TEMPERATURE FLUCTUATIONS OF COSMIC MICROWAVE BACKGROUND INDUCED BY GRAVITATIONAL LENSING

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Adopting the accepted interpretation of the cosmic microwave background (CMBR) as a relic of the early hot universe we show that any angular intensity variations existing in the background at the "last scattering surface" at the redshift of $\sim 10^3$ will induce bright fluctuations through gravitational lensing by intervening masses. The resulting temperature variations ΔT are estimated for gravitating masses like distant galaxies ($z \sim 1$) and local dark objects (e.g. population III stars) in our own Galaxy. It is found that the value of $\Delta T/T$ produced by the above mechanisms further constrains the theories of galaxy formation. The calculation also limits the amount of matter present in the form of population III objects in the galaxy.

1. Introduction

It is well known that the microwave background observed today should contain imprints of physical processes like galaxy formation in the early epochs. Such imprints should manifest themselves as fluctuations in the brightness profile of the radiation background. We wish to investigate how such fluctuations of the cosmic microwave background radiation are affected by any intercepting gravitational deflector.

The basic idea behind this calculation is simple and is illustrated in fig. 5 of ref. [1]. This picture of a grainy structure on a background screen was taken through an optical analogue of the gravitational lens. While the lens would not have produced any distortion of a smooth background, the situation in the abovementioned figure shows how it enhances the graininess inherent in the screen. In particular, a spherically symmetric lens produces a "gravity ring" around an inhomogeneity.

The CMBR may be considered as a screen located at redshifts $z \gtrsim 10^3$ while the lensing is provided by intervening massive objects. A totally smooth CMBR would of course be unaffected by gravity lensing (see e.g. ref. [2]). However, CMBR *cannot* be completely homogeneous down to the lowest length scale. One would definitely expect certain "graininess" at the length scales of the order of mean free path of the photon at the last scattering surface (LSS). Another source of inhomogeneity would be the adiabatic fluctuations which lead to galaxy formation. It is expected that an intervening lens would lend to produce brightness fluctuations in the background.

2. Gravity rings and CMBR

The importance of gravity rings has been emphasized in ref. [1], and we can use this feature of gravitational lensing to set limits on the temperature distortion of the microwave background caused by intervening deflector. In the notation of the above paper, the angular displacement of the image from the (unlensed) source-position is given by

$$\Delta\theta = \theta_0^2/\theta. \tag{1}$$

Here θ_0 is the angular radius of influence of the deflector:

$$\theta_0 = 2.9 \times 10^{-3} \left(\frac{M}{M_\odot}\right)^{1/2} \left[\frac{l_{DS}}{l_{OD} l_{OS}}\right]^{1/2}_{\mathrm{kpc}} \text{arcsec}, \tag{2}$$

M being the deflector mass, l_{DS} = source–deflector distance, l_{OD} = observer–deflector distance and l_{OS} = source–observer distance and θ is the angular displacement of the image from the sight-line of the deflector.

We can express the brightness distribution as

$$B(\theta) = B_0(1 + \theta/\Omega), \tag{3}$$

where Ω is the typical angular scale-length for brightness variation in the source, which for the CMBR screen we expect to be of the order of the angular scale of the inherent inhomogeneity. It is then straightforward to calculate the relative brightness contrast resulting from lensing as

$$\Delta B/B = \theta_0^2/\Omega\theta, \tag{4}$$

provided the telescope resolution is smaller than $\Delta\theta$.

Notice that the $\Delta B/B$ given by eq. (4) depends on Ω, the angular scale of the brightness fluctuation in the background screen, and not on the magnitude ($\Delta T/T$) of that fluctuation. Thus the $\Delta T/T$ arising from the above has to be added to whatever intrinsic $\Delta T/T$ that may be present in the background.

In this letter we apply the above ideas to two types of inhomogeneities of CMBR: (1) those associated with galaxy formation and (2) those with the characteristic scale of the mean free path of the photon at LSS. We will use z_R to denote the redshift of this (recombination) epoch.

