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GEOMETRIC PHASE IN VACUUM INSTABILITY: APPLICATIONS IN QUANTUM COSMOLOGY

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ABSTRACT

Three different methods viz. i) a perturbative analysis of the Schrödinger equation ii) abstract differential geometric method and iii) a semiclassical reduction of the Wheeler-Dewitt equation, relating Pancharatnam phase to vacuum instability are discussed. An improved semiclassical reduction is also shown to yield the correct zeroth order semiclassical Einstein equations with backreaction. This constitutes an extension of our earlier discussions on the topic.

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The study of geometric phases¹⁻³ seems to offer important insights in having a better understanding for a large class of physical problems. In quantum field theory for instance the Berry phase appears to play a significant role in elucidating several conceptual issues relating to anomalies and associated problems. It is shown^{4,5} that various gauge anomalies can be interpreted as due to a non-trivial holonomy on the second quantized (chiral) fermion Hilbert bundle over background static gauge fields. The non-trivial holonomy arises as a measure of topological obstructions in projecting the Fock vacuum in the physical sector of the gauge manifold (static gauge fields mod local gauge group). This in turn implies a loss of gauge invariance (global and non-abelian anomalies) and/or an induced symmetry breaking (axial anomaly).

Now the breakdown of the global $U(1)$ axial symmetry via an anomalous divergence of the axial current induces axial baryon-lepton non-conserving processes through the production of massless fermion excitations⁶. Nelson and Alvarez - Gaume⁴ have further shown that even the global and non-abelian anomalies could be explained in terms of pair productions. Although non-generic, the pair production occurs at the points of degeneracies of the background field dependent Dirac hamiltonian, inducing a twist in the pertinent Hilbert bundle.

Recently some applications of Berry phase are also discussed^{7,8} in the semiclassical gravity in the framework of a minisuperspace cosmological model. An improved Born-Oppenheimer analysis in the Wheeler-Dewitt (WD) equation is shown to yield the correct zeroth order semiclassical Einstein equations. The functional Schrödinger equation describing quantized matter fields in a background curved space is obtained at the next order of approximation. Further, the semiclassical backreaction of the matter fields is shown to be determined by the $U(1)$ Berry connection on the gravitational sector of the minisuperspace. An interesting consequence emerges in the Robertson-Walker (RW) minisuperspace which is one-dimensional with trivial R -topology. The relevant Hilbert bundle turns out also to be trivial, thereby reducing the induced Berry connection essentially to zero. As a consequence the WD equation corresponding to a gravitational action without a cosmological Λ -term yields at the semiclassical regime a matter Schrödinger equation essentially in the Minkowsky space⁸. However, for an action with a non-zero Λ , one gets zero matter equation in the DeSitter (DS) universe, although zeroth order analysis does not yield a suitable backreaction. One however, expects a finite rate of particle production in the DS background. It is, therefore, of interest to see how the semiclassical Einstein equations with a reasonable backreaction can be obtained through some modifications of the arguments in Refs.[7,8].

It is wellknown that the particle production in QFT in the presence of a classical external field is associated with the vacuum decay which is essentially a non-perturbative effect. Under the influence of a time varying external field the otherwise stable initial vacuum

evolves into an admixture of multiparticle states, thereby reducing the vacuum transition probability amplitude to a value less than the initially normalized value one.

Now, one naturally feels tempted to see if there is some intrinsic relationship between the particle production through vacuum decay and the particle production via symmetry breaking due to anomaly. It is therefore of interest to look for a description of particle creation through vacuum decay in the language of the geometric phase. This will also enable one with important insights as to how the modifications in the Born-Oppenheimer analysis is to be incorporated to get a consistent set of semiclassical Einstein equations.

The motivation of the present paper is exactly this. We discuss some wellknown examples of vacuum instability and show how a geometric phase can be associated with the decay width of the state. In sect. 2 we show in the context of a quantum mechanical decay model that the decay width Γ is related to the Pancharatnam phase³ between the initial and the final states. Pancharatnam phase is a generalized geometric phase which may be obtained even for a non-unitary, non-cycle evolution. In particular, Pancharatnam phase may be non-zero even for a case where Berry phase is zero or not sensible. Our method uses a perturbative argument although the result is exact and non-perturbative. The result also agrees with more abstract formulations⁵ of the geometric phase (and anomaly). However, we discuss this result here as a prelude to our main result (Sect. 3). (The author is however unaware of any prior explicit discussion of this example in literature.) In sect. 3, we present an extension of our earlier derivation^{7,8} of the semiclassical Einstein equations. This frees the earlier discussions from the necessity of a cyclic evolution. We show that for a non-cyclic evolution the backreaction can be related to the Pancharatnam phase. For topologically trivial minisuperspace where Berry phase is zero, this yields a set of semiclassical equations which describe gravity induced instabilities in the matter Fock vacuum. The method also offers another proof for the formula relating the Pancharatnam phase and the vacuum decay width.

