Crisis in Cosmology: Observational Constraints on Ω and H_0

This review of recent observations of cosmological interest seeks to take stock of how they constrain the standard hot big bang models with or without inflation. We look at two specific series indicative of this class of models. In one series the flatness condition of inflation requires that the density parameter shall be unity. Of late this statement has been relaxed somewhat to include the cosmological constant also as a contributor to the density parameter. Hence we have used this "generalised" flatness condition. The other series of models does not need the cosmological constant but assumes that the curvature parameter $k = -1$. Both these models are currently being pushed as "the" models of the universe.

The observational constraints used by us are the measurements of the Hubble constant and the deceleration parameter, the ages of globular clusters, the abundance of primordial deuterium, the abundance of rich clusters, the baryon content of galaxy clusters and the abundance of high redshift objects. These constraints essentially limit the allowed values of the cosmological parameters. Our findings are that with measurements within their quoted error bars, the available parameter space has shrunk to negligible proportions. For survival of the standard models, therefore, one needs to take recourse to two normally unpalatable steps: (i) to doubt the existing error bars and hope to expand them and (ii) to fine-tune the theoretical parameters so that they fall within the available space.

This is the essence of our perception of the crisis in cosmology.

KeyWords: *cosmology, age of the universe, cosmological constant, big bang nucleosynthesis*

Two decades ago, in an article in Nature, Gunn and Tinsley[1] reviewed the then available data in cosmology to conclude: "New data on the Hubble diagram, combined with constraints on the density of the

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universe and the ages of galaxies, suggest that the most plausible cosmological models have a positive cosmological constant, are closed, too dense to make deuterium in the big bang, and will expand for ever. . . ." Thanks to new technology of observations and fresh inputs from particle physics, cosmology has since advanced on both observational and theoretical fronts. The standard hot big bang model has, if at all, become more deeply rooted in cosmology today than in 1975. It is therefore opportune that we take fresh stock of the cosmological situation today and examine the observational and theoretical constraints as they are now. Not surprisingly, some of the issues discussed by Gunn and Tinsley (op. cit.) continue to be relevant today, whereas fresh ones have replaced the rest. The purpose of this article is to carry out a similar exercise in the modern cosmological framework. The bottom line in this review is that despite the availability of the cosmological constant as an extra parameter for flat Friedmann models, the allowed parameter space for such models has shrunk drastically. The observations that we will consider here include the ages of globular clusters, measurement of the Hubble constant, abundance of rich clusters of galaxies, fraction of mass contributed by baryons in rich clusters and abundance of high redshift objects. We begin with a brief description of the theoretical models in standard cosmology. For the notation the reader may refer to standard textbooks.[2,3]

THE STANDARD MODEL

The standard scenario in big bang cosmology assumes that at any given time the universe is homogeneous and isotropic when averaged over a sufficiently large scale. The expansion of the universe is described by the scale factor "a" that satisfies the following equation:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi}{3} G\rho + \frac{\Lambda}{3}. \qquad (1)$$

Here $k = 0, \pm 1$ represents curvature of the universe ($k = +1$ represents a closed universe, $k = -1$ an open universe and $k = 0$ is the transition or the flat universe), Λ is the cosmological constant and ρ is the density of matter. We can rewrite this equation for the present epoch as

$$H_0^2 + \frac{kc^2}{a_0^2} = \Omega_0 H_0^2 + \Omega_\Lambda H_0^2 \qquad (2)$$

where $H_0 = (\dot{a}/a)_{\text{today}}$, is the Hubble constant, Ω_0 is the density parameter for matter and Ω_Λ is the density parameter contributed by the cosmological constant.

Radiation contributes very little to the energy density of the universe at present, though it was the dominant constituent in the early universe. For standard values of the parameters, matter decoupled from radiation at a characteristic temperature of about 3000 K. The important relic from that epoch is believed to be the currently observed microwave background with a precise blackbody distribution with a temperature of about 2.7 K.

It is also believed in the standard scenario that structures like galaxies, clusters, etc. have formed out of growth of small scale inhomogeneities via gravitational instability. Observations also suggest that luminous (baryonic) matter forms only a small fraction of the total matter density, a much larger contribution coming from "dark" matter, which is likely to be nonbaryonic, noninteracting and collisionless.

