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# Dark Matter and Dark Energy

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**Abstract.** I briefly review our current understanding of dark matter and dark energy. The first part of this paper focusses on issues pertaining to dark matter including observational evidence for its existence, current constraints and the ‘abundance of substructure’ and ‘cuspy core’ issues which arise in CDM. I also briefly describe MOND. The second part of this review focusses on dark energy. In this part I discuss the significance of the cosmological constant problem which leads to a predicted value of the cosmological constant which is almost  $10^{123}$  times larger than the observed value  $\Lambda/8\pi G \simeq 10^{-47} \text{GeV}^4$ . Setting  $\Lambda$  to this small value ensures that the acceleration of the universe is a fairly recent phenomenon giving rise to the ‘cosmic coincidence’ conundrum according to which we live during a special epoch when the density in matter and  $\Lambda$  are almost equal. Anthropic arguments are briefly discussed but more emphasis is placed upon dynamical dark energy models in which the equation of state is time dependent. These include Quintessence, Braneworld models, Chaplygin gas and Phantom energy. Model independent methods to determine the cosmic equation of state and the Statefinder diagnostic are also discussed. The Statefinder has the attractive property  $\ddot{a}/aH^3 = 1$  for LCDM, which is helpful for differentiating between LCDM and rival dark energy models. The review ends with a brief discussion of the fate of the universe in dark energy models.

## 1 Dark Matter

Observations of the cosmic microwave background (CMB) and the deuterium abundance in the Universe suggest that  $\Omega_{\text{baryon}} h^2 \simeq 0.02$ , or  $\Omega_{\text{baryon}} \simeq 0.04$  if the current Hubble expansion rate is  $h = H_0/100\text{km/sec/Mpc} = 0.7$ . Although  $\Omega_{\text{baryon}}$  is much larger than the observed mass in stars,  $\Omega_{\text{stars}} \simeq 0.005$ <sup>1</sup>, it is nevertheless very much smaller than the total energy density in the universe inferred from the observed anisotropy in the cosmic microwave background [193]

$$\Omega_{\text{total}} \equiv \frac{8\pi G \rho_{\text{total}}}{3H^2} = 1.02 \pm 0.02 . \quad (1)$$

Both dark matter and dark energy are considered essential missing pieces in the cosmic jigsaw puzzle

$$\Omega_{\text{total}} - \Omega_{\text{baryons}} = ? \quad (2)$$

Although the nature of neither dark matter (DM) nor dark energy (DE) is currently known, it is felt that both DM and DE are non-baryonic in origin, and that DM is distinguished from DE by the fact that the former clusters on sub-Megaparsec scales (in order to explain galactic rotation curves) whereas the latter has a large negative pressure (and can make the universe accelerate). In addition there is strong evidence to suggest that

$$\Omega_m \simeq 1/3, \quad \Omega_{\text{DE}} \simeq 2/3 . \quad (3)$$

In these lectures I will briefly review some properties of both dark matter and dark energy.

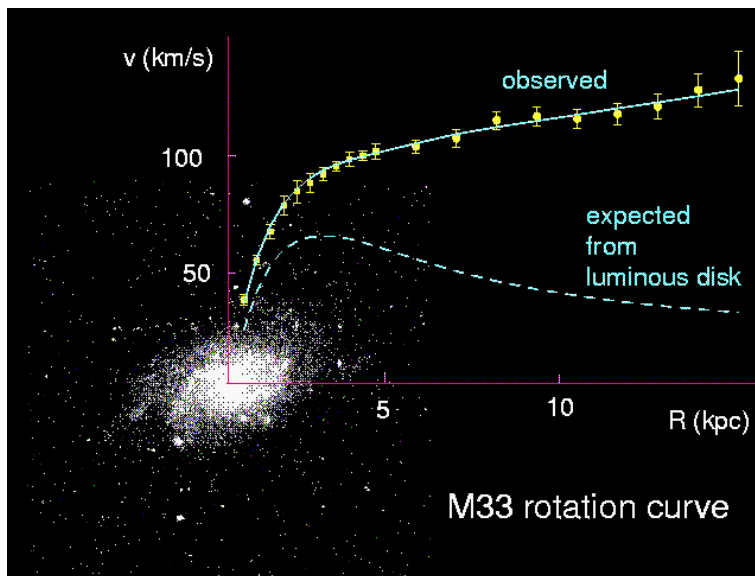
Though the observational evidence favouring a flat Universe with  $\Omega_{\text{total}} \simeq 1$  is fairly recent, the nature of the ‘unseen’ component of the universe (which dominates its mass density), is a long-standing issue in modern cosmology. Indeed, the need for dark matter was originally pointed out by Zwicky (1933) who realized that the velocities of individual galaxies located within the Coma cluster were quite large, and that this cluster would be gravitationally bound only if its total mass substantially exceeded the sum of the masses of its component galaxies. For clusters which have relaxed to dynamical equilibrium the mean kinetic and potential energies are related by the virial theorem [50]

$$K + \frac{U}{2} = 0 , \quad (4)$$

where  $U \simeq -GM^2/R$  is the potential energy of a cluster of radius  $R$ ,  $K \simeq 3M\langle v_r^2 \rangle/2$  is the kinetic energy and  $\langle v_r^2 \rangle^{1/2}$  is the dispersion in the line-of-sight velocity of cluster galaxies. (Clusters in the Abell catalogue typically have  $R \simeq 1.5h^{-1}$  Mpc.) This relation allows us to infer the mean gravitational

<sup>1</sup> This suggests that most of the baryonic matter at  $z = 0$  is not contained in stars but might be contained in hot gas [30].

potential energy if the kinetic energy is accurately known. The mass-to-light ratio in clusters can be as large as  $M/L \simeq 300M_{\odot}/L_{\odot}$ . However since most of the mass in clusters is in the form of hot, x-ray emitting intracluster gas, the extent of dark matter in these objects is estimated to be  $M/M_{\text{lum}} \simeq 20$ , where  $M_{\text{lum}}$  is the total mass in luminous matter including stars and gas.



**Fig. 1.** The observed rotation curve of the dwarf spiral galaxy M33 extends considerably beyond its optical image (shown superimposed); from Roy [164].

In individual galaxies the presence of dark matter has been convincingly established through the use of Kepler's third law

$$v(r) = \sqrt{\frac{GM(r)}{r}} \quad (5)$$

to determine the 'rotation curve'  $v(r)$  at a given radial distance from the galactic center. Observations of galaxies taken at distances large enough for there to be no luminous galactic component indicate that, instead of declining at the expected rate  $v \propto r^{-1/2}$  true if  $M \simeq \text{constant}$ , the velocity curves flattened out to  $v \simeq \text{constant}$  implying  $M(r) \propto r$  (see fig 1). This observation suggests that the mass of galaxies continues to grow even when there is no luminous component to account for this increase. Velocity curves have been compiled for over 1000 spiral galaxies usually by measuring the 21 cm emission line from neutral hydrogen (HI) [148, 191]. The results indicate that  $M/L = (10 - 20)M_{\odot}/L_{\odot}$  in spiral galaxies and in ellipticals, while this ratio can increase to  $M/L \simeq (200 - 600)M_{\odot}/L_{\odot}$  in low surface brightness galaxies

(LSB's) and in dwarfs. For instance, a recent measurement of the Draco dwarf spheroidal galaxy located at a distance of only 79 kpc from the Milky Way shows the presence of a considerable amount of dark matter  $M/L|_{\text{Draco}} = 440 \pm 240 M_{\odot}/L_{\odot}$  [97] ! It is interesting that the total mass of an individual galaxy is still somewhat of an unknown quantity since a turn around to the  $v \propto r^{-1/2}$  law at large radii has not been convincingly observed.

An important difference between the distribution of dark matter in galaxies and clusters needs to be emphasised: whereas dark matter appears to *increase* with distance in galaxies, in clusters exactly the reverse is true, the dark matter distribution actually *decreases* with distance. Indeed, for certain dwarfs (such as DD0154) the rotation curve has been measured to almost 15 optical length scales indicating that the dark matter surrounding this object is extremely spread out (see also figure 1). A foreground cluster, on the other hand, acts as a gravitational lens which focuses the light from background objects such as galaxies and QSO's thereby allowing us to determine the depth of the cluster potential well. Observations of strong lensing by clusters indicate that dark matter is strongly concentrated in central regions with a projected mass of  $10^{13} - 10^{14} M_{\odot}$  being contained within 0.2 - 0.3 Mpc of the central region. As we shall see later, this observation may prove to be problematic for alternatives to the dark matter hypothesis such as the Modified Newtonian Dynamics (MOND) approach of Milgrom [122].

As discussed earlier, the fact that only 4% of the cosmic density is baryonic suggests that the dark matter which we are observing could well be non-baryonic in origin. The need for non-baryonic forms of dark matter gets indirect support from the fact that baryonic models find it difficult to grow structure from small initial conditions and hence to reconcile the existence of a well developed cosmic web of filaments, sheets and clusters at the present epoch with the exceedingly small amplitude of density perturbations ( $\delta\rho/\rho \sim 10^{-5}$  at  $z \simeq 1, 100$ ) inferred from COBE measurements and more recent CMB experiments. Indeed, it is well known that, if the effects of pressure are ignored, linearized density perturbations in a spatially flat matter dominated universe grow at the rate  $\delta \propto t^{2/3} \propto (1+z)^{-1}$ , where  $1+z = a_0/a(t)$  is the cosmological redshift. (Contrast this relatively slow growth rate with the exponential 'Jeans instability' of a static matter distribution  $\delta \propto \exp\sqrt{4\pi G\rho}t$ .) In a baryonic universe, the large radiation pressure (caused by thompson scattering of CMB photons off electrons) ensures that density perturbations in the baryonic component can begin growing only after hydrogen recombines at  $z \simeq 1, 100$  at which point of time baryons and radiation decouple. Requiring  $\delta > 1$  today implies  $\delta > 10^{-3}$  at recombination, which contradicts CMB observations by over an order of magnitude ! In non-baryonic models on the other hand, the absence of any significant coupling between dark matter and radiation allows structure to grow much earlier, significantly before hydrogen in the universe has recombined. After recombination baryons simply fall into the potential wells created for them by the dominant non-baryonic component. As a result

a universe with a substantial non-baryonic component can give rise to the structure which we see today from smaller initial fluctuations.

The growth of structure via gravitational instability depends both upon the nature of primordial perturbations (adiabatic/isocurvature) and upon whether the dark matter species is hot or cold. The issue of density perturbations has been discussed in considerable detail by Ruth Durrer at this school and I shall not touch upon this important topic any further. Let me instead say a few words about hot and cold dark matter. Non-baryonic *Hot Dark Matter* (HDM) particles are assumed to have decoupled from the rest of matter/radiation when they were relativistic and so have a very large velocity dispersion (hence called ‘hot’). *Cold Dark Matter* (CDM) particles, on the other hand, have a very small velocity dispersion and decouple from the rest of matter/radiation when they are non-relativistic. The free-streaming (collisionless phase mixing) of non-baryonic particles as they travel from high density to low density regions (and vice versa) introduces an important length scale called the ‘free-streaming distance’  $\lambda_{\text{fs}}$  – which is the mean distance travelled by a relativistic particle species until its momentum becomes non-relativistic. In both HDM and CDM the processed *final* spectrum of density perturbations differs from its initial form. In the case of HDM this difference arises because fluctuations on scales smaller than  $\lambda_{\text{fs}}$  are wiped out due to free streaming with the result that the processed final spectrum has a well defined cutoff on scales smaller than  $\lambda \sim \lambda_{\text{fs}}$ . Perhaps the best example of HDM is provided by a light neutrino of mass about 30 eV. In this case  $\lambda_{\text{fs}} \simeq 41(30\text{eV}/m_\nu)$  Mpc with the result that large proto-pancakes having masses comparable to those of rich clusters of galaxies  $M \sim 10^{15}M_\odot$  are the first objects to form in HDM. Smaller objects (galaxies) are formed by the fragmentation of the proto-pancake. This *top-down* scenario for structure formation was originally suggested by Zeldovich and coworkers in connection with adiabatic baryonic models and subsequently applied to HDM. It has since fallen out of favour mainly due to the strong observational constraints on the mass of the neutrino  $\sum_{\nu_i} m_{\nu_i} < 0.7$  eV and on the relic neutrino density  $10^{-3} \lesssim \Omega_\nu h^2 \lesssim 10^{-1}$  [61, 193, 63, 123]. It also faces considerable difficulty in forming structure sufficiently early to explain the existence of galaxies and QSO’s at high redshifts.

In contrast to HDM, constituents of CDM have a much smaller free-streaming distance. Because of this small scales are the first to go non-linear and gravitational clustering proceeds in a *bottom up* fashion in this scenario. A key quantity defining gravitational clustering is the power spectrum of density perturbations  $P(k) \equiv |\delta_k|^2$ , which is related to the mean square density fluctuation via

$$\left\langle \left( \frac{\delta\rho}{\rho} \right)^2 \right\rangle = 4\pi \int_0^\infty P(k)k^2 dk . \quad (6)$$

Inflationary models predict  $P_i(k) \propto k^n$ ,  $n \simeq 1$ , at an early epoch. As the universe expands the power spectrum gets modified. The ‘processed’ final spectrum depends upon the nature of dark matter, the epoch of matter-radiation

equality and other cosmological quantities. The final and initial spectra are related through a transfer function

$$P_f(k) = P_i(k) \times T^2(k) . \quad (7)$$

CDM-type spectra have the following approximate form of the transfer function [165, 194, 166]

$$T(k) = \left( 1 + \frac{Ak^2}{\log(1+Bk)} \right)^{-1} . \quad (8)$$

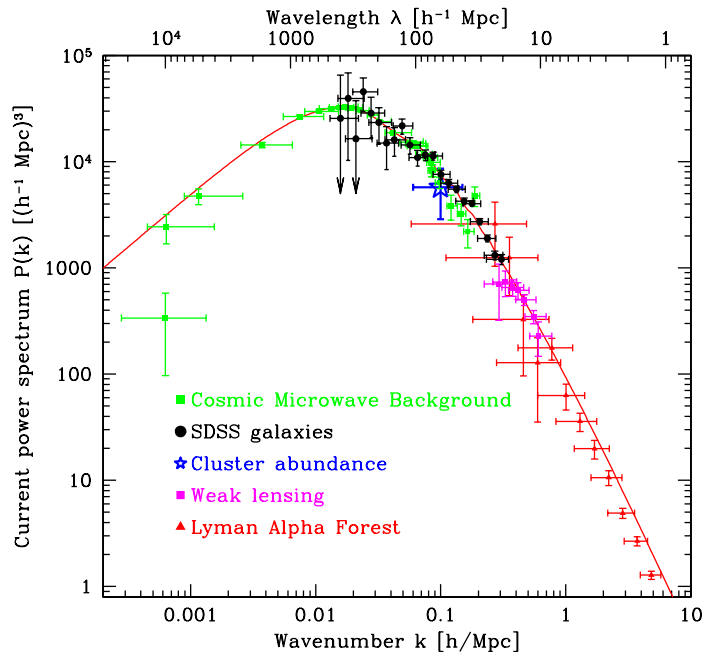
Equations (7) & (8) illustrate the ‘turn around’ of the power spectrum from its primordial scale invariant form  $P(k) \propto k$  on the largest scales to  $P(k) \propto k^{-3} \log^2 k$  on small scales. (The precise location of the turn-around and the amplitude of  $P(k)$  depend upon specific details of the cosmological model, see for instance [16].)

