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OBSERVATIONAL LIMITATIONS OF THE DOPPLER THEORY OF QUASARS

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ABSTRACT

This paper is mainly concerned with the hypothesis that the redshift of a quasar is entirely due to the Doppler effect arising from its high speed of ejection in a relatively nearby center of explosion. We examine the viability of this hypothesis in the light of the available data on the aligned triplets of quasars discovered by Arp and Hazard and by Saslaw. Before applying the Doppler hypothesis we consider in some detail, using computer simulations, whether such well aligned triplet configurations could have arisen by chance projection effects under the cosmological hypothesis. Even allowing for various uncertainties and selection effects, we find such a development rather unlikely.

Taking the view that the quasars in a triplet are physically associated, we show how to compute the various parameters of the Doppler problem and put constraints on quasar ejection scenarios. This enables us to examine critically Hoyle's recent hypothesis that quasars emit the bulk of their radiation in a specified backward cone. We find that the four triplets do provide prima facie evidence for such a hypothesis and suggest further checks on the Doppler model. In particular, our model predicts a very small but nonzero fraction of blueshifted quasars.

We also discuss the differences between this model and the hybrid Doppler model advanced recently by Narlikar and Edmunds, in which quasars with cosmological as well as Doppler redshifts are postulated.

Subject headings: cosmology — quasars

I. INTRODUCTION

Although a great majority of astronomers believe that the redshifts of quasars are of cosmological origin, the issue is still subject to controversy. Over the years the emphasis on the various lines of argument for and against the cosmological hypothesis (CH in brief hereafter) has changed as more and more data on quasars became available. For an evolution of the quasar controversy, see Field, Arp, and Bahcall (1973), Burbidge (1973), Rees (1977), and Burbidge (1979).

At the time the first quasars were discovered, CH was the only explanation which was known to have given large redshifts for astronomical objects, viz., the galaxies. The other explanations, namely, the gravitational and Doppler hypotheses, had yielded values of redshift $z < 10^{-3}$ for stars in the Galaxy. It was natural therefore for astronomers to opt for CH as the basis for the redshifts of quasars. Today the supporters of CH point out the following points in its favor:

1. The absorption lines in quasar spectra can be explained in terms of the absorption in foreground galaxies.
2. Very similar quasars with close separations have been interpreted as the images (of a single quasar) produced by a foreground galaxy acting as a gravitational lens.
3. Quasar galaxy associations in which quasars and galaxies close to each other have very nearly the same redshift (Stockton 1978).

The doubters of CH would consider point (1) nonproven and point (2) not necessarily the only way of understanding the observations. With regard to point (3), Arp and his coworkers have presented a number of examples of quasar-galaxy associations in which the members have discrepant redshifts (cf. Burbidge 1979). The opponents of CH argue that apart from the redshift anomalies, the geometry of the observed configurations and the distribution of redshifts suggest that such associations could not have arisen from chance projection of distant quasars against foreground galaxies.

Whatever the eventual outcome of these arguments, the present situation warrants that we do not entirely rule out alternatives to the CH. Two main alternatives have been in the field almost from the time the quasars were first discovered. In the Doppler theory (Terrell 1964; Hoyle and Burbidge 1966), the quasars are supposed to be traveling with relativistic speeds through the intergalactic medium. In the gravitational theory (Hoyle and Fowler 1967; Das 1976), the redshift arises from the deep potential well in the interior of the quasar. Recently it was argued by Narlikar and Das (1980) that even if CH were rejected, the data on quasar galaxy associations cannot be explained on the basis of either of these two alternatives. These authors suggested a third alternative which involves variable particle masses.

In the present paper, we wish to reconsider the Doppler alternative in some detail. The principal objection to this alternative rests on the result that if quasar redshifts are due to high recession velocities generated in some galactic ejection process, then we should also see some quasars with appreciable blueshifts. The absence of any blueshift in the quasar galaxy alignments seen by Arp and others therefore throws doubt on the viability of the Doppler theory (Narlikar and Das 1980). It is also well known that if quasars were ejected isotropically by galaxies in our neighborhood, then we should see far more blueshifted quasars than redshifted ones (Strittmatter 1967).

However, recently Hoyle (1980) has pointed out that if a quasar emits radiation in a narrow enough cone directed backward with respect to the direction of its motion, it is *never* seen blueshifted. More specifically, for a quasar moving with speed V the semivertical angle of this backward cone is given by

$$\theta_{\rm H}=\cos^{-1}\left[\frac{c-(c^2-V^2)^{1/2}}{V}\right]. \tag{1}$$

Here $c=$ the speed of light which we shall henceforth take as 1. We will refer to $\theta_{\rm H}$ as the Hoyle angle of the quasar. The Hoyle angle is suprisingly quite large unless V is very very close to 1 (for example, for $V=0.9$, $\theta_{\rm H}\approx 51°$). Thus, the requirement of backward emission is not a very stringent one.

Hoyle therefore suggested that if we adopt the above backward emission rule, the principal objection to the Doppler theory disappears.[1] The discovery by Arp and Hazard (1980) of two perfectly aligned triplets in nearby regions of the same photographic plate further seemed to suggest the origin of the members of a triplet in a linear ejection process. Edmunds and George (1981) have, however, contended on the basis of computer simulations that even on the basis of CH the appearance of such aligned triplets is not all that rare. In § II we will discuss this issue in some detail.

Taking the view that the Arp-Hazard triplets are not due to chance projection effects but arise from physical associations, Narlikar and Edmunds (1981, hereafter referred to as Paper I) examined a hybrid Doppler theory. In this theory it was assumed that there are two quasar populations; one population consisting of moderate cosmological redshifts ($z\lesssim 1$), and another of quasars with substantial Doppler redshifts. Thus in the quasar triplet, the redshift of the middle quasar was taken to be wholly cosmological, and it was assumed that the end quasars were ejected from the middle quasar with relativistic speeds. These authors showed that it is then possible to calculate all the kinematic parameters of the model. Further, assuming that the end quasars were of equal luminosity, it was possible to show by a comparison of their apparent magnitudes that these quasars emitted preferentially backward as Hoyle had assumed. A similar conclusion was suggested by the examination of two more triplets detected by W. C. Saslaw in another quasar field studied by Arp and Hazard (1980).

Is it necessary to have such a hybrid theory? Could we not assume instead that *all* quasars seen so far are local? Could we assume that the redshifts of all three quasars in a triplet are of Doppler origin? In §§ III and IV we examine these questions and show that while the pure Doppler theory starts off with more free parameters than the hybrid theory, the dynamical constraints on it are more severe. Finally in § V we discuss the overall limitations of the Doppler theory and how its viability can be tested by future observations.

II. STATISTICS OF QUASAR ALIGNMENTS

Suppose, to begin with, that CH is valid and that quasars follow Hubble's law. If we see three quasars of different redshifts almost linearly aligned on a photographic plate, can we claim that we are seeing a very rare event? To answer this question, it is necessary to compute the theoretical probability for this event to happen by chance.

