

Noise storm continua: power estimates for electron acceleration

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Abstract. We use a generic stochastic acceleration formalism to examine the power L_{in} (erg s^{-1}) input to nonthermal electrons that cause noise storm continuum emission. The analytical approach includes the derivation of the Green's function for a general second-order Fermi process, and its application to obtain the particular solution for the nonthermal electron distribution resulting from the acceleration of a Maxwellian source in the corona. We compare L_{in} with the power L_{out} observed in noise storm radiation. Using typical values for the various parameters, we find that $L_{\text{in}} \sim 10^{23-26} \text{ erg s}^{-1}$, yielding an efficiency estimate $\eta \equiv L_{\text{out}}/L_{\text{in}}$ in the range $10^{-10} \lesssim \eta \lesssim 10^{-6}$ for this nonthermal acceleration/radiation process. These results reflect the efficiency of the overall process, starting from electron acceleration and culminating in the observed noise storm emission.

Keywords:

1. Introduction

Solar noise storms are a very well studied phenomenon. They occur mostly at meter wavelengths, and comprise a long-lasting (1 hr – several days) broadband ($\delta f/f \sim 100\%$) component together with narrowband ($\delta f/f \sim \text{few}\%$) spiky bursts that last from 0.1 – 1 s. Elgaroy (1997) gives a thorough observational review of noise storms. Malik & Mercier (1996) present a comprehensive study of noise storms observed with the Nancay Radioheliograph (NRH). Klein (1998) presents a recent review of the role of suprathermal electrons in the solar corona that includes a broad discussion of the noise storm phenomenon.

In this study, we confine our attention primarily to type I noise storm *continua*, rather than the sporadic type I bursts, because we are interested in examining the basic energetics of the electron accel-

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eration processes responsible for producing the quasi-continuous radio emission.

Most theories of type I phenomena invoke nonthermal electrons as a crucial ingredient in producing the observed radiation. It is recognized that the nonthermal electrons that are involved in generating the noise storm continua are probably accelerated in closed coronal loops above active regions. Noise storm continua are sometimes accompanied by coincident (thermal) soft X-ray brightenings (e.g., Raulin & Klein 1994; Krucker et al. 1995), which are also signatures of coronal magnetic field evolution. When this occurs, it is clear that the electrons in the tail of the thermal distribution that produce the accompanying soft X-radiation cannot also produce the observed radio noise storm emission for more than a few minutes. This is inconsistent with the fact that noise storms are observed to persist for several hours to days (e.g., Raulin & Klein 1994; Malik & Mercier 1996 also present similar arguments). Since the X-ray emitting regions are typically situated at least one scale height below the layer in the corona where the noise storms originate, the two associated electron distributions cannot be co-spatial. Crosby et al. (1996) find that deka-keV X-ray emission is often observed towards the beginning of noise storms. The electrons producing this X-ray emission are energetic enough to power the noise storm simultaneously, and they could be transported to the noise storm emitting region either by turbulent diffusion or by direct transport along connecting magnetic field lines. However, the duration of the noise storm is much longer than the X-ray emission, and consequently continual electron acceleration is required. The acceleration is probably triggered by the same processes that give rise to the X-ray brightenings accompanying the onset of the noise storms.

Clearly an underlying acceleration mechanism is required in order to produce the nonthermal electron distribution implied by the noise storm emission. However, very little attention has been focused on this problem in the previous literature. The majority of the current theories simply assume that nonthermal electrons are present, and focus most of their attention on examining the wave-wave interaction processes through which observable radio emission is ultimately produced. It is fairly well established that noise storms are intimately connected with the temporal variation of the magnetic fields via the process of coronal evolution (e.g., Brueckner 1983; Stewart et al. 1986; Raulin & Klein 1994; Willson et al. 1997; Bentley et al. 2000). However, the precise physical processes that link the magnetic field evolution to the presence of nonthermal electrons is unclear. Spicer et al. (1981) adopted a specific driver model in order to examine the basic energetics of the process leading to type I bursts. Benz & Wentzel (1981) developed a similar

treatment that is also valid for type I bursts. The basic driver in both of these pictures is the process of coronal evolution, which causes magnetic fields to emerge into the corona and drive microinstabilities that spawn low-frequency turbulence. This turbulence in turn resonates with the electrons and stochastically accelerates them to form a nonthermal tail.

