GENERAL COVARIANCE, ACCELARATED FRAMES AND THE PARTICLE CONCEPT

T. PADMANABHAN
Tata Institute of Fundamental Research, Bombay, India

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Abstract. The definition of particle states in various accelerated frames is considered. It is shown that in any realistically accelerated system, quantum field theory can be formulated without any ambiguity. We further show that the definition of a particle based on Green's function techniques does not always agree with the definition based on explicit quantization. We analyse the standard accelerated detector results from this point of view and show that the uncertainty principle imposes a rigorous bound on these detection processes.

1. Introduction

Quantum fields in curved space-time is one of the areas well investigated in the previous decade (see DeWitt, 1979, for a review). This has led to the, by now famous, result that a black hole radiates as a black body (Hawking, 1975).

Thus, if one keeps a set of particle detectors at a large distance from the black hole, they will detect a Planck spectrum of particles emitted by the black hole. The question arises as to which source supplies the energy necessary for this emission. It is normally assumed that this process converts the mass of the black hole into energy, thereby decreasing the mass. This decrease in the mass of the black hole is a physical effect and we expect all the observers to agree on their predictions regarding this.

However, it is not clear how other observers would view the situation. For example, a freely-falling detector may see the particle spectrum quite differently (Davies, 1976). It is known that particle detectors in various states of geodesic motion will see the situation differently (Unruh, 1976). This is very surprising since we expect physical effects to be generally covariant. (One certainly expects an orbiting observer to notice any decrease in the mass of the black hole!).

These effects *per se* have nothing to do with gravity, and arise solely because of the ambiguity in the definition of positive and negative frequency components. A vast literature exists on the problem of a uniformly accelerated detector, which is also expected to see a Planck spectrum at a temperature proportional to the acceleration (see Gibbons, 1979, for a review). The uniformly accelerated frame (UAF) provides one with an extra time coordinate with reference to which positive and negative frequency modes can be defined. These modes involve a linear superposition of the standard Minkowski modes. This leads to the result that a uniformly accelerated detector will see a thermal spectrum of particles in the Minkowski vacuum.

Thus, the standard formalism of field theory leads to an observer-dependent particle concept even in flat space. (The energetics of the problem is not at all clearly understood.) It is only natural that these problems reappear in curved space-time cal-

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culations as well. By the same token, one may hope to cure this evil by understanding the flat space-time situation better. We shall attempt to do that in this paper.

More specifically, we shall approach the problem as follows:

In Part I of the paper we analyse the standard results for the uniformly accelerated frame. However, this UAF is not physically realizable; hence we extend our analysis to nonuniformly accelerated frames (NUAF) as well. Here we reach the surprising conclusion that no ambiguity arises in the particle definition. Thus, physically realizable detectors do respond properly to the Minkowski definition of particles.

In Part II we consider the problem through a discussion of Green's function. We show, by explicit construction, that the results obtained using Green's functions need not agree with the results obtained through direct quantisation of the field. In this respect, Green's function method is not a reliable indicator of the particle content of the field.

In Part III, we analyse the detection process in detail and show that the uncertainty principle rules out the possibility of detecting such a radiation. We conclude from all these that the UAF result is not physically relevant and represents an unobservable quantum fluctuation pattern.

PART I

2. Review of Standard Results

Consider a flat space-time described by the metric

$$ds^2 = -(d\bar{t}^2) + d\bar{x}^2 + d\bar{y}^2 + d\bar{z}^2. \tag{2.1}$$

Make a transformation to the new coordinates (t, x, y, z) by the equations

$$\begin{aligned}(1 + g\bar{x}) &= (1 + gx)\cosh gt, \\ g\bar{t} &= (1 + gx)\sinh gt, \quad \bar{y} = y, \bar{z} = z.\end{aligned} \tag{2.2}$$

This leads to the metric

$$ds^2 = -(1 + gx)^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{2.3}$$

As is well known, transformations (2.2) exist only in the wedge $|x| > |t|$, and represent the proper reference frame for the accelerated observer, moving with a uniform acceleration g along the x axis.

Within the framework of general covariance, arbitrary curvilinear coordinates can be chosen in flat space. But what is remarkable (and in some sense, causes the headache!) about (2.2) is that the metric is static in the new (t, x, y, z) coordinates. It can be shown that for any other type of coordinate transformation involving only rectilinear motion (especially any non-uniform acceleration), the metric will not be static in the new coordinates. We will comment more about this later.

Thus we have the freedom in flat space to choose either $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ or (t, x, y, z) in the domain of interest. As a mathematical step this is quite valid. However, physically the coordinates (t, x, y, z) can never be realised. No observer or detector can eternally be in the state of uniform acceleration (among other things this demands infinite energy on the part of the accelerating system). Hence, at best, Equations (2.2) and (2.3) can be physically realised for a finite interval of time. Thus, for a physically meaningful observer (or detector), the acceleration must be time-dependent and must go to zero at (large) early and late times ($|t| \to \infty$).

The proper reference frame of the non-uniformly accelerated observer can be derived by standard methods (see, e.g., Moller, 1969). The transformation equations (confining ourselves to motion along the x-axis) become

$$\bar{x} = \int^t \sinh \theta(t)\, \mathrm{d}t + x \cosh \theta(t), \tag{2.4}$$

$$\bar{t} = \int^t \cosh \theta(t)\, \mathrm{d}t + x \sinh \theta(t), \tag{2.5}$$

where the acceleration is given by

$$g(t) = \frac{\mathrm{d}\theta}{\mathrm{d}t}. \tag{2.6}$$

This leads to the metric

$$\mathrm{d}s^2 = -(1 + g(t)x)^2\, \mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2. \tag{2.7}$$

We keep $g(t)$ arbitrary except for the condition that $Lt_{|t| \to \infty} g(t) = 0$. It is quite possible for $g(t)$ to be constant in some finite interval (as long as it is sufficiently smooth). But note that (from Equation (2.7)) the metric cannot be static throughout.

The problems with the transformation (2.2) were first pointed out by Fulling (1973). The conventional formalism of quantum field theory requires the solution of a field equation to be separated out into positive and negative frequency solutions. The question arises whether to choose functions of the type (positive frequency with respect to $\bar{t}$.),

$$f_\omega = (\exp - i\omega\bar{t}) h_\omega(\bar{x}, \bar{y}, \bar{z}); \tag{2.8}$$

or those which are positive frequency with respect to t,

$$F_\omega = (\exp - i\omega t) g_\omega(x, y, z). \tag{2.9}$$

Since the transformations [Equation (2.2)] are quite non-trivial, either one of Equation (2.8) or (2.9), when expanded in terms of the other, involves both positive and negative frequency mixtures. This leads to nonequivalent quantisation schemes in flat space.*

* When one confines one's attention to rectilinear motion, uniformly accelerated motion alone is capable of leading to another static metric. But actually, this is only a special case. The most general motion satisfying this criterion involves a combination of 'rotation' and uniform acceleration and is specified by three arbitrary constants. In all those frames, there exists a time coordinate with respect to which the metric is stationary. In all these cases the ambiguity of quantisation will arise. In Part I, however, we will confine ourselves to rectilinear motion. Other cases of interest will be discussed in a later section.