The ‘standard’ cold dark matter (SCDM) paradigm, which assumed that  $\Omega_{\text{CDM}} = 1$ , was introduced during the early 1980’s at roughly the same time when HDM was perceived to be in trouble (see [101, 96, 50, 166] for references to earlier work on this subject). Although SCDM was very successful in explaining a host of observational details, it was clear already a decade ago, that the processed power spectrum of SCDM lacked sufficient power on large scales and so fell short of explaining the angular two point correlation function for galaxies on scales  $\sim 50$  Mpc [60]. The relevant cosmological quantity in this case is the shape of the power spectrum of density perturbations, which for CDM-like models, can be characterised by the ‘shape parameter’  $\Gamma = \Omega_m h$ . SCDM models with  $\Omega_m = 1$  and the HST-determined value  $h \simeq 0.7$  predict  $\Gamma \simeq 0.5$  which is much larger than the observed value  $\Gamma = 0.207 \pm 0.030$  inferred from observations of galaxy clustering in the sloan digital sky survey (SDSS) [154]. A modification of SCDM called LCDM assumes that, in addition to CDM the universe consists of a smoothly distributed component called a cosmological constant or a Lambda-term. LCDM models with  $h \simeq 0.7$  and  $\Omega_m = 0.3$  predict a smaller value for the shape parameter,  $\Gamma \simeq 0.2$ , and the resulting amplitude and shape of the power spectrum is in excellent agreement with several different sets of observations as demonstrated in figure 2.

From (6), (7) & (8) we find that on small scales, the contribution to the *rms* density fluctuation from a given logarithmic interval in  $k$  is

$$\left( \frac{\delta\rho}{\rho} \right)_k^2 \sim k^3 P_f(k) \propto \log^2 k , \quad (9)$$

which illustrates the fact that, although the smallest scales are the first to go non-linear, there is significant power to drive gravitational instability rapidly to larger scales in this model. Indeed, detailed N-body simulations of large scale structure show that filaments defining the cosmic web first form on the smallest scales. The scale-length characterizing the cosmic web grows as the



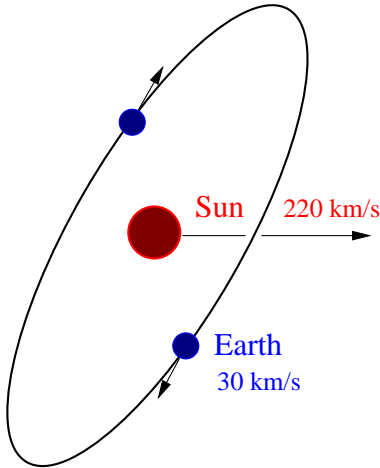
**Fig. 2.** The power spectrum inferred from observations of large scale structure, the Lyman $\alpha$  forest, gravitational lensing and the CMB. The solid line shows the power spectrum prediction for a flat scale-invariant LCDM model with  $\Omega_m = 0.28$ ,  $\Omega_b/\Omega_m = 0.16$ ,  $h = 0.72$ ; from Tegmark et al. [200].

universe expands, until at the present epoch the cosmic web consists of a fully developed supercluster-void network with a scale-length of several tens of Megaparsec [181, 183, 119, 206].

Promising candidates for cold dark matter include a 100 – 1000 GeV particle called a neutralino. The neutralino is a weakly interacting massive particle (WIMP). As its name suggests it is neutral and is a fermionic partner to the gauge and Higgs bosons (usually called the ‘bino, wino and higgsino’). It is believed that the lightest supersymmetric particle will be stable due to R-parity which makes the neutralino an excellent candidate for cold dark matter (see [163, 89] for reviews of particle dark matter). A radically different particle candidate for cold dark matter is an ultra-light pseudo-Goldstone boson called an axion with a mass of only  $m_a \sim 10^{-5\pm 1}$  eV. Although ultralight, the axion is ‘cold’ because it was created as a zero-momentum condensate. Its existence is a by-product of an attempt to resolve QCD of what is commonly called the ‘strong CP problem’ which arises because non-perturbative effects in QCD

give rise to an electric dipole moment for the neutron – in marked contrast with observations [101]. Other candidates for non-baryonic cold dark matter include string theory motivated moduli fields [32]; non-thermally produced super-heavy particles having a mass  $\sim 10^{14}$  GeV and dubbed Wimpzillas [100]; as well as axino’s and gravitino’s – superpartners of the axion and graviton respectively [163].

Since WIMP’s cluster gravitationally, one should expect to find a flux of these particles in our own solar system and attempts are being made to determine dark matter particles by measuring the scattering of WIMP’s on target nuclei through nuclear recoils. Now the earth orbits the sun with a velocity  $\simeq 30$  km/sec, even as the sun orbits the galaxy with  $v_{M_\odot} \simeq 220$  km/sec. Furthermore the plane of the Earth’s orbit is inclined at an angle of  $60^\circ$  to the galactic plane, because of which the dark matter flux on Earth is expected to be larger in June (when the Earth’s velocity and the Sun’s velocity add together) than in December (when these two velocities subtract). The resulting rate variation is about 7% between the flux measured during summer and winter. Precisely such a signal was reported by the DAMA experiment whose data (collected since 1996) appears to show a yearly modulation with greater events reported in June than in December [17]. However results obtained by the DAMA group remain controversial since they have not been substantiated by other groups which report negative results for similar searches (see [129, 95] for recent reviews on this subject).



**Fig. 3.** The Earth’s motion around the Sun; from Khalil and Munoz (2001).

Despite the excellent agreement of LCDM with large scale observations, some concerns have recently been expressed about the ability of this model to

account for a number of smaller scale observations which can be summarized as follows:

- *The substructure problem* is used to describe the fact that the cold dark matter model (with or without a cosmological constant) predicts an excessive number of dark matter subhaloes (or substructure) within a larger halo. If one (perhaps naively) associates each halo with a gravitationally bound baryonic object then the predicted number of dwarf-galaxy satellites within the local group exceeds the observed number by over an order of magnitude. Indeed, detailed N-body simulations as well as theoretical estimates predict around 1000 dark matter satellites in our local group which is much larger than the 40 or so observed at present [98, 126, 94, 35, 192, 24, 120, 199, 64].
- *The cuspy core problem*  
CDM predicts a *universal density profile* for dark matter halos in the wide range  $10^7 M_\odot - 10^{15} M_\odot$  which applies both to galaxy clusters as well as individual galaxies including dwarfs and LSB's.<sup>2</sup> The density profile originally suggested by Navarro, Frenk and White [133] is

$$\rho(r) = \rho_0 (r_s/r) \left[ 1 + \left( \frac{r}{r_s} \right) \right]^{-2}, \quad (10)$$

which gives  $\rho \propto r^{-1}$  for  $r \ll r_s$  and  $\rho \propto r^{-3}$  for  $r \gg r_s$ , where  $r_s$  is the scale radius and  $\rho_0$  is the characteristic halo density. (Other groups using higher resolution computations found somewhat steeper density profiles at small radii, such as  $\rho \propto r^{-1.5}$  [127, 87].)

*The cuspy core problem* refers to the apparent contradiction between N-body experiments – which show that the density profile in CDM halos has a  $1/r$  (or steeper) density cusp at the center, and observations – which appear to favour significantly shallower density cores in galaxy clusters as well as in individual dwarf and LSB galaxies (see [69, 36, 28, 37, 155, 199, 103, 162, 179, 109] for detailed discussions of this issue).

Although disconcerting, given the very considerable success of LCDM in explaining gravitational clustering on large scales, it may at this point be premature to condemn this model on the basis of small scale observations alone. It could be that the difficulties alluded to above are a result of an oversimplification of the complex physical processes involved and that a more careful analysis of the baryonic physics on small scales including the hydrodynamical effects of star formation and supernova feedback needs to be undertaken. For instance both dwarfs and LSB's have very shallow potential wells, a strong burst of star formation and supernova activity may therefore empty dark

<sup>2</sup> Low Surface Brightness Galaxies (LSB's) are dominated by their dark matter content and therefore provide particularly good astrophysical objects with which to test dark matter models.

matter halos of their baryonic content resulting in a large number of ‘failed galaxies’ and providing a possible resolution to the ‘satellite catastrophe’. (The failed galaxies will act as gravitational lenses and should therefore be detectable through careful observations.) Other explanations include the effects of tidal stripping recently discussed in [103]. Likewise issues involving beam smearing, the influence of bars and the interaction of baryons and dark matter in the central regions of galaxies and clusters could be intricately linked with the central cusp issue and must be better understood if one wishes to seriously test the CDM hypothesis on small scales.

In concluding this discussion on dark matter I would like to briefly mention Modified Newtonian Dynamics (MOND) which, in some circles, is regarded as an alternative to the dark matter hypothesis. As the name suggests, MOND is a modification of Newtonian physics which proposes to explain the flat rotation curves of galaxies without invoking any assumptions about dark matter. Briefly, MOND assumes that Newtons law of inertia ( $F = ma$ ) is modified at sufficiently low accelerations ( $a < a_0$ ) to

$$\mathbf{F} = m\mathbf{a}\mu(a/a_0) , \quad (11)$$

where  $\mu(x) = x$  when  $x \ll 1$  and  $\mu(x) = 1$  when  $x \gg 1$  [122, 180]. It is easy to see that this results in the modification of the conventional formula for gravitational acceleration  $\mathbf{F} = m\mathbf{g}_N$ , resulting in the following relation between the true acceleration and the Newtonian value:  $a = \sqrt{g_N a_0}$ . For a body orbiting a point mass  $M$ ,  $g_N = GM/r^2$ . Since the centripetal acceleration  $a = v^2/r$  now equals the *true* acceleration  $a$ , one gets

$$v^4 = GMa_0 , \quad (12)$$

*i.e.* for sufficiently low values of the acceleration the rotation curve of an isolated body of mass  $M$  does not depend upon the radial distance  $r$  at which the velocity is measured, in other words not only does this theory predict flat rotation curves it also suggests that the individual halo associated with a galaxy is infinite in extent ! (This latter prediction may be a problem for MOND since recent galaxy-galaxy lensing results [82] suggest that galaxy halo’s may have a maximum extent of about 0.5 Mpc.) The value of  $a_0$  needed to explain observations is  $a_0 \sim 10^{-8} \text{cm/s}^2$  which is of the same order as  $cH_0$  ! This has led supporters of this hypothesis to conjecture that MOND may reflect “the effect of cosmology on local particle dynamics” [180]. Although MOND gives results which are in good agreement with observations of individual galaxies, it is not clear whether it is as successful for explaining clusters for which strong gravitational lensing indicates a larger mass concentration at cluster centers than accounted for by MOND [180, 52]. Another difficulty with MOND is that it is problematic to embed this theory within a more comprehensive relativistic theory of gravity and hence, at present, it is not clear what predictions a MOND-type theory may make for gravitational lensing and other curved space-time effects. For some recent developments in this direction see [23].

To summarise, current observations make a strong case for clustered, non-baryonic dark matter to account for as much as a third of the total matter density in the Universe  $\Omega_m \simeq 1/3$ . The remaining two-thirds is thought to reside in a relative smooth component having large negative pressure and called Dark Energy.

## 2 Dark Energy

### 2.1 The cosmological constant and Vacuum energy

Type Ia supernovae, when treated as standardized candles, suggest that the expansion of the universe is speeding up rather than slowing down. The case for an accelerating universe also receives independent support from CMB and large scale structure studies. All three data sets can be simultaneously satisfied if one postulates that the dominant component of the universe is relatively smooth, has a large negative pressure and  $\Omega_{\text{DE}} \simeq 2/3$ .

The simplest example of dark energy is a cosmological constant, introduced by Einstein in 1917. The Einstein equations, in the presence of the cosmological constant, acquire the form

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi G}{c^4}T_{ik} + \Lambda g_{ik} . \quad (13)$$

Although Einstein originally introduced the cosmological constant ( $\Lambda$ ) into the left hand side of his field equations, it has now become conventional to move the  $\Lambda$ -term to the RHS, treating it as an effective form of matter. In a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe consisting of pressureless dust (dark matter) and  $\Lambda$ , the Raychaudhuri equation, which follows from (13), takes the form

$$\ddot{a} = -\frac{4\pi G}{3}a\rho_m + \frac{\Lambda}{3} . \quad (14)$$

Equation (14) can be rewritten in the form of a force law:

$$\mathcal{F} = -\frac{GM}{R^2} + \frac{\Lambda}{3}R, \quad (R \equiv a) \quad (15)$$

which demonstrates that the cosmological constant gives rise to a *repulsive force* whose value increases with distance. The repulsive nature of  $\Lambda$  could be responsible for the acceleration of the universe as demonstrated in (14).

