expansion. In models of inflation with oscillation the slow roll value of the field ϕ_s is " far away" from convex core given by ϕ_c such that $\langle U \rangle$ is positive before ϕ rolls below certain critical value of the field ϕ_{cr} Fig.1 . As a result the universe undergoes accelerated expansion ($\ddot{a} > 0$). This is because the average of intercept $U(\phi)$ is negative for $0 < \phi < \phi_c$ and the positive contribution to $\langle U(\phi) \rangle$ starts coming only as ϕ gets bigger than ϕ_c . Obviously one has to take ϕ further to some ϕ_{cr} in order to cancel the negative contribution to $\langle U(\phi) \rangle$ coming from the convex core . In certain sense the ϕ_{cr} defines the effective size of convex core . Clearly , as ϕ rolls below ϕ_{cr} , the average of $U(\phi)$ over one oscillation turns negative and consequently inflation ceases loosing to oscillations without accelerated expansion . Hence for inflation to occur during oscillations , the field amplitude ϕ_m should very between ϕ_{cr} and ϕ_s . We shall call this regime " oscillatory inflation regime " .

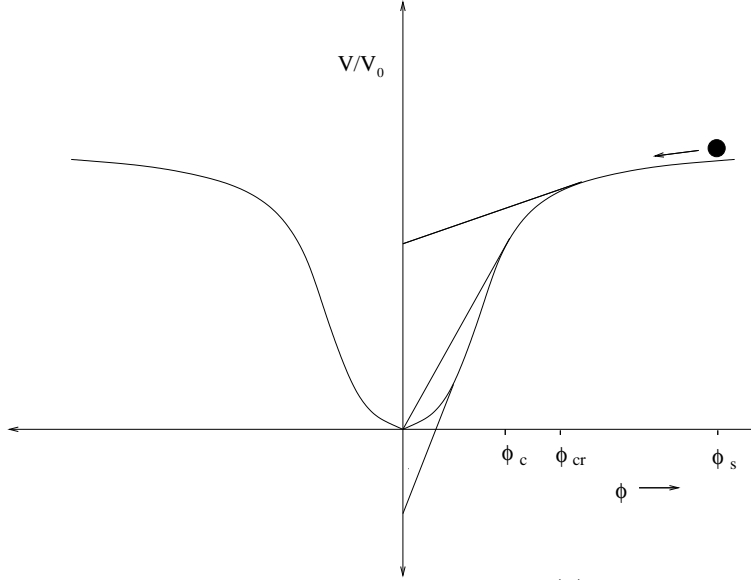


FIG. 1. Qualitative picture of an effective potential showing the intercept $U(\phi)$ of the tangent to the curve $V(\phi)$ at various values of the field ϕ . For $\phi < \phi_c$, the value of the intercept is negative. $U(\phi)$ is positive for $\phi > \phi_c$. The critical field ϕ_{cr} is fixed such that the positive contribution coming from the interval $\phi_c < \phi < \phi_{cr}$ to the average intercept $\langle U(\phi) \rangle$ exactly cancel the negative contribution of the convex core. $\langle U(\phi) \rangle$ is positive as ϕ varies between ϕ_{cr} and ϕ_s and the universe expands in this regime called the "oscillatory inflation regime".

III. MODEL:

We shall study here the logarithmic potential which may implement inflation with oscillations,

$$V(\phi) = V_0 \ln [1 + (\phi/\phi_c)^2] \quad (13)$$

This is the q tending to zero limit of the potential suggested by Damour and Mukhanov,

$$V(\phi) = \frac{V_0}{q} \left[\left(\frac{\phi^2}{\phi_c^2} + 1 \right)^{q/2} - 1 \right] \quad (14)$$

The potential has a convex core near the bottom of potential given by ϕ_c whereas $V_{,\phi\phi}$ is negative away from the core. For $\phi \gg \phi_c$, $V(\phi) = 2V_0 \ln(\phi/\phi_c)$ and consequently $\gamma \simeq 1/\ln(\phi_m/\phi_c) \equiv 1/\ln(\beta)$

A. critical value of field

In order to compute the size of inflation, it would be necessary to determine $\langle U \rangle$:

$$\langle U \rangle = \int_0^1 (V(x, \beta) - xV_{,x}(x, \beta)) dx \quad (15)$$

where $\beta \equiv \phi_m/\phi_c$, $x \equiv \phi/\phi_m$ and $V(x, \beta) = V_0 \ln(1 + \beta^2 x^2)$ in case of the potential given by (13). Since only the sign of $\langle U \rangle$ is important we have omitted several positive constants in the expression (15). The average intercept $\langle U \rangle$ is plotted in Fig. 2 in case of the logarithmic potential under consideration. The value of β where $\langle U \rangle$ changes sign precisely determines the critical value of the field or equivalently β_{cr} . For the logarithmic potential β_{cr} turns out to be nearly equal to 3.2.