It was soon suggested that a particle detector (originally tuned to detect particles defined wrt $\bar{t}$) when accelerated, will follow the evolution in the proper time t, and hence will detect particles in the Minkowski vacuum (Davies, 1975). All these works used only uniformly accelerated frames (hereafter called the UAF). The results were substantiated by various authors (DeWitt, 1975) especially after the works of Hawking (1975). It seems to be also generally believed that the result is due to the existence of a surface similar to the event horizon in the case of black holes.

However, for NUAF (Non-Uniformly Accelerated Frame), a second time coordinate with respect to which the metric is static is absent. This at once rules out a second expansion of the type given in Equation (2.9), and the conventional derivations of the particle detection in UAF breaks down. Also, for realistic observers (since $g(t) \to 0$ for large $|t|$), there does not exist any surface similar to the event horizon. Even derivations based on general properties, like periodicity of Green's function on imaginary time coordinate, are not applicable. Hence, it is not *a priori* evident what the situation would be in a NUAF. This is the problem we take up in the next section.

We would like to stress one more point before concluding ths section. Since the quantisation scheme for a field is non-local in character, one cannot use the results of UAF to deduce those for NUAF. After all, if the detection process depends on the acceleration, it may also depend on the derivatives of the acceleration in a non-trivial way. This is even more so because the coordinate structure of the space-time with asymptotically vanishing $g(t)$ is entirely different from that of UAF (with constant g).

3. Quantum Field Theory in NUAF

Consider a Klein-Gordan scalar field described by the Lagrangian

$$S = \tfrac{1}{2}\int (\partial_i\phi\partial^i\phi - m^2\phi^2)\sqrt{-g}\,\mathrm{d}^4x. \tag{3.1}$$

In the NUAF, with the metric given by Equation (2.7), this satisfies the standard field equation

$$\phi_{;\alpha}^{;\alpha} - m^2\phi = 0 \tag{3.2}$$

i.e.,

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\varphi) - m^2\phi = 0. \tag{3.3}$$

Putting in the various components and separating out the y, z dependences by

$$\phi(t, x, y, z) = f(t, x)\,\mathrm{e}^{ik_y y}\,\mathrm{e}^{ik_z z} \tag{3.4}$$

we get

$$-\frac{1}{(1 + g(t)x)}\frac{\partial}{\partial t}\left(\frac{1}{(1 + g(t)x)}\frac{\partial f}{\partial t}\right) + \\ + \frac{1}{(1 + g(t)x)}\frac{\partial}{\partial x}\left((1 + g(t)x)\frac{\partial f}{\partial x}\right) = \chi^2 f, \tag{3.5}$$

where $\chi^2 = m^2 + k_y^2 + k_z^2$. Since a separation $e^{i\omega t}$ will not work, let us try an ansatz of the form

$$f(x, t) = \exp i\left(\int \alpha(t)\,dt + \beta(t)x\right), \tag{3.6}$$

where the functions $\alpha(t)$ and $\beta(t)$ are to be determined. Substituting Equation (3.6) into (3.5) and separating the space and time variables we obtain a set of equations connecting $\alpha(t)$, $\beta(t)$ and $g(t)$, which can be simplified to

$$\alpha^2(t) - \beta^2(t) = \chi^2, \tag{3.7}$$

$$\frac{d\beta}{dt} = g(t)\alpha; \qquad \frac{d\alpha}{dt} = g(t)\beta. \tag{3.8}$$

This set can be solved uniquely at once to give

$$\alpha(t) = \chi \cosh[\theta(t) - \eta], \tag{3.9}$$

$$\beta(t) = \chi \sinh[\theta(t) - \eta], \tag{3.10}$$

where η is another constant, and $\theta(t) = \int g(t)\,dt$ is as defined in Equation (2.4) and (2.5). Therefore,

$$f(x, t) = f_{k_y k_z \eta}(x, t) = \exp -i\chi\left[\int \cosh(\theta - \eta)\,dt + x\sinh(\theta - \eta)\right]. \tag{3.11}$$

Though this solution looks strange, it has a trivial interpretation. This is simply the standard positive frequency solution, expressed in terms of $\bar{t}, \bar{x}$ as

$$f(\bar{t}, \bar{x}) = \exp i(k_x\bar{x} - \omega_k\bar{t}), \tag{3.12}$$

and transformed into the new coordinates (t, x) as a scalar, using Equations (2.4) and (2.5). We have made the substitutions

$$\omega_k = \chi\cosh\eta \quad \text{and} \quad k_x = \chi\sinh\eta, \tag{3.13}$$

and chosen the signs in Equations (3.9) and (3.10) to facilitate the identification. The complete set of basis solutions are parametrised by k_x, k_y, k_z or by η, k_y, k_z.

We have extended the normal time dependence of the form ($\exp i\omega t$) to ($\exp i\int\alpha(t)\,dt$) in Equation (3.6) to accommodate a variable $g(t)$. It is shown by the above analysis that this leads to a set of basic functions which are of positive frequency with respect to time $\bar{t}$, and not to any other complicated form, expressible only as linear superpositions of functions which are positive and negative frequency solutions wrt $\bar{t}$. In a way this has to be expected since the metric is not static in NUAF. The physical systems are, so to say, forced to recognise $\bar{t}$.

Now we shall show that the standard field quantisation in terms of 'in' and 'out' states can be carried through in this frame (almost trivially) and no spurious particle creation will occur. To do that, start with the general solution, now expressed as

$$\phi(t, x) = \sum_k (a_k f_k(x, t) \exp i(k_y y + k_z z) + \text{h.c.}) \tag{3.14}$$

(η is determined by Equation (3.13) in terms of k_x). We assume that $g(t)$ vanishes sufficiently fast, so that $\theta(t)$ is constant in the asymptotic region (since $g(t) = \mathrm{d}\theta/\mathrm{d}t$). Let

$$\underset{t \to -\infty}{Lt} \theta(t) = 0 \tag{3.15}$$

(this can be assumed without any loss of generality) and

$$\underset{t \to +\infty}{Lt} \theta(t) = \theta_0. \tag{3.16}$$

We define the 'in' and 'out' fields in the usual way – i.e., by

$$\phi_{\text{in}} = \underset{t \to -\infty}{Lt} \phi(x, t) \tag{3.17}$$

and

$$\phi_{\text{out}} = \underset{t \to +\infty}{Lt} \phi(x, t), \tag{3.18}$$

which in turn define the 'in' and 'out' creation, annihilation operators. Accordingly, the 'in' and 'out' vacua are defined as

$$a_k^{\text{in}} |0\rangle_{\text{in}} = 0 \quad \text{and} \quad a_k^{\text{out}} |0\rangle_{\text{out}} = 0. \tag{3.19}$$