2. VACUUM INSTABILITY IN QUANTUM MECHANICS

We consider the quantum mechanical decay of the ground state in the hump potential

$$V = x^2 - \lambda x^4, \lambda > 0 \quad (1)$$

This potential has a 'bounce' solution with a single negative mode in the Euclidean time $t_E = -it$. The standard instanton calculation⁹ yields the vacuum - vacuum amplitude.

$$\langle f|i \rangle \equiv \int \mathcal{D}x \exp \left(-i \int_0^T \left(\frac{1}{2} \dot{x}^2 - V \right) dx \right) \simeq e^{-\Gamma T} \quad (2)$$

where

$$\Gamma = |K|\sqrt{S_0} e^{-S_0} \quad (3)$$

is the decay width of the state, S_0 the Euclidean bounce action and K is a constant determinant factor. (By a suitable redefinition we absorb the harmonic oscillator ground state energy in the potential). The essential feature of the expression (2) is that the ground state energy of the corresponding hamiltonian which is defined via a suitable analytic continuation $-\lambda \rightarrow \lambda \bar{e}^{i\pi}$ picks up a small imaginary part Γ signalling the instability. In the instanton calculation this is taken care of by the negative mode in the bounce solution. Moreover, the basic object being the transition probability amplitude, the inquiry into the existence of an extra phase was not necessary in the standard discussion of the problem. However, we are here primarily interested in calculating the non-trivial phase of $\langle f|i \rangle$, if any.

For this purpose we use an adiabatic perturbation method to analyse the issue. Let us denote the relevant hamiltonian by $H(\lambda)$ and the corresponding ground state $\psi(\lambda)$. Introduce a Euclidean parameter τ periodic in $0 \leq \tau \leq 2$ and denote by $\lambda_\tau = \lambda(\tau)$ a slowly varying periodic function so that $\lambda(0) = 0$, $\lambda(1) = \lambda$. By slowly varying we mean $\lambda(\tau) \simeq \lambda$ almost everywhere in $[0, 1]$.

We now write

$$\psi = e^{-\Gamma t} \phi \quad (4)$$

in the real time Schrödinger equation

$$i \frac{\partial}{\partial t} \psi = H \psi \quad (5)$$

so that

$$H(\lambda)\phi(\lambda) = i\Gamma\phi(\lambda) \quad (6)$$

As stated already the energy is pure imaginary due to the analytic continuation $-\lambda \rightarrow \lambda \bar{e}^{i\pi}$. The important fact to note is that equation(6) can as well be obtained by treating λ_τ perturbatively via the Euclidean equation

$$H(\lambda_\tau)\chi_\tau = \frac{\partial}{\partial \tau} \chi_\tau \quad (7)$$

The ansatz

$$\phi_\tau = \bar{e}^{i \int_0^\tau \Gamma_\tau d\tau} \chi_\tau \quad (8)$$

then yields in the limit $\tau \rightarrow 1$, eqn.(6). The mechanism however defines a parallel transport which generates a phase Γ (for almost constant τ -dependence in Γ_τ for the state $\phi (\equiv \phi_1)$). Eqn.(4) then gives the intended phase relation[†]

$$\psi = e^{-\Gamma t} e^{-i\Gamma} \chi \quad (9)$$

Note that the states ψ and χ belong to different rays. The perturbatively generated phase Γ between them is by definition the Pancharatnam phase³ which signals an induced twist in the line bundle of states due to the perturbing potential λx^4 .

Further insights into the phase relation (9) can be obtained by letting τ to make a complete circuit in $0 \leq \tau \leq 2$. After a complete cycle through the classically forbidden region the final oscillator ground state $\psi(2)$ returns to the initial oscillator ground state $\psi(0)$ with however, an irreducible phase (-2Γ) ,

$$\psi(2) = e^{-i2\Gamma} \psi(0) \quad (10)$$

Although both the states are stable, the phase 2Γ carries an imprint of the twists in the perturbed line bundle. (The amplitude of ψ in eqn. (7) drops out since $\Gamma_\tau \rightarrow 0$ as $\tau \rightarrow 2$). Stretching the analogy too far, in the field theory language, the vacuum decays with associated particle creation in the first half of the circuit $0 \leq \tau \leq 1$. However, in the other half $1 \leq \tau \leq 2$ the annihilation of particles occurs restoring the initial vacuum. The whole process however leaves an imprint in the form of a non-trivial phase shift indicating particle creation (annihilation) in the intermediate stages. In the most of the physical situations though the two way processes cannot be realized leading to genuine particle production. In the case of gauge theories with chiral fermions the above cyclic process appears to occur; the final irreducible phase indicates the absence of a global symmetry/gauge invariance.