Although introduced into physics in 1917, the physical basis for a cosmological constant remained a bit of a mystery until the 1960's, when it was realised that zero-point vacuum fluctuations must respect Lorenz invariance and therefore have the form  $\langle T_{ik} \rangle = \Lambda g_{ik}$  [214]. As it turns out, the vacuum expectation value of the energy momentum is divergent both for bosonic and

fermionic fields, and this gives rise to what is known as ‘the cosmological constant problem’. Indeed the effective cosmological constant generated by vacuum fluctuations is

$$\frac{\Lambda}{8\pi G} = \langle T_{00} \rangle_{\text{vac}} \propto \int_0^\infty \sqrt{k^2 + m^2} k^2 dk, \quad (16)$$

since the integral diverges as  $k^4$  one gets an infinite value for the vacuum energy. Even if one chooses to ‘regularize’  $\langle T_{ik} \rangle$  by imposing an ultraviolet cutoff at the Planck scale, one is still left with an enormously large value for the vacuum energy  $\langle T_{00} \rangle_{\text{vac}} \simeq c^5/G^2 \hbar \sim 10^{76} \text{GeV}^4$  which is 123 orders of magnitude larger than the currently observed  $\rho_\Lambda \simeq 10^{-47} \text{GeV}^4$ . A smaller ultraviolet cut-off does not fare much better since a cutoff at the QCD scale results in  $\Lambda_{\text{QCD}}^4 \sim 10^{-3} \text{GeV}^4$ , which is still forty orders of magnitude larger than observed.

In the 1970’s the discovery of supersymmetry led to the hope that, since bosons and fermions (of identical mass) contribute equally but with opposite sign to the vacuum expectation value of physical quantities, the cosmological constant problem may be resolved by a judicious balance between bosons and fermions in nature. However supersymmetry (if it exists) is broken at the low temperatures prevailing in the universe today and on this account one should expect the cosmological constant to vanish in the early universe, but to reappear during late times when the temperature has dropped below  $T_{\text{SUSY}}$ . This is clearly an undesirable scenario and almost the very opposite of what one is looking for, since, a large value of  $\Lambda$  at an early time is useful from the viewpoint of inflation, whereas a very small current value of  $\Lambda$  is in agreement with observations [172, 171].

In the absence of a resolution to the cosmological constant problem the following possibility connecting the vacuum energy with the SUSY and Planck scales may be worth exploring [172, 171]. The mass scale associated with the scale of supersymmetry breaking in some models,  $M_{\text{SUSY}} \sim 1 \text{ TeV}$ , lies midway between the Planck scale and  $10^{-3} \text{ eV}$ . One could conjecture that the small observed value of the cosmological constant  $\rho_\Lambda \simeq (10^{-3} \text{ eV})^4$  is associated with the vacuum in a theory which had a fundamental mass scale  $M_X \simeq M_{\text{SUSY}}^2/M_{\text{Pl}}$ , such that  $\rho_{\text{vac}} \sim M_X^4 \sim (10^{-3} \text{ eV})^4$ .

The cosmological constant is also relevant from the perspective of models with spontaneous symmetry breaking [209]. Indeed, if one examines the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi), \\ V(\phi) &= V_0 - \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4, \end{aligned} \quad (17)$$

one notices that the symmetric state at  $\phi = 0$  is unstable and the system settles in the ground state  $\phi = +\sigma$  or  $\phi = -\sigma$ , where  $\sigma = \sqrt{\mu^2/\lambda}$ , thereby breaking the reflection symmetry  $\phi \leftrightarrow -\phi$  present in the Lagrangian. For

$V_0 = 0$  this potential gives rise to a large negative cosmological constant  $\Lambda_{eff} = V(\phi = \sigma) = -\mu^4/4\lambda$  in the broken symmetry state. This embarrassing situation can be avoided only if one chooses a value for  $V_0$  which almost exactly cancels  $\Lambda_{eff}$ , namely  $V_0 \simeq +\mu^4/4\lambda$  so that  $\Lambda_{eff}/8\pi G = V_0 - \mu^4/4\lambda \simeq 10^{-47}\text{GeV}^4$ .

The cosmological consequences of this rather ad-hoc ‘regularization’ exercise are instructive. Unless the value of  $\Lambda_{eff}$  lies in a very small window, the universe will be a very different place from the one we are used to. For instance a negative value of the  $\Lambda$ -term  $\Lambda_{eff}/8\pi G < -10^{-43}\text{GeV}^4$  will cause the universe to recollapse (the effect of  $\Lambda$  is attractive now instead of being repulsive) less than a billion years after the big bang – a period which is much too short for galaxies to form and for life (as we know it) to emerge. On the other hand a large positive  $\Lambda_{eff}/8\pi G > 10^{-43}\text{GeV}^4$  makes the universe accelerate much before the present epoch, thereby inhibiting structure formation and precluding the emergence of life.

The very small window in  $\Lambda$  which allows life to emerge has led some cosmologists to propose anthropic arguments for the existence of a small cosmological constant [20, 118, 76, 210]. One such possibility is the following “if our big bang is just one of many big bangs, with a wide range of vacuum energies, then it is natural that some of these big bangs should have a vacuum energy in the narrow range where galaxies can form, and of course it is just these big bangs in which there could be astronomers and physicists wondering about the vacuum energy” [210].

I will not discuss the anthropic argument any further in these lectures but will point the interested reader to [118, 76, 210] for further discussion of this issue.

It is important to note that there is no known fundamental symmetry in nature which will set the value of  $\Lambda$  to zero. In its absence, the small observed value of the dark energy remains somewhat of a dilemma which remains to be fully understood and resolved.<sup>3</sup>

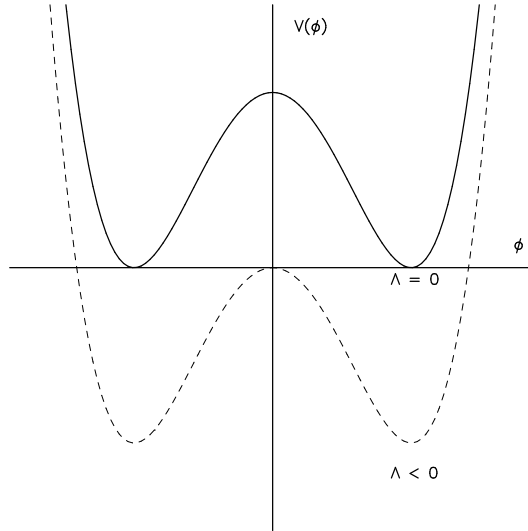
## 2.2 Dynamical models of Dark Energy

The cosmological constant is but one example of a form of matter (dark energy) which could drive an accelerated phase in the history of our universe. Indeed, (14) is easily generalised to

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i) = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i), \quad (18)$$

where the summation is over all forms of matter present in the universe with equation of state  $w_i = p_i/\rho_i$ . Eqn. (18) together with its companion equation

<sup>3</sup> The important role played by symmetries is illustrated by the U(1) gauge symmetry of electrodynamics whose presence implies a zero rest mass for the photon. No analogous symmetry exists for the neutrino and recent experiments do indicate that neutrinos could have a small mass.



**Fig. 4.** Spontaneous symmetry breaking in many field theory models takes the form of the Mexican top hat potential shown above. The dashed line shows the potential before the cosmological constant has been ‘renormalized’ and the solid line after. (From Sahni and Starobinsky 2000.)

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2} \quad (19)$$

completely describes the dynamics of a FRW universe ( $k/a^2$  is the Gaussian curvature of space).

Clearly a universe consisting of only a single component will accelerate if  $w < -1/3$ . Fluids satisfying  $\rho + 3p \geq 0$  or  $w \geq -1/3$  are said to satisfy the ‘strong energy condition’ (SEC). We therefore find that, in order to accelerate, ‘dark energy’ must violate the SEC. Another condition which is usually assumed to be sacrosanct, but has recently been called into question is the ‘weak energy condition’ (WEC)  $\rho + p \geq 0$  or  $w \geq -1$ . Failure to satisfy the WEC can result in faster-than-exponential expansion for the universe and in a cosmic ‘Big Rip’, which we shall come to in a moment.

It is often more convenient to rewrite (18) in terms of the ‘deceleration parameter’

$$q = -\frac{\ddot{a}}{aH^2} = \sum_i \left(\frac{4\pi G\rho_i}{3H^2}\right)(1 + 3w_i) = \frac{(1 + 3w_X\Omega_X)}{2}, \quad (20)$$

where  $\Omega_i = 8\pi G\rho_i/3H^2$  and we have assumed a flat universe with  $\Omega_m + \Omega_X = 1$  ( $\Omega_X \equiv \Omega_{DE}$ ). The condition for accelerated expansion ( $q < 0$ ) is equivalent

to

$$w_X < -\frac{1}{3(1 - \Omega_m)}, \quad (21)$$

which leads to

$$w < -\frac{1}{3} \text{ for } \Omega_m = 0, \quad (22)$$

$$w < -\frac{1}{2} \text{ for } \Omega_m = 1/3. \quad (23)$$

Eqn. (19) can be used to develop an expression for the Hubble parameter  $H \equiv \dot{a}/a$  in terms of the cosmological redshift  $z = a_0/a(t) - 1$ :

$$H(z) = H_0 \left[ \Omega_m(1+z)^3 + \Omega_X(1+z)^{3(1+w)} \right]^{1/2}, \quad (24)$$

where  $H_0 = H(z=0)$  is the present value of the Hubble parameter,  $\Omega_m = 8\pi G\rho_{0m}/3H_0^2$ ,  $\Omega_X = 8\pi G\rho_{0DE}/3H_0^2$ , describe the dimensionless density of matter and dark energy respectively, ( $w \equiv w_{DE}$ ), and we have made the assumption of a flat universe so that  $\Omega_m + \Omega_X = 1$ .

In LCDM cosmology  $w = -1$ ,  $\Omega_\Lambda = \Lambda/3H_0^2$ , and the expansion factor has the elegant form [172]

$$a(t) \propto \left( \sinh \frac{3}{2} \sqrt{\frac{\Lambda}{3}} ct \right)^{2/3}, \quad (25)$$

which smoothly interpolates between a matter dominated universe in the past ( $a \propto t^{2/3}$ ) and accelerated expansion in the future ( $a \propto \exp \sqrt{\frac{\Lambda}{3}} t$ ).

We are now in a position to appreciate the evidence for an accelerating universe which originates in observations of the light flux from high redshift type Ia supernovae. Type Ia supernovae are extremely bright objects, ( $M_B \simeq -19.5$ ) which makes them ideally suited for studying the properties of the universe at large distances.

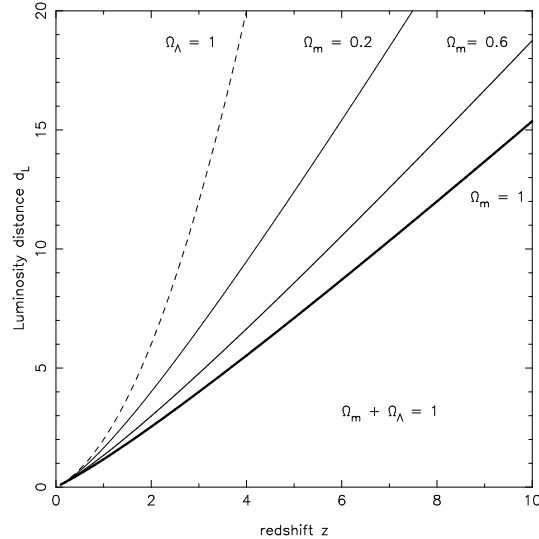
The light flux received from a distant supernova is related to its absolute luminosity  $\mathcal{L}$  and its ‘luminosity distance’  $d_L$  through the relation

$$F = \frac{\mathcal{L}}{4\pi d_L^2}. \quad (26)$$

If one views this problem from within the Newtonian perspective then, since the geometry of space is Euclidean,  $d_L = \sqrt{x^2 + y^2 + z^2}$ . In general relativity, on the other hand, the geometry of space can be non-Euclidean, and the luminosity distance to an object located at redshift  $z$  will, in general, depend both upon the geometry of space as well as the expansion history of the universe. Indeed, it can be shown that in a spatially flat and expanding FRW universe, the luminosity distance has the form

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}. \quad (27)$$

The luminosity distance is shown in Fig. 1 for a number of cosmological models with varying amounts of  $\Omega_m$  &  $\Omega_\Lambda$ . The limiting case  $\Omega_m = 1, \Omega_\Lambda = 0$  corresponds to standard cold dark matter (SCDM) in which the universe decelerates as a weak power law  $a(t) \propto t^{2/3}$ . The other extreme example  $\Omega_\Lambda = 1, \Omega_m = 0$  describes the de Sitter universe (also known as steady state cosmology) which accelerates at the steady rate  $a(t) \propto \exp\sqrt{\frac{1}{3}}t$ . From figure 1 we see that a supernova at redshift  $z = 3$  will appear 9 times brighter in SCDM than it will in de Sitter space !



**Fig. 5.** The luminosity distance  $d_L$  (in units of  $H_0^{-1}$ ) is shown as a function of cosmological redshift  $z$  for spatially flat cosmological models with  $\Omega_m + \Omega_\Lambda = 1$ . Heavier lines correspond to larger values of  $\Omega_m$ . The dashed line shows the luminosity distance in the spatially flat de Sitter universe ( $\Omega_\Lambda = 1$ ). From Sahni and Starobinsky [172].