Now we only have to relate the 'in' and 'out' state operators. This we can do easily from Equations (3.11), (3.15) and (3.16). The use of Equation (3.11) leads to the result disclosing that

$$\underset{t \to -\infty}{Lt} f(x, t) = \exp -i(\omega_k t - k_x x) \tag{3.20}$$

$$\begin{aligned} \underset{t \to +\infty}{Lt} f(x, t) &= \exp(-i\alpha) \exp i\chi(t \cosh(\theta_0 - \eta) + x \sinh(\theta_0 - \eta)) \\ &= \exp(-i\alpha) \exp -i(\omega_k' t - k_x' x), \end{aligned} \tag{3.21}$$

where

$$\alpha = \chi \int_{-\infty}^{+\infty} \cosh(\theta - \eta)\, \mathrm{d}t \tag{3.22}$$

and

$$\omega_k' = \frac{\omega_k - v k_x}{\sqrt{1 - v^2}}, \qquad k_x' = \frac{k_x - v\omega_k}{\sqrt{1 - v^2}}, \quad v = \tanh \theta_0. \tag{3.23}$$

The physics of Equations (3.20)–(3.23) is simple. From the transformation Equations (2.4) and (2.5) it is clear that $\mathrm{d}\bar{x}/\mathrm{d}\bar{t} = v(t) = \tanh \theta(t)$ is the instantaneous comoving velocity. After the acceleration has ended, the observer's (or detector's) frame will be moving with a uniform velocity $v = \tanh \theta_0$ with respect to the original Minkowski frame. We take this into account by a usual Lorentz transformation of ω and k. All the effect of acceleration is only to produce an extra phase, $e^{-i\alpha}$. Thus, essentially,

$$a_{\text{out}} = a_{\text{in}}\, e^{-i\alpha}, \qquad a_{\text{out}}^{+} = a_{\text{in}}^{+}\, e^{i\alpha}. \tag{3.24}$$

This means that the s is the unit matrix. The number operator is $N_k = a_k^{+} a_k$ and that remains invariant (except, of course that n particles which were originally seen with frequency ω will now have frequency ω'. This is just the effect of Lorentz transformation in which we are not interested.) Hence, in particular,

$${}_{\text{in}}\langle 0 | N_k^{\text{out}} | 0 \rangle_{\text{in}} = {}_{\text{in}}\langle 0 | N_{k'}^{\text{in}} | 0 \rangle_{\text{in}} = 0. \tag{3.25}$$

Thus, there is no spurious particle creation which arises out of acceleration. The usual formalism goes through. In fact, the theory can be extended to arbitrary interacting fields (for which 'in', 'out' states exist) in a straightforward manner and no new problems will arise. Thus, the formalism of quantum field theory is well defined and leads to expected results whenever the coordinate transformation is physically realisable.

Severall remarks are in order. First of all, note that we have treated the problem just as we will treat any other quantisation in a non-Lorentzian metric. We were given a metric $g_{\mu\nu} = g_{\mu\nu}(t, x)$ with the condition $Lt_{|t| \to \infty}\, g_{\mu\nu} = \eta_{\mu\nu}$. We define the 'in' and 'out' states and solve the field equation to find the relation between 'in' and 'out' operators. For an arbitrary $g_{\mu\nu}(t)$ (for example, adiabatically expanding RW universe) a_{out} will be a superposition of a_{in} and a_{in}^{+} and will not lead to Equation (3.25). This is a standard formalism to use when $g_{\mu\nu} \neq n_{\mu\nu}$. All references to the background $\bar{t}, \bar{x}$ coordinates are made only to facilitate physical interpretation.

Also notice that we are not saying anything about the response of detectors at this stage. All we are saying is this: If one uses the system of coordinates suited for a realistic accelerated observer, then one can formulate quantum field theory in such a way that no internal inconsistencies arise. In particular, there is no particle creation. This situation is quite different (as a mathematical problem) from the case of UAF, where there is an ambiguity, which arises because of non existence of asymptotic states.

Nothing was assumed about the detailed nature of the acceleration. In particular there may be an interval during which the acceleration was uniform. Our result shows that even in this case there is no *net* particle creation. We discuss this in more detail in the next section.

4. A Possibility of Uniform Acceleration

Our analysis given above crucially depends on the existence of 'in' and 'out' states and thus is not applicable directly to UAF. But some insights can be gained.

As we have stressed the whole problem arises in UAF because of the existence of an extra time coordinate in which the metric is static. This can be presented better through a discussion of the Hamiltonian.

In standard field theory, one proceeds to calculate the Hamiltonian as follows: the Lagrangian defines the energy-momentum tensor $T^{\mu\nu}$, from which one gets

$$P^{\mu} = \int T^{\mu\nu} \, d\sigma_{\mu} \tag{4.1}$$

and takes the zero component to define the Hamiltonian. When one generalises to arbitrary curvilinear coordinates there is no analogue for Equation (4.1) (which is not covariant since it adds vectors at different points). The Hamiltonian has to be defined as

$$H = \int T^{\mu\nu} \xi_{\mu} \, d\sigma_{\nu} \sqrt{-g}, \tag{4.2}$$

where ξ^{μ} is a timelike Killing vector field. (If no such Killing vectors are available, not even asymptotically, then it is a complicated question as to how to define H.)

In the UAF, there are two Killing vector fields: ξ_1^{μ} corresponding to translations in t coordinate, and ξ_2^{μ} corresponding to the original Minkowski time coordinate $\bar{t}$. Both of these are timelike in the region considered in UAF. (In contrast ξ_1^{μ} is *not* everywhere timelike in the Minkowski frame). These two lead to two nonequivalent Hamiltonians in the UAF. Explicitly, one will get, using Equation (4.2),

$$H_{\xi_1} = \tfrac{1}{2} \int dx \, dy \, dz [(1 + gx)] \left[\frac{1}{(1 + gx)^2} \left(\frac{\partial \phi}{\partial t} \right)^2 + (\nabla \phi)^2 \right] \tag{4.3}$$

$$H_{\xi_2} = \tfrac{1}{2} \int dx \, dy \, dz \left\{ (\cosh gt) \left[\frac{1}{(1 + gx)^2} \left(\frac{\partial \phi}{\partial t} \right)^2 + (\nabla \varphi)^2 \right] - \right.$$
$$\left. - 2 (\sinh gt) \left[\left(\frac{\partial \phi}{\partial x} \right) \left(\frac{\partial \phi}{\partial t} \right) \frac{1}{(1 + gx)} \right] \right\}. \tag{4.4}$$

In order to get the standard particle interpretation one has to choose the number operator which commutes with the Hamiltonian. Now if one chooses the basis functions to be of positive frequency with respect to t, then H_{ξ_1} is diagonal. If chosen with respect to $\bar{t}$, then H_{ξ_2} is diagonal.