The standard model described above has no clear mechanism for generating small inhomogeneities in the early universe. It is, however, possible to come up with such a mechanism if one invokes the hypothesis that the universe went through an inflationary phase at very high redshifts. The models involving inflation generically lead to two predictions: (i) the total density parameter $\Omega_0 + \Omega_\Lambda = 1$ and (ii) the initial power spectrum of inhomogeneities has the form $P_{\text{in}}(K) \propto K^n$ with $n \simeq 1$. Over the years, the idea of inflation has undergone several modifications to meet observational challenges, and it is now possible to find a model in the market that will provide almost any value for Ω_{total} and any form for $P_{\text{in}}(K)$. For the sake of definiteness we will only work with $n = 1$ models. Observations of microwave background radiation are consistent with the index n being equal to unity. As the fluctuations grow the power spectrum gets modified at small scales by different physical processes and this change is described by a transfer function. We shall work with the transfer function suggested by Efstathiou, Bond and White,[4] parametrised by $\Gamma \equiv \Omega_0 h$. The power spectrum is normalised with the COBE DMR observations[5] that give $Q_{\text{rms-ps}} = 20 \pm 3\mu\text{K}$. Here $Q_{\text{rms-ps}}$ is the amplitude of fluctuations in the quadrapole inferred from fluctuations in the higher moments.

Here we study constraints on two models, namely those with (i) $\Omega_0 + \Omega_\Lambda = 1$; $k = 0$ and (ii) $\Omega_0 < 1$; $\Omega_\Lambda = 0$; $k = -1$. The first one is consistent with the inflationary models though it requires an extreme fine-tuning of the cosmological constant which is contrary to the spirit of the inflationary scenario. (We shall comment more on this later.) The second model may be thought of as an "observer's model" in the sense that it tries to use what is known observationally. The amplitude of fluctuations for open models is obtained by rescaling the $\Omega_0 = 1$ model. Curvature effects are not important as we are interested only in scales much smaller than the curvature scale.

We next list the constraints arising from theory as well as observations, giving a brief description of methods that are used to obtain these and possible sources of error for each constraint. Then we merge constraints together to study the allowed regions in the parameter space defined by the density parameter for matter (Ω_0) and the Hubble constant (H_0).

AGES OF GLOBULAR CLUSTERS

It is axiomatic that the age of the universe must be larger than the ages of all its constituents. Therefore ages of the oldest objects provide lower bounds for the age of the universe. Stars in the globular clusters are the oldest known objects in the universe. Ages of these stars are computed by determining their mass, and by observing metallicity and the position off the main sequence turnoff point in the HR diagram. The uncertainties associated with these determinations are now believed to be reasonably small, and a fairly accurate estimate for ages of stars can be obtained by this method. Bolte and Hogan[6] compute the ages of stars in M 92 to be 15.8 ± 2.1 Gyr.

The theoretical age of the universe can be readily computed given the values of Ω_0, Ω_Λ and H_0. In Fig. 1 we have plotted curves for $t_0 = 12$, 15 and 18 Gyr (dashed lines); the allowed region for any age lies below the corresponding curve. The top frame shows these curves for flat models ($k = 0$) and the lower frame shows the same curves for open models ($k = -1$; $\Omega_\Lambda = 0$).