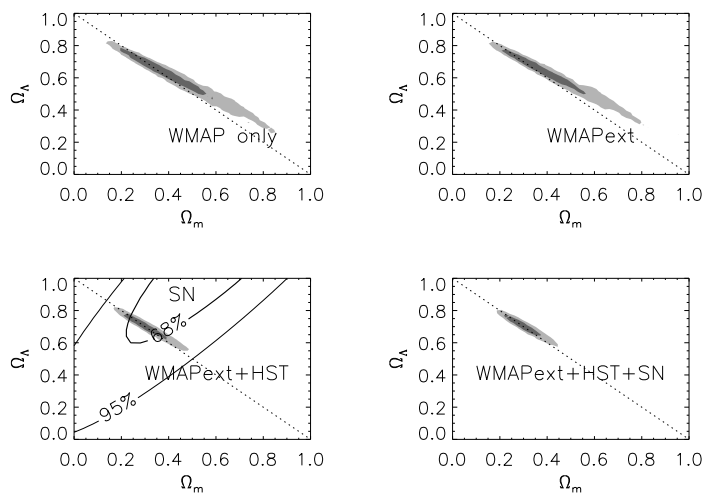
Systematic studies of type Ia supernovae have revealed that:

- Type Ia Sn are excellent standardized candles. The dispersion in peak supernova luminosity is small:  $\Delta m \simeq 0.3$ , and the corresponding change in intensity is about 25%. In addition the light curve of a type Ia supernova is correlated with its peak luminosity [149] to a precision of  $\sim 7\%$ , so that *brighter supernovae take longer to fade*. (Type Ia Sn take roughly 20 days to rise from relative obscurity to maximum light.) This allows

us to ‘standardize’ supernova light curves thereby reducing the scatter in their luminosities to  $\sim 12\%$  which turns type Ia supernovae into very good standard candles.

- Type Ia supernovae at higher redshifts are consistently dimmer than their counterparts at lower redshifts relative to what might be expected in  $\Lambda$ CDM cosmology. If type Ia supernovae are treated as standard candles then, assuming systematic effects such as cosmological evolution and dimming by intergalactic dust are either not vitally important or have been corrected for, the systematic dimming of high- $z$  Sn can be interpreted as evidence for an accelerated expansion of the universe caused by a form of ‘dark energy’ having large negative pressure.

The evidence for an accelerating universe from high redshift type Ia supernovae has now received independent support from an analysis of CMB fluctuations together with the HST key project determination of the Hubble parameter. Interestingly, the degeneracy in parameter space  $\{\Omega_m, \Omega_\Lambda\}$  arising from Sn observations is almost orthogonal to the degeneracy which arises from CMB measurements. This principle of ‘cosmic complementarity’ serves to significantly reduce the errors on  $\Omega_m$  &  $\Omega_\Lambda$  when the two sets of observations are combined, as shown in figure 6.



**Fig. 6.** Constraints on the density of dark matter  $\Omega_m$  and dark energy in the form of a cosmological constant  $\Omega_\Lambda$ , determined using WMAP (upper left), WMAP + other CMB experiments (WMAPext; upper right), WMAPext + HST key project data (lower left) and WMAPext + HST + supernova data (lower right); from Spergel et al (2003).

If dark energy is described by an unevolving equation of state  $w = p_X/\rho_X$ , then the transition between deceleration and acceleration ( $\ddot{a} = 0$ ) occurs at the redshift

$$(1 + z_a)^{-3w} = -(1 + 3w) \frac{\Omega_X}{\Omega_m} w < 0. \quad (28)$$

Another important redshift describes the epoch when the densities in dark matter and dark energy are equal

$$(1 + z_{\text{eq}})^{3w} = \left( \frac{\Omega_m}{\Omega_X} \right). \quad (29)$$

Substituting  $\Omega_\Lambda = 0.7$ ,  $\Omega_m = 0.3$  we find  $z_a \simeq 0.73$ ,  $z_{\text{eq}} \simeq 0.37$  for LCDM. The fact that the acceleration of the universe is a fairly recent phenomenon illustrates the ‘cosmic coincidence’ puzzle according to which we appear to live during a special epoch when the densities in dark energy and in dark matter are almost equal. A recent origin for the acceleration epoch is supported by supernova observations which suggest a decelerating universe at  $z \gtrsim 0.5$  [160]. It is important to note that dark energy models with an unevolving equation of state need to have their initial conditions properly ‘tuned’ in order to dominate the universe at precisely the present epoch. This problem is most acute for the cosmological constant. Since the cosmological constant does not evolve while both matter and radiation evolve rapidly ( $\rho_m \propto a^{-3}$ ,  $\rho_r \propto a^{-4}$ ), it follows that the small current value  $\rho_\Lambda = \Lambda/8\pi G \simeq 10^{-47} \text{ GeV}^4$  implies  $\rho_\Lambda/\rho_r \simeq 10^{-123}$  at the Planck time (when the temperature of the universe was  $T \sim 10^{19} \text{ GeV}$ ), or  $\rho_\Lambda/\rho_r \simeq 10^{-55}$  at the time of the electroweak phase transition ( $T \sim 100 \text{ GeV}$ ). Thus an extreme fine-tuning of initial conditions is required in order to ensure that  $\rho_\Lambda/\rho_m \sim 1$  today !

The fine tuning problem which plagues  $\Lambda$  also affects DE models in which  $w = \text{constant} \neq -1$ . A combined analysis of CMB, galaxy clustering and supernovae data indicates that a constant equation of state for dark energy must satisfy  $w < -0.82$  at the 95% confidence level [193, 201], and it is easy to show that for these models the fine tuning (and cosmic coincidence) problems are almost as acute as they are for the cosmological constant. This constraint on  $w$  also virtually rules out two interesting DE candidates based on topological defect models: a tangled network of cosmic strings  $w \simeq -1/3$  and domain walls  $w \simeq -2/3$ .

### 2.3 Quintessence

It is interesting that the fine tuning problem facing dark energy models with a constant equation of state can be alleviated if we assume that the equation of state is time dependent. An important class of models having this property are scalar fields (quintessence)<sup>4</sup> which couple minimally to gravity so that

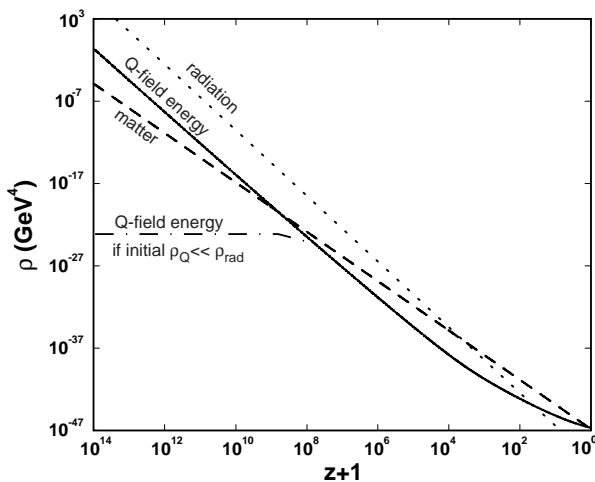
<sup>4</sup> Quintessence is named after the all pervasive fifth element of ancient philosophical thought. Note that the quintessence Lagrangian is the same as that used for Inflationary model building.

their Lagrangian density and energy momentum tensor is

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi) , \quad (30)$$

$$\rho \equiv T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p \equiv -T_\alpha^\alpha = \frac{1}{2}\dot{\phi}^2 - V(\phi) , \quad (31)$$

where we have assumed, for simplicity, that the field is homogeneous. Potentials which are sufficiently steep to satisfy  $\Gamma \equiv V''V/(V')^2 \geq 1$  have the interesting property that scalar fields rolling down such a potential approach a common evolutionary path from a wide range of initial conditions [217]. In these so-called ‘tracker’ models the scalar field density (and its equation of state) remains close to that of the dominant background matter during most of cosmological evolution.



**Fig. 7.** The quintessence Q-field while rolling an inverse power law potential tracks first radiation then matter, before coming to dominate the energy density of the universe at present. If the initial value of the Q-field density is small then  $\rho_Q$  remains constant until  $\rho_Q \sim \rho_{\text{rad}}$ , and then follows the tracker trajectory. From Zlatev, Wang and Steinhardt [217].

An excellent example of a tracker potential is provided by  $V(\phi) = V_0/\phi^\alpha$  [157]. During tracking the ratio of the energy density of the scalar field (quintessence) to that of radiation/matter gradually increases  $\rho_\phi/\rho_B \propto t^{4/(2+\alpha)}$  while its equation of state remains marginally smaller than the background value  $w_\phi = (\alpha w_B - 2)/(\alpha + 2)$ . For large values of  $\phi$  this potential becomes flat ensuring that the scalar field rolls sufficiently slowly ( $\dot{\phi}^2 \ll V(\phi)$ )

to allow the universe to accelerate. Note that for quintessence fields the conditions (22) & (23) translate into

$$\begin{aligned} w_\phi < -\frac{1}{3} &\Rightarrow \dot{\phi}^2 < V(\phi) , \\ w_\phi < -\frac{1}{2} &\Rightarrow \dot{\phi}^2 < \frac{2}{3}V(\phi) . \end{aligned} \quad (32)$$

(Current observations imply  $\alpha < 2$ .)

An extreme example of quintessence is provided by the exponential potential  $V(\phi) = V_0 \exp(-\sqrt{8\pi}\lambda\phi/M_p)$  [157, 213], where  $M_p = 1/\sqrt{G}$  is the Planck mass. In this case

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1 + w_B)}{\lambda^2} = \text{constant} < 0.2, \quad (33)$$

$\rho_B$  is the background energy density while  $w_B$  is the associated background equation of state. The lower limit  $\rho_\phi/\rho_{\text{total}} < 0.2$  arises because of nucleosynthesis constraints which prevent the energy density in quintessence from being large initially (at  $t \sim \text{few sec.}$ ). Equation (33) suggests that the exponential potential will remain subdominant if it was so initially. An interesting potential which interpolates between an exponential and a power law can however give rise to late time acceleration from tracker-like initial conditions [168]

$$V(\phi) = V_0[\cosh \lambda\phi - 1]^p, \quad (34)$$

has the property that  $w_\phi \simeq w_B$  at early times whereas  $\langle w_\phi \rangle = (p-1)/(p+1)$  at late times. Consequently (34) describes *quintessence* for  $p \leq 1/2$  and pressureless ‘cold’ dark matter (CDM) for  $p = 1$ . Thus the cosine hyperbolic potential (34) is able to describe both dark matter and dark energy within a tracker framework (also see [204, 12]).

Remarkably, quintessence can even accommodate a constant equation of state ( $w = \text{constant}$ ) by means of the potential [172, 173, 203]

$$V(\phi) \propto \sinh^{\frac{2(1+w)}{w}}(C\phi + D) , \quad (35)$$

with suitably chosen values of  $C, D$ .

Quintessence models can be divided into two categories: models which roll to large values of  $\phi/m_P \gtrsim 1$  and models for which  $\phi/m_P \ll 1$  at the present epoch. An important concern for the former is the effect of quantum corrections which, if large, could alter the shape of the quintessence potential [102, 33, 59, 182]. An important related issue is that the coupling between standard model fields and quintessence must be small in order to have evaded detection. Moreover even small couplings between quintessence and standard model fields can give rise to interesting changes in cosmology as shown in [6, 114].

Quintessence Potential	Reference
$V_0 \exp(-\lambda\phi)$	Ratra & Peebles (1988), Wetterich (1988), Ferreira & Joyce (1998)
$m^2\phi^2, \lambda\phi^4$	Frieman et al (1995)
$V_0/\phi^\alpha, \alpha > 0$	Ratra & Peebles (1988)
$V_0 \exp(\lambda\phi^2)/\phi^\alpha$	Brax & Martin (1999,2000)
$V_0(\cosh \lambda\phi - 1)^p$	Sahni & Wang (2000)
$V_0 \sinh^{-\alpha}(\lambda\phi)$	Sahni & Starobinsky (2000), Ureña-López & Matos (2000)
$V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$	Barreiro, Copeland & Nunes (2000)
$V_0(\exp M_p/\phi - 1)$	Zlatev, Wang & Steinhardt (1999)
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$	Albrecht & Skordis (2000)

Table 1.

I would like to end this section by mentioning that, due to the shortage of time I have not been able to cover all of the DE models suggested in the literature (a number that is growing rapidly !) For this reason these lectures will not discuss DE due to vacuum polarization [167, 141], k-essence [13], Cardassian expansion [73], Quasi-Steady State Cosmology [132], scalar-tensor models [5, 26, 45, 145, 205, 146, 161, 29, 147]. For other interesting approaches see [15, 83, 84, 62, 137, 80, 104, 105, 125, 187, 215]. A partial list of some popular quintessence models is given in Table 1, and the reader is also referred to the dark energy reviews in [172, 41, 143, 171, 140].

#### 2.4 Dark energy in braneworld models

Inspired by the Randall-Sundrum [156] scenario, braneworld cosmology suggests that we could be living on a three dimensional ‘brane’ which is embedded in a higher (usually four) dimensional bulk. According to such a scheme, all matter fields are confined to the brane whereas the graviton is free to propagate in the brane as well as in the bulk (see the lectures by Roy Maartens in this volume and [110] for a comprehensive discussion of Braneworld cosmology.) Within the RS setting the equation of motion of a scalar field propagating on the brane is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (36)$$

where [184]

$$\begin{aligned} H^2 &= \frac{8\pi}{3m^2}\rho\left(1 + \frac{\rho}{2\sigma}\right) + \frac{\Lambda_4}{3} + \frac{\mathcal{E}}{a^4}, \\ \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi). \end{aligned} \quad (37)$$

$\mathcal{E}$  is an integration constant which transmits bulk graviton influence onto the brane. The brane tension  $\sigma$  provides a relationship between the four dimensional Planck mass ( $m$ ) and the five-dimensional Planck mass ( $M$ )

$$m = \sqrt{\frac{3}{4\pi}} \left( \frac{M^3}{\sqrt{\sigma}} \right). \quad (38)$$

$\sigma$  also relates the four-dimensional cosmological constant  $\Lambda_4$  on the brane to the five-dimensional (bulk) cosmological constant  $\Lambda_b$  through

$$\Lambda_4 = \frac{4\pi}{M^3} \left( \Lambda_b + \frac{4\pi}{3M^3} \sigma^2 \right). \quad (39)$$

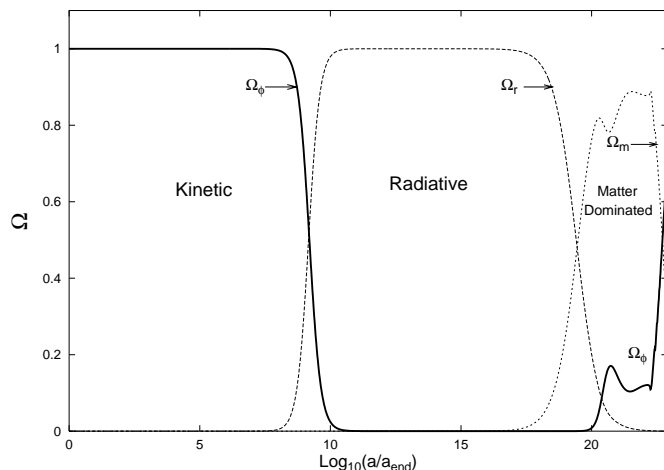
Note that (37) contains an additional term  $\rho^2/\sigma$  whose presence can be attributed to junction conditions imposed at the bulk-brane boundary. Because of this term the damping experienced by the scalar field as it rolls down its potential *dramatically increases* so that inflation can be sourced by potentials which are normally too steep to produce slow-roll. Indeed the slow-roll parameters in braneworld models (for  $V/\sigma \gg 1$ ) are [111]

$$\epsilon \simeq 4\epsilon_{\text{FRW}}(V/\sigma)^{-1}, \quad \eta \simeq 2\eta_{\text{FRW}}(V/\sigma)^{-1}, \quad (40)$$

illustrating that slow-roll ( $\epsilon, \eta \ll 1$ ) is easier to achieve when  $V/\sigma \gg 1$ . Inflation can therefore arise for the very steep potentials associated with quintessence such as  $V \propto e^{-\lambda\phi}$ ,  $V \propto \phi^{-\alpha}$  etc. This gives rise to the intriguing possibility that both inflation and quintessence may be sourced by one and the same scalar field. Termed ‘quintessential inflation’, these models have been examined in [142, 53, 85, 169, 116, 66, 106, 57, 185, 178, 177]. An example of quintessential inflation is shown in figure 8.