We fix ideas with the help of Figure 1*a*. There we have three quasars L, M, and N in an almost straight line. The central quasar M is off the line LN by a distance P, the length LN being d. In practice both P and d have angular measures, P being of the order of a few arcsec and d of the order of arcmin. Accordingly, we will write P'' and d' for P and d. Let Σ square degrees be the angular area of the photographic plate and suppose that there are N_0 randomly distributed quasars across it.

Given any of the N_0 points as point L, draw concentric circles or radii r' and $r'+dr'$ centered on L. The number of quasars in the circular shell will be

$$2\pi r'\,dr' N_0/(60)^2\Sigma.$$

[1] Geoffrey Burbidge informs us that P. A. Strittmatter was the first to point out in qualitative terms that backward emission would avoid the blueshift catastrophe (cf. Burbidge and Burbidge 1967, p. 173).

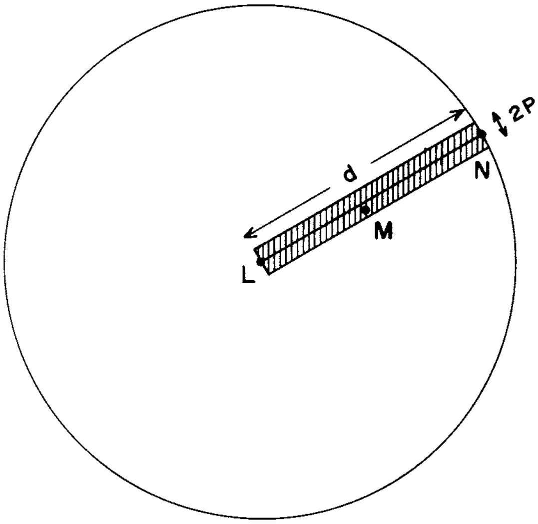

FIG. 1*a*

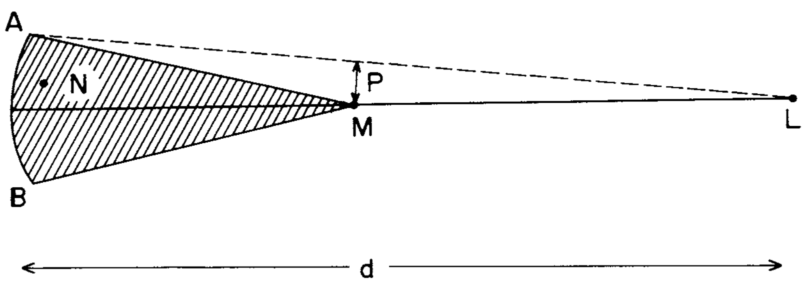

FIG. 1*b*

FIG. 1.—(*a*) An aligned triplet of quasars *L*, *M*, and *N* of length *d* and alignment accuracy of *P*. The figure illustrates the method used by Edmunds and George to calculate the number of aligned triplets. The middle quasar *M* has to fall within the shaded rectangle for the alignment to be good within $2P$. (*b*) The second method used in this paper to calculate the number of aligned triplets. The end quasar *N* has to fall within the shaded region for the triplet *L*, *M*, and *N* to be aligned to an accuracy *P*.

Taking any of these points as N draw a rectangle of width $2P''$ symmetrically about LN. To achieve a configuration of the type shown in Figure 1*a*, we need the third quasar M to lie in this rectangle. The mean number of quasars in this rectangular area is

$$\frac{N_0}{\Sigma}\frac{2r'P''}{(60)^3}.$$

The expected number of linear alignments of total length $LN \le d'$ is therefore

$$\int_0^{d'} \frac{N_0}{\Sigma}\frac{2r'P''}{(60)^3}\frac{2\pi r'\,dr' N_0}{(60)^2\Sigma} = \frac{4\pi}{3}\frac{d'^3P''N_0^2}{(60)^5\Sigma^2}.$$

This number has to be multiplied by N_0 to include all N_0 quasars as points L. To correct for our counting the same point twice (as LN and NL), divide the result by 2 to get the answer as

$$p = \frac{2\pi}{3}\frac{d'^3P''n_0^3\Sigma}{(60)^5}, \tag{2}$$

where $n_0 \equiv$ number of quasars per square degree on the photographic plate $= N_0/\Sigma$.

This was the answer obtained by Edmunds and George (1981). However, they themselves realized that the answer is not unambiguous. As first emphasized by the Bertrand paradox (cf. Kendal and Moran 1963), the calculation of p can give different answers according to how randomness is defined. For example, the simple estimate given by Hoyle differs from equation (2) by a factor ~ 0.3. Below we give a more elaborate version of Hoyle's calculation which in fact agrees with equation (2). This derivation (see Fig. 1b) is closer in spirit to the alignment problem than the derivation of Edmunds and George.

For $N_0 \gg 1$, the total number of distinct triplets on the plate is $\approx N_0^3/6$. Consider any such triplet and choose one member to be M, say. The chance that L lies within a distance x' and N within a distance y' from M is $(\pi x'^2)\times(\pi y'^2)/60^4\Sigma^2$. However, we want to calculate the chance to have linearly aligned triplets. For this we will impose the condition that the angle between the vectors LM and LN is small and is less than $(2P''/60)(x'^{-1}+y'^{-1})$. We will further impose the length condition that $x'+y'\leq d'$. We then have the probability for our one triplet given by

$$3\frac{4\pi P''}{(60)^5\Sigma^2}\iint_{x'+y'\leq d'} x'y'\left(\frac{1}{x'}+\frac{1}{y'}\right)dx'\,dy' = 4\pi\frac{P''d'^3}{(60)^5\Sigma^2}.$$

The extra factor of 3 comes in because we could have taken any of the three members of the triplet as M. Finally, to obtain p we multiply the above expression by $N_0^3/6$ to get

$$p=\frac{2\pi}{3}\frac{P''d'^3n_0^3\Sigma}{(60)^5}.$$

This is the same expected number of triplets per plate as given by equation (2).

Noting, however, the ambiguity of definition of randomness implicit in Bertrand's paradox, we write

$$p\sim 2\beta p''d'^3n_0^3\Sigma/(60)^5, \tag{3}$$

where β is a coefficient of order unity. For $N_0=200$, $\Sigma=36$ square degrees, $P''=1''$, $d'\approx 10'$, we get

$$p\sim 0.016\beta. \tag{4}$$

Thus for the values used by Arp and Hazard the probability of finding a triplet on a plate is less than $\sim 10^{-2}$ and the probability of finding two triplets on the plate is as low as $\sim 10^{-4}$. Further, if we include two more triplets noticed in an adjacent region on the same plate by Saslaw, the probability is reduced still further.