The characteristic angular scale for galaxy formation is given by (cf. e.g. ref. [3])

$$\theta_1 = 23\left(\frac{M_G}{10^{11}M_\odot}\right)^{1/3}\left(h_0 q_c^2\right)^{1/3}\ \text{arcsec}, \tag{5}$$

where q_0 is the deceleration parameter of the Friedmann model and the present Hubble constant is given by $100h_0$ (km/s)/Mpc.

The characteristic angle subtended by the mean free path for Thomson scattering of photons at z_R is similarly estimated at

$$\theta_2 = 2.6\left(\frac{q_0}{\Omega_B h_0}\right)\left(\frac{1+z_R}{10^3}\right)^{-2}\ \text{arcsec}, \tag{6}$$

where Ω_B is the present baryonic density expressed as a fraction of the present critical density.

We also consider gravity rings produced by two classes of objects: (a) dark objects like the massive remnants (population III stars) in our Galaxy and (b) galaxies intervening between us and the LSS.

For the first class of objects we take a typical mass M in the neighbourhood of $\sim 10^6 M_\odot$ at typical halo distances $l \sim 40$ kpc. The angular radius of the gravity ring, assuming $l_{OS} \gg l_{OD}$ will then be

$$\Theta_a = 0.46\left(\frac{M}{10^6 M_\odot}\right)^{1/2}\left(\frac{l}{40\ \text{kpc}}\right)^{1/2}\ \text{arcsec}. \tag{7}$$

For a typical intervening galaxy of mass $M \sim 10^{12} M_\odot$ at the distance of $l \sim 2000$ Mpc we similarly get the gravity ring radius to be

$$\Theta_b = 2.05\left(\frac{M}{10^{12} M_\odot}\right)^{1/2}\left(\frac{l}{2000\ \text{Mpc}}\right)^{-1/2}\ \text{arcsec}. \tag{8}$$

To compute the brightness contrast, which may be expressed as the temperature contrast $\Delta T/T$, we need to know the beam angle of the antenna. Denoting it by α we first relate it to the number n of gravitating objects that lie within it. Assuming that a fraction f of the total mass ($\sim 10^{12} M_\odot$) of our Galaxy in the form of population III objects, the number n is given by

$$n \lesssim 10^6\left(\frac{M}{10^6 M_\odot}\right)^{-1}\frac{\alpha^2}{16}f, \tag{9}$$

provided the dark massive halo is assumed to be spherically symmetric. From (9) we get

$$\alpha \gtrsim 13\left(\frac{M}{10^6 M_\odot}\right)^{1/2} n^{1/2} f^{-1/2}\ \text{arcmin}. \tag{10}$$

The brightness contrast at this beam angle is given by

$$\frac{\Delta B}{B} = \frac{\Delta T}{T} = \frac{\Theta_a^2}{\theta_i \alpha}, \tag{11}$$

where $i = 1, 2$ and Θ_a is given by (7). Using (5),

(7) and (10) we get for case (1)

$$\frac{\Delta T}{T} = 1.1 \times 10^{-5} \left(\frac{M}{10^6 M_\odot} \right)^{1/2} \left(\frac{l}{40\ \text{kpc}} \right)^{-1} \times \left(\frac{M_G}{10^{11} M_\odot} \right)^{-1} \left(h_0 q_0^2 \right)^{-1/3} n^{-1/2} f^{1/2}. \tag{12}$$

As pointed out earlier this $\Delta T/T$ is in addition to whatever $\Delta T/T$ may have existed in the primordial background.

For small values of q_0 (e.g. $q_0 \lesssim 0.05$) the above fluctuation will in principle be detectable by present techniques. For case (2) we get even higher values

$$\frac{\Delta T}{T} = 1.02 \times 10^{-4} \left(\frac{M}{10^6 M_\odot} \right) \left(\frac{l}{40\ \text{kpc}} \right)^{-1} \times \left(\frac{q_0}{\Omega_B h_0} \right)^{-1} \left(\frac{1 + z_R}{10^3} \right)^2 n^{-1/2} f^{1/2}, \tag{13}$$

which should have been detected (for $f \sim 1$) but have not. For example, Partridge [4] gives an upper bound 8×10^{-5} on the scale of 7 arc minutes at the frequency of 1.03 cm^{-1}.