A radically different way of making the Universe accelerate was suggested in [55, 170]. The braneworld model developed by Deffayet, Dvali and Gabadadze (DDG) was radically different from the RS model in that both the bulk cosmological constant and the brane tension were set to zero, while a curvature term was introduced in the brane action so that the theory was described by

$$S = M^3 \int_{\text{bulk}} \mathcal{R} + m^2 \int_{\text{brane}} R + \int_{\text{brane}} L_{\text{matter}}. \quad (41)$$



**Fig. 8.** The post-inflationary density parameter  $\Omega$  is plotted for the scalar field (solid line) radiation (dashed line) and cold dark matter (dotted line) in the quintessential-inflationary model described by (34) with  $p = 0.2$ . Late time oscillations of the scalar field ensure that the mean equation of state turns negative  $\langle w_\phi \rangle \simeq -2/3$ , giving rise to the current epoch of cosmic acceleration with  $a(t) \propto t^2$  and present day values  $\Omega_{0\phi} \simeq 0.7$ ,  $\Omega_{0m} \simeq 0.3$ . From Sahni, Sami and Souradeep [169].

The rationale for the  $\int_{\text{brane}} R$  term is that quantum effects associated with matter fields are likely to give rise to such a term in the Einstein action as discussed by Sakharov in his development of induced gravity [176].

The resulting Hubble parameter in the DDG braneworld is

$$H = \sqrt{\frac{8\pi G\rho_m}{3} + \frac{1}{l_c^2} + \frac{1}{l_c}}, \quad (42)$$

where  $l_c = m^2/M^3$  is a new length scale determined by the four dimensional Planck mass  $m$  and the five dimensional Planck mass  $M$  respectively. An important property of this model is that the acceleration of the universe is not caused by the presence of any ‘dark energy’. Instead, since gravity becomes five dimensional on length scales  $R > l_c = 2H_0^{-1}(1 - \Omega_m)^{-1}$ , one finds that the expansion of the universe is modified during *late times* instead of early times as in the RS model.

A more general class of braneworld models which includes RS cosmology and the DDG brane as subclasses was developed in [51, 189] and is described by the action

$$S = M^3 \int_{\text{bulk}} (\mathcal{R} - 2\Lambda_b) + \int_{\text{brane}} (m^2 R - 2\sigma) + \int_{\text{brane}} L_{\text{matter}}. \quad (43)$$

For  $\sigma = \Lambda_b = 0$  (43) reduces to the action describing the DDG model, whereas for  $m = 0$  it describes the Randall-Sundrum model.

As demonstrated by Sahni and Shtanov [170] the braneworld which follows from the action (43) describes an accelerating universe at late times with the Hubble parameter

$$\frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + \Omega_\sigma + 2\Omega_l \mp 2\sqrt{\Omega_l} \sqrt{\Omega_m(1+z)^3 + \Omega_\sigma + \Omega_l + \Omega_{\Lambda_b}}, \quad (44)$$

where

$$\Omega_l = \frac{1}{l_c^2 H_0^2}, \quad \Omega_m = \frac{\rho_{0m}}{3m^2 H_0^2}, \quad \Omega_\sigma = \frac{\sigma}{3m^2 H_0^2}, \quad \Omega_{\Lambda_b} = -\frac{\Lambda_b}{6H_0^2}. \quad (45)$$

(The  $\mp$  signs refer to the two different ways in which the brane can be embedded in the bulk, both signs give rise to interesting cosmology [170].) As in the DDG model  $l_c \sim H_0^{-1}$  if  $M \sim 100$  MeV. On short length scales  $r \ll l_c$  and at early times, one recovers general relativity, whereas on large length scales  $r \gg l_c$  and at late times brane-related effects begin to play an important role. Indeed by setting  $M = 0$  ( $\Omega_l = 0$ ) (44) reduces to the LCDM model

$$\frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + \Omega_\sigma, \quad (46)$$

whereas for  $\sigma = \Lambda_b = 0$  (44) reduces to the DDG braneworld. An important feature of the braneworld (44) is that it can lead to an effective equation of state of dark energy  $w_{\text{eff}} \leq -1$ . This is easy to see from the expression for the current value of the effective equation of state [170]

$$w_0 = \frac{2q_0 - 1}{3(1 - \Omega_m)} = -1 \pm \frac{\Omega_m}{1 - \Omega_m} \sqrt{\frac{\Omega_l}{\Omega_m + \Omega_\sigma + \Omega_l + \Omega_{\Lambda_b}}}, \quad (47)$$

we find that  $w_0 < -1$  when we take the lower sign in (47), which corresponds to choosing one of two possible embeddings of this braneworld in the higher dimensional bulk. (The second choice of embedding gives  $w_0 > -1$ .)

It is also possible, in this model, for the acceleration of the universe to be a transient phenomenon which ends once the universe returns to matter dominated expansion after the current accelerating phase. As discussed in [170] such a braneworld will not have an event horizon and may therefore help in reconciling an accelerating universe with the demands of string/M-theory. Other possibilities of obtaining dark energy from extra dimensions have been discussed in [7, 144, 44, 150, 151, 117, 34, 138]. The possibility that DE could arise due to modifications of gravitational physics has also been examined in [108, 40, 43, 58, 124, 135, 136].

## 2.5 Chaplygin gas

A completely different route to dark energy is provided by the Chaplygin gas [92] which obeys the equation of state

$$p_c = -A/\rho_c . \quad (48)$$

The conservation equation  $dE = -pdV \Rightarrow d(\rho a^3) = -pd(a^3)$  immediately gives

$$\rho_c = \sqrt{A + \frac{B}{a^6}} = \sqrt{A + B(1+z)^6} , \quad (49)$$

where  $B$  is a constant of integration. Thus the Chaplygin gas behaves like pressureless dust at early times and like a cosmological constant during very late times.

The Hubble parameter for a universe containing cold dark matter and the Chaplygin gas is given by

$$H(z) = H_0 \left[ \Omega_m(1+z)^3 + \frac{\Omega_m}{\kappa} \sqrt{\frac{A}{B} + (1+z)^6} \right]^{1/2} , \quad (50)$$

where  $\kappa = \rho_{0m}/\sqrt{B}$  and it is easy to see from (50) that

$$\kappa = \frac{\rho_{0m}}{\rho_c}(z \rightarrow \infty) . \quad (51)$$

Thus,  $\kappa$  defines the ratio between CDM and the Chaplygin gas energy densities at the commencement of the matter-dominated stage. It is easy to show that

$$A = B \left\{ \kappa^2 \left( \frac{1 - \Omega_m}{\Omega_m} \right)^2 - 1 \right\} . \quad (52)$$

It is interesting that the Chaplygin gas can be derived from an underlying Lagrangian in two distinct ways:

- One can derive it from a quintessence Lagrangian (30) with the potential [92]

$$V(\phi) = \frac{\sqrt{A}}{2} \left( \cosh 3\phi + \frac{1}{\cosh 3\phi} \right) . \quad (53)$$

- The Chaplygin gas can also be derived from the Born-Infeld form of the Lagrangian density

$$\mathcal{L} = -V_0 \sqrt{1 - \phi_{,\mu} \phi^{,\mu}} , \quad (54)$$

where  $\phi_{,\mu} \equiv \partial\phi/\partial x^\mu$ . For time-like  $\phi_{,\mu}$  one can define a four velocity

$$u^\mu = \frac{\phi^{,\mu}}{\sqrt{\phi_{,\alpha} \phi^{,\alpha}}} , \quad (55)$$

this leads to the standard form for the hydrodynamical energy-momentum tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} , \quad (56)$$

where [75]

$$\rho = \frac{V_0}{\sqrt{1 - \phi_{,\mu}\phi^{,\mu}}} , \quad p = -V_0\sqrt{1 - \phi_{,\mu}\phi^{,\mu}} , \quad (57)$$

*i.e.* we have recovered (48) with  $A = V_0^2$ .

The fact that the properties of the Chaplygin gas interpolate between those of CDM and a  $\Lambda$ -term led to the hope that the CG might provide a conceptual framework for a unified model of dark matter and dark energy. It should however be noted that in contrast to CDM and baryons, the sound velocity in the Chaplygin gas  $v_c = \sqrt{dp_c/d\rho_c} = \sqrt{A}/\rho_c$  quickly grows  $\propto t^2$  during the matter-dominated regime and becomes of the order of the velocity of light at present (it approaches light velocity asymptotically in the distant future). Thus, when one examines classical inhomogeneities, the properties of the Chaplygin gas during the matter-dominated epoch appear to be rather unusual and resemble those of hot dark matter rather than CDM, despite the fact that the Chaplygin gas formally carries negative pressure [2].

A ‘generalized Chaplygin gas’ has also been proposed for which  $p \propto -1/\rho^\alpha$ . The equation of state in this case is

$$w(a) = -\frac{|w_0|}{\left[|w_0| + \frac{1-|w_0|}{a^{3(1+\alpha)}}\right]} , \quad (58)$$

which interpolates between  $w = 0$  at early times ( $a \ll 1$ ) and  $w = -1$  at late times ( $a \gg 1$ );  $w_0$  is the current equation of state at  $a = 1$ . (The constant  $\alpha$  regulates the transition time in the equation of state.) WMAP, supernovae and large scale structure data have all been used to test Chaplygin gas models; see [27, 65, 78, 9, 14, 22, 115, 128, 56, 25].

## 2.6 Is Dark Energy a Phantom ?

In an influential paper Caldwell [38] noticed that a very good fit to the supernova-derived luminosity distance was provided by dark energy which violated the weak energy condition so that  $w < -1$ . He called this Phantom dark energy.<sup>5</sup> Indeed, a study of high- $z$  Sn [99] finds that the DE equation of state has a 99% probability of being  $< -1$  if no priors are placed on  $\Omega_m$ ! When these Sn results are combined with CMB and 2dFGRS the 95% confidence limits on an unevolving equation of state are  $-1.61 < w < -0.78$  [99], which is consistent with estimates made by other groups [193, 201].

A universe filled with Phantom energy has some interesting but bizarre properties.

<sup>5</sup> Phantom takes its name from Part I of the Star Wars movie series – the Phantom Menace.

- If  $t_{\text{eq}}$  marks the epoch when the densities in matter and phantom energy are equal then the expansion factor of a universe dominated by phantom energy grows as

$$a(t) \simeq a(t_{\text{eq}}) \left[ (1+w) \frac{t}{t_{\text{eq}}} - w \right]^{2/3(1+w)}, \quad w < -1, \quad (59)$$

and therefore *diverges in a finite* amount of cosmic time

$$a(t) \rightarrow \infty \quad \text{as } t \rightarrow t_{\text{BR}} = \left( \frac{w}{1+w} \right) t_{\text{eq}}. \quad (60)$$

By substituting  $w < -1$  into (24) we immediately find that the Hubble parameter also diverges as  $t \rightarrow t_{\text{BR}}$ , implying that an infinitely rapid expansion rate for the universe has been reached in a *finite* time. The divergence of the Hubble parameter is associated with the divergence of phantom density which grows without bound

$$\rho(t) \propto \left[ (1+w) \frac{t}{t_{\text{eq}}} - w \right]^{-2}, \quad (61)$$

and reaches a singular value in a finite interval of time  $\rho(t) \rightarrow \infty, t \rightarrow t_{\text{BR}}$ . Thus a universe dominated by Phantom energy culminates in a future curvature singularity ('Big Rip') at which the notion of a classical space-time breaks down. (See also [196, 47, 38, 121, 39, 42, 71, 72, 186, 88, 10, 93].)

- The ultra-negative phantom equation of state suggests that the effective velocity of sound in the medium  $v = \sqrt{|dp/d\rho|}$  can become larger than the velocity of light in this model.
- Although a dynamical model of phantom energy can be constructed with the 'wrong' sign of the kinetic term, see (31), such models are plagued with instabilities at the quantum level [49] which makes their existence suspect. It should be pointed out that phantom is not the only way to get  $w < -1$ . A model with similar properties but sharing none of phantom's pathologies is the braneworld model of [170, 1], which has  $w_{\text{eff}} < -1$  today but does not run into a 'Big Rip' in the future.