However, the formula (3) is very sensitive to d' and n_0. If both d' and n_0 were raised by a factor of 2, for example, the low value of equation (4) is changed to $p\sim\beta$ indicating that such triplets are not so uncommon after all! It is therefore essential to lay down well defined search criteria, e.g., the limits on d' and P'' for admissible triplets and also to specify what is the value of n_0. For example, in each of the two Arp-Hazard triplets the central quasar is bright ($m=17$). At this level of brightness the value of n_0 is ~ 0.2 (Koo and Kron 1981), and the theoretical probability changes from equation (4) to $\sim 6.5\times 10^{-4}\,\beta$.

It is also necessary to specify whether the whole area of the plate has actually been searched since this may affect the value of Σ chosen in equation (3). For instance, Arp and Hazard appear to have searched carefully only 15.75 deg^2 of the above photographic plate, which reduces the expected number of triplets still further. Clearly, controlled and systematic searches of photographic plates for such alignments can tell us whether the effect is real or not.

Meanwhile, we have carried through computer simulations of randomly distributed quasars on $6°\times 6°$ photographic plates. Our procedure, which differs from that of Edmunds and George both in its magnitude related search criteria and in the assumed quasar number densities, is briefly as follows.

For a given value of n_0 we distribute $N_0=36n_0$ points at random on a $6°\times 6°$ rectangle. We then search for alignments by demanding that (1) the farthest points L, N in a triplet are less than d' apart, and (2) the third (intermediate) point M lies at a perpendicular distance not exceeding P'' from the line LN. Several cases are obtained by assigning different values to n_0, d', and P''. Since for low values of n_0 and d' the expected number of triplets is much less than one per plate, 200 test cases were run. Table 1 summarizes the results for $P''=1''$, and $2''$ in the cases $n_0=2$, 6, and 15 and for $d'=12'$, $18'$, $24'$, and $30'$. The entry in each case gives the mean number of triplets per trial.

To see how sensitive these numbers are to magnitude limits, we made the following study. We assumed that while n_0 in Table 1 denotes the surface density of quasars brighter than 20 mag, the corresponding value for 19 mag is $n_0/3$. We then looked for those quasar alignments in which at least two quasars were brighter than 19 mag, for this is the

TABLE 1

MEAN NUMBER OF TRIPLETS PER PLATE[a] BRIGHTER THAN 20 MAG

n_0[b]	12′	18′	24′	30′
2	0.005	0.005	0.015	0.03
	0.01	0.02	0.04	0.065
6	0.025	0.07	0.185	0.445
	0.055	0.205	0.48	1.07
15[c]	0.056	2.06	4.48	8.64
	1.04	4.2	9	16.96

[a] The top value corresponds to $P''=1''$ and the bottom value to $P''=2''$.

[b] The surface density of quasars assumed at 20 mag.

[c] Average for 25 trials.

TABLE 2

MEAN NUMBER OF TRIPLETS PER PLATE[a] WITH AT LEAST TWO MEMBERS BRIGHTER THAN 19 MAG AND THE THIRD BRIGHTER THAN 20 MAG

n_0[b]	12′	18′	24′	30′
2	0.005	0.005	0.01	0.015
	0.005	0.01	0.015	0.02
6	0.005	0.01	0.03	0.11
	0.015	0.035	0.095	0.26
15[c]	0	0.36	0.8	2
	0.08	0.88	2.12	4.24

[a] The top value corresponds to $P''=1''$ and the bottom value to $P''=2''$.

[b] The surface density of quasars assumed at 20 mag.

[c] Average for 25 trials.

case with all the four triplets actually observed. The average number of alignments is given in Table 2 below with the same notation as for Table 1.

There is a significant drop in the expected number of triplets per plate between Table 1 and Table 2, thus emphasizing the effect of magnitude restriction as a selection criterion. This effect which can be important, was not explicitly discussed by Edmunds and George (1981). If in Table 2 we use $d'=30'$ and $P''=2''$, we get the mean number of triplets per plate as $\mu=0.26$ for the case $n_0=6$ of Arp and Hazard. By Poisson distribution, the probability of finding four triplets on the same plate then becomes

$$e^{-\mu}\frac{\mu^4}{4!}\approx 1.5\times 10^{-4}. \quad (5)$$

By any standards, this probability is low enough to rule out a null hypothesis that the observed quasar distribution is random.

Our numerical simulations suggest that a formula of the form of equation (3) is reliable provided the expected number of triplets per plate is large enough to be comparable to or to exceed unity. For low values, i.e., for $p \ll 1$, the fluctuations in numbers are likely to be large and thus make the formula unreliable. Also it is here that the uncertainty implicit in Bertrand's paradox becomes more important. Taking the largest entry in Table 1 and comparing it with equation (3) gives

$$\beta \sim 1$$

thus implying a very good agreement with the theoretical formula. (This was also noted by Edmunds and George in a different way.)

Since Arp and Hazard (1980) used the objective prism method to look for quasars, their estimate of $n_0 = 6$ (consistent with about 200 quasars on the plate) for quasars brighter than 20 mag is not unrealistic. If a different selection criterion is used for searching quasars, this value will, of course, change. For example, the present estimates of all quasars brighter than 20 mag suggest $n_0 \approx 12$ (Veron and Veron 1981; Koo and Kron 1981). On the other hand, a search procedure using radio quasars must reduce the effective n_0 by at least a factor of 10. In concluding whether an observed alignment is due to chance or not, it is incorrect to use a value of n_0 without reference to the method of selection.

Further, the dependence of equation (3) on the overall linear size of the triplet is also important. Very long alignments will surely always be found since p increases with d' as d'^3. The Arp-Hazard triplets have $d' \sim 10'$ and $15'$ while the Saslaw triplets have $d' \sim 20'$ and $25'$. So our limit of $d' = 30'$ is quite liberal, and the probability estimate in (5), an overestimate.

We therefore disagree with the conclusion of Edmunds and George that the evidence on alignments is not strong enough to warrant the consideration of a ballistic model. Rather, we would like to conclude that a prima facie case has been made by our above arguments that the observed alignments could not have come by chance projection effects.

III. SOME CONSIDERATIONS OF KINEMATICS AND DYNAMICS

a) The Basic Assumptions of a Ballistic Theory

Taking the view that the triplets of quasars discussed in § II do not arise from a chance projection on the sky but that the members of a triplet are physically associated, we proceed with the following scenario. We assume that in a typical triplet (L, M, N) the quasars L and N are at the two ends while the quasar M is in the middle. In the hybrid Doppler theory of Paper I, it was assumed that the redshift of M is entirely cosmological, while the excess redshifts of L and N are of Doppler origin. In contrast to this assumption we will suppose here that all the redshifts are of pure Doppler origin.

To fix ideas, refer to Figure 2 wherein we have two inertial frames. The frame S denotes the rest frame of the observer O, while the frame S' denotes the rest frame of the middle quasar M. Without loss of generality we take M to be moving in the Ox (or Mx') direction with speed V such that the distance between Mx' and Ox is D. The axes Oy and My' are in the plane of Mx' and Ox, while the axes Oz and Mz' are perpendicular to this plane. The Lorentz contraction occurs only in the x, x' direction.

