

PROBABILITY FOR PRIMORDIAL BLACK HOLES PAIR IN $1/R$ GRAVITY

Dilip Paul*and Bikash Chandra Paul†

Department of Physics, North Bengal University,
Siliguri, Dist. Darjeeling, Pin : 734 013, India

Abstract

The probability for quantum creation of an inflationary universe with a pair of black holes in $1/R$ - gravitational theory has been studied. Considering a gravitational action which includes a cosmological constant (Λ) in addition to δR^{-1} term, the probability has been evaluated in a semiclassical approximation with Hartle-Hawking boundary condition. We obtain instanton solutions determined by the parameters δ and Λ satisfying the constraint $\delta \leq \frac{4\Lambda^2}{3}$. However, we note that two different classes of instanton solutions exists in the region $0 < \delta < \frac{4\Lambda^2}{3}$. The probabilities of creation of such configurations are evaluated. It is found that the probability of creation of a universe with a pair of black holes is strongly suppressed with a positive cosmological constant except in one case when $0 < \delta < \Lambda^2$. It is also found that gravitational instanton solution is permitted even with $\Lambda = 0$ but one has to consider $\delta < 0$. However, in the later case a universe with a pair of black holes is less probable.

PACS No(s) : 04.70.Dy, 98.80.Bp, 98.80.Cq.

To appear in Phys. Rev. D 72 (2005)

*permanent address : Khoribari H.S. School, Khoribari, Dist. : Darjeeling

†e-mail : bcpaul@iucaa.ernet.in

I. INTRODUCTION :

It is now generally believed from the recent observations that the present universe is accelerating [1]. It is also believed that our universe might have emerged from an inflationary era in the past. In the last two decades a number of literature appeared which explored early universe with an inflationary phase [2]. Inflation not only opens up new avenues in the interface of particle physics and cosmology but also solves some of the outstanding problems not understood in the standard bigbang cosmology. Most successful theory developed so far is based on the dynamics of scalar field with a suitable potential which behaves like a cosmological constant. In order to explain present acceleration in the universe a number of literature appeared with a new gravitational physics considering theories other than scalar field e.g., Chaplygin gas [3], phantom fields [4] etc. Moreover, it has been realized that modification of the Einstein-Hilbert action with higher order terms in curvature invariants that are become effective in the high curvature region are important in cosmological model building. The modified theories permit inflation in the early epoch [5]. In the same way it is important to explore modification of the Einstein gravitational action with terms that might be important at extremely low curvature region to explain the present cosmic acceleration. There are attempts to explain the cosmic speed up with a modification of the Einstein Hilbert action. Recently, Carroll *et al.* [6] suggested a gravitational action of the form

$$I = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R - \frac{\mu^4}{R} \right], \quad (1)$$

where R is the Ricci Scalar, $\kappa^2 = 8\pi G$, and μ is a mass scale of the order of Hubble scale. Cosmological models with a modification of the above action have been used to construct an alternative to dark energy and dark matter models [7]. It is also important to explore the cosmological issues in the framework of new theories which are important

at the present epoch. The important astrophysical objects, for example black holes need to be investigated in this theory. The mass of these objects may be greater than the solar mass or even less. It is known in stellar physics that a blackhole is the ultimate corpse of a collapsing star when its mass exceeds twice the mass of the Sun. Another kind of black holes are important in cosmology which might have formed due to quantum fluctuations of matter distribution in the early universe. These are termed as topological blackholes having mass many times smaller than the the solar mass. In particular these are important from the view point of Hawking radiation [8]. Bousso and Hawking [9] (in short, BH) calculated the probability of the quantum creation of a universe with a pair of primordial black holes in $(3 + 1)$ dimensional universe. In the paper they considered two different Euclidean space-time : (1) a universe with space-like sections with S^3 - topology and (2) a universe with space-like section with $S^1 \times S^2$ - topology, as is obtained in the Schwarzschild- de Sitter solution. The first kind of spatial structure describes an inflationary (de Sitter) universe without black hole while the second kind describes a Nariai universe [10], an inflationary universe with a pair of black holes. BH considered in their paper a theory with a massive scalar field which provided an effective cosmological constant for a while through a slow-rolling potential (mass-term). Chao [11] has studied the creation of a primordial black hole (in short, PBH) and Green and Malik [12] studied the primordial black holes production during reheating. Paul *et al.* [13] following the approach of BH studied the probability of creation of PBH including R^2 -term in the Einstein action and found that the probability is very much suppressed in the R^2 -theory. Paul and Saha [14] studied probability of creation of a pair of black hole with higher order Lagrangian i.e., considering higher loop contributions into the effective action that are higher than quadratic in R . In this paper we investigate a pair of black holes in the modified gravity given by action (1) in the presence of a cosmological constant.

