

MHD modes of solar wind flow tubes

Suresh Chandra¹

School of Physical Sciences, S.R.T.M. University,
Nanded 431 606, India
(email: suresh492000@yahoo.co.in)

Abstract. Nakariakov & Roberts (1995) and Nakariakov et al. (1996) investigated the linear magnetosonic waves trapped within solar wind flow tubes by accounting for a slab having boundaries at $x = \pm d$ and extended up to infinity in y and z directions. They obtained dispersion relations for sausage and kink surface waves in incompressible plasma. Following the approach of Nakariakov & Roberts (1995), we have obtained dispersion relations for sausage and kink surface waves in a compressible plasma. Values of the parameter $a \equiv \omega/kC_{Ao}$ are found to vary in the ranges 0.755 – 0.849 and 1.080 – 1.356 for the sausage waves and in the ranges 0.850 – 0.882 and 1.358 – 1.538 for the kink waves in the compressible plasma. It shows an interesting feature that the sausage and kink surface waves are exclusive from each other.

1 Introduction

HELIOS spacecraft observations (Thieme et al., 1990) supported Parker’s assumption (1963) that the solar wind could be fine-structured in the form of flow tubes, sometimes also called ‘spaghetti structures’. These flow tubes are considered to originate and extending for large distances in the solar wind. Two adjacent flow tubes would have plasma having different values for parameters. They may be separated by tangential discontinuities in flows and in magnetic fields. The total (gas plus magnetic) pressure has to be continuous across the structure boundary (Tu and Marsch, 1995). In these flow tubes, the magnetosonic waves may be excited assuming the Alfvén speed to be less inside the tube than that outside. Nakariakov & Roberts (1995) and Nakariakov et al. (1996) investigated one-dimensional problem by considering a slab having boundaries at $x = \pm d$ and extended up to infinity in y and z directions (Figure 1) and obtained dispersion relations for sausage and kink surface waves in incompressible plasma. For incompressible plasma, the speed of sound is taken much larger than all other velocities.

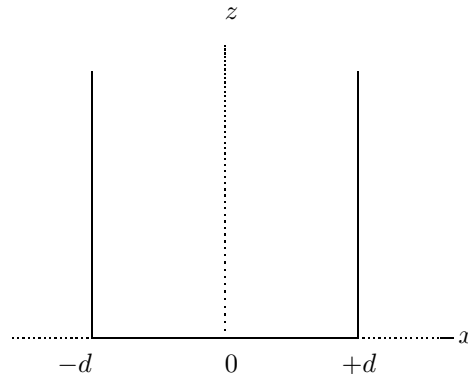


Figure 1: A slab having boundaries at $x = \pm d$ and extended up to infinity in the y and z directions.

As the speed of sound may not be much larger than than all other velocities, the plasma may not

¹Visiting Associate of IUCAA, Pune, India

be considered incompressible. In the present communication, following the approach of Nakariakov & Roberts (1995), we have obtained dispersion relations for sausage and kink surface waves in the compressible plasma.

2 Waveguide model

Nakariakov & Roberts (1995) and Nakariakov et al. (1996) considered a slab having boundaries at $x = \pm d$ and extended up to infinity in y and z directions (Figure 1). Further, both the magnetic field (inside B_o and outside B_e) and steady flows (inside U_o and outside U_e) are directed along the z -axis. The sound speeds are C_{S_o} inside and C_{S_e} outside, whereas the Alfvén speeds are C_{A_o} inside and C_{A_e} outside. The unperturbed gas densities ρ_o inside and ρ_e outside are connected by the condition of total pressure balance for an ideal gas:

$$\frac{\rho_e}{\rho_o} = \frac{2C_{S_o}^2 + \gamma C_{A_o}^2}{2C_{S_e}^2 + \gamma C_{A_e}^2} \quad (1)$$

where $\gamma = 5/3$ is adiabatic index. For the regions inside and outside the slab, the linearized equations of ideal MHD can be expressed as the following differential equations (Nakariakov & Roberts, 1995; Nakariakov et al., 1996)