Next we assume that the quasars L and N were ejected from M in opposite directions in its rest frame so that all three quasars move in a straight line in the frame S'. It is easy to verify that the same straight line ejection will be seen in the rest frames of L or N. Let L move in S' with speed V_L in the direction specified by the direction cosines (l, m, n). Similarly V_N denotes the speed of N in the direction $(-l, -m, -n)$ as measured in S'. It is not clear whether the observer O will see all the three quasars exactly aligned. In fact, we will calculate in § III*e* the extent of misalignment seen by O.

We will further assume that the respective clocks in S and S' measure times t and t' which were set equal to zero when M was on OY. Let the time when L and N became separated from M, be denoted by $\bar{t}'$ in S'. Suppose the ray of light leaving M at $t' = t'_M$ reaches O at $t = t_0$. Define R' and θ' and η_0 by

$$R' = \left(D^2 + \frac{V^2 t_0^2}{1 - V^2} \right)^{1/2}, \qquad D = R' \sin\theta', \qquad \eta_0 = (t'_M - \bar{t}')/R'. \tag{6}$$

θ' is the angle made by the backward direction of motion of M with the direction of emission to O, as seen in S'. The angle θ between the directions OM and OX as measured in S is related to θ' by the relation

$$\cos\theta = \frac{\cos\theta' - V}{1 + V\cos\theta'}. \tag{7}$$

It is easy to verify that when $\theta' = \theta_H, \theta = \pi - \theta'$. In fact the case $\pi - \theta = \theta' = \theta_H$ corresponds to emission with a zero redshift, as can be seen from the redshift formulae

$$1 + z_M = \frac{1 + V\cos\theta}{(1 - V^2)^{1/2}} = \frac{(1 - V^2)^{1/2}}{1 - V\cos\theta'}. \tag{8}$$

The fact that $z_M > 0$ signifies $\theta' < \theta_H$, i.e., emission inside the Hoyle angle.

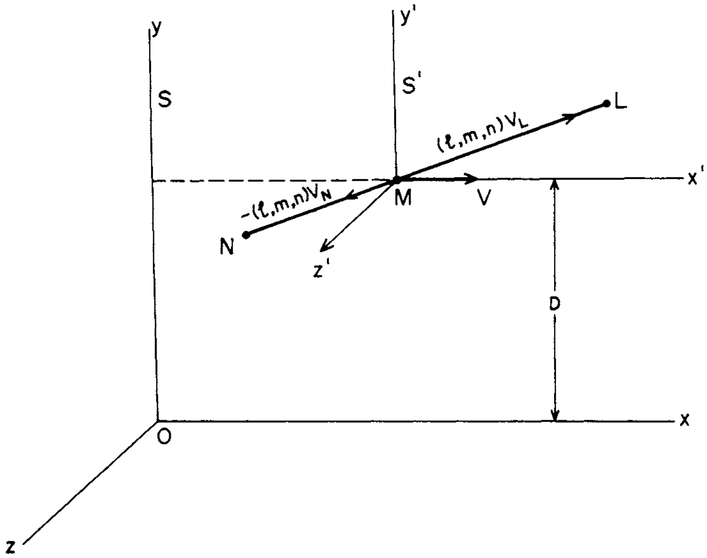

FIG. 2*a*

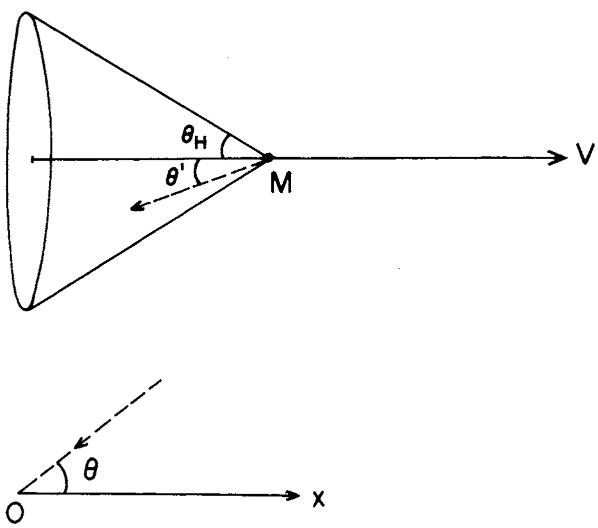

FIG. 2*b*

FIG. 2.—(*a*) The frame S of the observer and the rest frame S' of the middle quasar M are shown. M moves with velocity V in the frame S' in the OX direction. The end quasars L and N move in opposite directions $\pm$ (l, m, and n) in frame S' with velocities V_L and V_N. D is the distance between MX' and OX. (*b*) The Hoyle cone for a quasar moving with velocity V. Emission within this cone is necessary for there to be a redshift. The angle of emission in frame S' is θ' while the observer sees it as θ due to relativistic aberration.

b) *The Redshifts of End-Quasars*

To calculate the redshifts of L and N, let us assume that light left these quasars respectively at $t'=t'_L$ and $t'=t'_N$ in S'. Consider L first. Define $\eta_L=(t'_L-t'_M)/R'$. Then after some algebra, it can be shown that the condition for the signal emitted by L at $t'=t'_L$ to reach O at $t=t_0$ is given by

$$\eta_L^2(1-V_L^2)-2\eta_L(1+fV_L+\eta_0V_L^2)-(\eta_0^2V_L^2+2\eta_0fV_L)=0, \tag{9}$$

where

$$f=l\cos\theta'+m\sin\theta'. \tag{10}$$

So far our analysis is exact. We now introduce the approximation that quasar separations are small compared to their distances from O. This is justified since the angular separations are $<10^{-2}$. Thus, we ignore η_0^2, η_L^2 and higher powers of η_0 and η_L in comparison with unity. We then get from equation (9)

$$\eta_L \approx -\frac{fV_L}{1+fV_L}\eta_0, \tag{11}$$

and hence,

$$\frac{dt'_L}{dt'_M} = -\frac{1}{1+fV_L}. \tag{12}$$

Since the proper time of L corresponding to the interval dt'_L is $dt'_L(1-V_L^2)^{-1/2}$, we get the following relation between the redshifts z_L, z_M of L and M:

$$1+z_L = \frac{1+fV_L}{(1-V_L^2)^{1/2}}(1+z_M). \tag{13}$$

Similarly, we have

$$1+z_N = \frac{1-fV_N}{(1-V_N^2)^{1/2}}(1+z_M). \tag{14}$$

A simpler but somewhat similar relation was derived in Paper I. However, in the hybrid theory z_M was the cosmological redshift and f was replaced by $\cos\alpha$, α being the angle made by the line LMN with the radial direction OM. Since f contains three parameters, l, m, θ', it appears that the present theory has more degrees of freedom than the hybrid theory. We will reconsider this point at a later stage.

c) *The Angular Separations*

To evaluate the angles θ_{LM} and θ_{MN} subtended by the segments LM and MN, respectively, at the observer O, it is convenient to use the three-vector notation. In the frame S denote by $\boldsymbol{r}_L, \boldsymbol{r}_M, \boldsymbol{r}_N$ the position vectors of L, M, and N. Applying the Lorentz transformation from S' to S and using equation (11), we get

$$\boldsymbol{r}_L = \boldsymbol{r}_M + \frac{V_L}{1+fV_L}\boldsymbol{q}^{(1)}, \tag{15}$$

where

$$\boldsymbol{q}^{(1)} = R'\eta_0\left[\frac{l-fV}{(1-V^2)^{1/2}}, m, n\right]. \tag{16}$$

The superscript (1) in $\boldsymbol{q}^{(1)}$ indicates that we are concerned here with quantities like η_0 of the first order of smallness. Later, we will investigate the effect of retaining second order terms in η_0.