Consequently it is also important to study the effects of these terms to study quantum creation of a universe with a pair of PBH. We calculate the probabilities for the creation of a universe with two types of topology namely, $R \times S^3$ - topology and $R \times S^1 \times S^2$ - topology, where the later accommodates a pair of primordial black holes. To calculate the probabilities for these spatial topologies, we use a semiclassical approximation for the evaluation of the Euclidean path integrals. The condition that a classical spacetime should emerge, to a good approximation, at a large Lorentzian time was selected by a choice of the path of the time parameter τ along the τ^{Re} axis from 0 to $\frac{\pi}{2H}$ and then continues along the τ^{im} axis. The wave-function of the universe in the semiclassical approximation is given by

$$\Psi_o[h_{ij}, \Phi_{\partial M}] \approx \sum_n A_n e^{-I_n} \quad (2)$$

where the sum is over the saddle points of the path integral, and I_n denotes the corresponding Euclidean action. The probability measure of the creation of PBH is

$$P[h_{ab}, \Phi_{\partial M}] \sim e^{-2 I^{Re}}$$

where h_{ab} is the boundary metric and I^{Re} is the real part of the action corresponding to the dominant saddle point, i.e. the classical solution satisfying the Hartle-Hawking (henceforth, HH) boundary conditions [15]. It was believed that all inflationary models lead to $\Omega_o \sim 1$ to a great accuracy. This view was modified after it was discovered that there is a special class of inflaton effective potentials which may lead to a nearly homogeneous open universe with $\Omega_o \leq 1$ at the present epoch. Cornish *et al.* [16, 17] studied the problem of pre-inflationary homogeneity and outlines the possibility of creation of a small, compact, negatively curved universe. We show that a universe with S^3 -topology may give birth to an open inflation similar to that obtained in *Ref.* 18.

The paper is organised as follows : in sec. II we write the gravitational action for a

higher derivative theories and obtain gravitational instanton solutions and in sec. III we use the action to estimate the relative probability of the two types of the universes and in sec. IV we give a brief discussion.

II. GRAVITATIONAL INSTANTON SOLUTIONS WITH OR WITHOUT A PAIR OF PRIMORDIAL BLACK HOLES :

We consider a Euclidean action which is given by

$$I_E = -\frac{1}{16\pi} \int d^4x \sqrt{g} f(R) - \frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{h} K f'(R) \quad (3)$$

where g is the 4-dimensional Euclidean metric, $f(R) = R + \frac{\delta}{R} - 2\Lambda$, R is the Ricci scalar, δ is a dimensional parameter and Λ is the 4-dimensional cosmological constant. In the gravitational surface term, h_{ab} is the boundary metric and $K = h^{ab}K_{ab}$ is the trace of the second fundamental form of the boundary ∂M in the metric. The second term is the contribution from $\tau = 0$ back in the action. It vanishes for a universe with S^3 topology, but gives a non vanishing contribution for $S^1 \times S^2$ -topology, with the gravitational instanton discussed here.

(A) Topology S^3 , the de Sitter spacetime :

In this section we study vacuum solution of the modified Euclidean Einstein action with a cosmological constant in four dimensions. We now look for a solution with spacelike section having S^3 - topology and the corresponding four dimensional metric ansatz which is given by

$$ds^2 = d\tau^2 + a^2(\tau) \left[dx_1^2 + \sin^2 x_1 d\Omega_2^2 \right] \quad (4)$$

where $a(\tau)$ is the scale factor of a four dimensional universe and $d\Omega_2^2$ is a line element on the unit two sphere. The scalar curvature is given by

$$R = -6 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2} \right) \right],$$

where an overdot denotes differentiation with respect to τ . We rewrite the action (3), including the constraint through a Lagrangian multiplier β and obtain

$$I_E = -\frac{\pi}{8} \int \left[f(R)a^3 - \beta \left(R + 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2 - 1}{a^2} \right) \right] d\tau - \frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{h} K f'(R). \quad (5)$$