We also note that the introduction of the Euclidean parameter τ alongwith the analytic continuation in the Schrödinger eqn.(5) provides a complex structure⁵ in the quantum system. No such natural complex structure is available in the case of tunnelling between degenerate vacua. So one does not expect a geometric phase in this case.

It is comforting to see that the non-trivial phase in eqns. (9) and (10) can also be obtained from a more abstract formalism⁵. The present quantum system actually corresponds to a hermitian holomorphic bundle over the punctured complex plane $C - \{0\} : z = \pi(t + i\tau)$. The hermitian metric on this bundle is defined by the norm of the state ψ

$$\gamma \equiv \langle \psi | \psi \rangle = \exp \left(-\frac{1}{\pi} \int \Gamma (dz + d\bar{z}) \right) \quad (11)$$

The unique holomorphic connection corresponding to the metric (11) is given by

$$A = \gamma^{-1} \partial \gamma = -\frac{1}{\pi} \Gamma - \frac{1}{\pi} \int \frac{\partial \Gamma}{\partial z} d\bar{z} \quad (12)$$

The corresponding curvature $F = \bar{\partial}\gamma^{-1}\partial\gamma$ vanishes identically because of the reality of Γ . However the connection has a non-vanishing holonomy

$$\oint Adz = -\oint \Gamma \frac{dz}{\pi} - \frac{1}{\pi} \oint \int \frac{d\Gamma}{dz} dz d\bar{z} \quad (13)$$

The second integral vanishes for a suitable choice of $\Gamma(\tau)$ (\Rightarrow vanishing residue of $\partial\Gamma/\partial z$ at $z = 0$). Again for almost constant Γ along the cycle we have the desired phase (-2Γ) .

We close this section with the following remark. The general method of holomorphic line bundle can be applied to the QFT vacuum instabilities. As in the quantum mechanical model, the probability of pair creations in a finite volume is equal to the Pancharatnam phase between the out and in vacuum states. An application of the phase in gravity is discussed in the next section.

3. BACKREACTION AND PARTICLE PRODUCTION IN GRAVITY

We consider a minisuperspace gravity-matter system described by the hamiltonian^{7,8,10}

$$\begin{aligned} H &= H_g + H_m \\ H_g &= -\frac{1}{2M} \nabla_g^2 + MV(g) \end{aligned} \quad (14)$$

where H_g stands for the gravitational and H_m for the matter hamiltonian. We represent the matter fields by the symbol φ . The WD equation assumes the form

$$\left(-\frac{1}{2M} \nabla_g^2 + MV(g) + H_m \right) \Psi(g, \varphi) = 0 \quad (15)$$

In Ref [7,8], it is shown that an improved Born-Oppenheimer approximation¹¹ with the inclusion of a non-trivial Berry phase yields the effective gravitational equation

$$\left[-\frac{1}{2M} Dg^2 + MV(g) + \langle \psi | H_m | \psi \rangle \right] \phi(g) = 0 \quad (16)$$

where

$$\Psi(g, \varphi) = \phi(g) \psi(g, \varphi) \quad (17)$$

$D_g = \nabla_g - iA$ denotes the covariant derivative due to the induced $U(1)$ adiabatic connection

$$A = i \langle \psi | \nabla_g \psi \rangle \quad (18)$$

Further, using standard semiclassical analysis¹⁰ around the expanding WKB state $\phi(g) \sim \exp(-iS(g))$ the curved space equation is obtained in the Schrödinger picture at the order $0(M^0)$:

$$i \frac{d}{dt} \bar{\psi} = -H_m \psi \quad (19)$$

where the 'WKB time' t is defined by

$$\frac{d}{dt} = \nabla_g S \cdot \nabla_g \quad (20)$$

The zeroth order Einstein equation is retrieved to order $0(M)$

$$\frac{1}{2M} P_{eff}^2 + V(g) + \langle \psi | H_m | \psi \rangle = 0 \quad (21)$$

