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# Nonlinear Evolution of Density Perturbations

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**Abstract.** From the epoch of recombination ($z \approx 10^3$) till today, the typical density contrasts have grown by a factor of about $10^6$ in a Friedmann universe with $\Omega = 1$. However, during the same epoch the typical gravitational potential has grown only by a factor of order unity. This fact can be exploited to provide a new, powerful, approximation scheme to study the formation of nonlinear structures in the universe by evolving the initial distribution of matter using a gravitational potential frozen in time. We carry out this scheme for several standard models and discuss the results.

*Key words*: Galaxies: formation—large scale structure of Universe.

It is believed that structures like galaxies and clusters of galaxies formed out of small inhomogeneities via gravitational instability. In this paper we discuss a new approximation scheme called Frozen Potential Approximation (hereafter FPA) (Bagla & Padmanabhan 1994a; Brainerd *et al.* 1993) for studying the nonlinear growth of inhomogeneities and to summarise the results.

The exact trajectory of a particle in a Friedmann universe is described by the equations:

$$\frac{d^2\mathbf{x}}{da^2} + \frac{3}{2a}\frac{d\mathbf{x}}{da} = -\frac{3}{2a}\nabla\psi; \quad \nabla^2\psi = \frac{\delta}{a}, \tag{1}$$

where $a(t)$ is the expansion factor, $\mathbf{x}$ is the comoving coordinate, $\rho_b$ is the background matter density, and $\psi$ is the scaled gravitational potential $\psi = (2/3H_0^2)\phi$. In Zeldovich approximation (ZA) (Zeldovich 1970; Shandarin & Zeldovich 1989), it is assumed that the velocity of a particle remains constant along its trajectory thereby leading to the trajectory $\mathbf{x}(a) = \mathbf{x}_{in} - a\nabla\psi|_{in}$. Zeldovich ansatz can be used to show that the first structures to form in a generic gravitational collapse are planar surfaces of high density, the so called pancakes. The unrealistic part of dynamics in ZA is that particles continue to move (after collapsing and forming a pancake) with the same velocity leading to thickening of pancakes.

It is possible to generate a better approximation (Bagla & Padmanabhan 1994a; Brainerd *et al.* 1993) by freezing the potential at the initial value and evolving the density perturbations to mildly non-linear density contrasts with much more realistic dynamics. The trajectories are now evolved using (1) but with a fixed potential:

$$\frac{d^2\mathbf{x}}{da^2} + \frac{3}{2a}\frac{d\mathbf{x}}{da} = -\frac{3}{2a}\nabla\psi(a_{in}). \tag{2}$$

In FPA, the particles move towards the minima of the potential and then oscillate about it with a decaying amplitude due to the expansion of the Universe. We now discuss several results of this approximation.

The two approximations are compared visually in Fig. 5 which gives slices [with dimensions $(96 \times 96 \times 15)$ $(h^{-1}\,\mathrm{MPc})^3$] of the universe in these schemes at different epochs for CDM normalised to COBE. It is obvious that at early stages ($z \gg 1$) of structure formation, all approximations give similar results, while at later stages ($z \simeq 0$) the differences are quite prominent. Pancakes do not thicken in FPA and, at late stages, clumpy structures dominate. In Fig. 1 we have shown the density contrast $\sigma(r)$ defined as $\sigma^2 \equiv \bar{\xi}$ with $\bar{\xi}(r) = 3J_3(r)/r^3$ for three redshifts for standard CDM using FPA. For comparison we have also plotted the N-body result at $z = 0$.