Varying the action w.r.t. R , we determine

$$\beta = a^3 f'(R). \quad (6)$$

Substituting the above equation, treating a and R as independent variables we get

$$I_E = -\frac{\pi}{8} \int_{\tau=0}^{\tau=\frac{\pi}{2H}} \left[a^3 f(R) - f'(R) \left(a^3 R - 6a\dot{a}^2 - 6a \right) + 6a^2 \dot{a} R f''(R) \right] d\tau - \frac{3\pi}{4} \left[\dot{a} a^2 f'(R) \right]_{\tau=0}, \quad (7)$$

here we have eliminated \ddot{a} term in the action by integration by parts. The field equations are now obtained by varying the action with respect to a and R respectively, giving

$$f''(R) \left[R + 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2 - 1}{a^2} \right] = 0, \quad (8)$$

$$2f'''(R)\dot{R}^2 + 2f''(R) \left[\ddot{R} + 2\frac{\dot{a}}{a}\dot{R} \right] + f'(R) \left[4\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} - \frac{2}{a^2} + R \right] - f(R) = 0. \quad (9)$$

We now consider $f(R) = R + \frac{\delta}{R} - 2\Lambda$ and obtain an instanton solution which is given by

$$a = \frac{1}{H} \sin H\tau \quad (10)$$

where $R = 12H^2$ and H is determined from the constraint equation

$$48H^4 - 16\Lambda H^2 + \delta = 0. \quad (11)$$

We note that (i) two unequal values of H are permitted when $0 < \delta < \frac{4\Lambda^2}{3}$, (ii) $H^2 = \frac{\Lambda}{3}$ for $\delta = 0$, and (iv) $\Lambda = 0$ permits a real value for H^2 when $\delta < 0$. It is evident that the instanton solution obtained here satisfies the HH no boundary conditions viz., $a(0) = 0$,

$\dot{a}(0) = 1$. One can choose a path along the τ^{Re} axis to $\tau = \frac{\pi}{2H}$, the solution describes half of the Euclidean de Sitter instanton S^3 . Analytic continuation of the metric (3) to Lorentzian region, $x_1 \rightarrow \frac{\pi}{2} + i\sigma$, gives

$$ds^2 = d\tau^2 + a^2(\tau) \left[-d\sigma^2 + \cosh^2 \sigma d\Omega_2^2 \right] \quad (12)$$

which is a spatially inhomogeneous de Sitter like metric. However, if one sets $\tau = it$ and $\sigma = i\frac{\pi}{2} + \chi$, the metric becomes

$$ds^2 = -dt^2 + b^2(t) [d\chi^2 + \sinh^2 \chi d\Omega_2^2]$$

where $b(t) = -i a(it)$. The line element (12) now describes an open inflationary universe. The real part of the Euclidean action corresponding to the solution calculated by following the complex contour of τ suggested by BH is given by

$$I_{S^3}^{Re} = -\frac{2\pi}{3H^2} \left[\frac{12\Lambda H^2 - \delta}{16\Lambda H^2 - \delta} \right]. \quad (13)$$

With the chosen path for τ , the solution describes half the de Sitter instanton with S^4 topology, joined to a real Lorentzian hyperboloid of topology $R^1 \times S^3$. It can be joined to any boundary satisfying the condition $a_{\partial M} > 0$. For $a_{\partial M} > H^{-1}$, the wave function oscillates and predicts a classical space-time. We note the following cases :

- $\delta = 0$, the action reduces to that obtained by BH.
- $0 < \delta < \frac{4\Lambda^2}{3}$, a realistic solution corresponds to $H_o^2 > \frac{\delta}{12\Lambda}$.
- $\Lambda = 0$, the action is decided by δ which is $I = -\frac{8\pi}{\sqrt{-3\delta}}$, it is real for $\delta < 0$.

(B) Topology $S^1 \times S^2$:

In this section we consider Euclidean Einstein equation and look for a universe with $S^1 \times S^2$ -spacelike sections as this topology accommodates a pair of black holes. The corresponding ansatz for (1 + 1 + 2) dimensions is given by

$$ds^2 = d\tau^2 + a^2(\tau) dx^2 + b^2(\tau) d\Omega_2^2 \quad (14)$$

where $a(\tau)$ is the scale factor of S^1 -surface and $b(\tau)$ is the scale factor of the two sphere.