But what is *the* Hamiltonian for UAF? Which Hamiltonian 'really' governs the physical process? One way of answering the question is to look at NUAF again. There, of course, ξ_1^{μ} is not a Killing vector. Only timelike Killing vector field is ξ_2^{μ} and the Hamiltonian is unique. (Our explicit analysis (in last section) also showed that it is this which is followed.) Even if the acceleration is uniform for a finite interval in the middle of the trajectory, nothing happens because one cannot switch from one Hamiltonian to another.

In this sense, our might say, that H_{ξ_2} is more physical than H_{ξ_1}. Certainly, physically realisable systems (which should be unaccelerated as $|t| \to \infty$) follow only H_{ξ_2}. Because of this, one might suggest that, even in the case of UAF one should take H_{ξ_2} as *the* Hamiltonian. This would correspond to using the Minkowski time coordinate for the definition of positive frequency, even in UAF. One might feel that this is not correct since t is the proper time and $\bar{t}$ has no *locus standi* in UAF. This is quite true within the framework of UAF. But our suggestion is based on the fact that this time coordinate is a mathematical construction and is not realisable by a physical system. Also the metric for NUAF will not be static in the coordinate system using proper time as 'the time'.

PART II: OTHER STATIONARY TRAJECTORIES

5. Introduction

We have concluded in the previous section, (based on a field theory formalism) that quantum field theory can be formulated in a meaningful way for any realistic detector. We have also tried to explain what happens in the UAF case. However, there is another way of approaching the problem, viz. through Green's functions. One can consider the observed spectra to be the Fourier transform of the autocorrelation function of the field. This has a well-defined meaning only for stationary space-times. However, in an approximate sense (this is something like adiabatic approximation), one can treat a slowly varying acceleration in exact analogy with UAF. This will lead to the result of a Planck spectrum with a slowly varying temperature.

Two questions arise immediately. (i) Will the standard method of quantisation, calculation of Bogoliubov coefficients etc. lead to a result consistent with that obtained by the 'detector'? In other words does the so called 'model detector' detect particles as defined by the standard canonical quantization? (ii) What is the nature of the spectra corresponding to the various trajectories? The first can be settled only by detailed study. This is important to decide, whether the occurrence of the Planck spectrum in the UAF is accidental or a result of some physical significance.

We shall investigate the above questions in this part. Our major results are as follows:

(i) In general, there is no correspondence between the particles detected by the 'model detector' and the particles as defined through canonical quantisation procedure. This we shall show by explicit construction.

(ii) The existence of Planck spectrum for the UAF is a mere accident. Various types of detector motions lead to very different kinds of spectra. In particular, no natural definition of temperature exists for these trajectories.

These results are discussed in detail in the later sections of the paper. We have also attempted to understand the detection process and seek the reason for the above results.

6. Detector Trajectories for Static Spectra

Consider a scalar field ($\varphi(x)$ interacting with a detector linearly, as described by DeWitt (1979; p. 693). It can be shown that (we will say more about this in a later section) the spectrum is given by

$$P(\omega) = 2\pi\rho(\omega)\int_0^\infty \mathrm{d}\tau\, \mathrm{e}^{-i\omega\tau}\langle 0|\,\phi(\chi^\mu(\tau+s))\phi(\chi^\mu(s))\,|0\rangle. \tag{6.1}$$

Now consider the right-hand side. It is the Fourier transform of the correlation function for the scalar field taken between two events at proper times $\tau + s$ and s. (Note that $x^\mu(\tau)$ is the trajectory of the detector with τ parametrised to be the proper time.) In general this will depend on τ as well as **s**, so that $P(\omega)$ will also depend on **s**. The spectra has a time independent, intrinsic meaning only when it is independent of **s**. This can happen only when the correlation function depends only on the differences in proper time.

The correlation function for the massless scalar field has a very simple structure, (it is essentially the reciprocal of geodesic interval) and is given by

$$\begin{aligned} G(\tau_1,\tau_2) &= \langle 0|\,\phi[x^\mu(\tau_1)]\phi[x^\mu(\tau_2)]\,|0\rangle \\ &= \frac{1}{2\pi^2}[(x_\mu(\tau_1) - x_\mu(\tau_2))(x^\mu(\tau_1) - x^\mu(\tau_2))]^{-1}. \end{aligned} \tag{6.2}$$

Our first problem is to find the set of trajectories $x^\mu(\tau)$, such that $G(\tau_1, \tau_2)$ depends only on $(\tau_1 - \tau_2)$. Uniform velocity motion and uniformly accelerated motion certainly belongs to this class. However, much more general trajectories are admissible. For each one of these trajectories, one gets, in principle, a spectrum $P(\omega)$ (see, for a similar analysis by Letaw, 1980).

It turns out, however, that the integral in Equation (6.1) cannot be evaluated in closed form for most of these trajectories. Also many of these trajectories are not physically very interesting. So we shall indicate below how the general trajectory can be constructed and will confine our attention to some special cases which are of importance.

That Green's function should not depend on the instants at which it is evaluated (and only on the difference) means that, in the local region, any event in the trajectory is indistinguishable from any other event. This can be achieved if the trajectories are chosen to be the integral curves of a timelike Killing vector field in the space-time. As is well known there are 10 independent Killing vector fields in the flat space-time. They are of three types – translation, boost and rotation. By choosing various linear combinations and constructing their integral curves, one can generate various trajectories. However, nothing much is gained by treating, say, rotations about the z axis and rotations about the x axis separately. So, one has to only consider a suitable linear combination of Killing vectors incorporating the effects of translation, boost and rotation and consider its trajectory. Such a sufficiently general Killing vector can be taken to be

$$\xi^\mu(x) = (1 + gx, gt - \alpha y, \alpha x - \beta z, \beta y), \tag{6.3}$$

where (g, α, β) are constants and (t, x, y, z) are the standard Minkowski coordinates. Consider now a transformation from (t, x, y, z) to another frame $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ such that the Killing vector has the new components

$$\bar{\xi}^{\mu} = N^{-1}(1, 0, 0, 0). \tag{6.4}$$

This set of transformation can be found by simply integrating the equations

$$\xi^{\mu}(x(\bar{x})) = \frac{\partial x^{\mu}(\bar{x})}{\partial \bar{x}^{\nu}} \xi^{\nu} \tag{6.5}$$

$$= N^{-1}\left(\frac{\partial t}{\partial \bar{x}^0}, \frac{\partial x}{\partial \bar{x}^0}, \frac{\partial y}{\partial \bar{x}^0}, \frac{\partial z}{\partial \bar{x}^0}\right). \tag{6.6}$$

Once this transformation to the coordinate system suited for the observer is known, we have solved the problem. The trajectory $x^{\mu}(\tau)$ can be expressed in terms of the new proper time by simple algebra.