Our goal here is to develop a model-independent approach to the estimation of the noise storm energetics that avoids focusing on a specific mechanism for accelerating the electrons. Instead, we prescribe a generic second-order (stochastic) Fermi mechanism. Electrons from the tail of the thermal distribution are injected into the acceleration process, forming a nonthermal distribution. Only electrons above a critical energy in the tail of the thermal distribution are subjected to net acceleration; the rest remain thermal due to collisions. The basic parameters of the acceleration mechanism are constrained as follows. First we borrow from the literature estimates of the nonthermal electron fraction needed to produce the observed noise storm continua. By combining this estimate with approximate expressions for the dominant loss timescales influencing the electrons, we derive an estimate for the power that drives the electron acceleration process. This yields an approximate determination of the efficiency of the process starting from nonthermal electron acceleration and culminating in the observed noise storm emission. The efficiency so obtained is a relatively well-defined quantity that provides a general, model-independent constraint on the acceleration/radiation mechanisms, which in turn serves as a useful guide in the subsequent development of more detailed models. Furthermore, we expect that a good understanding of the efficiency of electron acceleration in the context of the solar corona will also provide useful insights into similar phenomena occurring in other astrophysical environments.

2. Electron acceleration

2.1. WHY ARE NONTHERMAL ELECTRONS IMPORTANT?

There is considerable observational evidence for the presence of nonthermal electrons in nonflaring regions of the solar corona (e.g., Klein 1998). The high brightness temperatures and significant positive spectral slopes in multi-frequency observations of noise storms strongly suggest an underlying nonthermal electron population (e.g., Thejappa & Kundu 1991; Sundaram & Subramanian 2004). An anisotropy in velocity or physical space causes these nonthermal electrons to spontaneously emit Langmuir/upper hybrid waves (e.g., Robinson 1978;

Wentzel 1985), which can coalesce with a suitable low-frequency wave population to produce the observed electromagnetic emission.

Melrose (1980) argues that low-frequency turbulence (including ion-acoustic, lower-hybrid, or a variety of other waves) will be generated as a natural product of coronal heating. Thejappa (1991) considers lower-hybrid waves excited by protons (as in Wentzel 1986) or by a series of weak shocks (as in Spicer et al. 1981) as candidates for the low-frequency wave population. Benz & Wentzel (1981) investigate the possibility that the turbulence is composed of ion-acoustic waves excited by evolving magnetic fields in the corona.

While most of the theories for noise storm continua do not specify the source of the nonthermal electrons powering the observed emission, Benz & Wentzel (1981) propose that these electrons are leftover particles from the population that produced the previous type I bursts. However, Krucker et al. (1995) note that the type I continuum and bursts are spatially separated. Furthermore, Malik & Mercier (1996) pointed out that solar noise storms are often not bursty in the beginning, and the continuum exists alone. These observational results contradict Benz & Wentzel's (1981) picture for the production of the nonthermal electrons. On the other hand, the model proposed by Spicer et al. (1981) for type I bursts considers the electrons to be accelerated via the modified two-stream instability, which in turn is spawned by random weak shocks caused by the emerging magnetic flux. However, this suggestion is contradicted by the work of Krucker et al. (1995).

Although there is currently no theoretical consensus regarding the fundamental mechanism powering type I phenomena, there is no question that nonthermal electrons are responsible for the observed emission. This situation leads us to suggest that a careful examination of the energy budget and the associated constraints on the efficiency of the acceleration process may provide the best route towards enhanced physical understanding.