The metric for the two-sphere is given by the metric

$$d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$$

The scalar curvature is given by

$$R = - \left[2\frac{\ddot{a}}{a} + 4\frac{\ddot{b}}{b} + 2\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right) + 4\frac{\dot{a}\dot{b}}{ab} \right]. \quad (15)$$

The Euclidean action (3) becomes

$$I_E = -\frac{\pi}{2} \int \left[f(R)ab^2 - \beta \left(R + 2\frac{\ddot{a}}{a} + 4\frac{\ddot{b}}{b} + 4\frac{\dot{a}\dot{b}}{ab} + 2\frac{\dot{b}^2}{b^2} - \frac{2}{b^2} \right) \right] d\tau - \frac{1}{8\pi} \int_{\partial M} \sqrt{h} d^3x K f'(R). \quad (16)$$

One can determine β as is done before and obtains

$$I_{S^1 \times S^2} = -\frac{\pi}{2} \int_{\tau=0}^{\tau_{\partial M}} \left[f(R) - f'(R) \left(R - 4\frac{\dot{a}\dot{b}}{ab} - 2\frac{\dot{b}^2}{b^2} - \frac{2}{b^2} \right) + 2f''(R)\dot{R} \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \right] ab^2 d\tau - \pi \left[(\dot{a}b^2 + 2abb) f'(R) \right]_{\tau=0}. \quad (17)$$

Variation of the action with respect to R , a and b respectively are given by the following equations

$$f''(R) \left[R + 2\frac{\ddot{a}}{a} + 4\frac{\ddot{b}}{b} + 4\frac{\dot{a}\dot{b}}{ab} + 2\frac{\dot{b}^2}{b^2} - \frac{2}{b^2} \right] = 0, \quad (18)$$

$$2f'''(R)\dot{R}^2 + 2f''(R) \left[\ddot{R} + 2\dot{R}\frac{\dot{b}}{b} \right] + f'(R) \left[R + 4\frac{\ddot{b}}{b} + 2\frac{\dot{b}^2 - 1}{b^2} \right] - f(R) = 0, \quad (19)$$

$$2f'''(R)\dot{R}^2 + 2f''(R) \left[\dot{R} \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) + \ddot{R} \right] + f'(R) \left[R + 2\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} + 2\frac{\dot{a}\dot{b}}{ab} \right] - f(R) = 0. \quad (20)$$

Let us now consider $f(R) = R + \frac{\delta}{R} - 2\Lambda$. Equations (18)-(20) admit an instanton solution which is given by

$$a = \frac{1}{H_o} \sin(H_o\tau), \quad b = H_o^{-1},$$

$$R = 4H_o^2, \quad (21)$$

where H_o satisfies the constraint equation

$$16H_o^4 - 16\Lambda H_o^2 + 3\delta = 0. \quad (22)$$

We note the following : (i) $\delta = 0$, one gets $H_o^2 = \Lambda$, (ii) $0 < \delta < \frac{4\Lambda^2}{3}$, one obtains two unequal real values of H_o^2 are permitted, (iii) $\delta = \frac{4}{3}\Lambda^2$ leads to $H_o^2 = \frac{\Lambda}{2}$ and (iii) $\Lambda = 0$, gives $H_o^2 = \sqrt{-\frac{3\delta}{16}}$ which is real if $\delta < 0$. The above instanton solution satisfies the HH boundary conditions $a(0) = 0$, $\dot{a}(0) = 1$, $b(0) = b_o$, $\dot{b}(0) = 0$. Analytic continuation of the metric (14) to Lorentzian region, i.e., $\tau \rightarrow it$ and $x \rightarrow \frac{\pi}{2} + i\sigma$ yields

$$ds^2 = -dt^2 + c^2(t) d\sigma^2 + H_o^{-2} d\Omega_2^2, \quad (23)$$

where $c(t) = -i a(it)$. In this case the analytic continuation of time and space do not give an open inflationary universe. The corresponding Lorentzian solution is given by

$$\begin{aligned} a(\tau^{Im})|_{\tau^{Re} = \frac{\pi}{2H_o}} &= H_o^{-1} \cosh H_o \tau^{Im}, \\ b(\tau^{Im})|_{\tau^{Re} = \frac{\pi}{2H_o}} &= H_o^{-1} \end{aligned} \quad (24)$$