$$\frac{d^2 V_{xi}}{dx^2} - m_i^2 V_{xi} = 0 \quad (2)$$

where the index i is either o (for inside the slab) or e (for outside the slab). The transversal plasma velocity is $V_{xi} \exp i(\omega t - kz)$ and

$$m_i^2 = \frac{[a_{Ai}^2 - (a - M_i)^2][a_{Si}^2 - (a - M_i)^2]}{a_{fi}^2[a_{Ti}^2 - (a - M_i)^2]} k^2 \quad (3)$$

where

$$\begin{aligned} a &= \frac{\omega}{k C_{A_o}} & a_{Ai}^2 &= \frac{C_{Ai}^2}{C_{A_o}^2} & a_{Si}^2 &= \frac{C_{Si}^2}{C_{A_o}^2} \\ M_i &= \frac{U_i}{C_{A_o}} & a_{Ti}^2 &= \frac{C_{Ai}^2 C_{Si}^2}{C_{A_o}^2 (C_{Ai}^2 + C_{Si}^2)} & a_{fi}^2 &= \frac{C_{Ai}^2 + C_{Si}^2}{C_{A_o}^2} \end{aligned}$$

Here, all the variables are normalized on C_{A_o} . M_i is the Alfvén Mach number and a the phase speed in the units of the Alfvén speed C_{A_o} .

Following Nakariakov & Roberts (1995) and Nakariakov et al. (1996), equations (2) are to be solved subject to the boundary conditions relating the perturbations across the slab boundaries and also the conditions at infinity. The first boundary condition is the continuity

$$\frac{V_{xe}(x = \pm d)}{\omega - kU_e} = \frac{V_{xo}(x = \pm d)}{\omega - kU_o} \quad (4)$$

and the second boundary condition is the continuity of the total (gas plus magnetic) pressure perturbation

$$p_{Te}(x = \pm d) = p_{To}(x = \pm d) \quad (5)$$

A relation between the total pressure $p_{Ti}(x)$ and velocity $V_{xi}(x)$ is

$$p_{Ti}(x) = \frac{iC_{A_o}\rho_i a_{fi}^2 [a_{Ti}^2 - (a - M_i)^2]}{k(a - M_i) [a_{Si}^2 - (a - M_i)^2]} \frac{dV_{xi}}{dx} \quad (6)$$

where ρ_i is the unperturbed gas density. The transversal structure of the modes outside the slab are

$$V_{xe}(x) = \begin{cases} A_1 \exp[-m_e(x-d)] & x > d \\ A_2 \exp[+m_e(x+d)] & x < -d \end{cases} \quad (7)$$

where A_1 and A_2 are constants. Equation (7) gives zero wave energy as $x \rightarrow \pm\infty$. Here, m_e is positive, so that there is no leakage from the slab and the slab behaves as a wave guide. The transversal structure of the modes inside the slab are

$$V_{xo}(x) = \begin{cases} A \sinh(m_o x) & \text{for sausage surface modes} \\ A \cosh(m_o x) & \text{for kink surface modes} \\ A \sin(n_o x) & \text{for sausage body modes} \\ A \cos(n_o x) & \text{for kink body modes} \end{cases} \quad (8)$$

where A is a constant and $n_o^2 = -m_o^2$. After connecting equations (7) and (8) by means of the boundary conditions (4) and (5), we get

$$\frac{\rho_e m_o [a_{Ae}^2 - (a - M_e)^2]}{\rho_o m_e [1 - (a - M_o)^2]} = \begin{cases} -\tanh(m_o d) \\ -\coth(m_o d) \end{cases} \quad (9)$$

for surface waves. The upper case corresponds to the kink waves whereas the lower to the sausage waves. The similar relations for body waves are

$$\frac{\rho_e n_o [a_{Ae}^2 - (a - M_e)^2]}{\rho_o m_e [1 - (a - M_o)^2]} = \begin{cases} -\tan(n_o d) \\ \cot(n_o d) \end{cases} \quad (10)$$

Here, also the upper case corresponds to the kink waves whereas the lower to the sausage waves.