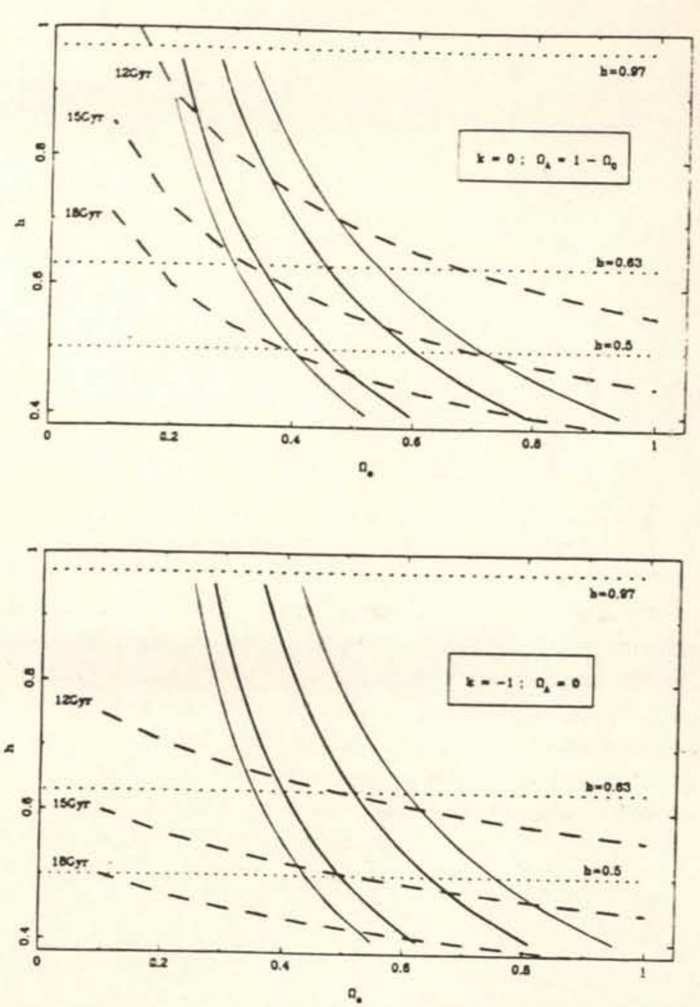

FIGURE 1 This figure shows the constraints on the density parameter contributed by matter, Ω_0, and the Hubble constant h arising from: (i) ages of globular clusters, (ii) measurements of the Hubble constant, and (iii) abundance of rich clusters. Top frame shows the constraints for a model with $k = 0$, $\Omega_\Lambda \neq 0$ and $\Omega_0 + \Omega_\Lambda = 1$. The lower frame is for the $k = -1$, $\Omega_\Lambda = 0$ and $\Omega_0 \leq 1$ model. Lines of constant age are shown as dashed lines for specified values of Ω_0 and h. Dotted lines mark the band enclosing value of the local Hubble constant ($0.63 < h < 0.97$) obtained from HST measurements. We have also shown the assumed lower limit for its global value ($h = 0.5$). Thick unbroken lines enclose the region which is permitted by the observed abundance of clusters. Thin unbroken lines show the extent to which this region can shift due to uncertainty in the COBE normalisation of the power spectrum. Note that these three constraints rule out large regions in the parameter space. In particular, it is clear that the $\Omega_0 = 1$ model is ruled out.

THE HUBBLE CONSTANT

We will use the parametrization $h = H_0/100$ km s^{-1} Mpc^{-1}. To measure h, we must measure distance and recession velocity of a galaxy, or a group of galaxies. Uncertainity in measurement of recession velocity of galaxies comes mainly from their peculier motions. These can be reduced by going to large recession velocities where the fractional error arising from peculier velocities is small. Error in the distance estimate depends upon the method that is used, and in general it increases with distance. Distance indicators can be divided into three classes. Primary indicators like Cepheid variables can be calibrated within our galaxy, and therefore the uncertainity associated with these is small. Secondary indicators like the Tully–Fisher relation are based on properties of galaxies as a whole, and these have to be calibrated with the help of galaxies to which distance has been measured using primary indicators. This extra step involved tends to increase errors in the computed distance. There is a class of "physics"-based indicators which are independent of the entire distance ladder, such as those using supernovae, the Sunyaev–Zeldovich effect, etc.

Recent measurement[7] of distance to M 100 (a galaxy in the Virgo cluster) by the Hubble Space Telescope, with the use of the Cepheid period luminosity relation, gives the value $h = 0.80 \pm 0.17$. This is the "local" value of the Hubble constant which may differ somewhat from its global value. Turner, Cen and Ostriker[8] and Nakamura and Suto[9] have computed the probability distribution for the Hubble constant given a local value. They show that values smaller than $h = 0.5$ are ruled out at 94% confidence level. The global value of the Hubble constant can be measured with methods like the Sunyaev–Zeldovich effect. The value determined by this method[10] for Abell 2218 is $h = 0.65 \pm 0.25$. Sandage and Tamman, on the other hand, consistently obtain values of h in the range 0.5–0.6 from a variety of methods (see, for example Ref. 11).