θ_{LM} is the angle between $\boldsymbol{r}_L$ and $\boldsymbol{r}_M$. Since θ_{LM} is small, we get, with the help of equations (15) and (16)

$$\theta_{LM} \approx \frac{|\boldsymbol{r}_L \times \boldsymbol{r}_M|}{|\boldsymbol{r}_M^2|}$$

$$= \eta_0 V_L \frac{[(1-f^2)(1-V^2)]^{1/2}}{(1-V\cos\theta')(1+fV_L)}. \tag{17}$$

Similarly, we will have

$$\theta_{MN} = \eta_0 V_N \frac{[(1-f^2)(1-V^2)]^{1/2}}{(1-V\cos\theta')(1-fV_N)}. \tag{18}$$

Notice that both θ_{LM} and θ_{MN} are proportional to the unknown quantity η_0. Observations of θ_{LM} and θ_{MN} therefore tell us the ratio of equations (17) and (18) which does not contain η_0:

$$k \equiv \frac{\theta_{LM}}{\theta_{MN}} = \frac{V_L(1-fV_N)}{V_N(1+fV_L)}. \tag{19}$$

An exactly similar result was obtained in the hybrid theory of Paper I with f replaced by $\cos\alpha$.

d) *Solution*

We have four equations (8), (13), (14), and (19) which relate the observed quantities z_L, z_M, z_N, and k to the six independent unknowns V, V_L, V_N, l, m, and θ'. Because of the similarity in form (though not in interpretation) of these equations with the corresponding ones in the hybrid theory we will not go into the details of their solution but will simply state the answer.

Define the parameters λ_L and λ_N by

$$\lambda_L = \frac{1}{(1+k)}\frac{(1+z_M)}{(1+z_L)}, \qquad \lambda_N = \frac{k}{(1+k)}\frac{(1+z_M)}{(1+z_N)}. \tag{20}$$

Then we have

$$V_L = \left[1 - \left(\frac{2\lambda_L}{1+\lambda_L^2-\lambda_N^2}\right)^2\right]^{1/2}, \tag{21}$$

$$V_N = \left[1 - \left(\frac{2\lambda_N}{1+\lambda_N^2-\lambda_L^2}\right)^2\right]^{1/2}, \tag{22}$$

$$f = \frac{(1-k)(1+k)^{-1} + \lambda_N^2 - \lambda_L^2}{\left\{\left[1-(\lambda_N+\lambda_L)^2\right]\left[1-(\lambda_N-\lambda_L)^2\right]\right\}^{1/2}}. \tag{23}$$

Because we have four equations and six unknowns, we do not have the complete solution which was possible in the hybrid theory. Equation (8) relates V to θ'. If we have one more relation between these two quantities, we will be able to determine them both. Even so, the complete Doppler problem is not determined; for the parameter f depends on l and m as well as on θ'. Thus one more relation will be needed to connect l and m, even when θ' is known.

There are two possible inputs into the problem which could help in determining l, m, V, and θ'. One input is in the form of special assumptions about how the initial explosion took place. This involves dynamical considerations and will be discussed in § III*f*. The second input is from the measurement of misalignment in the triplet. Even though in the rest frame of M, L and N are ejected in exactly opposite directions, the observer O may not see them in a straight line. In § III*e* we calculate this effect.

For completeness we give in Table 3 our solutions for the four triplets. Triplets I and II are those discovered by Arp and Hazard, while III and IV are the triplets found by Saslaw.

e) *Misalignment in the Observer's Reference Frame*

Lorentz contaction and relativistic aberration may "spoil" the perfect alignment of the quasars in a triplet when viewed from S. Since linear alignment was the main motivation for a Doppler theory, it is important to evaluate the magnitude of this effect. We calculate this effect with the help of Figure 3.

In Figure 3 we see L, M, and N projected as L', M', and N' on the celestial sphere centered on the observer O. If $\boldsymbol{r}_L$, $\boldsymbol{r}_M$, and $\boldsymbol{r}_N$ are the position vectors of L, M, and N, we have the position vectors of L', M', and N' as

$$R_0\frac{\boldsymbol{r}_L}{|\boldsymbol{r}_L|}, \qquad R_0\frac{\boldsymbol{r}_M}{|\boldsymbol{r}_M|}, \qquad R_0\frac{\boldsymbol{r}_N}{|\boldsymbol{r}_N|},$$

where R_0 is the radius of the celestial sphere. The angle χ between the arcs $L'M'$ and $M'N'$ denotes the extent of misalignment.

TABLE 3

KINEMATIC PARAMETERS OF QUASAR TRIPLETS I–IV

Parameter	I	II	III	IV
Observed Quantities				
z_M	0.51	0.54	1.01	1.01
z_L	2.15	2.12	2.22	1.67
z_N	1.72	1.61	1.93	2.12
$\theta_{LM}\times 10^3$	1.66	0.83	3.64	5.03
$\theta_{MN}\times 10^3$	3.08	2.03	3.58	1.01
Computed Quantities				
V_L	0.8	0.76	0.77	0.94
V_N	0.91	0.93	0.74	0.54
f	0.279	0.397	0.021	−0.576

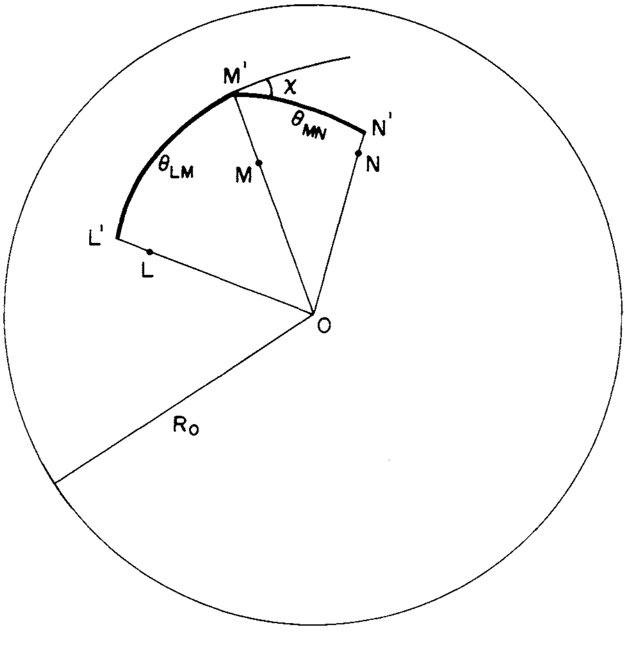

FIG. 3.— L, M, and N are projected on the celestial sphere centered on the observer O as L', M', and N'. R_0 is the radius of the celestial sphere. χ is the angle between the planes OLM and OMN and is a measure of the misalignment of the triplet (L, M, N) as seen by O.