Its space like sections can be visualised as three spheres of radius H_o^{-1} with a *hole* of radius $b = H_o^{-1}$ punched through the north and south poles. The physical interpretation of the solution is that of two - spheres containing two black holes at opposite ends. The black holes have the radius H_o^{-1} which accelerates away from each other with the expansion of the universe. The real part of the action can now be determined following the contour suggested by BH [2], and it is given by

$$I_{S^1 \times S^2}^{Re} = -\frac{4\pi}{H_o^2} \left[\frac{4\Lambda H_o^2 - \delta}{16\Lambda H_o^2 - 3\delta} \right].$$

The solution (24) describes a universe with two black holes at the poles of a two sphere. It may be pointed out here that the contribution of the integrand in (17) for the instanton

vanishes and the non-zero contribution of the action here arises from the boundary term only. We note the following :

- $\delta = 0$, the action reduces to that obtained by BH.
- $\Lambda = 0$, the action is decided by δ which is $I = -\frac{16\pi}{3\sqrt{-3}\delta}$, it is real for $\delta < 0$.
- $\delta = \frac{4\Lambda^2}{3}$, the action becomes $I = -\frac{4\pi}{3\Lambda}$.
- $\delta < \frac{4\Lambda^2}{3}$, either $H_o^2 > \frac{\delta}{4\Lambda}$ or $H_o^2 < \frac{3\delta}{16\Lambda}$ allows a negative definite action corresponding

to the instanton solution.

III. PRIMORDIAL BLACK HOLES PAIR CREATION PROBABILITY :

In the previous section we have calculated the actions for inflationary universe with or without a pair of black holes. We now compare the probability measure in the two cases. The probability for creation of a de Sitter universe is determined from the action (7). The probability for nucleation of an inflationary universe without a pair of Black holes is given by

$$P_{S^3} \sim e^{\frac{\pi}{36H^6} [12\Lambda H^2 - \delta]}. \quad (25)$$

However for an inflationary universe with a pair of black holes the corresponding probability of nucleation can be obtained from the action (17). The corresponding probability is

$$P_{S^1 \times S^2} \sim e^{\frac{\pi}{2H_o^6} [4\Lambda H_o^2 - \delta]} \quad (26)$$

We now describe spacial cases :

- $\delta = 0$, one recovers the result obtained by Bousso and Hawking [9]

$$P_{S^3} \sim e^{\frac{3\pi}{\Lambda}}, \quad P_{S^1 \times S^2} \sim e^{\frac{2\pi}{\Lambda}}. \quad (27)$$

Thus with a positive cosmological constant the probability for a universe with PBH is less than that without PBH.

- $\delta \neq 0$, since eqs. (11) and (22) are quadratic in H^2 and H_o^2 , there are two branches of solutions admitting two different instanton solutions. In one case the probabilities are

given by

$$P_{S^3} \sim e^{\frac{6\pi}{(\Lambda + \sqrt{\Lambda^2 - \frac{3\delta}{4}})^3} [2\Lambda(\Lambda + \sqrt{\Lambda^2 - \frac{3\delta}{4}}) - \delta]}, \quad P_{S^1 \times S^2} \sim e^{\frac{4\pi}{(\Lambda + \sqrt{\Lambda^2 - \frac{3\delta}{4}})^3} [2\Lambda(\Lambda + \sqrt{\Lambda^2 - \frac{3\delta}{4}}) - \delta]}$$
(28)

for $\delta < \frac{4\Lambda^2}{3}$, and in the other case the probabilities are

$$P_{S^3} \sim e^{\frac{6\pi}{(\Lambda - \sqrt{\Lambda^2 - \frac{3\delta}{4}})^3} [2\Lambda(\Lambda - \sqrt{\Lambda^2 - \frac{3\delta}{4}}) - \delta]}, \quad P_{S^1 \times S^2} \sim e^{\frac{4\pi}{(\Lambda - \sqrt{\Lambda^2 - \frac{3\delta}{4}})^3} [2\Lambda(\Lambda - \sqrt{\Lambda^2 - \frac{3\delta}{4}}) - \delta]}$$
(29)

when $0 < \delta < \frac{4\Lambda^2}{3}$.