In Fig. 1 we have plotted dotted lines bounding the region allowed by the value obtained for M 100 ($0.63 < h < 0.97$) and also for $h = 0.5$ as the lower limit for the global value of the Hubble constant. If we assume that $h = 0.63$ (the lowest value allowed by HST observations), then $\Omega_0 = 1$ will require the age of globular clusters to be as low as 10.6 Gyr. If $h = 0.5$, we get $t_0 = 13.3$ Gyr. If the age is greater than 15 Gyr then we need $\Omega_\Lambda > 0.3$ for $h = 0.5$ and $\Omega_0 + \Omega_\Lambda = 1$. Thus a nonzero cosmological constant is needed to allow for globular clusters as old as 15 Gyr.

ABUNDANCE OF RICH CLUSTERS

Mass per unit volume contained in rich clusters can be estimated from the observed number density of such clusters and their average mass. Clusters are identified from x-ray observations by requiring the central temperature to exceed 7 keV. The mass of these clusters can be estimated by a variety of methods, such as assuming virial equilibrium and using the velocity dispersion of galaxies, gravitational lensing, etc. One way of representing the observed number is to state the contribution of mass in these clusters to the density parameter, $\Omega^{\rm obs}_{\rm clusters}$. This number can be computed for any theoretical model using the Press–Schechter method,[12] and successful models should satisfy the equality $\Omega(> M_{\rm clusters}) = \Omega^{\rm obs}_{\rm clusters}$, within the errors of observations.

A comparison of observations with theory can also be carried out in a more involved manner by converting the number density of clusters into the amplitude of density fluctuations at their mass scale. This amplitude is then scaled to $8h^{-1}$ Mpc assuming power law form for σ, the rms fluctuations in density perturbations. The index is chosen to match that expected in the model being considered.[13] The result is expressed as a constraint on σ_8, the rms fluctuations at 8h^{-1} Mpc.

Errors in the determination of $\Omega^{\rm obs}_{\rm clusters}$ are related mostly to the determination of mass for clusters. Various considerations show that masses higher than those used for calculation are completely ruled out, and in fact we may be overestimating the mass of these objects. A change towards lower masses will tend to lower the allowed values of σ_8. This will shift the allowed region towards lower values of $\Gamma = h\Omega_0$.

We have used observational constraints given by Viana and Liddle.[14] For a flat model the constraints are similar to those given by White, Efstathiou and Frenk.[13] We have (in Fig. 1) plotted thick lines showing the region within one sigma of the mean. Thin lines show the bounds if the uncertainity in COBE normalisation is taken into account. The top frame shows these curves for the flat model while the corresponding curves for open models are shown in the lower frame. These figures leave very little room in the parameter space for open models. One may like to relax (i.e., lower) the globular cluster ages and/or (lower) the Hubble constant value somewhat to widen the allowed region, but all values have to be pushed to their extreme limits for this purpose. The allowed region for flat models is somewhat larger than in the $k = -1$; $\Omega_\Lambda = 0$ case. Only three contraints have been used so far and all of these are fairly robust.

BARYON CONTENT OF GALAXY CLUSTERS

Rich clusters of galaxies like the Coma cluster have been studied in great detail. It is possible to determine the fraction of mass contributed by baryons to rich clusters by assuming the Coma cluster to be a prototype. It is found that[15]

$$\frac{M_B}{M_{tot}} = \frac{\Omega_B}{\Omega_0} \geq 0.009 + 0.050h^{-3/2} \tag{3}$$

with 25% uncertainity in the right-hand side. This can be combined with the value of Ω_B determined from primordial nucleosynthesis to further constrain Ω_0.

Light nuclei form in the early universe as it cools from a very dense high temperature phase. The relative abundance of different elements is a function of $\Omega_B h^2$. The observed relative abundance of elements can be used to put limits on this parameter (see Ref. 16). We use the values[17] $0.01 \leq \Omega_B h^2 \leq 0.02$. (There is no consensus on the allowed range; therefore we are using a conservative set of values.) By combining this value with the fraction of mass contributed by baryons in clusters we can constrain Ω_0. Plotted in Fig. 2 are the lowest and the highest bounds on matter density after the uncertainity in the observations of fraction of mass contributed by baryons has been taken into account. The permitted region lies to the left of the curve. The reduction in uncertainity associated with these observations can restrict the allowed region in parameter space very effectively. Generalising to inhomogeneous primordial nucleosynthesis does not help as that tends to reduce the value of the baryon density,[18] leading to a tighter bound on Ω_0.