It is easy to verify that if we use equation (15) and work to first order in η_0, the angle $\chi=0$. To second order, however, χ is nonzero, and we will therefore work to this order of approximation. Going back to the quadratic equation (9), we solve for η_L to second order in η_0 and then repeat the analysis leading to the calculation of $\boldsymbol{r}_L$. We get this way

$$\boldsymbol{r}_L=\boldsymbol{r}_M+\frac{V_L}{1+fV_L}\boldsymbol{q}^{(1)}+\frac{(1-f^2)V_L^3}{2(1+fV_L)^3}\boldsymbol{q}^{(2)}, \tag{24}$$

where $\boldsymbol{q}^{(1)}$ is as given in equation (16) and

$$\boldsymbol{q}^{(2)}=-R'\eta_0^2\left[\frac{V+lV_L}{V_L(1-V^2)^{1/2}}, m, n\right]. \tag{25}$$

$\boldsymbol{r}_N$ is similarly obtained by writing V_N for V_L and changing (l, m, n) to $(-l, -m, -n)$.

The angle χ is obtained through the following steps:

$$|\boldsymbol{p}_L|\,|\boldsymbol{p}_N|\sin\chi = |\boldsymbol{p}_L \times \boldsymbol{p}_N|,$$

where

$$\boldsymbol{p}_L = R_0^2(\boldsymbol{r}_L \times \boldsymbol{r}_M)/|\boldsymbol{r}_L|\,|\boldsymbol{r}_M|, \qquad \boldsymbol{p}_N = \boldsymbol{R}_0^2(\boldsymbol{r}_N \times \boldsymbol{r}_M)/|\boldsymbol{r}_N|\,|\boldsymbol{r}_M|.$$

In general, we expect χ to be small so that $\sin\chi \approx \chi$, and our manipulation finally gives

$$\chi \approx \frac{n}{(1-f^2)^{1/2}} \frac{V\sin\theta'}{(1-V^2)^{1/2}} \frac{\theta_{LM}+\theta_{MN}}{2}, \tag{26}$$

where we have used equation (17) to express χ in terms of the directly observed θ_{LM} and θ_{MN}.

The first factor on the right hand side is always less than unity, being zero for $n=0$. The second factor is usually of order unity for relativistic velocities, although it is possible for it to be arbitrarily large as $V \to 1$ and $\theta' \to \pi$. The last quantity is small, as we have seen in Table 3.

Thus, for the last factor of order, say, 3×10^{-3} we expect $\chi \sim 3\times10^{-3}$. Note, however, that the misalignment assumed in our statistical calculations of § II was denoted by P. In the present context χ is of the order P/d. Thus, the above value of χ with $d \sim 3\times10^{-3}$ gives $P \sim 10^{-5} \sim 2''$. The expected misalignment, is, therefore, of the same order as the finite size quoted for quasar images. Thus, greater positional accuracy is needed to decide whether the above misalignments exist or not. With the improved resolution of the Space Telescope it should be possible to measure misalignments of this order.

In a private communication, Cyril Hazard gave us positions of quasars in the triplets I and II from which we were able to compute χ in both the cases. We find $\chi \sim 8.9\times10^{-3}$ for I and $\chi \sim 10.6\times10^{-3}$ for II. These values are somewhat higher than what is expected from equation (26), thus suggesting some misalignment in the ejection process in the frame S'. However, the value of χ is very sensitive to the positional accuracy of L, M, and N and we suspect that the (unknown) errors in the above values of χ may be large. Hazard has suggested that the positional accuracy of the images is $\sim 2''$.

Finally, if χ is accurately measurable, then equation (26) gives us another relation connecting the unknowns l, m, θ', and V. [Note that $n=(1-l^2-m^2)^{1/2}$]. This was the additional equation referred to in § III*d*.

f) *Conservation of Linear Momentum*

So far, we have not imposed any dynamical constraints on our Doppler model. The velocities V, V_L, and V_N are therefore arbitrary. Of these, we have been able to determine V_L and V_N uniquely from observations. Can we hope to determine V by considerations of dynamics?

For example, the simplest possibility is to suppose that L, M, and N were ejected when a single object, at rest in S, broke up into three bits. In that case, the law of conservation of linear momentum determines the velocity of one quasar in terms of the velocities of the other two. To see how this constraint operates, suppose that m_L, m_M, and m_N are the rest masses of L, M, and N, and let their velocities of ejection in S be $\tilde{\boldsymbol{V}}_L$, $\boldsymbol{V}$, and $\tilde{\boldsymbol{V}}_N$ respectively. We then have from the conservation of linear momentum of the system

$$\frac{m_L\tilde{\boldsymbol{V}}_L}{(1-\tilde{V}_L^2)^{1/2}} + \frac{m_N\tilde{\boldsymbol{V}}_N}{(1-\tilde{V}_N^2)^{1/2}} + \frac{m_M\boldsymbol{V}}{(1-V^2)^{1/2}} = \boldsymbol{0}. \tag{27}$$

A simple algebra then shows that when expressed in terms of V, V_L, and V_N the only possible solution of equation (27) for general values of (l, m, n) is

$$V=0, \tag{28}$$

which clearly is impossible since the quasar M has a redshift.

The only way out of the above conclusion is that both m and n are zero and $l=1$. In that case we have $f=\cos\theta'$. However, we then know θ' from equation (23) and should be able to determine V from equation (8). But we find that for the values of f given in Table 3, no solutions are possible for V from equation (10). That is, the value of $\theta' = \cos^{-1} f$ lies outside the range of θ' for which the quadratic equation (8) for V has real roots in the range $0 \le V < 1$. We conclude, therefore, that in spite of larger number of parameters available in the pure Doppler theory, the simple assumption of momentum conservation of the three-body system is ruled out by the observed features of the triplets.

It is, of course, possible to solve the dynamical equations consistently by enlarging the scope of our initial condition. Thus, if we assume that a fourth body, e.g., a galaxy, much more massive than the quasars takes up the balance of the recoil momentum, then the right-hand side of equation (27) is nonzero. Alternatively, we can assume that one quasar was ejected first by a massive galaxy and that it subsequently broke up into a triplet of quasars which were ejected linearly in its rest frame. This was the scenario mentioned in § III*a*. However, in neither case are we able to determine the unknown quantity V, the speed of the quasar M.