2.2. ENERGY LOSS MECHANISMS

It is expected that the electron momentum distribution, f , will have essentially a two-part structure, comprising a thermal component along with a nonthermal, high-energy (but still nonrelativistic) electron "tail" that is responsible for producing the noise storm emission. In view of the considerable uncertainty surrounding the precise physical mechanism that results in the formation of the nonthermal portion of the electron distribution, we shall work in terms of a generic, stochastic (second-order) Fermi process. The total electron number density, n_e , including both thermal and nonthermal particles, is related to the momentum

distribution f via

$$n_e (\text{cm}^{-3}) = \int_0^\infty p^2 f dp, \quad (1)$$

where p is the electron momentum. The associated total electron energy density is given by

$$U_e (\text{erg cm}^{-3}) = \int_0^\infty \epsilon p^2 f dp = \frac{1}{2m_e} \int_0^\infty p^4 f dp, \quad (2)$$

where m_e is the electron mass and $\epsilon = p^2/(2m_e)$ is the electron kinetic energy. In the present application, we are mainly interested in the nonthermal electrons, which are picked up from the thermal population and subsequently accelerated to high energies.

At marginal stability, the loss timescale due to the emission of Langmuir waves by electrons is similar to the Coulomb loss timescale (Melrose 1980). The mean Coulomb energy loss rate for nonthermal electrons with energy ϵ colliding with ambient electrons and protons in a fully-ionized hydrogen plasma is given in cgs units by (e.g., Brown 1972b)

$$\left\langle \frac{d\epsilon}{dt} \right\rangle \Big|_{\text{loss}} = \frac{1.57 \times 10^{-23} \Lambda n_e}{\epsilon^{1/2}} \left(1 + \frac{m_e}{m_p} \right), \quad (3)$$

where Λ is the Coulomb logarithm and m_p is the proton mass. The first and second terms in parentheses on the right-hand side of equation (3) refer to electron-electron and electron-proton collisions, respectively. The factor of m_e/m_p clearly indicates that the cooling rate due to electron-proton bremsstrahlung is negligible compared with the effect of electron-electron collisions. Ignoring the factor of m_e/m_p in equation (3), we find that the characteristic timescale for Coulomb losses is given by

$$t_{\text{loss}} \equiv \epsilon \left\langle \frac{d\epsilon}{dt} \right\rangle^{-1} \Big|_{\text{loss}} = \frac{6.37 \times 10^{22} \epsilon^{3/2}}{\Lambda n_e} \quad (4)$$

in cgs units.

2.3. STOCHASTIC ACCELERATION OF ELECTRONS

For low electron energies, $\epsilon \lesssim kT$, collisions between electrons are expected to efficiently maintain a Maxwellian distribution. However, we shall demonstrate below that for electron energies exceeding the critical energy ϵ_c , stochastic acceleration dominates over collisional losses on average (see eq. [10]). In this situation, the high-energy tail of the electron distribution function is governed by a rather simple transport equation that describes the diffusion of electrons in momentum space

due to collisions with magnetic scattering centers. The time evolution of the Green's function for this process, f_G , is described by (e.g., Becker 1992; Schlickeiser 2002)

$$\frac{\partial f_G}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 \mathcal{D} \frac{\partial f_G}{\partial p} \right) + \frac{\dot{N}_0 \delta(p - p_0)}{p_0^2} - \frac{f_G}{\tau}, \quad (5)$$

where \mathcal{D} is the (as yet unspecified) diffusion coefficient in momentum space and τ is the mean residence time for electrons in the acceleration region. The source term in equation (5) corresponds to the injection into the acceleration region of \dot{N}_0 particles per unit volume per unit time, each with momentum p_0 . Although we do not explicitly include losses due to the emission of Langmuir/upper hybrid waves by the accelerated electrons, it is expected that these waves will be generated as a natural consequence of the spatial anisotropy of the electron distribution (e.g., Thejappa 1991). We will demonstrate below that our neglect of energy losses due to Coulomb collisions and the emission of Langmuir/upper hybrid waves is reasonable for the nonthermal electrons treated by equation (5). Note that in writing equation (5), we have ignored spatial transport so as to avoid unnecessary mathematical complexity.