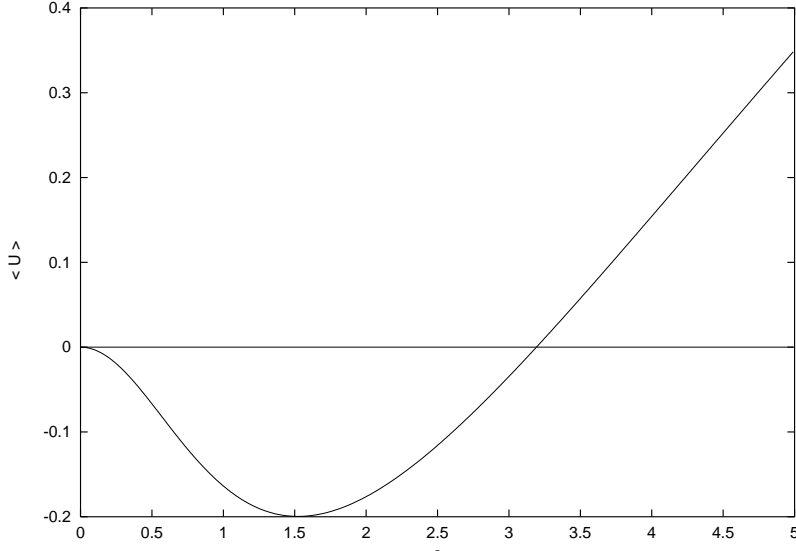


FIG. 2. Plot of the average intercept $\langle U(\phi) \rangle$ versus $\beta \equiv \phi_m/\phi_c$ for the potential given by Eq.(13). $\langle U(\phi) \rangle$ changes sign as ϕ_m becomes nearly equal to $3.2\phi_c$ indicating the beginning of oscillatory inflation regime.

B. slow roll field:

The slow roll parameter ϵ is defined as :

$$\epsilon = \frac{M^2}{16\pi} \left[\frac{V_{,\phi}}{V} \right]^2 \quad (16)$$

The slow roll ends when $\epsilon \simeq 1$. Using (16) yields the slow roll value of the field ϕ_s ,

$$\sqrt{16\pi} \times \beta_s \ln \beta_s \simeq M/\phi_c \quad (17)$$

where $\beta_s = \frac{\phi_s}{\phi_c}$ and ϕ_s depends upon ϕ_c . It should be emphasized that β_s or equivalently ϕ_s would always depend upon ϕ_c in case of non-power law potentials.

C. Number of e-folds:

The number of e-folds is defined by ,

$$N = \ln \frac{a_f}{a_i}, \quad (18)$$

Since the scale factor "a" depends upon ϕ_m or equivalently upon β , (18) should be used carefully . We divide the interval $[\beta_{cr}, \beta_s]$ into small slices , $\Delta\beta_j = \beta_{j+1} - \beta_j$ with $j = 1, M$ such that $\beta_1 = \beta_{cr}$ and $\beta_M = \beta_s$. The number of e-foldings obtained for the interval $\Delta\beta_j$ is given by ,

$$N(\beta_j)\Delta\beta_j = -\ln \frac{a(\beta_{j+1})}{a(\beta_j)}$$

where $a_{j+1} < a_j$ for universe rolling down from β_s to β_{cr} using (9) and (10) leads to the following expression ,

$$N_j\Delta\beta_j = \frac{1}{3\gamma(\beta)} \ln (V(\beta_{j+1})/V(\beta_j))$$

Expanding $V(\beta_{j+1})$ in Teller series in the neighborhood of β_j and using $\ln(1+x) \simeq x$ for $x \ll 1$ we get ,

$$N_j \Delta\beta_j = \left(\frac{1}{3\gamma(\beta_j)} \right) \frac{V'(\beta_j)}{V(\beta_j)} \Delta\beta_j \quad (19)$$

Integrating (19) from β_{cr} to β_s one obtains the total number of e-foldings ,

$$N = \int_{\beta_{cr}}^{\beta_s} N(\beta) d\beta$$

where

$$N(\beta) = \frac{V'(\beta)/V(\beta)}{3\gamma(\beta)}$$

$N(\beta)$ can be interpreted as the density of e-foldings with regard to β . The correct definition , however , should take into account the change in the comoving Hubble length [7] ,

$$\tilde{N} = \ln \left(\frac{a_f H_f}{a_i H_i} \right) \quad (20)$$

Similar arguments as above can be used to obtain the expression for \tilde{N} ,

$$\tilde{N} = \int_{\beta_{cr}}^{\beta_s} \tilde{N}(\beta) d\beta \quad (21)$$

where

$$\tilde{N}(\beta) = \left(\frac{1}{3\gamma(\beta)} - \frac{1}{2} \right) \frac{V'(\beta)}{V(\beta)} \quad (22)$$