IV. EVIDENCE FOR ANISOTROPIC EMISSION

To avoid the blueshift problem, Hoyle introduced the concept of backward emission within the angle θ_H. In practice, the restriction on the mode of emission from a quasar must come from the physics of the emission process. Hoyle had argued that a fast moving quasar would tend to pile up intergalactic medium in its forward direction of motion, and the resulting increased opacity would lead to a damping of its radiation. In the backward direction, on the other hand, the medium will be rarefied in the wake of the quasar, and this would make the passage of radiation easier. In a future paper this astrophysical aspect of the problem will be examined.

For the present purpose we will assume as in Paper I that in its rest frame, the typical quasar emits preferentially *backward*. Thus, if $\mathcal{L}$ is the total luminosity of the quasar, we will assume that it emits an energy $\mathcal{L}Q(\theta')J(\nu')\,d\nu'\,d\Omega'$ per unit time in the frequency range $(\nu', \nu'+d\nu')$ and in a solid angle $d\Omega'$ centered on a direction making an angle θ' with the backward direction. Therefore

$$\int_0^\infty J(\nu')\,d\nu'=1, \qquad \int_0^\pi 2\pi Q(\theta')\sin\theta'\,d\theta'=1. \tag{29}$$

In Paper I we took $J(\nu')\alpha\nu'^{-\beta}$, with β a constant in the range (0, 1) relevant for quasars. We will use the same form for $J(\nu')$ here and also, following Paper I we will take for Q

$$Q(\theta')=\frac{\lambda}{2\pi(1-\cos\theta_H)}\exp\left[-\frac{\lambda(1-\cos\theta')}{(1-\cos\theta_H)}\right]. \tag{30}$$

(In Paper I we had used a slightly different notation with $\lambda=1/2n^2$.) For $\lambda>0$ equation (30) shows the desired feature of more emission in the backward direction. In fact the luminosities in the forward and backward direction are in the ratio

$$\frac{\mathcal{L}_{\text{forward}}}{\mathcal{L}_{\text{backward}}}=\exp\left[-\frac{\lambda}{1-\cos\theta_H}\right]. \tag{31}$$

Thus for a substantial effect we require $\lambda \gg 1-\cos\theta_H$.

The application of equation (30) to the Arp-Hazard and Saslaw triplets was carried out in Paper I within the framework of the hybrid Doppler theory, and it was found that $\lambda=4$ gives a good agreement for all triplets. The fact that λ turned out positive was taken as support for Hoyle's hypothesis that quasars emit preferentially backwards.

To carry out a similar analysis in our pure Doppler theory, we have to calculate the flux density of radiation from a typical quasar and then apply the result to L, M, and N. Denoting the flux density at the observed frequency ν at O by $S(\nu)$ we get

$$S_M(\nu)=\frac{\mathcal{L}_M Q(\theta'_M)J[\nu(1+z_M)]}{R'^2(1+z_M)}, \tag{32}$$

$$S_L(\nu)=\frac{\mathcal{L}_L Q(\theta'_L)J[\nu(1+z_L)]}{R'^2(1+z_L)^2}(1+z_M), \tag{33}$$

$$S_N(\nu)=\frac{\mathcal{L}_N Q(\theta'_N)J[\nu(1+z_N)]}{R'^2(1+z_N)^2}(1+z_M), \tag{34}$$

where θ'_M, θ'_L, and θ'_N are angles of emission in the rest frame of M, L, and N, respectively.

TABLE 4

DAMPING[a] PARAMETER λ CALCULATED FOR TRIPLET I FOR A RANGE OF V, l, AND FOR $\beta = 1$

V	−0.4	−0.3	−0.2	−0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.4......	...	...	...	...	...	...	2	1.85	1.61	1.22	...	...	...	...
0.5......	...	...	2.48	2.44	2.37	2.24	2.06	1.77	1.27	0.18	−5.05	9.26	4.50	3.30
0.6......	...	2.81	2.81	2.75	2.63	2.43	2.12	1.58	0.48	−3.62	14.29	5.43	3.85	2.98
0.7......	3.27	3.31	3.29	3.19	3.00	2.67	2.14	1.18	−1.20	−33.33	9.09	5.38	3.91	2.80
0.8......	4.17	4.17	4.10	3.91	3.55	2.99	2.12	0.47	−4.10	166.67	10.87	6.58	4.55	2.99
0.9......	...	5.95	5.68	5.21	4.51	3.52	1.99	−0.60	−6.67	−62.5	23.81	11.36	7.14	4.13

[a] The thick lines enclose the small range of values of V and l for which there is preferential emission in the forward direction.

Although the unknown quantity R' enters these formulae, it is eliminated when we compare the apparent magnitudes of the quasars. Thus,

$$\frac{S_L(\nu)}{S_N(\nu)} = \frac{1-\cos\theta_{NH}}{1-\cos\theta_{LH}} \frac{J[\nu(1+z_L)]}{J[\nu(1+z_N)]} \left(\frac{1+z_N}{1+z_L}\right)^2 \exp\left[-\lambda\left(\frac{1-\cos\theta'_L}{1-\cos\theta_{LH}} - \frac{1-\cos\theta'_N}{1-\cos\theta_{NH}}\right)\right], \tag{35}$$

and the difference in the apparent magnitudes of the quasars is given by

$$m_L(\nu) - m_N(\nu) = -2.5 \log \frac{S_L(\nu)}{S_N(\nu)}. \tag{36}$$

We have assumed here, as in Paper I that L and N are equally luminous in their rest frames.

A fresh difficulty arises now which was absent in Paper I. We need to know the Hoyle angles for L and N and hence their velocities in S. Although we know these velocities in S', we still do not know V, l, and θ'. By contrast in Paper I, the ratio S_L/S_N depended on only one unknown quantity, λ. Thus, we are not able to determine λ uniquely in the present theory from the observed parameters of a triplet.

In Table 4 we determine λ for a range of values of V and l. Note that if V is known, θ' is determined from equation (10) and hence the ratio (35) can be calculated for given λ and β. For a given β we can therefore determine λ by demanding that the calculated ratio equals the observed one. This is what is done in Table 4 for triplet I for $\beta = 1$ (steep spectrum case).

Similar tables can be prepared for all the four triplets and for the range $0 \leq \beta \leq 1$. The kinematical restrictions do not permit all pairs of values (V, l) in the range $0 \leq V < 1$, $0 \leq l \leq 1$. In the permissible range, it is clear from the example of Table 4 that positive values of λ predominate over negative ones. Thus although we cannot rule out the possibility of preferential forward emission, it seems less favored compared to the possibility of preferential backward emission. Also, large positive λ predominate, suggesting that the backward emission is severely curtailed outside the Hoyle cone.

Our approach could be criticized as being ad hoc on two counts: in our choice of the damping function, and in our assumption of equal luminosities for L and N. Our procedure, however, is no different from that followed elsewhere in astronomy where for lack of information the simplest assumptions are tried first. For example, equal luminosity was assumed for all quasars in Strittmatter's (1967) calculation which led to the blueshift catastrophe. As we shall see in the following section, equal luminosity coupled with damped forward emission drastically alters that conclusion.