Although the specific form for \mathcal{D} as a function of p depends on the spectrum of the turbulent waves that accelerates the electrons (Smith 1977), it is possible to make some fairly broad generalizations that help to simplify the analysis. In particular, we point out that a number of authors have independently suggested that $\mathcal{D} \propto p^2$. Examples include the treatment of particle acceleration by large-scale compressible magnetohydrodynamical (MHD) turbulence (Ptuskin 1988; Chandran & Maron 2003); analysis of the acceleration of electrons by cascading fast-mode waves in flares (Miller, LaRosa & Moore 1996); and the energization of electrons due to lower hybrid turbulence (Luo et al. 2003). Hence we shall write

$$\mathcal{D} = D_0 p^2, \quad (6)$$

where D_0 is a constant with the units of inverse time.

We can obtain an expression for the mean rate of change of the electron momentum p due to stochastic acceleration by focusing on the instantaneous evolution of a localized δ -function distribution in momentum space. Following the procedure described by Subramanian, Becker, & Kazanas (1999), we find based on equation (5) that the mean rate of change of the particle momentum due to stochastic acceleration is given by

$$\left\langle \frac{dp}{dt} \right\rangle \Big|_{\text{accel}} = \frac{1}{p^2} \frac{d}{dp} \left(p^2 \mathcal{D} \right) = 4 D_0 p. \quad (7)$$

Since $\epsilon = p^2/(2m_e)$, we conclude that the corresponding mean rate of change of the electron energy is

$$\left\langle \frac{d\epsilon}{dt} \right\rangle \Big|_{\text{accel}} = \frac{p}{m_e} \left\langle \frac{dp}{dt} \right\rangle \Big|_{\text{accel}} = 8 D_0 \epsilon . \quad (8)$$

The associated timescale for stochastic acceleration is therefore

$$t_{\text{accel}} \equiv \epsilon \left\langle \frac{d\epsilon}{dt} \right\rangle^{-1} \Big|_{\text{accel}} = \frac{1}{8 D_0} . \quad (9)$$

On average, acceleration dominates over losses for a particle with energy ϵ if $t_{\text{accel}} < t_{\text{loss}}$. Comparison of equations (4) and (9) establishes that acceleration is dominant if $\epsilon > \epsilon_c$, where the critical energy ϵ_c is given in cgs units by

$$\epsilon_c \equiv 1.57 \times 10^{-16} \left(\frac{\Lambda n_e}{D_0} \right)^{2/3} . \quad (10)$$

It should be noted that due to the stochastic nature of the acceleration process, some particles with energy $\epsilon > \epsilon_c$ will lose energy, although on average such particles will gain energy. Our assumption that the mean acceleration rate dominates over losses for the nonthermal particles is self-consistent provided the nonthermal particles all have $\epsilon > \epsilon_c$. The corresponding result for the critical momentum is

$$p_c \equiv (2 m_e \epsilon_c)^{1/2} = 1.64 \times 10^{-21} \left(\frac{n_e}{D_0} \right)^{1/3} , \quad (11)$$

where we have set the Coulomb logarithm $\Lambda = 29.1$ (see Brown 1972a). Particles with $p > p_c$ experience net acceleration on average. Note that the electron source term appearing in equation (5) must have $p_0 > p_c$ in order to validate our neglect of collisional losses in that equation. When this condition is satisfied, the injected electrons are accelerated to form a nonthermal distribution. While we have considered only Coulomb losses in this calculation, we remind the reader that the loss timescale due to the emission of Langmuir waves at marginal stability is similar to the Coulomb loss timescale (see § 2.2).