We shall now present some special cases which arise from Equation (6.3).

Case 1.

$$g = \alpha = \beta = 0 \quad \therefore \xi^{\mu}(x) = (1, 0, 0, 0). \tag{6.7}$$

Nothing much need be said here. The coordinate transformation to $\bar{x}^{\mu}$ contains all the curvilinear coordinates.

Case 2.

$$\alpha = \beta = 0, \qquad \xi^{\mu}(x) = (1 + gx, gt, 0, 0). \tag{6.8}$$

This leads to trajectory of the form $x^{\mu}(\tau) = 1/g\,(\sinh g\tau, \cosh g\tau, 0, 0)$ which corresponds to a uniformly accelerated motion along the x axis. Here also curvilinear coordinate transformations are allowed among the space axes.

Case 3. $\beta = 0$. This case would lead to a Killing vector of the form, $\xi^{\mu}(x) = (1 + gx, gt - \alpha y, \alpha x, 0)$. However using case 2, and case 1, one can consider a simplified Killing vector of the form

$$\xi^{\mu}(x) = (c, -\alpha y, \alpha x, 0). \tag{6.9}$$

This corresponds to a rotational motion given by the trajectory

$$x^{\mu}(\tau) = (C\tau, R\cos\alpha\tau, R\sin\alpha\tau, 0); \quad R = \alpha\sqrt{c^2 - 1}. \tag{6.10}$$

Case 4.

$$|g| = |\alpha|, \qquad \beta = 0 \qquad \xi^{\mu}(x) = (1 + gx, gt - gy, gx, 0).$$

This gives rise to a peculiar type of motion with a trajectory

$$x^{\mu}(\tau) = (\tau + \tfrac{1}{6}g^2\tau^3, \tfrac{1}{2}g\tau^2, \tfrac{1}{6}g^2\tau^3, 0). \tag{6.11}$$

These give a sufficiently representative sample for our analysis. The coordinate transformation equations for cases 1, 2, 3 are well-known. For the case 4 it turns out to be, (Minkowski (T, X, Y, Z); New frame (t, x, y, z))

$$T = \tfrac{1}{6}g^2t^3 + t(gx + \tfrac{1}{2}) + y, \qquad X = \tfrac{1}{2}gt^2 + (x - \tfrac{1}{2}g),$$
$$Y = \tfrac{1}{6}g^2t^3 + t(gx - \tfrac{1}{2}) + y, \qquad Z = z; \tag{6.12}$$

leading to a metric

$$ds^2 = 2gx\,dt^2 + 2\,dy\,dt - dx^2 - dz^2. \tag{6.13}$$

Now we shall consider the behaviour of a massless scalar field as viewed by these observers.

7. Spectra Found by the Detector

To do this is now straightforward. One has to compute the proper geodesic interval between any two events in the trajectory for various cases. Case 1, of course, is trivial. For case 2, Green's function is of the form

$$G(\tau_1 - \tau_2) = \frac{g^2}{8\pi^2}\,\mathrm{cosech}^2\,\frac{g}{2}(\tau_1 - \tau_2). \tag{7.1}$$

The Fourier transform can be performed to give the energy spectrum

$$P(\omega) = \frac{\omega^3}{2\pi^2 g^3 (e^{2\pi\omega/g} - 1)}, \tag{7.2}$$

which is the (by now famous) accelerated-frame Planck spectrum.

For case 3, the trajectory is given by Equation (6.10). Green's function turns out to be

$$G(\tau_1 - \tau_2) = \frac{1}{2\pi^2}\,\frac{1}{c^2(\tau_1 - \tau_2)^2 - 4R^2\sin^2(\alpha/2)(\tau_1 - \tau_2)}. \tag{7.3}$$

Unfortunately, the integral cannot be evaluated in closed form (in terms of simple functions). However, two points are clear: (i) A detector following this trajectory *does* see a spectrum (ii) It is not a Planckian spectrum.

For the last one, case 4, we get (using the trajectory in Equation (6.11)) the Green's function

$$G(\tau_1 - \tau_2) = \frac{1}{2\pi^2}\,\frac{1}{(\tau_1 - \tau_2)^2 + (g^2/12)(\tau_1 - \tau_2)^4}. \tag{7.4}$$

This can be Fourier-transformed to give the spectrum defined by

$$P(\omega) = \frac{\omega^2}{8\pi^2\sqrt{3}\,g^2}\left[\exp\left(-3\sqrt{2}\left(\frac{\omega}{g}\right)\right)\right], \tag{7.5}$$

which obviously has nothing to do with the Planck spectrum. In this way, one can calculate the spectra for various such trajectories. These cases are enough for our discussion. Similar results were obtained in Letaw (1980) by a different method.

We shall now proceed to quantise the scalar field in the coordinate systems appropriate to these trajectories and will show that the results are *incompatible* with those obtained by the analysis of the detector response.

8. Quantization in the Non-inertial Frame

The standard procedure for quantisation in a curved space-time or in a non-inertial frame is discussed in detail in literature (Isham, 1977). One imposes a dot product

$$\langle f_1, f_2 \rangle_\Sigma = i \int_S f_1^* \sqrt{-g}\, g^{\mu\nu} \partial_\nu f_2 \, d\Sigma_\mu \tag{8.1}$$

in the space of solutions of the wave equation

$$\phi^{;\alpha}_{;\alpha} = 0. \tag{8.2}$$

Whenever the metric is static, the solution can be separated into positive and negative frequencies wrt the time coordinate as

$$\phi = \sum (a_i f_i + a_i^+ f_i^*), \tag{8.3}$$

where f_i is a set of complete, orthonormal, positive frequency solutions. The operator a_i is the annihilation operator and defines the vacuum to be

$$a_i |0\rangle = 0. \tag{8.4}$$

The problem of inequivalent quantisation arises when there exists another set of basis functions $\bar{f}_i$, which is positive frequency wrt some other time coordinate. It can be shown that, then (Isham, 1977) $\bar{f}_i$ and f_i are related by a Bogoliubov transformation

$$\bar{f}_i = \sum_j (\alpha_{ij} f_j + \beta_{ij} f_j^*), \tag{8.5}$$

which mixes up the positive and negative frequency solutions. The coefficients α, β satisfy the standard Bogoliubov relations

$$\sum_k \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}_{ik} \begin{pmatrix} \alpha^+ & -\tilde{\beta} \\ -\beta^+ & \tilde{\alpha} \end{pmatrix}_{kj} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \delta_{ij}. \tag{8.6}$$

One can expand the field in terms of the new basis as

$$\phi = \sum_i (\tilde{a}_i \bar{f}_i + \tilde{a}_i^+ \bar{f}_i^*), \tag{8.7}$$

where the new operator is related to the old by the relation

$$\tilde{a}_i = \sum_j (\alpha_{ij}^* a_j - \beta_{ij}^* a_j^+). \tag{8.8}$$

Because of the non-zero β_{ij} it is clear that the 'old' vacuum will contain 'new' particles whose number is given by

$$_{\text{old}}\langle 0| \sum_i \tilde{a}_i^+ a_i |0\rangle_{\text{old}} = \sum_{ij} (\beta_{ij}\beta_{ji}^+), \tag{8.9}$$

which is what we seek.