Expression (22) formally written for oscillatory regime presents a general result for the number e-foldings . In fact ,in slow roll approximation ,

$$\gamma = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} + 1 \simeq \frac{\dot{\phi}^2}{V} \simeq \left(\frac{M^2}{24\pi} \right) \left(\frac{V'}{V} \right)^2$$

Substituting this expression in (22) leads to the standard formula for the number of e-foldings [2].Needless to say that \tilde{N} nearly coincides with N in case of the slow role regime .In case γ is a slowly varying function of β , \tilde{N} assumes the following form ,

$$\tilde{N} = \left(\frac{1}{3\gamma} - \frac{1}{2} \right) \ln \left[\frac{V(\beta_s)}{V(\beta_{cr})} \right] \quad (23)$$

For a power law potential (23) readily reduces to the expression obtained by Liddle and Majumdar [6].

IV. NUMERICAL SIMULATION AND ANALYTICAL RESULTS

We will show that a simple and elegant analytical formula for the number of e-folding can be obtained in case of the logarithmic potential (13). As mentioned earlier that $\gamma \simeq 1/\ln(\beta)$ in the present case [4,5].Substituting the approximate value of γ in (22) leads to a simple expression for $\tilde{N}(\beta)$,

$$\tilde{N}(\beta) = \frac{1}{3\beta} - \frac{1}{2\beta \ln \beta} \quad (24)$$

Using (21) we obtain the formula for the total number of e-foldings ,

$$\tilde{N} = \frac{1}{3} \ln \frac{\beta_s}{\beta_{cr}} - \frac{1}{2} \ln \left(\frac{\ln \beta_s}{\ln \beta_{cr}} \right) \quad (25)$$

It should be mentioned that we have used the approximate value of γ to obtain the above expression for \tilde{N} . Since the approximation holds good away from the core and little contribution to the number of e-foldings comes from the regions near the core, we would expect that the expression (25) gives the correct value of \tilde{N} . We have evolved the field equations numerically and studied the evolution of the number of e-foldings. Fig2 displays the numerical evolution of \tilde{N} for two different values of ϕ_c . The dependence of β_s on ϕ_c can in general be red off from (17). The heights of maxima of \tilde{N} are described by (25). We see the remarkable agreement of our analytical result with the numerical simulation. As emphasized earlier [4, 5] the number of e-foldings \tilde{N} is small. Even if ϕ_c is pushed to electro-weak scale \tilde{N} does not exceed 10. One might think that brane world inflation would be helpful here as the brane corrections enhances the prospects of inflation [8, 9, 10]. Unfortunately , this is not so as the brane induced term increases the Hubble expansion rate which is not relevant during oscillations.

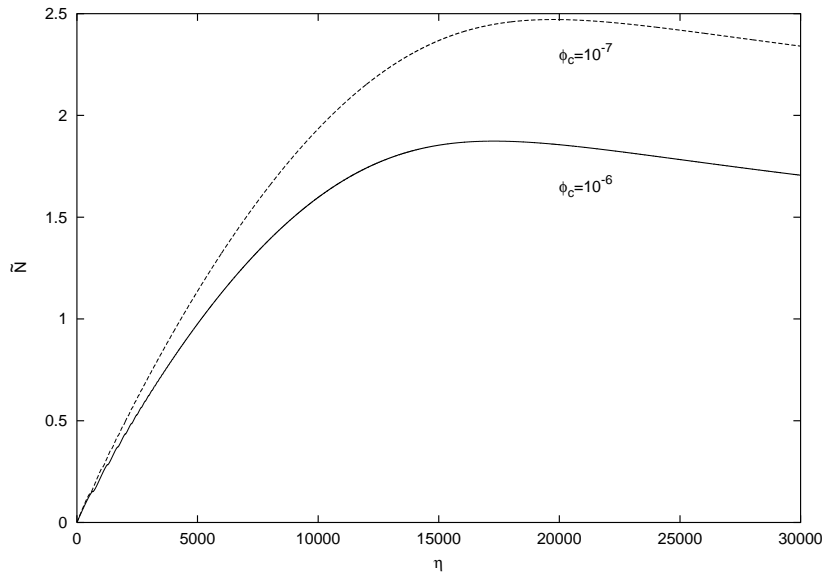


FIG. 3. Plot of \tilde{N} versus $\eta \equiv tM$ based upon numerical simulation for two choices of ϕ_c . The evolution takes place from the end of slow roll and counts the number of e-foldings during the oscillations. The heights of maxima give the actual number of e-foldings described by (25).

We have described the general features of oscillatory inflation. We have been able to obtain a general expression for the number of e-foldings and demonstrated its consistency. Applying the same to the logarithmic potential we obtained an elegant and simple analytical formula for the number of e-foldings. We evolve the field equations numerically and observe a remarkable agreement of the analytical result with the numerical simulation.

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VI. REFERENCES

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