It may be pointed out here that the values of H^2 and H_o^2 corresponding to the probability measure given by eq.(28) are higher than that in eq.(29). It is evident from eqs. (28) and (29) that the creation of a universe without PBH is more probable than with PBH for $\delta < \frac{4\Lambda^2}{3}$ with a positive cosmological constant. It admits negative δ also. However, the probability of creation of a universe with a pair of PBH is favoured in the first case when one considers a negative cosmological constant. In the second case, the probability of creation of a universe with a pair of PBH is favoured with a positive cosmological constant for $0 < \delta < \Lambda^2$, but it is suppressed for $\Lambda^2 < \delta < \frac{4\Lambda^2}{3}$. In the second case no instanton solution exists with a negative cosmological constant.

- $\delta = \frac{4\Lambda^2}{3}$, the probabilities are given by

$$P_{S^3} \sim e^{\frac{4\pi}{\Lambda}}, \quad P_{S^1 \times S^2} \sim e^{\frac{8\pi}{3\Lambda}}.$$
(30)

In the above only one kind of instanton solution is permitted with $H = \sqrt{\frac{\Lambda}{6}}$ in the S^3 -topology and $H_o = \sqrt{\frac{\Lambda}{2}}$ in the $S^1 \times S^2$ -topology.

- $\Lambda = 0$, the probabilities are given by

$$P_{S^3} \sim e^{\frac{16\pi}{\sqrt{-3\delta}}}, \quad P_{S^1 \times S^2} \sim e^{\frac{32\pi}{3\sqrt{-3\delta}}}.$$
(31)

in this case only negative values are permitted for the parameter δ in the action which reduces to the action that considered by Carrol *et al.* [6].

IV. DISCUSSIONS :

In this paper we have evaluated the probability for primordial black holes pair creation in a modified theory of gravity. In section II, we have obtained the gravitational instanton solutions in the two cases : (i) a universe with $R \times S^3$ - topology and (ii) a universe with $R \times S^1 \times S^2$ - topology respectively. The Euclidean action is then evaluated corresponding to the instanton solutions. For a non zero δ in the gravitational action, we obtain two classes of instanton solutions differing in the values of the radius of the sphere H^{-1} and H_o^{-1} respectively. We found that the probability of a universe with $R \times S^3$ topology turns out to be much lower than a universe with topology $R \times S^1 \times S^2$ in the modified theory unless $0 < \delta < \frac{4\Lambda^2}{3}$ with a positive cosmological constant in the case of lower instantonic radius (H^{-1}, H_o^{-1}) and in the case of higher instantonic radius (H^{-1}, H_o^{-1}) with $\Lambda^2 < \delta < \frac{4\Lambda^2}{3}$ with a positive cosmological constant. It may be mentioned here that one gets a regular instanton in S^4 - topology if there are no black holes. The existence of black holes restricts such a regular topology. The results obtained here on the probability of creation of a universe with a pair of primordial black holes are found to be strongly suppressed depending on the parameter δ determined by the cosmological constant in some cases which are presented here. We note an interesting solution here in the framework of the modified gravitational action with inverse power of R -theory, which admits de Sitter instantons with S^3 and $S^1 \times S^2$ topologies even without a cosmological constant. In this case the action is similar to that considered by Carroll *et al.* [6]. It is interesting to note here that analytic continuation of a $R \times S^3$ metric considered here to Lorentzian region leads to an open 3 - space. One obtains Hawking-Turok [18] type open inflationary universe in this case. In the other type of topology an open inflation section of the universe is not permitted.

A detailed study of an open inflationary universe will be discussed elsewhere. Thus in a modified Lagrangian with inverse power in R -theory, quantum creation of PBH seems to be suppressed in the minisuperspace cosmology for some values of the parameters in the action, which are determined here. Another new result obtained here is that a gravitational instanton solution may be obtained even with a negative cosmological constant which is not permitted in the case considered by BH [9].

Acknowledgement : BCP gratefully acknowledges valuable discussions with Naresh Dadhich. DP would like to thank S. Mukherjee for providing facilities of the IUCAA Reference Centre at North Bengal University to carry out a part of the work. This work is partially supported by University Grants Commission, New Delhi.