## 2.7 Reconstructing Dark Energy and the Statefinder diagnostic

In view of the considerable number of dark energy models suggested in the literature, it becomes meaningful to ask whether we can reconstruct the properties of DE from observations in a model independent manner. This indeed may be possible if one notices that the the Hubble parameter is related to the luminosity distance [195, 175]

$$H(z) = \left[ \frac{d}{dz} \left( \frac{d_L(z)}{1+z} \right) \right]^{-1}, \quad (62)$$

and that, in the case of quintessence, the scalar field potential as well as its equation of state can be directly expressed in terms of the Hubble parameter and its derivative [195, 175]

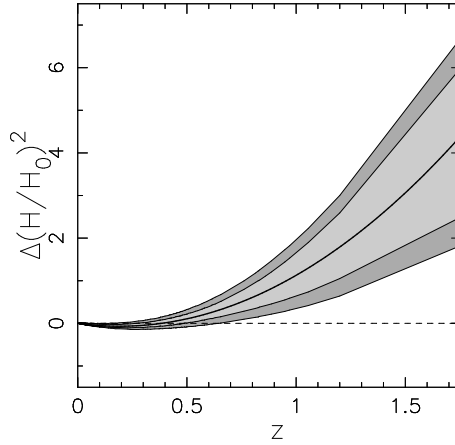
$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2} \frac{dH^2}{dx} - \frac{1}{2}\Omega_m x^3, \quad (63)$$

$$\frac{8\pi G}{3H_0^2} \left( \frac{d\phi}{dx} \right)^2 = \frac{2}{3H_0^2 x} \frac{d \ln H}{dx} - \frac{\Omega_m x}{H^2}, \quad x = 1 + z, \quad (64)$$

$$(65)$$

$$w_\phi(x) \equiv \frac{p}{\varepsilon} = \frac{(2x/3)d \ln H/dx - 1}{1 - (H_0^2/H^2) \Omega_m x^3}. \quad (66)$$

Both the quintessence potential  $V(\phi)$  as well as the equation of state  $w_\phi(z)$  may therefore be reconstructed provided the luminosity distance  $d_L(z)$  is known to reasonable accuracy from observations.



**Fig. 9.** The relative difference between the Hubble parameter reconstructed from Sn data and the LCDM value is shown as a function of redshift. Sn data from Tonry et al (2003) were used for the reconstruction. The best-fit is represented by the thick solid line assuming  $\Omega_m = 0.3$ . The light (dark) grey contours represents the  $1\sigma$  ( $2\sigma$ ) confidence levels around the best-fit. The dashed horizontal line shows LCDM. From Alam, Sahni, Saini and Starobinsky [4].

In practice it is useful to have an ansatz for either one of three cosmological quantities:  $d_L(z)$ ,  $H(z)$  or  $w(z)$ , which can then be used for cosmological reconstruction [175, 131, 112, 211]. Popular fitting functions discussed in the literature include:

- (i) An ansatz for the dark energy [173]

$$\rho_{\text{DE}}(x) = \sum_{i=0}^N A_i x^i, \quad x = 1 + z. \quad (67)$$

(ii) Fitting functions to the dark energy equation of state [212, 107]:

$$w(z) = \sum_{i=0}^N w_i z^i, \\ w(z) = w_0 + \frac{w_1 z}{1+z}. \quad (68)$$

The fitting parameters  $w_i$ ,  $A_i$  are obtained by matching to observations. In practice the first few terms in either series (67), (68) is sufficient since the current Sn data is quite noisy; see [46, 212, 54, 77, 113, 2] for a discussion of these issues. An example of cosmological reconstruction of the Hubble parameter from Sn data is shown in figure (9); see also [208, 134].

The Sn inventory is increasing dramatically every year and so are increasingly precise measurements of galaxy clustering and the CMB. To keep pace with the better quality observational data which will soon become available and the increasing sophistication of theoretical modelling, a new diagnostic of DE called ‘Statefinder’ was introduced in [173].

The statefinder probes the expansion dynamics of the universe through higher derivatives of the expansion factor  $\ddot{a}$  and is a natural companion to the deceleration parameter which depends upon  $\ddot{a}$  (20). The statefinder pair  $\{r, s\}$  is defined as follows:

$$r \equiv \frac{\ddot{a}}{aH^3} = 1 + \frac{9w}{2}\Omega_X(1+w) - \frac{3}{2}\Omega_X \frac{\dot{w}}{H}, \quad (69)$$

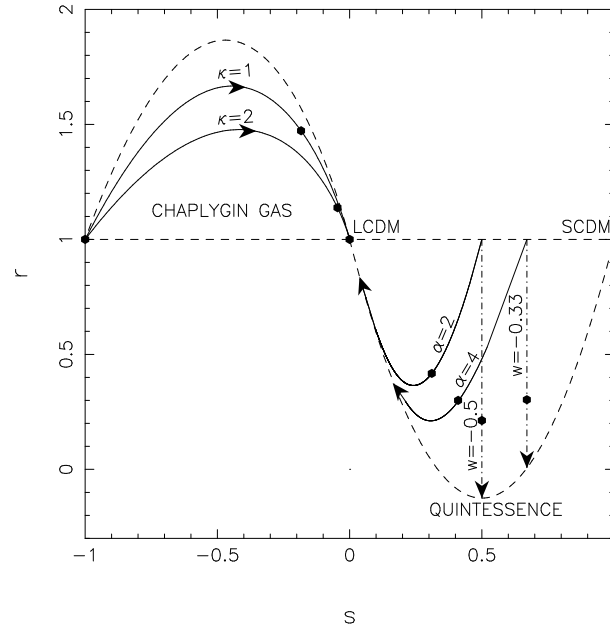
$$s \equiv \frac{r-1}{3(q-1/2)} = 1 + w - \frac{1}{3} \frac{\dot{w}}{wH}. \quad (70)$$

Inclusion of the statefinder pair  $\{r, s\}$ , increases the number of cosmological parameters to four<sup>6</sup>:  $H, q, r, s$ . The Statefinder is a ‘geometrical’ diagnostic in the sense that it depends upon the expansion factor and hence upon the metric describing space-time. An important property of the Statefinder is that spatially flat LCDM corresponds to the fixed point

$$\{r, s\} \Big|_{\text{LCDM}} = \{1, 0\}. \quad (71)$$

Departure of a given DE model from this fixed point provides a good way of establishing the ‘distance’ of this model from LCDM [2]. As demonstrated in [173, 2, 78, 216] the Statefinder can successfully differentiate between a wide variety of DE models including the cosmological constant, quintessence, the Chaplygin gas, braneworld models and interacting DE models; an example is provided in figure 10.

<sup>6</sup>  $r$  has also been called ‘cosmic jerk’ in [207]



**Fig. 10.** The time evolution of the statefinder pair  $\{r, s\}$  for quintessence models and the Chaplygin gas. Solid lines to the right of LCDM represent tracker potentials  $V = V_0/\phi^\alpha$ , while those to the left correspond to the Chaplygin gas. Dot-dashed lines represent DE with a constant equation of state  $w$ . Tracker models tend to approach the LCDM fixed point ( $r = 1, s = 0$ ) from the right at  $t \rightarrow \infty$ , whereas the Chaplygin gas approaches LCDM from the left. For Chaplygin gas  $\kappa$  is the ratio between matter density and the density of the Chaplygin gas at early times. The dashed curve in the lower right is the envelope of all quintessence models, while the dashed curve in the upper left is the envelope of Chaplygin gas models (the latter is described by  $\kappa = \Omega_m/1 - \Omega_m$ ). The region outside the dashed curves is forbidden for both classes of dark energy models. The ability of the Statefinder to differentiate between dark energy models is clearly demonstrated. From Alam, Sahni, Saini and Starobinsky [2].

## 2.8 Big Rip, Big Crunch or Big Horizon ? – The fate of the Universe in dark energy models

The nature of dark energy affects the future of our Universe in a very significant way. If DE is simply the cosmological constant, then the universe will accelerate for ever. Of great importance is the fact that an accelerating LCDM universe develops an event horizon similar to the one surrounding a black hole [196]. Consider an event  $(r_1, t_1)$  which we wish to observe at our location at  $r = 0$ . Setting  $ds^2 = 0$  we get

$$\int_0^{r_1} \frac{dr}{\sqrt{1 - \kappa r^2}} = \int_{t_1}^t \frac{cdt'}{a(t')}. \quad (72)$$

Any event in the universe will one day be observed by us if the integral in the RHS of (72) *diverges* as  $t \rightarrow \infty$ . For power law expansion this clearly implies  $a \propto t^p$ ,  $p < 1$ , *i.e.* a *decelerating universe*. In an accelerating universe exactly the opposite is true, the integral in the RHS converges signalling the presence of an event horizon. In this case our civilization will receive signals only from those events which satisfy [172]

$$\int_0^{r_1} \frac{dr}{\sqrt{1 - \kappa r^2}} \leq \int_{t_1}^{\infty} \frac{cdt'}{a(t')}. \quad (73)$$

For de Sitter-like expansion  $a = a_1 \exp H(t - t_1)$ ,  $H = \sqrt{\Lambda/3}$ , we get  $r_1 = c/a_1 H$ , so that the *proper distance* to the event horizon is  $R_H = a_1 r_1 = c/H$ . In LCDM cosmology,

$$H \equiv H(t \rightarrow \infty) = \sqrt{\Lambda/3} = H_0 \sqrt{1 - \Omega_m}, \quad (74)$$

and the proper distance to the horizon is

$$R_H = \frac{c}{H_0 \sqrt{1 - \Omega_m}} \simeq 3.67h^{-1} \text{ Gpc}, \quad (75)$$

if  $\Omega_m \simeq 1/3$ . Thus our observable universe will progressively shrink as astrophysical bodies which are not gravitationally bound to the local group get pushed to distances beyond  $R_H$ . (More generally, horizons exist in a universe which begins to perpetually accelerate after a given point of time [81, 68, 171]. To this category belong models of dark energy with equation of state  $-1 < w < -1/3$ , as well as ‘runaway scalar fields’ [198] which satisfy  $V, V', V'' \rightarrow 0$  and  $V'/V, V''/V \rightarrow 0$  as  $\phi \rightarrow \infty$ .)

The presence of an event horizon implies that, at any given moment of time  $t_0$ , there is a ‘sphere of influence’ around our civilization. This sphere has an associated redshift  $z_H$ , and a celestial body having  $z > z_H$  will be unreachable by any signal emitted by our civilization now or in the future;  $z_H \simeq 1.8$  in LCDM cosmology with  $\Omega_\Lambda \simeq 2\Omega_m \simeq 2/3$ . Thus all celestial bodies with  $z > 1.8$  lie beyond our event horizon and there is no possibility of causal contact with any of them.

Interestingly, an N-body simulation tracking the future of an LCDM universe has shown that  $\sim 100$  billion years from now the observable universe will consist of only a single massive galaxy within our event horizon – the merger product of the Milky Way and Andromeda galaxies [130]. Furthermore, since the growth of large scale structure freezes in an accelerating universe, the mass distribution of bound objects will cease to evolve after about 30 billion years.

This somewhat gloomy future scenario is not absolutely essential and can be avoided if the currently observed acceleration of the universe is a transient phenomenon.<sup>7</sup> Just such a possibility exists in a class of braneworld models

<sup>7</sup> Accelerating cosmologies without event horizons are important in a different context. Since the conventional S-matrix approach may not work in a universe with an event horizon, models with horizons may pose a serious challenge to a fundamental theory of interactions such as string/M-theory.

[170] in which the current accelerating phase is succeeded by a decelerating matter dominated regime. Quintessence potentials can also have this property, as discussed in [21]. An interesting class of transiently accelerating DE models is constructed around a scalar field potential which decays with time and becomes negative at late times [74, 48, 139, 90, 91, 3]. An example is  $V = V_0 \cos \phi/f$  which describes axionic quintessence [74, 48, 139, 3]. Such a universe recollapses in the future when  $H(t_0 + \Delta T) = 0$ , and contracts thereafter towards a ‘Big Crunch’ singularity. Supernova observations indicate that, for typical decaying potentials, the universe will not collapse for at least  $\Delta T \simeq 20$  Gyrs [3].

DE models have also been proposed which encounter a ‘quiescent singularity’ while expanding. At the ‘quiescent singularity’ the second derivative of the expansion factor diverges while its first derivative remains finite [190, 79] (*i.e.*  $\ddot{a} \rightarrow -\infty$ ,  $\dot{a} \simeq \text{constant}$ ). In such models the expansion of the universe ‘brakes’ to a virtual standstill as the universe approaches the singular regime at which invariants of the space-time metric diverge ( $R_{iklm}R^{iklm} \rightarrow \infty$ ) while, curiously, the Hubble parameter and the energy density remain finite. Cosmological consequences of models which encounter a future quiescent singularity (or a ‘Big Break’ [79]) have been briefly discussed in [1, 190, 79] but need to be examined in more detail.

Finally, as discussed in section 2.6, Phantom models with  $w < -1$  expand towards a Big Rip, at which the density and all curvature invariants become infinite. As in the case of the Big Crunch singularity, the Big Rip will occur only in the very distant future (if it occurs at all). For instance, if  $w = \text{constant} \geq -1.5$ ,  $H_0 = 70\text{km/sec/Mpc}$  and  $\Omega_m = 0.3$ , the time to the Big Rip exceeds 22 Gyr [196].