2.4. SOLUTION FOR THE GREEN'S FUNCTION

In a steady state, it is straightforward to show based on equation (5) that when $\mathcal{D} = D_0 p^2$ as assumed here, the solution for the Green's function is given by (Subramanian, Becker, & Kazanas 1999)

$$f_G(p, p_0) = A_0 \begin{cases} (p/p_0)^{\alpha_1} , & p \leq p_0 \quad , \\ (p/p_0)^{\alpha_2} , & p \geq p_0 \quad , \end{cases} \quad (12)$$

where the exponents α_1 and α_2 are related to D_0 and τ via

$$\alpha_1 \equiv -\frac{3}{2} + \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{1/2}, \quad \alpha_2 \equiv -\frac{3}{2} - \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{1/2}, \quad (13)$$

and the normalization parameter A_0 has the value

$$A_0 = \frac{\dot{N}_0}{2 D_0 p_0^3} \left(\frac{9}{4} + \frac{1}{D_0 \tau}\right)^{-1/2}. \quad (14)$$

Since the second-order Fermi acceleration process is stochastic in nature, the particles diffuse away from the injection momentum p_0 , and the $p \geq p_0$ and $p \leq p_0$ branches of the Green's function appearing in equation (12) describe the acceleration and deceleration of the source particles, respectively. However, on average, acceleration wins out over deceleration, as indicated by equation (8) which demonstrates that the mean acceleration rate is positive. Substitution into the integral $\int_0^\infty p^2 f_G dp$ using equation (12) confirms that the number density of the Green's function is equal to $\dot{N}_0 \tau$, as expected in this steady-state situation. The values of D_0 , τ , and \dot{N}_0 will be constrained later using observational data.

The Green's function f_G describes the response to the injection of \dot{N}_0 electrons per unit volume per unit time with momentum p_0 , and therefore it is easy to show based on the linearity of the transport equation (5) that the particular solution for a general source term j is obtained via the convolution (e.g., Becker 2003)

$$f(p) = \int_0^\infty \frac{p_0^2 j(p_0)}{\dot{N}_0} f_G(p, p_0) dp_0, \quad (15)$$

where the quantity $p_0^2 j(p_0) dp_0$ represents the number of electrons injected per second per cm^3 with momenta between p_0 and $p_0 + dp_0$. In the physical application of interest here, the injected particles are supplied by the high-energy ($p_0 > p_c$) portion of the Maxwellian distribution in the corona. Assuming that the characteristic timescale for the thermal electrons to enter the acceleration region is equal to the mean residence time τ , it follows that the source term is given by

$$j(p_0) = \begin{cases} \frac{4\pi n_e \tau^{-1}}{(2\pi m_e kT)^{3/2}} e^{-p_0^2/2m_e kT}, & p_0 \geq p_c, \\ 0, & p_0 < p_c. \end{cases} \quad (16)$$

2.5. PARTICULAR SOLUTION FOR MAXWELLIAN SOURCE

In the case of a Maxwellian source, which is our focus here, we can combine equations (12) through (16) to compute the particular solution for the nonthermal electron distribution. The lower bound for the integration over p_0 in equation (15) is set equal to p_c since the source $j(p_0)$ vanishes for $p_0 < p_c$ according to equation (16). The result obtained for the nonthermal electron distribution is therefore given by

$$f(p) = \frac{n_e \left\{ \xi^{\alpha_1/2} \Gamma\left(-\frac{\alpha_1}{2}, \xi\right) - \xi^{\alpha_2/2} \left[\Gamma\left(-\frac{\alpha_2}{2}, \xi\right) - \Gamma\left(-\frac{\alpha_2}{2}, \xi_c\right) \right] \right\}}{\sqrt{2\pi} (m_e k T)^{3/2} (\alpha_1 - \alpha_2) D_0 \tau}, \quad (17)$$

where we have introduced the dimensionless electron energy ξ and the dimensionless critical energy ξ_c , defined by

$$\xi \equiv \frac{p^2}{2 m_e k T}, \quad \xi_c \equiv \frac{p_c^2}{2 m_e k T}. \quad (18)$$