We shall always choose the first set of basis functions to be the standard Minkowski positive frequency functions. Thus, our old vacuum is just the Minkowski vacuum. As the new basis functions, we shall use the basis functions suited for the new coordinate system. This can be done by writing, and separating, the wave equation in the new coordinates. From Equation (8.5) and the orthonormality of the basis functions it follows that

$$\beta_{ij} = -\langle f_j^*, \tilde{f}_i \rangle, \tag{8.10}$$

where f_j are the Minkowski plane waves. This can be at once used in Equation (8.9). Non-zero density of particles is signalled by non-zero value of β_{ij}. We shall now evaluate it for the three previous cases.

Case 2. Uniform acceleration

This has been done in literature so many times that we are not going to repeat it! The basis functions turn out to be Bessel functions of imaginary order and argument and evaluation of the vacuum density by Equation (8.9) leads to the Planckian spectrum. This agrees with what is seen by the accelerated detector.

Case 3. Uniform rotation

The metric for this case can be expressed most conveniently in terms of cylinderical coordinates as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\theta + \Omega\,\mathrm{d}t)^2 - \mathrm{d}z^2 \tag{8.11}$$

where Ω is related to the parameters in Equation (8.10) by $\Omega = \alpha/c$. The Klein-Gordan equation assumes the form

$$\left\{\left(\frac{\partial}{\partial t} - \Omega\frac{\partial}{\partial\theta}\right)^2 - \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\partial^2}{\partial z^2}\right\}\phi = 0. \tag{8.12}$$

This has the positive frequency orthonormal set of solutions given by

$$\bar{\psi}_{\omega m k_z} = \frac{1}{2\pi[2(\omega + m\Omega)]^{1/2}}\,\mathrm{e}^{-i\omega t}\,\mathrm{e}^{im\theta}\,\mathrm{e}^{ik_z z}\,J_m(\alpha r), \tag{8.13}$$

where $(\omega + m\Omega)^2 - k_z^2 = \alpha^2$ and J_m are the Bessel functions.

One can expand the field as

$$\phi = \sum_m \int_0^\infty q\,\mathrm{d}q \int_{-\infty}^{+8} \mathrm{d}k_z[\bar{a}(q, m, k_z)\bar{\psi}_{qmk_z} + \text{h.c.}]. \tag{8.14}$$

The Bogoliubov coefficients can now be evaluated between these modes and the plane waves modes. A detailed computation gives (when Minkowski modes are labeled by a vector $\bar{k}$)

$$\alpha(q, m, k_z; \bar{k}) = \frac{1}{(2\pi)^{1/2}}\left[\frac{\bar{k}_x - i\bar{k}_y}{q}\right]^m \frac{\delta(q - (\bar{k}_x^2 + \bar{k}_y^2)^{1/2})}{q}\,\delta(k_z - \bar{k}_z), \tag{8.15}$$

$$\beta(q, m, k_z; \bar{k}) = 0. \tag{8.16}$$

Equation (8.16) expresses the crucial fact that there is no ambiguous particle density. The vacua of rectangular and rotating frames are the same even though a rotating 'model detector' might see a spectrum. This shows the incompatibility of the two definitions.

Actually this result may be anticipated in the following sense. The β's vanish when one goes from rectangular (xyz) to cylindrical (r, θ, z) coordinates in the Minkowski frame, since there is no mixing of positive and negative frequencies. The normal plane wave modes for the (r, θ, z) coordinates can be obtained from Equation (8.13) by simply putting $\Omega = 0$. Now it is easy to see that β vanishes between the set in Equation (8.13) and a new set obtained with $\Omega = 0$. This is a physical way of looking at the result.

Case 3. In this coordinate frame also the result persists. The metric is given in Equation (8.13) as

$$ds^2 = 2gx\,dt^2 + 2\,dy\,dt - dx^2 - dz^2. \tag{8.17}$$

The Klein-Gordon equation is than of the form

$$\left(2\frac{\partial^2}{\partial t\,\partial y} - \frac{\partial^2}{\partial x^2} - 2gx\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right)\phi = 0. \tag{8.18}$$

The positive frequency, orthonormal solutions to this equation can be represented in terms of Airy functions as

$$\bar{\psi}_{pk_zk_y} = \frac{1}{4\pi}\left[\frac{4}{gk_y^2}\right]^{1/6} e^{-i\omega t}\,e^{+ik_y y}\,e^{ik_z z}\,Ai(p + qx), \tag{8.19}$$

where

$$p = \frac{1}{q^2}(k_z^2 - k_y\omega); \quad q = (2gk_y^2)^{1/3}; \tag{8.20}$$

and $A_i(z)$ is the Airy function. The field is expanded as

$$\phi = \int dk_y\,dk_z\,dp[a(k_y, k_z, p)\bar{\psi}_{pk_yk_z} + \text{h.c.}].$$

The Bogoliubov coefficients (again found by direct integration), are

$$\alpha(k_y, k_z, p; \bar{Q}) = \tfrac{1}{2}\left|\frac{\omega}{\pi g k_y}\right|^{1/2} \exp\left|i\frac{\bar{Q}_x}{2gk_y^2}(\bar{Q}_x^2 + k_y^2 + pq^2)\right| \times$$
$$\times\ \delta(\bar{Q}_z - k_z)\delta[k_y - \omega + \bar{Q}_y]$$

and

$$\beta(k_y, k_z, p; \bar{Q}) = 0.$$

These results, unfortunately, have no simple interpretation and, in fact, come somewhat as a surprise. It is clear that the vacua are the same in this case also. We have explicitly calculated the spectra seen by the 'model detector' in Equation (4.5). Once again, it is clear that the detector is not detecting the particles corresponding to the standard canonical quantisation.

9. Implications of the Result

The concept of a particle, carrying well defined quantum numbers arises in the theory from the canonical formalism. When the UAF results were known, conflict arose between the definition of particles based on various coordinate systems. The recourse to the 'model detector' was made only to settle this question.

However, implicit in such a method is the assumption that what the detector sees actually corresponds to the 'particle' in the standard sense of the field theory. We have shown here that this is not true in general. The detector essentially measures a different pattern of vacuum fluctuation and may not correspond to the particles of the field theory. Thus one is forced to consider the results based on the detector model with suspicion – they have to be confirmed by actual quantisation calculations.