The FPA can be quite useful in studying the evolution of voids in the universe. One way of estimating the sizes of voids is to compute the rms displacement of particles in a simulation (Shandarin 1992) which can be related to the average diameter of voids by $D_{\mathrm{void}} = 2kd_{\mathrm{rms}}$ where (for spherical voids) $k = 1.3$. The thick line in Fig. 2 shows the evolution of $d_{\mathrm{rms}}$ for CDM and HDM. For the standard CDM model, we find that $d_{\mathrm{rms}} \simeq 9h^{-1}$ MPc at $z = 0$. The behaviour of $d_{\mathrm{rms}}$ can also be studied analytically in some special cases (Bagla & Padmanabhan 1994b). Consider, for example, the motion of particles near a local density minima. The density profile away from the centre of a low density void can be approximated as $\delta = -(r/L)^{-n}$ where $n$ is a positive constant. In this case, equation of motion has one simple solution:

$$r = \left[\frac{3n^2}{(3-n)(n+2)}\right]^{1/n} La^{1/n}. \tag{3}$$

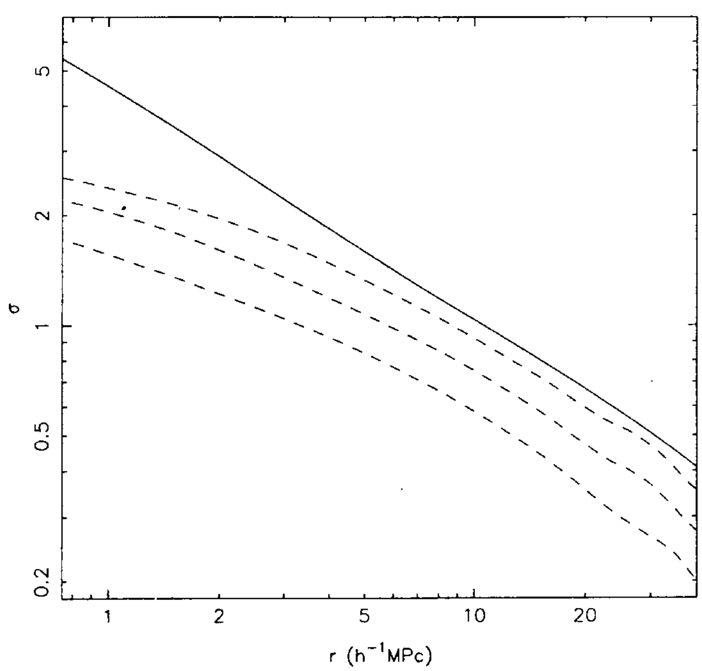


**Figure 1.** Evolution of density contrast in the Frozen Potential Approximation. Also drawn here is the curve for density contrast obtained from $N$-Body for $z = 0$. Here the density contrast is defined as $\sigma \equiv \sqrt{\bar{\xi}}$.

which holds for $n < 3$. For a gaussian random field, the profile of density around a density extrema is a power law with the same index as the correlation function (Bardeen *et al.* 1986 (BBKS)). For our universe, the index of the correlation function is about 1.8, and we expect the voids to grow as $a^{1/1.8}$ or $(1+z)^{-1/1.8}$ in the late stages. However, in the linear limit, we expect the void radius to grow as $a$. While calculating $d_{rms}$ from simulations, we average over regions with different index $n$ and hence expect $d_{rms}$ to grow at a rate intermediate between $a$ and $a^{1/1.8}$. These features are clearly seen in Fig. 2. The two dot-dash lines indicate growth proportional to $a$ and $a^{1/1.8}$ and have intercepts chosen to match with $d_{rms}$ for CDM.

Finally, let us consider the accuracy and limitations of FPA. It can be shown that the relation between true density contrast $\sigma^2$ and the linear density contrast $\sigma_L^2$ (Hamilton *et al.* 1991; Nityananda *et al.* 1993) is well approximated by:

$$\sigma^2(x,a) \propto \begin{cases} \sigma_l^2(l,a) & (\text{for } \sigma^2 \lesssim 1) \\ [\sigma_L^2(l,a)]^3 & (\text{for } 3 \lesssim \sigma^2 \lesssim 50) \\ [\sigma_L^2(l,a)]^{3/2} & (\text{for } 50 \lesssim \sigma^2), \end{cases} \quad (4)$$