where P_{eff} is the effective gravitational momentum. We also note the relation

$$\nabla_g S \cdot A = -\langle \psi | H_m | \psi \rangle \quad (22)$$

It thus follows that the backreaction in the form of an energy expectation value gets determined by the Berry connection A . However for a simply connected minisuperspace with flat geometry the connection A can be gauged away $A \equiv 0$, yielding instead of eqn.(21) the source-free equation⁸

$$\frac{1}{2M} P_{free}^2 + V(g) = 0. \quad (23)$$

Thus the zeroth order backreaction cannot be obtained from the above argument. For example, in a RW minisuperspace without a Λ -term, $V(g) = g^2$ [g = scalefactor] and a self consistent solution of eqns. (15) and (23) is a flat Minkowsky space obtained via a Euclidean-continuation⁸. This means that the semiclassical reduction of WD equation for a Friedmann-like model yields only a Minkowski space matter Schrödinger equation. However, for a non-zero Λ -term, the reduction yields a DS space as a solution of eqn (23). The matter eqn. (18) is therefore a DS space Schrödinger equation. The exponential expansion of the scale factor must therefore produce particles through DS vacuum instability which, it is expected, should backreact to gravity.

One must therefore look for this backreaction either at a higher order of the semiclassical approximation or by a modification of the Born-Oppenheimer scheme applied to eqn.(15). We prefer the later to get a zeroth order backreaction even for a simply connected flat minisuperspace.

We again start from eqn.(17)

$$\Psi(g, \phi) = \phi(g) \psi(g, \varphi) \quad (24)$$

Choose two different values of $g : g_i$ and g_f corresponding to two ‘instants’ of quasiclassical evolution. Define, for convenience, $|\psi_i\rangle = \psi(g_i, \varphi)$ the initial normalized Fock vacuum : $\langle \psi_i | \psi_i \rangle = 1$. We also assume that all excited states in the initial Fock column are empty: $|\psi\rangle \langle \psi_i| = \mathbf{1}$. Now instead of projecting the whole WD eqn(15) on $|\psi_i\rangle$ itself we choose to project on a final Fock state $|\psi_f\rangle = \psi(g_f, \varphi)$:

$$\left[-\frac{1}{2M} (\nabla_g - iA)^2 + MV + \frac{\langle \psi_f | H_m | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} \right] \phi = 0 \quad (25)$$

where A is given by

$$A = i \langle \psi_i | \nabla_g \psi_i \rangle \quad (26)$$

For an adiabatically evolving gravitational mode, A is the Berry connection(18). However, the derivation of eqn.(25) does not require the need of a cyclic evolution and may be applied even to a non-cyclic case. Using the definition of time eqn.(20) one gets

$$\nabla_g S.A = i \langle \psi_i | \frac{d}{dt} \psi_i \rangle \quad (27)$$

Further using the Schrödinger eqn(19) and the inverse of the time definition $\frac{dg}{dt} = \nabla_g S$, one obtains the Pancharatnam phase¹¹ for the transition $|\psi_i\rangle \rightarrow |\psi_f\rangle$ in the form

$$\int_{g_i}^{g_f} A.dg = - \int_{t_i}^{t_f} \frac{\langle f | H_m | i \rangle}{\langle f | i \rangle} dt \quad (28)$$

A comment is in order here.

- i) The unitarity condition $|\psi\rangle \langle \psi| = \mathbf{1}$ asserts that the parallel transport of states is to be done along the horizontal subspace^{2,3} only. The condition for such a horizontal transport is $\langle \psi | \frac{d}{dt} \psi \rangle = 0 \Rightarrow A = 0$ in the intermediate stage $t_i \leq t < t_f$. However, the condition of horizontal transport fails at $t = t_f$. Consequently a mixing of the initial Fock states is allowed in the final Fock vacuum yielding a non-zero value for the phase integral (28).

Continuing with the discussion of the phase integral (28) we note that the instantaneous vacuum energy of the Hamiltonian H_m in the presence of an induced instability assumes a small imaginary part $E_0 + i\Gamma$. Here E_0 is a possible non-zero energy due to vacuum polarization and Γ is related to the vacuum decay width. The integral in the r.h.s. of eqn.(28) has the formal expression $\int_i^f (E_0 + i\Gamma) dt$. To get a real value one must evaluate the integral for Γ along the Euclidean time $\tau = it$. Further, in the case of a simply-connected flat minisuperspace the integral $\int E_0 dt$ can be safely gauged away. Eqn.(28) together with the remark (i) therefore yield the Pancharatnam phase associated with an instability :

$$\int_i^f A.dg = -\Gamma \quad (29)$$

which agrees with the phase obtained in sect. 2. Infact. eqn(29) is another derivation of the fact that the Pancharatnam phase indicating an instability has to be pinned via a transport along a Euclidean time¹³. The result is exact (modulo adiabatic condition) and appears to have a general validity.