Comparing Fig. 2 with Fig. 1, we notice that this constraint supplements that given by the abundance of rich clusters for high values of h. For small h it is a stronger constraint and rules out more region from the parameter space. A reduction in uncertainty in Ω_B/Ω_0 or a lowering of the upper bound on $\Omega_B h^2$ can further reduce the allowed region. For example, if we use the mean values of observations that combine to give this constraint, we will rule out about one half of the region that survives in the parameter space in Fig. 1.

Recent observations[19] of deuterium in a high redshift absorption system suggest that Ω_B is much smaller than previously thought ($\Omega_B h^2 = 6.2 \pm 0.8 \times 10^{-3}$). This makes the constraint discussed above very strong and rules out large regions from the parameter space. However, more

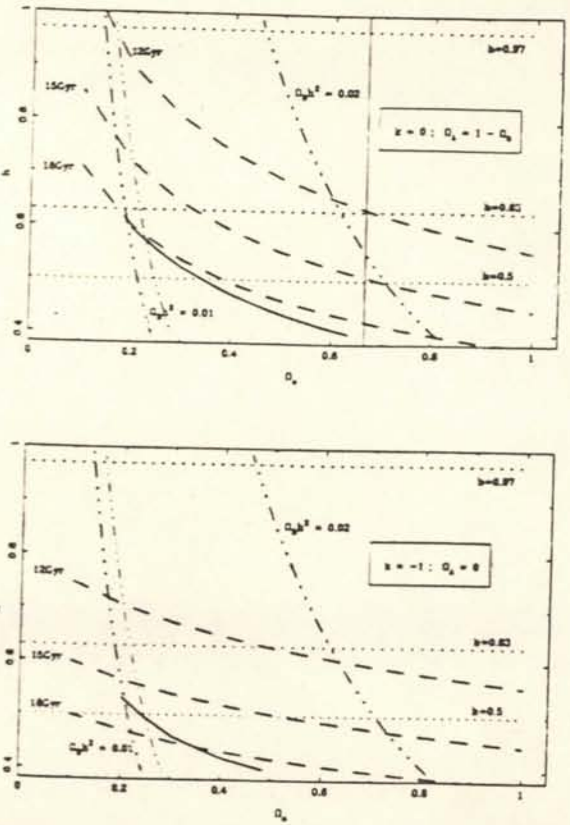

FIGURE 2 This figure shows the constraints on the density parameter contributed by matter, Ω_0, and the Hubble constant h arising from: (i) ages of globular clusters, (ii) measurements of the Hubble constant, (iii) abundance of high redshift objects, (iv) fraction of mass contributed by baryons in clusters and primordial nucleosynthesis, and, (v) measurment of the deceleration parameter. Top frame shows the constraints for a model with $k = 0$, $\Omega_\Lambda \neq 0$ and $\Omega_0 + \Omega_\Lambda = 1$. The lower frame is for the $k = -1$, $\Omega_\Lambda = 0$ and $\Omega_0 \leq 1$ model. Lines of constant age are shown as dashed lines for specified values of Ω_0 and h. Dotted lines mark the band enclosing value of the local Hubble constant ($0.63 < h < 0.97$) obtained from HST measurements. We have also shown the assumed lower limit for its global value ($h = 0.5$). The thick unbroken line is a lower bound on permitted values of h from the abundance of high redshift objects. This line depicts $\sigma(10^{11} M_\odot, z = 2) = 1$. Note that this constraint implies that a $k = 0$ universe can not be much older than 18 Gyr. Dot-dashed lines mark the extreme upper limits allowed by primordial nucleosynthesis and the fraction of mass contributed by baryons in clusters. For a given Ω_0 allowed values of h lie below this curve; conversly, for a given h, allowed values of Ω_0 lie to the left of this curve. The thin dot dashed line shows the upper bound implied by observation of the deuterium abundance at high redshifts. The thin vertical unbroken line marks the lower bound on Ω_0 implied by measurment of the deceleration parameter. The permitted region lies to the right of this curve. Uncertainties in all observations have been included while plotting this curve.

detailed observations are required to confirm these numbers. (See Ref. 20 for a discussion of reliability of the present observations.) In Fig. 2 we have plotted a thin dot dashed line for the upper bound implied by this observation. The permitted region lies to the left of this curve.