3 Dispersion relations

Equation (3) shows that m_i tends to $\rightarrow k$ in two situations: (i) when $a = M_o = M_e$ and (ii) when the speed of sound is much larger than all other velocities (the case of incompressible plasma).

(i) For $a = M_o = M_e$, the steady shear flows are equal inside as well as outside the slab. Under such situation, equation (9) reduces to

$$\frac{\rho_e C_{Ae}^2}{\rho_o C_{Ao}^2} = \begin{cases} -\tanh(kd) & \text{kink surface waves} \\ -\coth(kd) & \text{sausage surface waves} \end{cases}$$

It shows that there is no dispersion relation which relates a and k for this situation.

(ii) When the speed of sound is much larger than all other velocities (the case of incompressible plasma), the dispersion relation is

$$\frac{\rho_e [a_{Ae}^2 - (a - M_e)^2]}{\rho_o [1 - (a - M_o)^2]} = \begin{cases} -\tanh(kd) & \text{kink surface waves} \\ -\coth(kd) & \text{sausage surface waves} \end{cases} \quad (11)$$

This relation for surface waves in an incompressible plasma was obtained by Nakariakov & Roberts (1995), Nakariakov et al. (1996) and Srivastava & Dwivedi (2006). No such relation is available for body waves. However, Srivastava & Dwivedi (2006) has given a dispersion relation for body waves also, which is absolutely wrong (Chandra, 2007). Equation (11) can be expressed as

$$kd = \frac{1}{2} \begin{cases} \ln\left(\frac{1-y}{1+y}\right) & \text{for kink surface waves} \\ \ln\left(\frac{y-1}{y+1}\right) & \text{for sausage surface waves} \end{cases}$$

where

$$y = \frac{\rho_e [a_{Ae}^2 - (a - M_e)^2]}{\rho_o [1 - (a - M_o)^2]}$$

Using the values $M_e = 0$, $M_o = 1.46$, $C_{Ao} = 65$ km/s, $C_{Ae} = 100$ km/s, $\rho_e/\rho_o = 0.583$, as adopted by Nakariakov & Roberts (1995) and Nakariakov et al. (1996) for solar wind flow tube, the variation of kd versus a for sausage and kink surface waves is shown in figure 2. Values of the parameter a are found to vary in the ranges $0.00 - 0.46$ and $2.00 - 2.46$ for the sausage surface waves and in the range $1.54 - 1.92$ for the kink surface waves in the incompressible plasma. It shows that in the incompressible plasma, the sausage and kink waves are exclusive from each other.

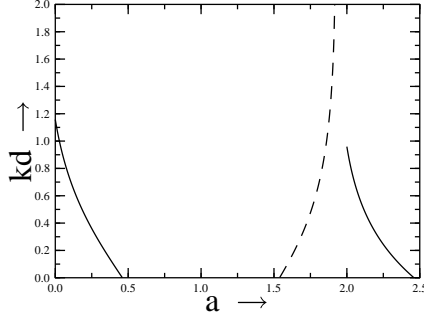


Figure 2: Variation of kd versus a for sausage (solid line) and kink (dashed line) surface waves for incompressible plasma.

3.1 Compressible plasma

When the speed of sound is not much larger than all other velocities, the plasma can not be treated as incompressible. By using m_o and m_e in equation (9), the dispersion relation is

$$kd = \frac{1}{2A} \begin{cases} \ln\left(\frac{1+B}{1-B}\right) & \text{for kink surface waves} \\ \ln\left(\frac{B+1}{B-1}\right) & \text{for sausage surface waves} \end{cases} \quad (12)$$

where

$$A = \frac{1}{a_{fo}} \sqrt{\frac{[1 - (a - M_o)^2] [a_{So}^2 - (a - M_o)^2]}{[a_{To}^2 - (a - M_o)^2]}}$$

and

$$B = \frac{\rho_e a_{fe}}{\rho_o a_{fo}} \sqrt{\frac{[a_{Ae}^2 - (a - M_e)^2] [a_{So}^2 - (a - M_o)^2] [a_{Te}^2 - (a - M_e)^2]}{[1 - (a - M_o)^2] [a_{Se}^2 - (a - M_e)^2] [a_{To}^2 - (a - M_o)^2]}}$$

The values of a_{fe} and a_{fo} may be positive as well as negative. Thus, A and B can assume positive as well as negative values. For getting positive value of k , A and B both should be either positive or negative. For the compressible plasma also, we have only surface waves. It shows that the magnitude of B is larger than 1 for the sausage surface waves whereas is smaller than 1 for the kink surface waves. Hence these two types of waves are exclusive from each other.