In the absence of an instability eqn(29) yields a vanishing Pancharatnam phase. This agrees with the result of Ref[8] that for a Lorentzian evolution emerging from a flat simply connected minisuperspace the corresponding Berry phase (infact, connection) is zero. (For a curved minisuperspace the real time energy integral (28) might yield a meaningful geometric phase. This issue will be taken up seperately).

The semiclassical backreaction equation corresponding to eqn.(25) thus assumes the form

$$\frac{P_g^2}{2M} + MV(g) + \frac{\langle f|H_m|i\rangle}{\langle f|i\rangle} = 0 \quad (30)$$

where P_g is the source-free gravitational momentum. Note, however, that the gravitational component $\frac{P_g^2}{2M} + MV(g)$ in eqn.(30) is of order $0(M)$ whereas that of the backreaction one order less $0(M^\circ)$. Thus a reasonable set of Einstein equations obtained via a semiclassical reduction must assume the following iterative form :

$$0(M) : \frac{P_{g_0}^2}{2M} + M V(g_0) = 0 \quad (31a)$$

$$0(M^\circ) : i \frac{d}{dt} \psi(g_0, \varphi) = H_m \psi(g_0, \varphi) \quad (31b)$$

and the backreaction eqn.(30) is obtained only as a 2nd order iterated equation:

$$\frac{P_g^2}{2M} + MV(g) = R_e \frac{\langle f|H_m|i\rangle_{g_0}}{\langle f|i\rangle_{g_0}}, \quad (31c)$$

the imaginary part in the r.h.s. being exponentially small is neglected in the adiabatic approximation.

In the case of a closed RW minisuperspace with a pure Einstein-matter action eqn.(31a) does not have a reasonable solution in the Lorentzian sector (because momentum P_{g_0} becomes imaginary). However, the equation yields a flat Euclidean solution which via an analytic continuation implies in turn that the evolution of matter is essentially described by a Minkowsky space Schrödinger equation. In this case no gravitationally induced instability is possible and hence eqn.(31c) reduces to eqn.(31a) [normal ordered vacuum energy expectation value vanishes in Minkowsky space].

However, for an action with a positive cosmological term, eqn. (31a) yields a DS space as a solution. The matter eqn. (31b) is thus a DS vacuum Schrödinger equation which is supposed to produce particles (modulo technicalities in defining appropriate Fock states) because of an instability¹⁴ induced by the exponential expansion. Eqn.(31c) then describes a possible modification in the DS metric by an appropriate backreaction.

Before closing the discussion we note that the relevant phase integral in the presence of a Euclidean wormhole structure assumes the form

$$\int_i^f A.dg = i \int_{\tau_i}^{\tau_f} \frac{\langle f | \tilde{H}_m | i \rangle}{\langle f | i \rangle} d\tau \quad (32)$$

Here τ is a Euclidean time parametrizing the wormhole handle and \tilde{H}_m denotes an appropriate matter Hamiltonian. It is wellknown¹⁵ that the occurrence of a wormhole needs a complex matter field. Existence of a non-trivial geometric phase along a non-contractible wormhole handle now suggests that \tilde{H}_m must be realizable as an anti-hermitian operator on a Euclidean Schrödinger energy eigenstate. (This particular point has not been explicitly stated in Ref. 7).

4. FINAL REMARKS

Three different methods are shown to yield the same phase relation between the in and out vacua in the presence of an instability. It is clear that an instability occurs whenever the Hilbert bundle associated to a given quantum system has a natural complex structure. In the examples discussed here the complex structure arises from the punctured complex plane of the analytically continued physical time. Although the moduli space of a punctured plane (which is topologically a torus) is non-trivial, the present discussion suggests that the Pancharatnam phase for an unstable vacuum is insensitive to the class of inequivalent complex structures. In any case, it is however of interest to study any possible relationship between the vacuum instability and the moduli space of a torus.

The present discussion also suggests a general unambiguous method of obtaining semiclassical Einstein equations from a fully quantized system: It is however unclear the precise sense how the energy expectation values capture the backreaction of the particles produced in the cosmological background. In any case, it is of much interest to see how this argument applies to a more general superspace. It is also of interest to substantiate the general results discussed here by explicit calculations.

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† The phase $-\Gamma$ is dimensionless. The period of the Euclidean parameter τ is determined by the intrinsic time scale of the problem fixed by the harmonic oscillator groundstate energy. We also set $\hbar = 1$.