This result, in some sense, strengthens a previous conclusion. It was shown in Part I that any realistic detector whose acceleration vanishes initially and finally will not see any net particles. This was derived using the formalism of field theory and is independent of detector models. However, a detector of the type discussed above might see a complicated time-dependent pattern. The present results certainly indicate that the results based on the formalism are more reliable. Hence, one might take it that as far as the formalism goes no real difficulty arises in defining particle concept for any physically realisable observer motion.

Some more aspects of the conventional particle detector' will be discussed in the next section. Before concluding this section we wish to point out one more feature. It is clear from the previous discussion that the spectra seen by the model detector is Planckian only for a particular case. Thus associating the concept of temperature with the accelerated observer has only a very limited (if any!) validity.

PART III: THE MODEL DETECTOR

10. What Happens with the 'Model Detector'?

We have now shown that the detector does not always respond to the particles as defined by the field theory. The question arises as to what exactly happens with our 'model detector'. Also it is now doubtful whether this phenomenon is completely quantum mechanical. We shall first show that a simple classical analogue exists.

Consider a classical scalar field $\phi(x)$ which is monochromatic, represented in the usual Minkowski coordinates as

$$\phi(\bar{x}, t) = f(\bar{x})\, e^{-i\omega t}. \tag{10.1}$$

Any detector which detects variations wrt the time **t** will attribute to ϕ a frequency ω (when the detector is at rest; constant $\bar{x}$.) So the power spectrum seen by the detector goes as $\sim \delta(E - \omega)$. Now consider another detector with a trajectory $x^\mu = x^\mu(\tau)$, where τ is the proper time. Let us assume that this detector measures variation of ϕ with respect to its own proper time τ. Taking the measurements to be local, the field varies with τ as (remembering it to be a scalar)

$$\phi(\tau) = f[\bar{x}(\tau)]\, e^{-i\omega t(\tau)}. \tag{10.2}$$

This complicated function τ can be Fourier transformed with respect to τ as

$$\phi(\tau) = \int_{-\infty}^{+\infty} \beta(\omega)\, e^{-i\omega\tau} \frac{d\omega}{2\pi}, \tag{10.3}$$

with β given by

$$\beta(\omega) = \int_{-\infty}^{+\infty} \phi(\tau)\, e^{i\omega\tau}\, d\tau. \tag{10.4}$$

Quite evidently the detector will see a complicated spectra, far from monochromatic, given by $|\beta|^2$ (since the power spectrum is the square of the amplitude). Thus, the effect can arise even in a purely classical setting.

If the form of $f(\bar{x})$ is fixed the calculation can be performed for a given trajectory $x^\mu(\tau)$. This is not at all difficult since we expect $\phi(x)$ to be a scalar under Lorentz transformations. Therefore, $f(\bar{x})$ must be $\sim e^{ik\cdot x}$ with $|k| = \omega$: i.e.,

$$\phi(\bar{x}, t) = A \exp i(\bar{k}\cdot\bar{x} - \omega t). \tag{10.5}$$

For simplicity consider only two dimensions (x, t). If the trajectory of the detector is taken to be that of a uniformly accelerated one (viz. $x(\tau) = 1/g \cosh g\tau$; $t(\tau) = 1/g \sinh g\tau$) the field varies with τ as

$$\begin{aligned} \phi(\tau) &= A \exp i\omega(x - t) \quad \text{(2 dimensions)} \\ &= A \exp\left(\frac{i\omega}{g} e^{-g\tau}\right). \end{aligned} \tag{10.6}$$

The Fourier decomposition gives

$$\beta(E) = \int_{-\infty}^{+\infty} \phi(\tau)\, e^{iE\tau}\, d\tau \tag{10.7}$$

$$= A \int_{-\infty}^{+\infty} d\tau \exp\left(i\left[\frac{\omega}{g} e^{-g\tau} + E\tau\right]\right). \tag{10.8}$$

By making a substitution $e^{-g\tau} = x$ this can be reduced to the form

$$\beta(E) = \frac{A}{g}\int_0^\infty \frac{dx}{x^{1+(iE/g)}}\exp\left(i\frac{\omega}{g}x\right). \tag{10.9}$$

This can be evaluated by rotating the contour to the imaginary axis, leading to

$$\beta(E) = \frac{A}{g}\exp\left(-\frac{\pi E}{2g}\right)\left(\frac{g}{\omega}\right)^{-iE/g}\Gamma\left(-\frac{iE}{g}\right), \tag{10.10}$$

so that

$$|\beta|^2 = \frac{A^2}{g^2}\exp\left(-\frac{\pi E}{g}\right)\Gamma\left(-\frac{iE}{g}\right)\Gamma\left(\frac{iE}{g}\right)$$

$$= \frac{\pi A^2}{g}\frac{E}{(\exp(2\pi E/g) - 1)}, \tag{10.11}$$

which is Planckian. Thus this spectrum arises purely from transforming a plane wave to the accelerated frame.

This derivation is based on the assumption that any accelerated detector responds to variation wrt its proper time. The task before us is to consider an inertial detector and check whether it performs like this under acceleration. Unfortunately, this is put in by hand in the standard model of the detector, in the following manner.

Consider (this is essentially a rederivation based on DeWitt, 1979) a detector with a monopole moment (operator) $m(\tau)$ interacting with the scalar field through the potential

$$L_1 = m(\tau)\phi[x^\mu(\tau)]. \tag{10.12}$$

Let the detector make the transition from $|i\rangle$ to $|f\rangle$ when the field goes from $|0\rangle$ to $|\alpha\rangle$, where $|0\rangle$ in the Minkowski vacuum state and $|\alpha\rangle$ is arbitrary. To first order in ε, the *amplitude* for this process is given by

$$\mathscr{A}_{\alpha,if} = \varepsilon\int_{-\infty}^{\tau_0} d\tau\,\langle f|\,m(\tau)\,|i\rangle\,\langle\alpha|\,\phi(x^\mu(\tau))\,|0\rangle. \tag{10.13}$$

Since ϕ can connect only one particle states to vacuum, $|\alpha\rangle$ is one of states labelled by momentum $|k\rangle$. At this stage a crucial assumption is made by setting

$$\langle f|\,m(\tau)\,|i\rangle = \langle f|\,m(0)\,|i\rangle\exp(-i(E_f - E_i)\tau). \tag{10.14}$$

This is valid iff there exists a set of eigenstates for the Hamiltonian of the accelerated detector (which must, incidentally, be static) which varies with proper time as $e^{-iE\tau}$. This is completely equivalent to assuming that an accelerated detector measures the variation wrt its proper time. This assumption is built-in in this model.