where $l = x(1+\sigma^2)^{1/3}$ (Bagla & Padmanabhan 1994b). Taking $\sigma_L^2(l,a) \propto a^2 l^{-(n+3)}$ where $n$ is a local index, we get for $(\sigma/a)$:

$$\frac{\sigma(x,a)}{a} \propto \begin{cases} a^{-[(n+1)/(n+4)]} & 10 \lesssim \sigma^2 \lesssim 50 \\ a^{-[(n+2)/(n+5)]} & 50 \lesssim \sigma^2. \end{cases} \quad (5)$$

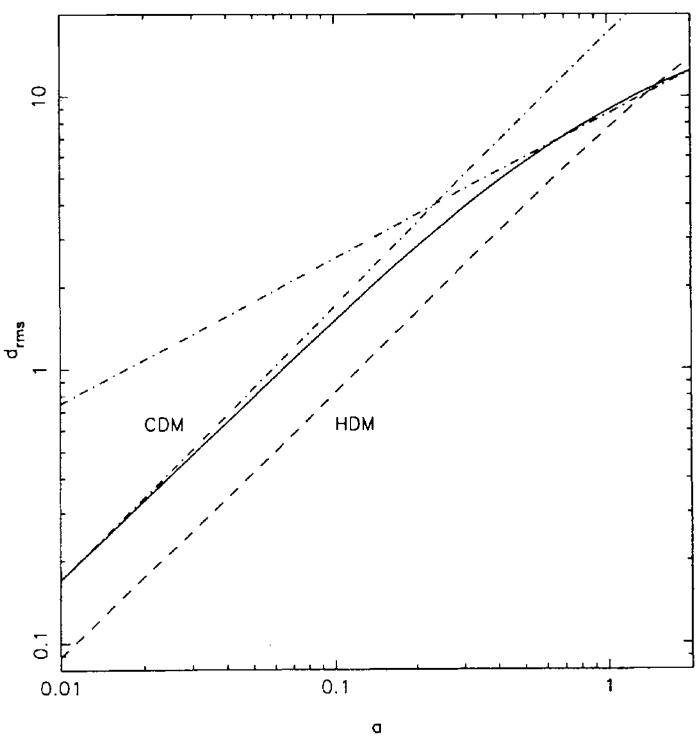


**Figure 2.** Evolution of $d_{rms}$. Here $d_{rms}$ is plotted as a function of the scale factor, the continuous line is for CDM and the dashed line corresponds to HDM model. Dot-dashed lines have been drawn for the power law solutions, $a$ and $a^{1/1.8}$ for early and late times respectively, as discussed in the text.