The number and energy densities associated with the particular solution f above the critical momentum p_c can be computed using

$$n_* (\text{cm}^{-3}) \equiv \int_{p_c}^{\infty} p^2 f(p) dp, \quad (19)$$

$$U_* (\text{erg cm}^{-3}) \equiv \int_{p_c}^{\infty} \epsilon p^2 f(p) dp. \quad (20)$$

By combining equations (17), (19), and (20), we find that the exact solutions for n_* and U_* are given by

$$\begin{aligned} \frac{n_*}{n_e} &= \frac{2 \xi_c^{(3+\alpha_1)/2} \Gamma\left(-\frac{\alpha_1}{2}, \xi_c\right)}{\sqrt{\pi} (3+\alpha_1)(\alpha_2-\alpha_1) D_0 \tau} \\ &+ 2 e^{-\xi_c} \left(\frac{\xi_c}{\pi}\right)^{1/2} + \text{Erfc}\left(\xi_c^{1/2}\right), \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{U_*}{n_e k T} &= \frac{2 \xi_c^{(5+\alpha_1)/2} \Gamma\left(-\frac{\alpha_1}{2}, \xi_c\right)}{\sqrt{\pi} (5+\alpha_1)(\alpha_2-\alpha_1) D_0 \tau} \\ &+ \frac{2\sqrt{\pi} \xi_c (3+2\xi_c) e^{-\xi_c} + 3\pi \text{Erfc}\left(\xi_c^{1/2}\right)}{2\pi(1-10 D_0 \tau)}. \end{aligned} \quad (22)$$

Based on equations (8) and (20), we conclude that the rate of change of the energy density of the nonthermal electrons due to second-order Fermi acceleration is given by

$$\left. \frac{dU_*}{dt} \right|_{\text{accel}} = \int_{p_c}^{\infty} p^2 \left\langle \frac{d\epsilon}{dt} \right\rangle \Big|_{\text{accel}} f(p) dp = 8 D_0 U_*, \quad (23)$$

with U_* computed using equation (22).

3. Estimate of power input to electron acceleration process

Assuming that the nonthermal electrons spontaneously emit Langmuir waves, which then coalesce with lower hybrid waves to produce the observable electromagnetic radiation, Thejappa (1991) gives estimates of the fraction of nonthermal electrons, n_*/n_e , that are needed to produce a given noise storm continuum brightness temperature T_b at a particular observing frequency. In this model, noise storm radiation exhibits broadband continuum characteristics provided n_*/n_e is small enough so that collisional damping (due to ambient thermal electrons) dominates over negative damping due to nonthermal electrons. When n_*/n_e exceeds a certain threshold, the negative damping due to nonthermal electrons dominates, the brightness temperature increases steeply and type I bursts are produced. The threshold value of n_*/n_e at 169 MHz is $n_*/n_e = 2.2 \times 10^{-7}$, corresponding to $T_b \sim 10^{10}$ K. Statistics of noise storm continuum brightness temperatures are most abundant at 169 MHz, and Kerdraon & Mercier (1983) find that the brightest noise storm continua at that frequency have $T_b \sim 5 \times 10^9 - 10^{10}$ K. Accordingly, we consider the value $n_*/n_e = 2.2 \times 10^{-7}$ in our calculations. However, some authors (e.g., Klein 1995 and references therein) use a frequency-independent value of $n_*/n_e = 10^{-5}$, and consequently we also consider this value in our calculations (see Table 1).