ABUNDANCE OF HIGH REDSHIFT OBJECTS

The existence of high redshift objects like radio galaxies and damped lyman alpha systems (DLAS) allows us to conclude that the amplitude of density perturbations is of order unity at $M \simeq 10^{11}\, M_\odot$ at redshift $z = 2$. We have plotted this lower bound in the top frame of Fig. 2 for flat models and in the lower frame for open models. For flat models, the curve runs almost parallel to lines of constant age and thus provides *an upper bound for the age of the universe*. If this constraint becomes stronger or we discover globular clusters with age greater than 18 Gyr, very little region will be left in the parameter space we are considering. Similar results follow for open models.

A more rigorous calculation can be done along the same lines as that described for the abundance of clusters. However, in the case of DLAS, theoretically computed value of the density parameter $\Omega(> M, z)$ should be greater than or equal to the observed value, as not all systems in that mass range host a DLAS. Observations of DLAS give us the mean column density ($\langle \bar{N} \rangle$) of neutral hydrogen and the number of DLAS per unit redshift (dN/dz). Using these and the estimated neutral fraction for gas ($f_N \sim 0.5$), we can estimate the density parameter contributed by DLAS (for more details, see, e.g., Ref. 21). It is also possible to compute the density contributed by collapsed objects at a given redshift using the Press–Schechter formalism. It is important to ensure that collapsed objects of the relevant mass scale, in a given model, are produced with the required abundance. It turns out that this constrant is satisfied if DLAS are associated with masses less than $10^{12}\, M_\odot$.

DECELERATION PARAMETER

Observations of H_0, q_0 and Ω_0 can quantify the background Friedmann model completely. Unfortunately, there is a large uncertainty in values of the cosmological deceleration and density parameters. Most observations used for measuring the deceleration parameter are affected by evolution in number density and luminosity of test objects, and it is difficult to deliniate these effects from the geometrical effects.

Recently, Perlmutter *et al.*[22] have used a supernova at $z = 0.458$ to estimate the value of the deceleration parameter. Later observations of a larger number of supernovae of type Ia at high redshift have led to a more stringent bound[23] $q_0 = 0.3 \pm 0.3$. We have shown the the lower bound on Ω_0 as a thin unbroken line in Fig. 2. The permitted region lies to the right of this curve.

DISCUSSION

These constraints rule out large regions, and the surviving region shrinks further or may even disappear if observational uncertainty is reduced. In Fig. 3 we have shaded regions that are allowed after taking all the constraints into account. We have assumed that globular clusters are not older than 12 Gyr and assumed $h > 0.5$. A somewhat less conservative interpretation of observations will lead to a much smaller allowed region, shown here as the cross-hatched area. We have not used the bounds coming from q_0 and observation of deuterium abundance at high redshifts as we feel that a more detailed analysis may be required before these can be used for ruling out models.

This figure clearly shows that present observations rule out large regions in the space of cosmological parameters. Flat models with cosmological constant have a better chance of surviving as compared to open models. We have not used other constraints coming from detailed structure formation models like velocity fields, the shape of the correlation function, etc. One reason for not considering them is the large uncertainty associated with values derived from these. Another is that we are able to rule out large regions in the parameter space with only a handful of fairly robust constraints. Lastly, all the constraints we use can be scaled trivially if the observational uncertainties or values of some input parameters change.

The allowed region in the parameter space can be widened if we allow tilted spectra, i.e., spectra with the index of power spectrum $n \neq 1$. This does not allow considerably greater freedom, and we should keep in mind that this puts strong limits on another parameter, namely the index of the power spectrum. We have not specifically discussed any mixed