### 3 Conclusions and future directions

From the theoretical standpoint the single most important question to be asked of dark energy is

$$\text{Is } w = -1 ?$$

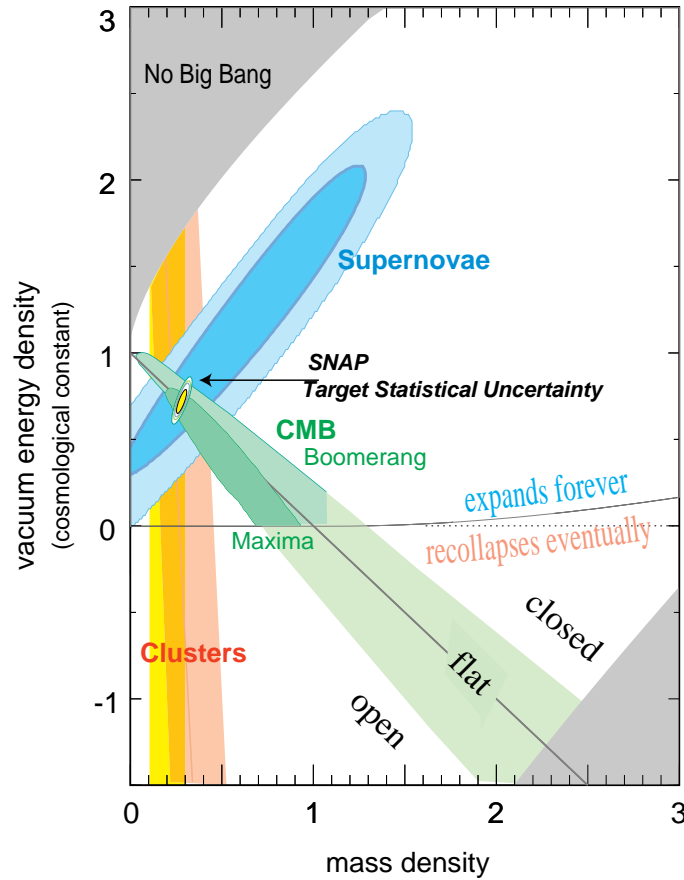
Rephrased in terms of the Statefinder diagnostic the question is:

$$\text{Is } \ddot{a} / aH^3 = 1 ?$$

If future observations do answer this question in the affirmative<sup>8</sup> then, in all likelihood the cosmological constant is the vacuum energy, and one will need to review the cosmological constant problem again, in order to fathom why the formally infinite quantity  $\langle T_{ik} \rangle$  is in fact so very small.

If on the other hand, either  $w \neq -1$  or if the DE density is shown to be time dependent, then the cosmological constant problem may need to be decoupled from the DE conundrum and searches for evolving DE models which produce

<sup>8</sup> *i.e.* if  $w = -1$  is measured to satisfyingly high accuracy



**Fig. 11.** Target statistical uncertainty of the SNAP experiment is shown overlaid with current results from CMB and LSS observations. From Aldering [11].

$\rho_{\text{DE}} \simeq 10^{-47} \text{GeV}^4$  without exacerbating ‘cosmic coincidence’ will need to be examined deeply in the light of developments both in high energy physics and in gravitation theory (superstring/M-theory, extra dimensions etc.). In either case the key to determining the properties of DE to great precision clearly lies with ongoing and future astrophysical experiments and observations.

Since the original discovery of an accelerating universe [152, 153, 159] the Sn data base has grown considerably and data pertaining to  $\sim 200$  type Ia supernovae are available in the literature [202, 99, 19, 160]. Although systematic effects such as luminosity evolution, dimming by intervening extragalactic material (alternatively brightening due to gravitational lensing) continue to

be a cause of some concern – recall that a luminosity evolution of  $\sim 25\%$  over a lookback time of  $\sim 5$  Gyr is sufficient to nullify the DE hypothesis [158] – it is reassuring that recent observations of CMB anisotropies and estimates of galaxy clustering in the 2dF and SDSS surveys, make a strong and independent case for dark energy [193, 200, 201]. Indeed, a joint analysis of CMB data from WMAP + HST Key Project determination of  $H_0$  imply  $w_{\text{DE}} < -0.5$  at the 95% confidence level [193].

It is of paramount importance that Sn observations continue to be supplemented by other investigations which are sensitive to the geometry of space and can be used as independent tests of the DE hypothesis. The volume-redshift test, Sunyaev-Zeldovich surveys, the Alcock-Paczynski test, the angular size-redshift test and gravitational lensing have all been suggested as possible probes of dark energy, and will doubtless enrich the theory vs observations debate in the near future. In addition, the proposed SNAP satellite which aims to measure light curves of  $\sim 2000$  supernovae within a single year [219], should provide a big step forward in our understanding of type Ia supernovae and help determine the cosmological parameters to great precision, as shown in figure 11.

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## References

## References

- \*. E-mail: varun@iucaa.ernet.in,
1. Alam, U. and Sahni, V., astro-ph/0209443.
  2. Alam, U., Sahni, V., Saini, T.D. and Starobinsky, A.A., *Mon. Not. Roy. ast. Soc.*, **344**, 1057 (2003) [astro-ph/0303009].
  3. Alam, U. Sahni, V. and Starobinsky, A.A., JCAP **0304**, 002 (2003) [astro-ph/0302302]
  4. Alam, U., Sahni, V., Saini, T.D. and Starobinsky, A.A., astro-ph/0311364.
  5. Amendola, L. (1999) *Phys. Rev. D* **60**, 043501.
  6. Amendola, L. and Tocchini-Valentini, D. *Phys. Rev. D* **64**, 043509 (2001); Tocchini-Valentini, D. and Amendola, L. *Phys. Rev. D* **65**, 063508 (2002)
  7. Albrecht, A., Burgess, C.P., Ravndal, F. and Skordis, C., *Phys. Rev. D* **65**, 123507 (2002)
  8. Albrecht, A. and Skordis, C. *Phys. Rev. Lett.* **84**, 2076 (2000).

9. Alcaniz, J.S., Jain, D. and Dev, A., *Phys. Rev. D* **67**, 043514 (2003) [[astro-ph/0210476](#)].
10. Alcaniz, J.S., [astro-ph/0312424](#)
11. Aldering, G., [astro-ph/0209550](#)
12. Arbey, A., Lesgourgues, J and Salati, P., *Phys. Rev. D* **68** 023511 (2003)
13. Armendariz-Picon, C., Mukhanov, V. and Steinhardt, P.J., *Phys. Rev. Lett.* **85**, 4438 (2000).
14. Avelino, C., Beca, L.M.G., de Carvalho, J.P.M., Martins, C.J.A.P. and Pinto, P., *Phys. Rev. D* **67**, 023511 (2003) [[astro-ph/0208528](#)].
15. Axenides, M. and Dimopoulos, K., [hep-th/0401238](#).
16. Bardeen, J.M., Bond, J.R., Kaiser, N. and Szalay, A.S., *Astroph. J.* **304**, 15 (1986)
17. Barnebei, R. et al, *Phys. Lett. B* **480**, 23 (2000)
18. Barreiro, T., Copeland, E.J. and Nunes, N.J., *Phys. Rev. D* **61**, 127301 (2000);
19. Barris, B. J. *et al.*, [astro-ph/0310843](#).
20. Barrow, J.D. and Tipler, F.J. (1986) *The Anthropic Cosmological Principle*, New York: Oxford University Press.
21. Barrow, J.D., Bean, R. and Magueijo, J., *MNRAS* **316**, L41 (2000).
22. Bean, R. and Dore, O., *Phys. Rev. D* **68** 023515 (2003) [[astro-ph/0301308](#)].
23. Bekenstein, J.D., [astro-ph/0403694](#).
24. Benson, A.J. et al. *MNRAS* **333**, 177 (2002)
25. Bento, M., Bertolami and Sen, A.A., *Phys. Lett. B* **575** 172, (2003) [[astro-ph/0303538](#)]
26. Bertolami, O. and Martins, P.J. (1999) [gr-qc/9910056](#).
27. Bilic, N., Tupper, G.B. and Viollier, R., *Phys. Lett. B* **535**, 17 (2002).
28. Bolatto, A.D, *et al. Astroph. J.* **565**, 238 (2002)
29. Boisseau, B., Esposito-Farese, G., Polarski, D. and Starobinsky, A.A., *Phys. Rev. Lett.* **85**, 2236 (2000)
30. Bosma, A., [astro-ph/0312154](#)
31. Brax, P. and Martin, J. *Phys. Rev. D* **61**, 103502 (2000); *Phys. Lett. B* **468**, 40 (1999).
32. Brustein, R., in [hep-ph/9810526](#)
33. Burgess, C.P., Grenier, P. and Hoover, D., [hep-ph/0308252](#);
34. Burgess, C.P., [hep-th/0402200](#).
35. Bullock, J.S., Kravtsov, A.V. and Weinberg, D.H., *Astroph. J.* **539**, 517 (2000)
36. Burkert, A., *Astroph. J.* **447**, L25 (1995)
37. de Blok, W.J.G. and McGaugh, S.S., *MNRAS* **290**, 533 (1997)
38. Caldwell, R.R., *Phys. Lett. B* **545**, 23 (2002) [[astro-ph/9908168](#)].
39. Caldwell, R.R., Kamionkowski, M. and Weinberg, N.N., *Phys.Rev.Lett.* **91** 071301 (2003) [[astro-ph/0302506](#)].
40. Capozziello, S., Carloni, S. and Troisi, A., [astro-ph/0303041](#)
41. Carroll, S.M., *Living Rev.Rel.* **4** 1 (2001) [[astro-ph/0004075](#)].
42. Carroll, S.M., Hoffman, M. and Trodden, M., *Phys. Rev. D* **68**, 023509 (2003)
43. Carroll, S.M., Duvvuri, V., Trodden, M. and Turner, M.S., [astro-ph/0306438](#)
44. Chen, J.-W., Luty, M.A. and Ponton, E., *JHEP* **0009**, 012 (2000)
45. Chiba, T. *Phys. Rev. D* **D60**, 083508 (1999).
46. Chiba, T. and Nakamura, T., *Phys. Rev. D* **62**, 121301(R) (2000).
47. Chiba, T., Okabe, T. and Yamaguchi, M, 2000, *Phys. Rev. D* **62**, 023511.
48. Choi, K., *Phys.Rev. D* **62** 043509 (2000) [[hep-ph/9902292](#)].

49. Cline, J.M., Jeon, S. and Moore, G.D., hep-ph/0311312.
50. Coles, P. and Lucchin, F. *Cosmology, The origin and evolution of cosmic structure*, Wiley.
51. Collins, H. and Holdom, B., *Phys. Rev. D* **62**, 105009 (2000) [hep-ph/0003173].
52. Combes, F., astro-ph/0206126
53. Copeland, E.J., Liddle, A.R. and Lidsey, J.E. *Phys. Rev. D* **64** 023509 (2001).
54. Corasaniti, P.S. and Copeland, E.J., *Phys. Rev. D* **67** 063521 (2003) [astro-ph/0205544].
55. Deffayet, C., Dvali, G. and Gabadadze, G., *Phys. Rev. D* **65**, 044023 (2002) [astro-ph/0105068]; Deffayet, C., Landau, S.J., Raux, J., Zaldarriaga, M. and Astier, P., *Phys. Rev. D* **66**, 024019 (2002) [astro-ph/0201164].
56. Dev, A., Jain, D. and Alcaniz, J. astro-ph/0311056
57. Dimopoulos, K., *Phys. Rev. D* **68**, 123506 (2003) [astro-ph/0212264]
58. Dolgov, A.D. and Kawasaki, M., *Phys. Lett. B* **573**, 1 (2003) [astro-ph/0307285]
59. Doran M. and Jaeckel, J., *Phys. Rev. D* **66**, 043519 (2003), astro-ph/0203018.
60. Efstathiou, G., Sutherland, W. and Maddox, S.J., *Nature* **348**, 705 (1990).
61. Elgaroy, O. et al., *Phys. Rev. Lett.* **90**, 021802 [hep-ph/0204152]
62. Elizalde, E., Lidsey, J., Nojiri, S. and Odintsev, S.D., *Phys. Lett.* **B574**, 1 (2003) [hep-th/0307177].
63. Ellis, J., astro-ph/0304183.
64. Evans, N.W. astro-ph/0102082
65. Fabris, J.S., Goncalves, S.V. and de Souza, P.E., astro-ph/0207430.
66. Feng, B. and Li, M., *Phys. Lett.*, **B** 564, 169 (2003) [hep-ph/0212213].
67. Ferreira, P.G. and Joyce, M. *Phys. Rev. Lett.* **79**, 4740 (1997); Ferreira, P.G. and Joyce, M. *Phys. Rev. D* **58**, 023503 (1998);
68. Fischler, W., Kashani-Poor, A., McNeese, R. and Paban, S., *JHEP* 0107 003 (2001) [hep-th/0104181].
69. Flores, R.A. and Primack, J.R. *Astroph. J.* **427**, L1 (1994)
70. Frampton, P.H. and Takahashi, T., *Phys. Lett. B* **557**, 135 (2003) [astro-ph/0211544].
71. Frampton, P., *Phys. Lett. B* **555**, 139 (2003).
72. Frampton, P. and Takahashi, T., *Phys. Lett. B* **557**, 135 (2003).
73. Freese, K. and Lewis, M., *Phys. Lett. B* **540** 1 (2002) [astro-ph/0201229]
74. Frieman, J., Hill, C.T., Stebbins, A. and Waga, I., *Phys. Rev. Lett.* **75**, 2077 (1995).
75. Frolov, A., Kofman, L. and Starobinsky, A.A., *Phys. Lett. B* **545**, 8, (2002) [hep-th/0204187].
76. Garriga, J. and Vilenkin, A., *Phys. Rev. D* **61** 083502 (2000).
77. Gerke, B., & Efstathiou, G., *Mon. Not. Roy. ast. Soc.* **335** 33 (2002), [astro-ph/0201336].
78. Gorini, V., Kamenshchik, A. and Moschella, U., astro-ph/0209395.
79. Gorini, V., Kamenshchik, A., Moschella, U., Pasquier, V. hep-th/0311111
80. Gu, P., Wang, X. and Zhang, X., *Phys. Rev. D* **68**, 087301 (2003) [hep-ph/0307148].
81. Hellerman, S., Kaloper, N. and Susskind, L., *JHEP* 0106 003 (2001) [hep-th/0104180].
82. Hoekstra, H., yee, H.K.C. and Gladders, M.D., astro-ph/0109514
83. Hsu, S. and Murrey, B., astro-ph/0402541.