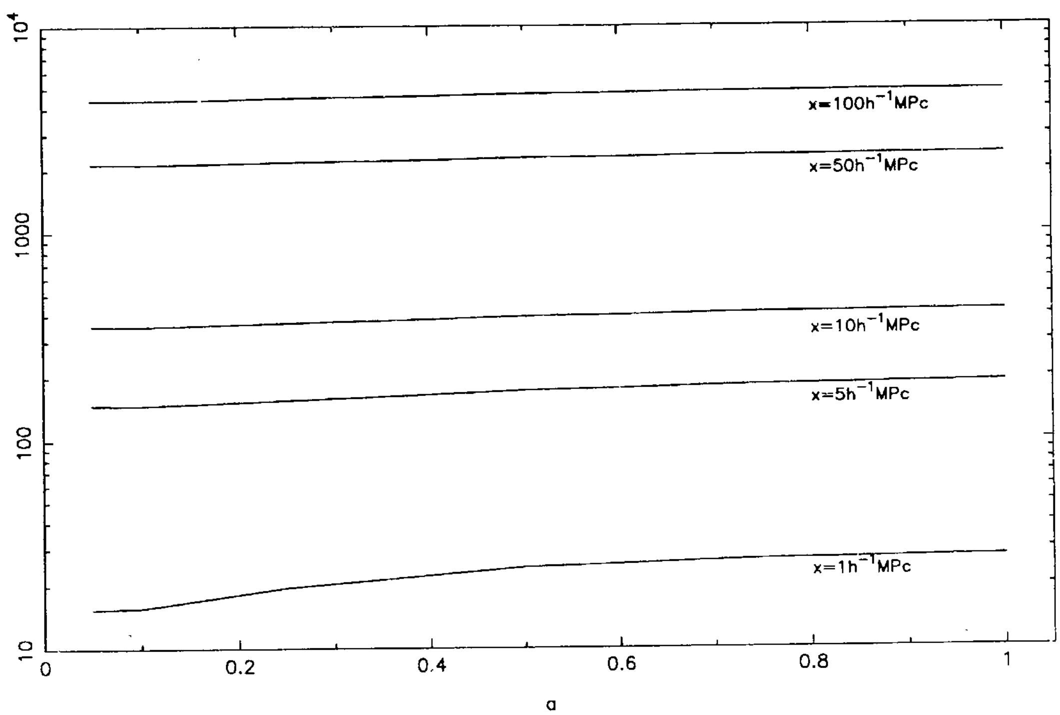


**Figure 3.** Variation in potential with scale factor. We have plotted the variance of potential $\zeta$ as a function of the scale factor for $r = 1, 5, 10, 50$, and $100\,\mathrm{h}^{-1}$ MPc.