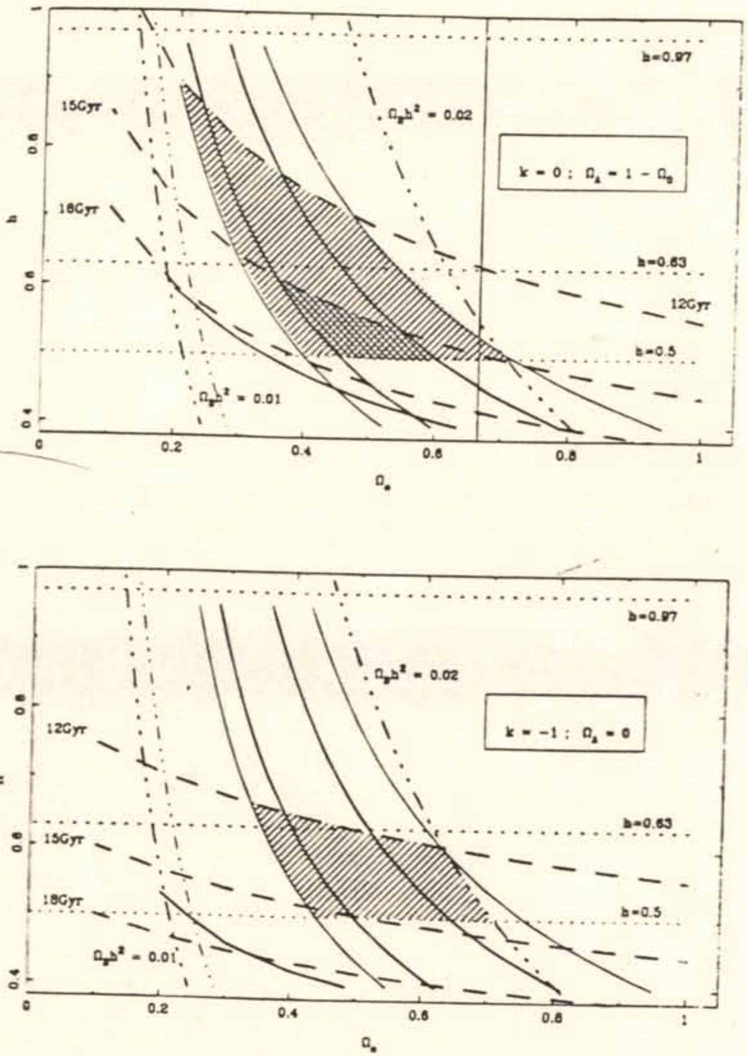

FIGURE 3 This figure summarises all the constraints plotted in Figs. 1 and 2. The shaded region is permitted for $t_0 > 12$ Gyr, $h > 0.5$ and other constraints being satisfied. The cross-hatched area shows the region with $t_0 > 15$ Gyr and cluster abundance in the allowed region without taking uncertainity in COBE normalisation into account. If the uncertainities in the observations are pushed to the extreme limits, then the allowed parameter space corresponds to the shaded region. A somewhat less conservative interpretation of observations will lead to a much smaller allowed region, shown here as the cross-hatched area. Here we have not used the bounds arising from values of the deceleration parameter and observation of deuterium abundance at high redshift.

dark matter models as $\Omega_0 = 1$ models are ruled out by constraints discussed above.

CONCLUSIONS

This brief review highlights the new developments in cosmology since the review of Gunn and Tinsley.[1] Although the constraints of "age" have been with the big bang cosmology for several decades, only now are they coming into focus, thanks to the greater precision in the measurements of the Hubble constant and an improved understanding of stellar evolution. Even allowing for errors on both fronts, the conclusion today is inescapable that the standard big bang models *without* the cosmological constant are effectively ruled out.

The constraints from structure formation, abundances of clusters, primordial nucleosynthesis and high redshift objects are all relatively recent; but they additionally constrain the models *even with the cosmological constant*. Indeed, with the present understanding of extragalactic astronomy very little parameter space is now left for the standard model with or without the cosmological constant.

It is still possible to defend the standard models by taking refuge behind larger error bars than the "one-sigma" limits used here. However, we would consider this as a last-ditch response, for, even granting the theoretician's customary skepticism of the observeving accuracy, the limits quoted by the observers (and used here) reflect some measure of ground reality that cannot be pushed under a rug. While we look forward to improved reliability of observations in the future, we feel that the above analysis should already start to ring warning bells for the theoreticians.