If one accepts this observational bound on $\Delta T/T$, then (13) can be used to set bounds on the fractional mass contributed by population III objects to dark matter in our Galaxy (within ~ 40 kpc). Inverting (13) and taking $l \sim 40$ kpc, $n \sim 1$, $1 + z_R \sim 10^3$, we get,

$$f < 0.62 \left(\frac{M}{10^6 M_\odot} \right)^{-2} \left(\frac{q_0}{\Omega_B h_0} \right)^2. \tag{14}$$

Assuming that population III objects are only clustered around galaxies, this would lead to $\Omega_{\text{pop III}} < 0.62\ \Omega_{\text{tot}}$. In other words, population III objects alone are unlikely to produce $\Omega = 1$. Note that this constraint is completely independent of considerations like deuterium abundance and primordial nucleosynthesis.

We next consider deflectors due to intervening galaxies. A galaxy of size ~ 10 kpc at a distance of ~ 2000 Mpc subtends an angle $\eta \approx 5 \times 10^{-6}$ rad ~ 1 arcsec, if we ignore the small cosmological correction. The beam angle α should be larger than this, a condition that is adequately satisfied. More importantly the radius of the gravity ring Θ_b should also exceed η. This condition is easily satisfied for the above galaxy parameters. However, since $\Theta_b/\eta \sim l^{1/2}$ the condition becomes harder to satisfy as l becomes smaller and is barely satisfied for $l = 500$ Mpc. The validity of point-mass approximation is suspect under such circumstances and needs further investigation.

With these restrictions in mind we apply the corresponding formula for $\Delta T/T$ to the gravity rings resulting from galaxies acting as deflectors. We get for Θ_B given by (8) and θ_1 given by (5)

$$\frac{\Delta T}{T} = 5.0 \times 10^{-4} \left(\frac{M}{10^{12} M_\odot} \right) \left(\frac{l}{2000\ \text{Mpc}} \right)^{-1} \times \left(\frac{M_G}{10^{11} M_\odot} \right)^{-1/3} \left(h_0 q_0^2 \right)^{-1/3} \left(\frac{\alpha}{4\ \text{arcmin}} \right)^{-1}. \tag{15}$$

Similarly (8) and (6) give

$$\frac{\Delta T}{T} = 4.5 \times 10^{-3} \left(\frac{M}{10^{12} M_\odot} \right) \left(\frac{l}{2000\ \text{Mpc}} \right)^{-1} \times \left(\frac{\alpha}{4\ \text{arcmin}} \right)^{-1} \left(\frac{q_0}{\Omega_B h_0} \right)^{-1} \left(\frac{1 + z_R}{10^3} \right)^2. \tag{16}$$

3. Discussion and conclusions

The estimated values of $\Delta T/T$, particularly in the case (2), are far above the limits down to which no fluctuations in CMBR have been found. It is possible to lower the values of $\Delta T/T$ in (15) or (16) down to $\sim 10^{-5}$ (i.e., below the present levels of observations) by supposing that the condensates in the galaxy formation process that left any traces on CMBR at $z_R \sim 10^3$ were much more massive than galaxies, with $M \sim 10^{14}$–$10^{15} M_\odot$. Perhaps the distortion in $\Delta T/T$ given by (12) might just be possible if the massive remnants of population III objects in the galaxy halo act as deflectors for a certain set of parameters. We conclude therefore that the following assumptions, which are usually accepted as reasonable when taken in isolation, may be mutually inconsistent.

(1) CMBR is of primordial origin with the LSS at redshifts $z_R \gtrsim 10^3$.

(2) Fluctuations due to the process of Thomson scattering or due to galaxy formation should have left some inherent angular graininess in CMBR at z_R.

(3) There exist galaxies and massive objects en route to LSS that bend light rays according to Einstein's general theory of relativity.

(4) The observational upper limits on $\Delta T/T$ are not totally unreliable.

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