V. DISCUSSION

The following picture emerges from our calculations of the Doppler framework in the preceding sections.

If we wish to argue that the three quasars in a linear triplet arose out of a single explosion ejecting the three objects in a straight line, then we cannot claim that the linear momentum of the three-body system was conserved. We need a fourth body, e.g., a galaxy, to take over the excess momentum of recoil. The galaxy would be much more massive than the quasars. The most likely scenario is one in which the galaxy first emitted on object which subsequently broke up into three quasars moving linearly in its rest frame. The discovery of a galaxy in the neighborhood of the triplet would be an observation in favor of these ideas.

It is interesting that strict linearity is not preserved in the observer's rest frame. A slight misalignment is to be expected. The extent of misalignment in the case of the four triplets is at present of the same order as the accuracy in locating quasar images.

Further, if more aligned triplets are seen in the future, according to equation (26) we expect to see greater misalignment for longer triplets. This trend is opposite to that expected if all the triplets are due to chance projection.

Next, we consider the question of blueshifts. Strittmatter's calculation leading to a preponderence of blueshifts has to be modified to take into account the anisotropic emission function (30). The modified calculation briefly proceeds as follows.

Suppose a survey is limited by a flux density S_0. Let there be n sources of quasars per unit volume in our local neighborhood and suppose each source emits isotropically $F(V)\,dV$ quasars in the velocity range $V, V+dV$. Suppose further that each quasar has a luminosity $\mathfrak{L}$ but that its emission function is given by equation (30). Also, we will assume that the spectral function $J(\nu')$ for each quasar in its rest frame has the form $K\nu'^{-\beta}$ where $K=$ constant in the relevant frequency range of observations. A quasar with relative speed V will have maximum and minimum redshifts given by $z_{\max}$ and $z_{\min}$, where

$$1+z_{\max}=(1+z_{\min})^{-1}=\left(\frac{1+V}{1-V}\right)^{1/2}. \tag{37}$$

The range $0>z\geq z_{\min}$ in fact corresponds to blueshifted quasars.

The flux density of a typical quasar is given by equation (32), which can be rewritten in terms of $z, z_{\max}$ as

$$S(\nu)=\frac{\mathfrak{L}\lambda K\nu^{-\beta}}{2\pi(1+z_{\max}/2)R^2}(1+z)^{-3-\beta}\exp\left[-\frac{\lambda(z_{\max}-z)}{z_{\max}(1+z)}\right]. \tag{38}$$

The flux density limit $S(\nu)\geq S_0$ thus limits the distance up to which such quasar would be seen:

$$R\leq R(z,z_{\max}),$$

$$R(z,z_{\max})=\left(\frac{\mathfrak{L}\lambda K\nu^{-\beta}}{2\pi S_0}\right)^{1/2}(1+z_{\max}/2)^{-1/2}(1+z)^{-(3+\beta)/2}\exp\left[-\frac{\lambda(z-z_{\max})}{2z_{\max}(1+z)}\right]. \tag{39}$$

Following Strittmatter's method the total number of redshifted quasars in the survey is given by

$$N_r=\frac{4\pi}{3}n\int_0^1 F(V)\,dV\int_0^{z_{\max}}\frac{(1+z)[R(z,z_{\max})]^3}{(z_{\max}-z_{\min})}dz. \tag{40}$$

Similarly, the number of blueshifted quasars is given by

$$N_b=\frac{4\pi}{3}n\int_0^1 F(V)\,dV\int_{z_{\min}}^0\frac{(1+z)[R(z,z_{\max})]^3}{(z_{\max}-z_{\min})}dz. \tag{41}$$

Strittmatter considered the case $\lambda=0$ (isotropic emission) and $F(V)\alpha\delta(V-V_0)$ (all quasars ejected with the same velocity). To illustrate the difference that anisotropic emission makes to the ratio N_b/N_r we will consider the case $F(V)\alpha\delta(V-V_0)$ but with $\lambda>0$. We then have

$$\frac{N_b}{N_r}=\frac{\Phi(0,z_{\min})}{\Phi(z_{\max},0)}, \tag{42}$$

where

$$\Phi(z_1,z_2)=\int_{z_1}^{z_2}(1+z)[R(z,z_{\max})]^3\,dz. \tag{43}$$

In Table 5 we show how for $\beta=1$ the ratio N_b/N_r varies with $z_{\max}$ and λ. The case $\lambda=0$ is that discussed by Strittmatter in which case $N_b/N_r=(1+z_{\max})^4$.

The entries in the table show how rapidly the ratio N_b/N_r drops as λ increases. Although more than 1500 quasars are now known, there are not many large flux limited samples. In the discussion of luminosity volume test for quasars,

TABLE 5

EXPECTED RATIO OF BLUESHIFTED QUASARS TO REDSHIFTED QUASARS

λ	z_{max} 2	3	4	5
0	81	256	625	1296
1	3.894	5.773	7.163	9.166
2	0.577	0.7915	0.951	1.070
3	0.134	0.193	0.240	0.276
4	0.034	0.052	0.068	0.080
5	0.0085	0.014	0.019	0.023

Wills and Lynds (1978) have 226 radio quasars in flux limited samples. Corresponding to this number, the case $\lambda=4$, $z_{max}=3$ gives an expected number $\lesssim 12$ of blueshifted quasars.

That *no* blueshifted quasars have been found may be taken as a disproof of the Doppler theory even in its present form. However, we would question the statement that no blueshifted quasars exist and suggest that a more thorough search of possible spectral lines in the red or near-infrared might yield candidate lines with moderate blueshifts. In Paper I the suggestion was made that the BL Lac objects might be quasars with blueshifts.

Finally, we comment on the apparent drop-off in the number of quasars beyond $z \approx 3.5$. From equation (38) we see that the maximum flux density occurs at a redshift given by

$$\bar{z}=\frac{\lambda(1+z_{max})}{(3+\beta)z_{max}}-1. \tag{44}$$

Denoting by $\bar{S}$ the flux density at this value of $\bar{z}$, the ratio of the flux densities

$$\frac{S(z_{max})}{\bar{S}}=\left(\frac{\lambda_e}{(3+\beta)z_{max}}\right)^{3+\beta}\exp(-\lambda/z_{max}). \tag{45}$$

This quantity has a maximum value for $z_{max}=\lambda/(3+\beta)$. Now, to be able to observe large redshifts z_{max} must be large, i.e., we must have very high ejection velocities. For $\lambda=4$, $3+\beta=4$, equation (45) has a maximum for $z_{max}=1$. For $z_{max}>1$ the ratio (45) steadily decreases, thus making it comparatively more and more difficult to observe quasars at the maximum Doppler redshifts. The fact that quasars in the redshift range 2–3 are quite numerous suggests that the value of $\lambda=4$ may be an underestimate and that $\lambda \approx 8$ may be closer to reality. This implies a more rapid cut-off beyond the Hoyle cone, and an even smaller ratio N_b/N_r.

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