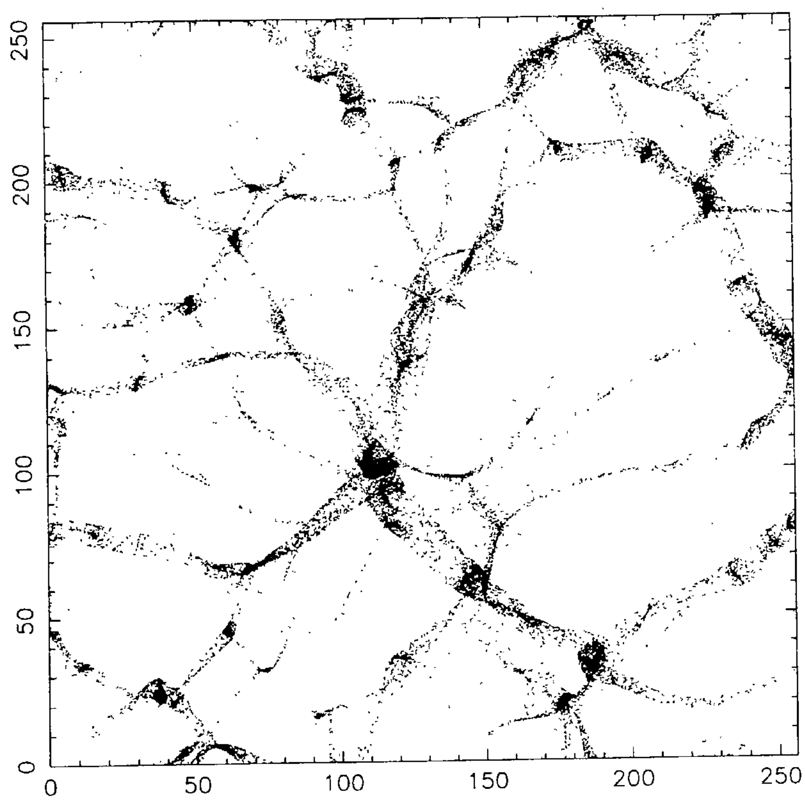


**Figure 4a.** An FPA simulation in two dimensions.

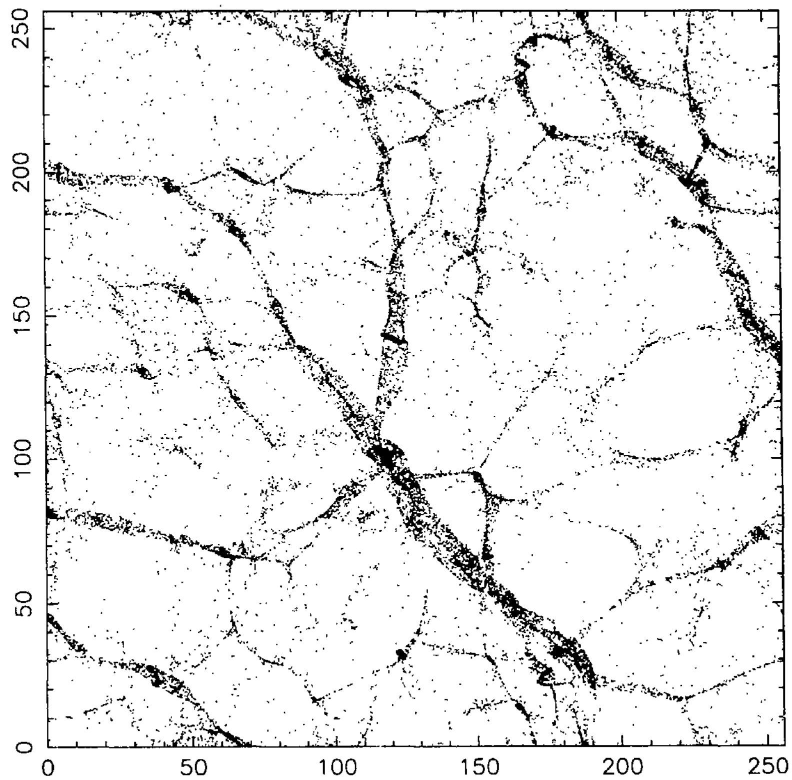


**Figure 4b.** An $N$-body frame from a 2-d simulation for the same initial potential as that used in Fig. 5a. The pancakes are much thinner than the ones in the FPA simulation. Also notice the shift of pancakes in this case.

In the standard CDM model $n \simeq -2$ in the nonlinear domain ($\sigma^2 \gtrsim 50$) and $n \simeq (-1$ to $-2)$ in the quasilinear regime. From (5) we see that $(\sigma/a)$ remains constant for $n = -1$ and $n = -2$ in the quasilinear and nonlinear regimes respectively. Hence in CDM like spectra, there is a conspiracy of indices ensuring that there is very little change in the gravitational potential. Fig. 3 shows $\psi$ at four different length scales in standard CDM model, as a function of scale factor. It is clear that constancy of $\psi$ is a reasonable approximation for CDM like spectra.

The FPA, however, is not accurate at small scales, for a different reason. If the streaming velocity at a (large) length scale $L$ is $v(L)$ then structures at this scale would have, on the average moved a distance of $H_0^{-1}v(L)$. In CDM models at scales $L \simeq 50\,\mathrm{h}^{-1}$ MPc, $v \simeq 200\,\mathrm{kms}^{-1}$; hence these structures could have moved about $H_0^{-1}v \simeq 2\,\mathrm{h}^{-1}$ MPc by now. Thus at scales smaller than about $1\,\mathrm{h}^{-1}$ MPc we would expect the approximation to show some inaccuracy due to motion of pancakes.

These aspects can be clearly seen in Fig. 4a, b which compares FPA with exact $N$-body simulations in 2D. The pancakes in $N$-body simulations are thinner than those in FPA (due to deepening of potential wells) and are also slightly shifted with respect to those in FPA.

The FPA is a powerful approximation which can be used to obtain approximate results, numerically as well as analytically. A very strong point in favour of this approximation is that it contains full information about velocities and therefore it can be used to study various models in the redshift space.