Using the values $M_e = 0$, $M_o = 1.46$, $C_{Ao} = 65$ km/s, $C_{So} = 65$ km/s, $C_{Ae} = 100$ km/s, $C_{Se} = 70$ km/s, $U_o = 750$ km/s, $U_e = 655$ km/s, $\rho_e/\rho_o = 0.583$, as adopted by Nakariakov &

Roberts (1995) and Nakariakov et al. (1996) for solar wind flow tube, the variation of kd versus a for sausage and kink surface waves is shown in figure 3. Values of the parameter a are found to vary in the ranges $0.755 - 0.849$ and $1.080 - 1.356$ for the sausage waves and in the ranges $0.850 - 0.882$ and $1.358 - 1.538$ for the kink waves in the compressible plasma. It shows an interesting feature that for the compressional plasma also the sausage and kink waves are exclusive from each other.

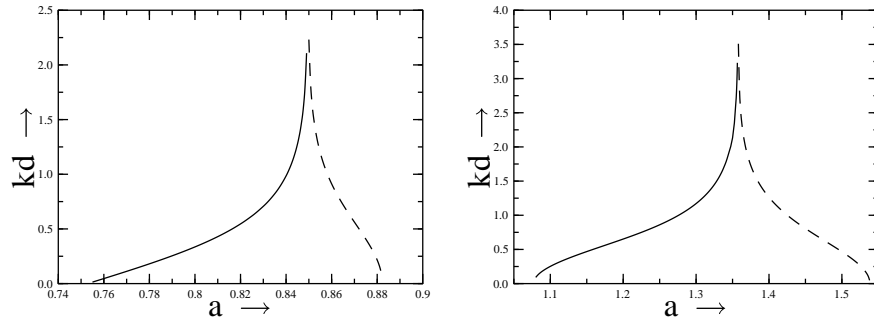


Figure 3: Variation of kd versus a for sausage (solid line) and kink (dashed line) surface waves for compressible plasma.

Comparison of figures 2 and 3 that the ranges for a are different for compressible and incompressible plasmas, as expected.

4 Conclusions

We found that for incompressible plasma, the sausage and kink surface waves are exclusive from each other. We have obtained dispersion relations for compressible plasma under the same situation as considered by Nakariakov & Roberts (1995) and Nakariakov et al. (1996). It is interesting to note that the sausage and kink surface waves are exclusive from each other for the compressible plasma as well. As per expectations, the values of a for the compressible plasma are different from those for incompressible plasma. The interesting feature obtained is the exclusive behaviour of the sausage and kink surface waves.

Acknowledgments

Our sincere thanks are due to the learned referees for constructive comments; one of them pointed out a mistake in the calculations. We heartily thank Prof. E.R. Pekünü for cooperative and encouraging correspondence. Financial support from the Department of Science & Technology (DST), New Delhi and the Indian Space Research Organization (ISRO), Bangalore in the form of research projects is thankfully acknowledged.

References

- Chandra, S. : 2007, *J. Astrophys. Astron.* (In press)
- Nakariakov, V.M. and Roberts, B. 1995, *Solar Phys.* **159**, 213.
- Nakariakov, V.M., Roberts, B. and Mann, G. 1996, *Astron. Astrophys.* **311**, 311.

Parker, E.N. : 1963, *Interplanetary Dynamical Processes*, Interscience, New York.

Srivastava, A.K. and Dwivedi, B.N. : 2006, *J. Astrophys. Astron.* **27**, 353.

Thieme, K.M., Marsh, E. and Schwenn, R. : 1990, *Ann. Geophys.* **8**, 713.

Tu, C.-Y. and Marsch, E. : 1995, *Space Sci. Rev.* **73**, 1.