References

- [1] D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003); L. Verde *et al.*, *Astrophys. J. Suppl.* **148**, 195 (2003); S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999); *Bull. Am. Astron. Soc.* **29**, 1351 (1997), *Astrophys. J.* **517**, 565 (1999), *Measuring cosmology with Supernovae*, astro-ph/0303428.
- [2] A. A. Starobinsky, *Phys. Lett.* **99**, 24 (1980); A. H. Guth, *Phys. Rev.* **D 23**, 347 (1981); A. D. Linde, *Phys. Lett.* **B 108**, 389 (1982); A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [3] N. Bilic, R. J. Lindebaum, G. B. Tupper and R. D. Viollier, *Inhomogeneous Chaplygin gas cosmology*, astro-ph/0310181; L.P. Chimento, *Phys. Re.* **D 69**, 123517 (2004); O. Bertolami, *Challenges on the generalized chaplygin gas cosmology*, astro-ph/0403310; T. Barreiro and A. A. Sen, *Phys. Rev.* **D 70**, 124013 (2004).
- [4] P. Singh, M. Sami and N. Dadhich, *Phys. Rev.*, **D 68**, 023522 (2003).
- [5] T. V. Ruzmaikina and A. A. Ruzmaikin, *Sov. Phys. JETP* **30**, 372 (1970); A. A. Starobinsky, *Phys. Lett.*, **99**, 24 (1980); A. A. Starobinsky, *Sov. Astron. Lett.*, **9**, 302 (1983); R. Fabbri and M. D. Pollock, *Phys. Lett.* **125 B** 445, (1983); S. W. Hawking and J. C. Luttrell, *Nuc. Phys.*, **B 247**, 250 (1984); A. A. Starobinsky, *JETP Lett.*, **42**, 15 (1985); L. A. Kofman, A. D. Linde and A. A. Starobinsky, *Phys. Lett.*, **157 B**, 361 (1985); A. Vilenkin, *Phys. Rev.* **D 32**, 2511 (1985); M. B. Mijic, M. S. Morris and W. Suen, *Phys. Rev.* **D 34**, 2934 (1986); S. Gottlober and V. Muller, *Class. Quantum Grav.* **3**, 183 (1986); B. C. Paul, D. P. Datta and S. Mukherjee, *Mod. Phys. Lett.* **A 3**, 843 (1988).

- [6] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. turner, *Phys. Rev.* **D 70**, 043528 (2004), astro-ph/0306438.
- [7] S. Capozziello, V. F. Cardone and A. Troisi, *Phys. Rev.* **D 71** , 043503 (2005); I. Brevik, S. Nojiri, S. D. Odintsov and I Vanzo, *Phys. Rev.* **D 70**, 043520 (2004); S. Nojiri and S. D. Odintsov, *Phys. Lett.* **B 576**, 5 (2003); S. Nojiri and S. D. Odintsov, *Phys. Rev.* **D 68**, 123512 (2003); M. Abdalla, S. Nojiri and S. D. Odintsov, *Class. Quantum Grav.* **22**, L 35 (2005); S. Das, N. Banerjee and N. Dadhich, *Curvature driven acceleration : an utopia or a reality ?*, astro-ph/0505096.
- [8] S. W. Hawking, *Comm. Math. Phys.* **43**, 199 (1975).
- [9] R. Bousso and S. W. Hawking, *Phys. Rev.* **D 52**, 5659 (1995).
- [10] H. Nariai, *Sci. Rep. Tohoku Univ.* **35**, 62 (1951).
- [11] W. Z. Chao, *Int. J. Mod. Phys.* **D6**, 199 (1997).
- [12] A. M. Green and K. A. Malik, *Phys. Rev.* **D 64**, 021301 (2001).
- [13] B. C. Paul, G. P. Singh, A. Beesham and S. Mukherjee, *Mod. Phys. Letts.* **A 13**, 2289 (1998).
- [14] B. C. Paul and A. Saha, *Int. J. Mod. Phys.* **D 11**, 493 (2002)..
- [15] J. B. Hartle and S. W. Hawking, *Phys. Rev.* **D 28**, 2960 (1983).
- [16] N. J. Cornish, D. N. Spergel and G. D. Starkman, *Phys. Rev. Letts.* **77**, 215 (1996) ; *Class. Quantum Grav.* **15**, 2657 (1998).
- [17] N. J. Cornish and D. N. Spergel, ‘ *A small Universe after all ?* ’ - astro-ph/9906401.

- [18] S. W. Hawking and N. G. Turok, *Phys. Lett.* **B 425**, 25 (1998), hep-th/9802030; N. G. Turok and S. W. Hawking, *Phys. Lett.* **B 432**, 271 (1998), hep-th/9803156.