It is convenient to treat the high-energy power law index, α_2 , as a free parameter in our model. According to equation (20), in order to obtain a finite value for the nonthermal electron energy density U_* , we must have $\alpha_2 < -5$. Combining this constraint with equation (13), we find that

$$D_0 \tau < \frac{1}{10}. \quad (24)$$

The same condition can also be derived by noting that the second term in equation (22) for U_* diverges in the limit $D_0 \tau \rightarrow 1/10$. Using equation (9) to substitute for D_0 in terms of the stochastic acceleration timescale t_{accel} yields the equivalent result

$$\frac{\tau}{t_{\text{accel}}} < \frac{4}{5}. \quad (25)$$

Any steady-state physical configuration governed by the transport equation (5) must satisfy this condition. Once the value of α_2 is selected, then α_1 and the product $D_0 \tau$ can be computed using equations (13). By combining this information with observational estimates for the

coronal temperature T and the ratio n_*/n_e , we can compute p_c using equation (21), which is an implicit equation for $\xi_c = p_c^2/(2m_e kT)$. We remind the reader that p_c signifies the critical momentum above which stochastic acceleration dominates over losses on average (see eq. [11]). In our example calculations we use $T = 10^6$ K for the coronal temperature and either $n_*/n_e = 2.2 \times 10^{-7}$ or $n_*/n_e = 10^{-5}$ for the fraction of nonthermal electrons.

Although noise storm continua are observed between $\sim 50 - 300$ MHz, observations are most abundant at 169 MHz (Kerdran & Mercier 1983). In what follows, we will be using observations at 169 MHz to define the typical source size of noise storm continua. We therefore utilize a value for the thermal electron density n_e that corresponds to a plasma frequency of 169 MHz (e.g., Krall & Trivelpiece 1986),

$$n_e = 3.54 \times 10^8 \text{ cm}^{-3} . \quad (26)$$

With p_c already determined using equation (21) along with selected values for n_*/n_e and T as explained above, we can now solve for the Fermi acceleration constant D_0 by using equation (11) to write

$$D_0 = 4.41 \times 10^{-63} n_e p_c^{-3} . \quad (27)$$

Finally, the stochastic energization rate per unit volume is obtained by combining equations (22) and (23).

The typical size of a noise storm continuum source at 169 MHz is $3' \sim 10^{10}$ cm (Kerdran & Mercier 1983). The vertical extent of a noise storm continuum source is not directly known, but the typical relative bandwidth of the observed emission is $\delta f/f \sim 100\%$. If $H = n_e (\nabla n_e)^{-1}$ is the scale height of density variation in the corona, the noise storm continuum emission must emanate from a range of heights $(\delta f/f) H/2$ (e.g., Melrose 1980). Using $H \sim 10^5$ km, this yields an estimate of

$$V \sim 10^{30} \text{ cm}^3 \quad (28)$$

for the volume of the acceleration region. This estimate assumes that the noise storm emitting region is a cylinder of height $H/2$ and cross section $3' \times 3'$, which is consistent with the data at 169 MHz. However, the cross section of the emitting region may vary with frequency, and therefore with height in the corona, in which case the emission volume would not be a simple cylinder. With this caveat in mind, we shall adopt the simple cylindrical picture here and use a value of n_e referenced to 169 MHz because noise storm continua statistics are most abundant at this frequency.

The power input to the nonthermal electron acceleration process is computed using

$$L_{\text{in}} = V dU_*/dt \text{ erg s}^{-1} , \quad (29)$$

Table I. Representative results

n_*/n_e	α_2	ϵ_c	D_0	L_{in}	L_{out}	η
2.2×10^{-7}	-5.1	1.43	0.185	5.62×10^{24}	$10^{17}-10^{18}$	$\sim 10^{-8}-10^{-7}$
2.2×10^{-7}	-6.0	1.42	0.186	8.00×10^{23}	$10^{17}-10^{18}$	$\sim 10^{-7}-10^{-6}$
1.0×10^{-5}	-5.1	1.09	0.270	3.00×10^{26}	$10^{17}-10^{18}$	$\sim 10^{-10}-10^{-9}$
1.0×10^{-5}	-6.0	1.08	0.280	4.2×10^{25}	$10^{17}-10^{18}$	$\sim 10^{-9}-10^{-8}$