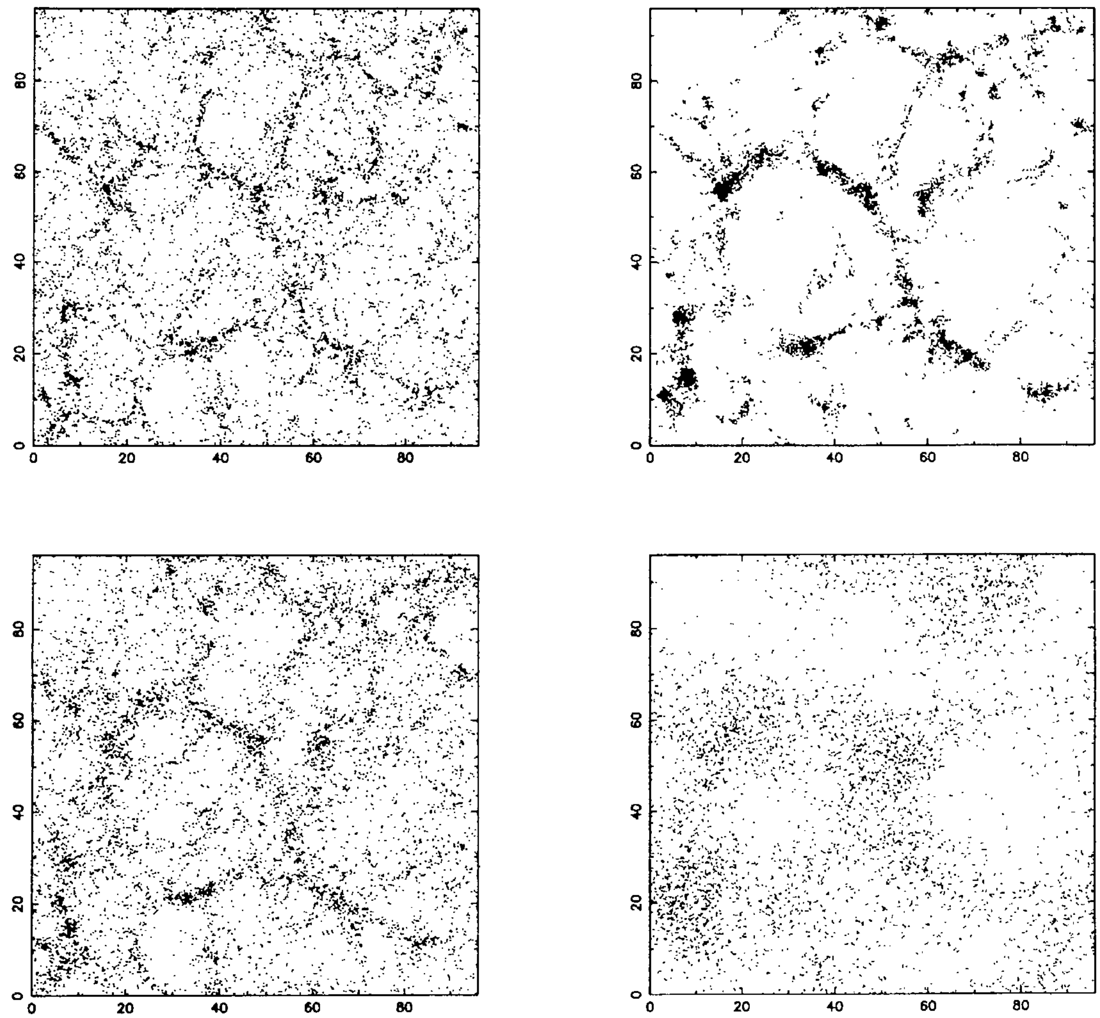


**Figure 5.** Evolution of density perturbations in CDM for Frozen Potential and Zeldovich approximation. These frames, from left to right, correspond to $a = 0.25$ and 1.0 respectively. The top row of frames is for FPA and the bottom row is for ZA.

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