Finally, we would like to comment on the issue of "fine-tuning". If we take the absence of fine-tuning to imply the dictum "all dimensionless parameters should be of order unity," then we would consider $\Omega_{\rm total} = 1$ models as natural. (Any other model would require fine-tuning of this parameter in the early universe, a difficulty usually called the "flatness problem".) By the same token we would have insisted that $\Omega_\Lambda = 0$. Such a model is clearly ruled out by the observations. It is indeed hard to understand why the leftover cosmological constant is such as to exactly conform to the flatness condition. As pointed out by Weinberg[24] this requires fine-tuning to one part in 10^{108}. There have been attempts in the past to invoke a dynamically evolving cosmological constant to circumvent this difficulty; however, none of these models have any compelling features about them. At present, we must conclude that there is indeed a crisis in cosmology.

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References

1. J. E. Gunn. and B. M. Tinsley, Nature, **257**, 454 (1975)
2. J. V. Narlikar, Introduction to Cosmology, Cambridge (1992)
3. T. Padmanabhan, Structure Formation in the Universe, Cambridge (1993)
4. G. Efstathiou, J. R. Bond, and S. D. M. White, MNRAS, **258**, 1p (1992)
5. K. M. Gorski, *et al.*, ApJ, **430**, L89 (1994)
6. M. Bolte. and C. J. Hogan, Nature, **376**, 399 (1995)
7. W. L. Freedman, *et al.*, Nature, **371**, 757 (1994)
8. E. L. Turner, R. Cen, and J. P. Ostriker, Astron. J. **103**, 1427 (1991)
9. T. Nakamura, and Y. Suto, Preprint UTAP—202/95 (1995)
10. M. Birkinshaw, and J. P. Hughes, ApJ, **420**, 33 (1994)
11. A. Saha, *et al.*, ApJ, **438**, 8 (1995)
12. W. H. Press, and P. Schechter, ApJ, **187**, 452 (1974)
13. S. D. M. White, G. Efstathiou, and C. S. Frenk, MNRAS, **262**, 1023 (1993)
14. P. T. P. Viana, and A. R. Liddle, Sussex preprint SUSSEX-AST 95/11-1 (1995)
15. S. D. M. White, J. F. Navarro, A. E. Evrard, and C. S. Frenk, Nature, **366**, 429 (1993)
16. K. A. Olive, and S. T. Scully, University of Minnesota preprint UMN-TH-1341/95 (1995)
17. C. J. Copi, D. N. Schramm, and M. S. Turner, Science, **267**, 192 (1995)
18. R. E. Leonard, and R. J. Scherrer, Preprint OSU-TA-222/95 (1995)
19. A. Songaila, L. L. Cowie, C. J. Hogan, and M. Rugers, Nature, **368**, 599 (1994)
20. C. J. Hogan, To appear in Cosmic Abundances, ed. G. Sonneborn and S. S. Holt, PASP conference series, 1996
21. K. Subramanian, and T. Padmanabhan, IUCAA-5/94; astro-ph/9402006 (1994)
22. S. Perlmutter, *et al.*, ApJ, **440**, L41 (1995)
23. R. Ellis, Talk given at the International Conference on Gravitation and Cosmology in IUCAA, Pune, India (1995)
24. S. Weinberg, Revs. Mod. Phys., **66**, 1 (1989)

The Impact of the Cosmic Microwave Background on Large-Scale Structure

The *COBE* detection of microwave anisotropies provides the best way of fixing the amplitude of cosmological fluctuations on the largest scales. We discuss the impact of this new, precise normalization and give fitting formulae for the horizon-crossing amplitude as a function of Ω_0 and n for both open and flat cosmologies. We also discuss the relevant normalization (σ_8) at galaxy-clustering scales. Already it is clear that the inferred σ_8 can be unnaccepatably high for some of the simplest inflationary models, although many minor variants give an adequate fit. Generic topological defect models appear to fare rather badly, and it is unclear whether minor variants or improved calculations will help much. The detection and mapping of structure in the CMB anisotropy spectrum on smaller scales in the near future will enable us to achieve much stronger constraints on models.

Key Words: *cosmic microwave background, cosmology: theory, large-scale structure*

1. INTRODUCTION

The study of fluctuations in cosmology has two distinct branches, the Cosmic Microwave Background (CMB) and Large-Scale Structure (LSS). Any theory which purports to explain phenomena in one field must also be able to withstand observational scrutiny from the other. Thus any advances in the study of CMB anisotropies impact upon LSS studies.

Perhaps the most immediate impact that the CMB has made upon LSS is in the area of normalization. In order to make firm predictions, a

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