The critical energy ϵ_c (eq. [10]) is expressed here in keV. The quantity D_0 (eq. [27]) is in units of s^{-1} . The input power to the electrons L_{in} (eq. [29]) and the power observed in electromagnetic radiation L_{out} are expressed in units of erg s^{-1} .

where dU_*/dt is given by equations (22) and (23). Representative results are presented in Table 1. We find that $L_{\text{in}} \sim 10^{23-26} \text{ erg s}^{-1}$. Expectedly, L_{in} is larger for the calculations where we have used the relatively larger value of n_*/n_e . From independent observational considerations, the power in noise storm continua is estimated to be $L_{\text{out}} \sim 10^{17-18} \text{ erg s}^{-1}$ (e.g., Elgaroy 1977; Raulin & Klein 1994). The associated efficiency,

$$\eta \equiv \frac{L_{\text{out}}}{L_{\text{in}}}, \quad (30)$$

is therefore estimated to be in the range $10^{-10} \lesssim \eta \lesssim 10^{-6}$.

4. Summary and Discussion

We have stipulated a generic stochastic Fermi acceleration mechanism for generating the nonthermal electrons responsible for noise storm continua. The mathematical approach is based on a rigorous derivation of the Green's function describing the acceleration of (initially) monoenergetic electrons, which achieve a power-law distribution at high energies (see eq. [12]). The Green's function is convolved with the high-energy portion of the electron Maxwellian in the corona to obtain the particular solution for the momentum distribution of the nonthermal electrons responsible for producing the observed noise storm emission (eq. [17]). Integration of the particular solution in turn yields exact solutions for the number and energy densities of the nonthermal electrons, as well as an expression for the rate of change of their energy density (eqs. [21] – [23]).

Our work utilizes estimates for the ratio of the nonthermal to thermal electron densities n_*/n_e from Thejappa (1991) (implied by the typically observed values for the noise storm continuum brightness temperature T_b at $\sim 169 \text{ MHz}$) and Klein (1995). Since we do not have

reliable observational estimates for the residence time τ of the electrons in the acceleration region, we instead parametrize the model using the high-energy power-law index α_2 of the electron distribution function. We must restrict ourselves to values of $\alpha_2 < -5$ in order to obtain a finite value for the nonthermal electron energy density. By combining an observational value for n_*/n_e with a selected value for α_2 , we can use equation (21) to determine the critical momentum p_c , which separates the thermal and nonthermal portions of the electron distribution. The analytical solution for the energy density of the nonthermal electrons is then used to compute the energization rate due to second-order Fermi acceleration (see eqs. [22] and [23]).

Typical observational values for the total electron number density n_e and the volume of the noise storm emitting region V were used to arrive at the results presented in Table 1. These results demonstrate that the power input to the electron acceleration process is around 10^{23-26} erg s⁻¹, which is 6–10 orders of magnitude larger than the power that is ultimately observed in the noise storm continuum radiation. The efficiency of the process, starting from the acceleration of the nonthermal electrons and culminating in the observable noise storm continuum radiation, is therefore in the range $10^{-10} \lesssim \eta \lesssim 10^{-6}$. Using data from type III decametric storms and impulsive 2–10 keV electron events at 1 AU associated with type I noise storms, Lin (1985) estimates that the energy release rate for these electrons is around 10^{23} erg s⁻¹ (cf. also Jackson & Leblanc 1991). Klein (1995) estimates that the energy supply to the energetic electrons is of the order of 10^{23-24} erg s⁻¹. Prior to the calculations presented here, these were the only (indirect, and rather approximate) estimates of the power input to nonthermal electrons required to drive noise storm radiation.

We believe that the general analysis of the energy budget presented in this paper, based on a generic second-order Fermi energization mechanism, will help to guide subsequent work on the detailed acceleration processes responsible for powering the noise storm continua. Our results also provide an important quantitative data point for discussions of electron acceleration in other high-temperature astrophysical environments.

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