84. Hsu, S., hep-th/0403052.
85. Huey, G. Huey Lidsey, J. *Phys. Lett. B* **514**, 217 (2001).
86. Huterer, D. and Turner, M. S., *Phys. Rev. D* , **60** 81301 (1999)
87. Jing, Y.P. and Suto, Y., *apj* **529**, L69 (2000)
88. Johri, V.B., 2003, astro-ph/0311293.
89. Jungman, J., Kamionkowski, M. and Griest, K., *Phys. Rep.* **267**, 195 (1996)
90. Kallosh,R., Linde, A., Prokushkin, S. and Shmakova, M., *Phys. Rev. D* **66** 123503 (2002) [arXiv:hep-th/0208156].
91. Kallosh, R. and Linde,A. *JCAP* **02** 02 (2003) [astro-ph/0301087].
92. Kamenshchik, A., Moschella, U. and Pasquier, V., *Phys. Lett. B* **511** 265 (2001) [gr-qc/0103004].
93. Kaplinghat, M. and Bridle, S., astro-ph/0312430
94. Kauffmann, G., White, S.D.M. and Guiderdoni, B., *MNRAS* **264**, 201 (1993)
95. Khalil, S. and Munoz, C., *Contemp.Phys.* **43**, 51 (2002) [hep-ph/0110122].
96. Khlopov, M. Yu., *Cosmoparticle physics*, World Scientific, 1999.
97. Klenya, J., et al, *Astroph. J.* **563**, L115 (2001)
98. Klypin, A. et al. *Astroph. J.* **522**, 82 (1999)
99. Knop, R.A., et al., 2003, astro-ph/0309368.
100. Kolb, E., *Phys. Rev. Lett.* **81**, 4048 (1998)
101. Kolb, E.W. and Turner, M.S. (1990) *The Early Universe*, Addison Wesley.
102. Kolda, C. and Lyth, D.H., *Phys. Lett.* **B458** 197 (1999)
103. Kravtsov, A.V., Gnedin, O. and Klypin, A., astro-ph/0401088
104. Li, M., Wang, X., Feng, B. and Zhang, X.,
105. Li, M., and Zhang, X., *Phys.Lett.* **B573**, 20 (2003) [hep-ph/0209093] *Phys. Rev. D* **65**, 103511 (2002) [hep-ph/0112069]
106. Liddle, A. R. and Urena-Lopez, L.A., astro-ph/0302054.
107. Linder, E.V., *Phys. Rev. Lett.* **90** 091301, (2003) [astro-ph/0208512].
108. Lue, A., Scoccimaro, R. and Starkman, G.D., *Phys. Rev. D* **69**, 044005 (2004) [astro-ph/0307034].
109. Ma, C.P. and Boylan-Kolchin, M., astro-ph/0403102
110. Maartens, R., gr-qc/0312059
111. Maartens, R., Wands, D., Bassett, B.A. and Heard, I.P.C., *Phys. Rev. D* **62**, 041301 (2000).
112. Maor, I., Brustein, R. and Steinhardt, P.J., *Phys. Rev. Lett.* **86**, 6 (2001)
113. Maor, I. et al., *Phys. Rev. D* **65** 123003 (2002) [astro-ph/0112526].
114. Maccio, A.V. et al., astro-ph/0309671
115. Mackler, M., de Oliviera, Q. S. and Waga, I., *Phys. Rev. D* **68** 123521 (2003) [astro-ph/0306507].
116. Majumdar, A. S. *Phys. Rev. D* **64**, 083503 (2001).
117. Maroto, A.L., hep-ph/0402278
118. Martel, H., Shapiro, P., and Weinberg, S., *Astroph. J.* **492**, 29 (1998).
119. Martinez, V.J. and Saar, E. "Statistics of the galaxy distribution", Chapman and Hall, 2002.
120. Mateo, M.L., *A.R.A.A.* **36**, 435 (1998)
121. McInnes, B., *JHEP* **0208**, 029 (2002) [hep-th/0112066].
122. Milgrom, M., *Astroph. J.* **270**, 365; **270**, 371; **270**, 384 (1983).
123. Minakata, H. and Sugiyama, H., hep-ph/0212240
124. Mofatt, J.W., astro-ph/0403266
125. Mongan, T.R., *Gen.Rel.Grav.* **33**, 1415 (2001) [gr-qc/0103021].

126. Moore, B. *et al.*, *Astroph. J.* **524**, L19 (1999)
127. Moore, B. *et al.*, *Astroph. J.* **310**, 1147 (1999)
128. Multimaki, T., Manera. M. and Gaztanaga, E., *Phys. Rev. D* **69** 023004, (2004) [astro-ph/0307533]
129. Munoz, C., IJMPA (in press) hep-ph/0309346
130. Nagamine, K. and Loeb, A., *New Astron.* **8**, 439 (2003) [astro-ph/0204249]
131. Nakamura, T. and Chiba, T., *Mon. Not. Roy. ast. Soc.* , **306**, 696 (1999);
132. Narlikar, J.V. *et al.*, *Astroph. J.* **585** 1, (2003) [astro-ph/0211036], and references therein.
133. Navarro, J.F., Frenk, C.S. and White, S.D.M. *MNRAS* **275**, 720 (1995); *apj* **462**, 562 (1996); *apj* **490**, 493 (1997)
134. Nesseris, S. and Perivolaropoulos, L., astro-ph/0401556.
135. Nojiri, S. and Odintsev, S.D., *Phys. Rev. D* **68**, 123512 (2003) [hep-th/0307288].
136. Nojiri, S. and Odintsev, S.D., *Phys. Lett.* **B576**, 5 (2003) [hep-th/0307071].
137. Nojiri, S. and Odintsev, S.D., *Phys. Lett.* **B562**, 147 (2003) [hep-th/0303117]
138. Nojiri, S. and Odintsev, S.D., *Phys. Lett.* **B565**, 1, (2003) [hep-th/0304131]
139. Ng, S.C. and Wiltshire, D.L., *Phys.Rev. D* **64** 123519 (2001) [astro-ph/0107142].
140. Padmanabhan, T., *Phys. Rep.* **380**, 235 (2003). [hep-th/0212290].
141. Parker, L. and Raval, A., *Phys. Rev. D* **60**, 063512, 123502 (1999).
142. Peebles, P.J.E. and Vilenkin, A., *Phys. Rev. D* **59** 063505 (1999).
143. Peebles, P.J.E. and Ratra, B., *Rev.Mod.Phys.* **75**, 559 (2002). [astro-ph/0207347].
144. Perivolaropoulos, L. and Sourdis, C., *Phys. Rev. D* **66**, 084018 (2002).
145. Perrotta, F., Baccigalupi, C. and Matarrese, S. *Phys. Rev. D* **61**, 023507 (1999).
146. Perrotta, F. and Baccigalupi, C. *Phys. Rev. D* **59**, 123508 (1999).
147. Perrotta, F., Matarrese, S., Pietroni, M. and Schimd, C., astro-ph/0310359.
148. Persic, M., Salucci, P. and Stel, F., astro-ph/9506004
149. Phillips, M.M. *Astroph. J.* **413**, L105 (1993).
150. Peitroni, M., *Phys. Rev. D* **67**, 103523 (2003)
151. Peloso, M. and Poppitz, E., *Phys. Rev. D* **68**, 125009 (2003)
152. Perlmutter, S.J. et al., *Nature* **391**, 51 (1998).
153. Perlmutter, S.J. et al., *Astroph. J.* **517**, 565 (1999).
154. Pope, A.C., et al, astro-ph/0401249.
155. Primack, J., astro-ph/0112255
156. Randall, L. and Sundrum, R., *Phys. Rev. Lett.* **83**, 4690 (1999).
157. Ratra, B. and Peebles, P.J.E., *Phys. Rev. D* **37**, 3406 (1988);
158. Riess, A.G., Filipenko, A.V., Li, W. and Schmidt, B.P. *Astron.J.* **118** 2668 (1999) [astro-ph/9907038].
159. Riess, A.G. et al., *Astron. J.* **116**, 1009 (1998).
160. Riess, A.G. et al., [astro-ph/0402512]
161. de Ritis, R., Marino, A.A., Rubano, C. and Scudellaro, P. *Phys.Rev. D* **62** 043506 (2000)
162. Rhee, G., Klypin, A. and Valenzuela, O., astro-ph/0311020
163. Roszkowski, L., hep-ph/9903467
164. Roy, D.P. physics/0007025; also see E. Corbelli and P. Salucci, astro-ph/9909252
165. Sahni, V., PhD thesis, Moscow State University, Moscow, 1984.

166. Sahni, V. and Coles, P., *Phys. Rept.*, **262**, 1 (1995).
167. Sahni, V. and Habib, S., *Phys. Rev. Lett.* **81**, 1766, (1998) [[hep-ph/9808204](#)].
168. Sahni, V. and Wang, L., *Phys. Rev. D* **62**, 103517 (2000);
169. Sahni, V., Sami, M. and Souradeep, T., *Phys. Rev. D* **65** 023518 (2002).
170. Sahni, V. and Shtanov, Yu.V., JCAP 0311,014, (2003) [astro-ph/0202346](#);
171. Sahni, V., *Class.Quant.Grav.* **19** 3435 (2002) [[astro-ph/0202076](#)].
172. Sahni, V. and Starobinsky, A.A. *IJMP D* **9**, 373 (2000).
173. Sahni, V., Saini, T.D., Starobinsky, A.A. and Alam, U., *JETP Lett.* **77** 201 (2003) [[astro-ph/0201498](#)].
174. Sahni, V., *Chaos, Solitons and Fractals* **16**, 527 (2003)
175. Saini, T.D., Raychaudhury, S., Sahni, V. and Starobinsky, A.A., *Phys. Rev. Lett.* **85**, 1162 (2000).
176. Sakharov, A.D., *Dokl. Akad. Nauk SSSR. Ser. Fiz.* **177**, 70 (1967) [*Sov. Phys. Dokl.* **12**, 1040 (1968)]; reprinted in: *Usp. Fiz. Nauk* **161**, 64 (1991) [*Sov. Phys. Usp.* **34**, 394 (1991)]; *Gen. Rel. Grav.* **32**, 365 (2000).
177. Sami, M., Dadhich, N. and Shiromizu, T., *Phys.Lett. B* **568** 118 (2003) [[hep-th/0304187](#)]
178. Sami, M. and Sahni, V., [hep-th/0402086](#).
179. Sand, D.J., Treu, T, Smith, G.P. and Ellis, R.E., [astro-ph/0309465](#)
180. Sanders, R.H. and McGaugh, S.S., [astro-ph/0204521](#)
181. Sathyaprakash, B.S., Sahni, V. and Shandarin, S.F., *ApJL*, **462**, L5 (1996)
182. Seery, D. and Bassett, B.A., [astro-ph/0310208](#)
183. Shandarin, S.F., Sheth, J.V. and Sahni, V., [astro-ph/0312110](#)
184. Shiromizu, T., Maeda, K. and Sasaki, M., *Phys. Rev. D* **62**, 024012 (2000).
185. Shiromizu, T., Torii, T. and Uesugi, T., [hep-th/0302223](#)
186. Singh, P., Sami, M. and Dadhich, N.K., *Phys. Rev. D* **68**, 023522 (2003) [[hep-th/0305110](#)].
187. Shapiro, I. and Sola, J., *Phys. Lett.* **B475**, 236 (2000) [[hep-ph/9910462](#)]
188. Shapiro, I., Sola, J., Espana-Bonet, C. and Ruiz-Lapuente, P., *Phys. Lett.* **B574**, 149 (2003) [[astro-ph/0303306](#)].
189. Shtanov, Yu., [hep-th/0005193](#).
190. Shtanov, Yu. and Sahni, V., *Class.Quant.Grav.* **19**, L101 (2003) [[gr-qc/0204040](#)]
191. Sofue, Y. and Rubin. V., [astro-ph/0010594](#).
192. Somerville, R., *Astroph. J.* **572** L23 (2002)
193. Spergel, D.N., et al, *Astrophys.J.Suppl.* **148**, 175 (2003) [[astro-ph/0302209](#)]
194. Starobinsky, A. A. and Sahni, V., in *Modern Theoretical and Experimental Problems of General relativity* MGPI Press, Moscow, 1984, p. 77.
195. Starobinsky, A.A., *JETP Lett.* **68**, 757 (1998).
196. Starobinsky, A.A. *Grav. Cosmol.* **6**, 157 (2000).
197. Steinhardt, P.J., Wang, L., and Zlatev, I., *Phys. Rev. D* **59**, 123504 (1999);
198. Steinhardt, P.J., “Quintessential Cosmology and Cosmic Acceleration”, <http://feynman.princeton.edu/~steinh/>
199. Tasitsiomi, A. [astro-ph/0205464](#)
200. Tegmark, M. et al, [astro-ph/0310725](#)
201. Tegmark, M. et al, [astro-ph/0310723](#)
202. Tonry, J.L., et al., 2003, *Astroph. J.* **594**, 1, [[astro-ph/0305008](#)].
203. Ureña-López L.A. and Matos, T. , *Phys. Rev. D* **62**, 081302 (2000).

204. Ureña-López, L.A. and Liddle, A., *Phys. Rev. D* **66**, 083005, (2002) [astro-ph/0207493]
205. Uzan, J. *Phys. Rev. D* **59**, 123510 (1999).
206. van de Weygaert, R., 2002, “Froth across the Universe, Dynamics and Stochastic Geometry of the Cosmic Foam”, arXiv: astro-ph/0206427
207. Visser, M. gr-qc/0309109
208. Wang, Y. and Mukherjee, P., astro-ph/0312192.
209. Weinberg, S., *Rev. Mod. Phys.* **61**, 1 (1989).
210. Weinberg, S., astro-ph/0104482.
211. Weller, J. and Albrecht, A. *Phys. Rev. Lett.* **86**, 1939 (2001).
212. Weller, J. and Albrecht, A., *Phys. Rev. D* **65**, 103512 (2002). [astro-ph/0106079].
213. Wetterich, C., *Nuclear Physics B* **302**, 668 (1988).
214. Zel’dovich, Ya.B., *Sov. Phys. – Uspekhi* **11**, 381 (1968).
215. Zhu, Z.-H., Fujimoto, M.-K. and Tatsumi, D. *Astron. Astrophys.* **372**, 377 (2001). [astro-ph/0107234]
216. Zimdahl, W. and Pavon, D. gr-qc/0311067.
217. Zlatev, I., Wang, L. and Steinhardt, P.J., *Phys. Rev. Lett.* **82**, 896 (1999).
218. Zwicky, F., *Helv. Phys. Acta* **6**, 110 (1933).
219. <http://snap.lbl.gov/>