Once this is granted, the derivation goes through. The probability is given by the modulus

$$\mathscr{P}(0\to\alpha; i\to f) = |A_{\alpha,if}|^2 = \varepsilon^2\int_{-\infty}^{\tau_0} d\tau_1\int_{-\infty}^{\tau_0} d\tau_2\bar{m}_{if}^2\,\times$$

$$\times\exp(-i\omega(\tau_1-\tau_2)\langle 0|\,\phi(\tau_2)\,|\alpha\rangle\,\langle\alpha|\,\phi(\tau_1)\,|0\rangle, \tag{10.15}$$

The probability for detector excitation irrespective of the state of the field is

$$P[i \to f] = \sum_{\alpha} |A_{\alpha, if}|^2$$

$$= \varepsilon^2 \int_{-\infty}^{\tau_0} \mathrm{d}\tau_1 \int_{-\infty}^{\tau_0} \mathrm{d}\tau_2 \bar{m}_{if}^2 \times$$

$$\times \exp(-i\omega(\tau_1 - \tau_2) \sum_{\alpha} \langle 0| \phi(\tau_2) |\alpha\rangle \langle\alpha| \phi(\tau_1) |0\rangle$$

$$= \varepsilon^2 \int_{-\infty}^{\tau_0} \mathrm{d}\tau_1 \int_{-\infty}^{\tau_0} \mathrm{d}\tau_2 \bar{m}_{if}^2 \exp(-i\omega(\tau_1 - \tau_2) G(\tau_1, \tau_2). \quad (10.16)$$

Since $\sum_\alpha |\alpha\rangle \langle\alpha| = I$. This is essentially the form used before. (When $G = G(\tau_1 - \tau_2)$, one integral can be removed to give rate of transitions.)

What is the final state? The detector is in state $|f\rangle$ while the field is in some one particle state $|\alpha\rangle$. It is evidently a process (as far as the Minkowski observer is concerned) where one quanta has been *emitted* by the 'detector' while going from $|i\rangle$ to $|f\rangle$. With respect to the accelerated frame $|f\rangle$ may be a higher energy state than $|i\rangle$. This certainly is not true with respect to the Minkowski time. Thus as far as the Minkowski observer is concerned it can *release* energy going from $|i\rangle$ to $|f\rangle$ which manifests itself as the scalar quanta.

The moral of the story is that this does not conform to the conventional concept of a detector where the detection is treated as an irreversible invariant process.

Before concluding, we wish to take up one more important aspect of the problem. Can this theory be put to test in the laboratory? In other words, is there any theoretical limitations for observing these spectra? We show below that, because of uncertainty principle, a large portion of the spectra is unobservable.

The idea essentially follows from the commutation relation, (in the Heisenberg picture)

$$ih\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = ihm\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = ihm\mathbf{A} = [H, \mathbf{P}]; \quad (10.17)$$

which implies that

$$\Delta E |\Delta\mathbf{P}| \geq mh |\mathbf{A}|, \quad (10.18)$$

where $\mathbf{A}$ is the operator corresponding to the acceleration. For the sake of clarity we present a detailed model analysis below.

For a detector to detect a particle it has to interact with the field corresponding to the particle. But in order to accelerate a detector of mass M to an acceleration g, the detector has to be placed in a potential of strength Mgx, with which it will be interacting. A detection process breaks down when these interactions are of equal strength so that the accelerating potential itself will start causing transitions in the levels.

Consider a model detector of mass M and spatial extent L which is used to detect a particle of energy E and momentum p, within the uncertainties ΔE and Δp. Let the detection process take a time Δt during which the average velocity and average acceleration of the detectors are v and g respectively. In order to give a detector of mass M an acceleration g, it must be interacting with a potential, which around the event of interaction varies as

$$V \simeq Mgx. \tag{10.18a}$$

In the time interval Δt taken for the detection, the detector moves through a distance $v\Delta t$ (to first order). So the total uncertainty in the position of the detected particle is $\Delta x = L + v\Delta t$, resulting in the relations,

$$\Delta p \geqslant \frac{h}{(L + v\,\Delta t)}, \tag{10.19}$$

$$\Delta E \geqslant \frac{h}{\Delta t} + Mg(L + v\,\Delta t). \tag{10.20}$$

The second term in Equation (10.20) arises because of the interaction of the detector with the accelerating mechanism represented by the potential in Equation (10.18a). Macroscopically one can look at it as the uncertainty in the potential felt in the detector. Microscopically the potential is capable of creating virtual pairs of particles seen by the detector as a noise. Normally the detection is assumed to be an instantaneous process (which is wrong) and this term is not considered.

The rest is simple mathematics. Multiplying Equation (10.19) and (10.20) we get in agreement with Equation (10.18)).

$$\Delta p\,\Delta E \geqslant \frac{h}{L\,\Delta t} + h\,Mg + 0((\Delta t)^2), \tag{10.21}$$

Obviously the concept of detection breaks down when $\Delta p \sim p$ and $\Delta E \sim E$, since $L\,\Delta t$ is somewhat adjustable, it is better to write a strict inequality in the resolution limit $p \simeq \Delta p$ $E \simeq \Delta E$ as

$$pE \gtrsim Mg\,h. \tag{10.22}$$

This leads to the limit on the acceleration as

$$g \ll \left(\frac{m_e}{M}\right)\frac{c^2}{\lambda} \quad \text{for detection of massive particles} \tag{10.23}$$

where λ is the de Broglie wavelength of the particle, and $m_e = m_0/\sqrt{1 - u^2/c^2}$ is the effective mass of the detected particle. The heavier the detector the less freedom we have. Even when $M \gtrsim m_e$(!)-(ideal case) we must have

$$g \ll \frac{c^2}{\lambda}. \tag{10.24}$$

If it is massless radiation which is detected then the formula replacing (10.23) becomes

$$g \ll \frac{\lambda_M}{\lambda}\frac{c^2}{\lambda}, \tag{10.25}$$

where $\lambda_M = h/Mc$ and $\lambda =$ the wavelength of detected radiation. One will notice that most of the Planck spectrum claimed to be observed comes from wavelengths of this order and clearly violates these inequalities. Anything to be detected will be completely mixed up with the noise caused by the transitions arising out of the external accelerating potential.

This is the general outline. To make an explicit model of a detector in order to verify the above conclusions (as well as results about NUAF) is a difficult task. The pioneering work in the field by Unruh does not take into account the transitions caused in the detector by the accelerating potential. When a Schrödinger particle interacts with a potential this does not happen. But if one uses a Klein–Gordan particle as a detector, an external potential will cause transitions involving negative energy sea. (Essentially it is the Klein's paradox repeated again.) Further in NUAF, the field to be detected cannot be separated into Rindler modes (no Rindler time exists) but must be done in terms of 'in' and 'out' formalism. This analysis, when completed should give the detailed mechanism by which results of Section 3 are realised.

11. Conclusions

We do not have today a rigorous theory for quantum mechanics in an accelerated frame. We have tried here to bring out some salient features which must form part of any complete theory. Two major points emerge from this analysis.

(i) The equivalence of various formalisms for defining the particle concept, breaks down when curvilinear coordinates are used. (ii) Uncertainty principle leads to non-trivial limitations in the accelerated coordinate system which must be kept in mind while discussing the measurement processes.

It would be of interest to study the implications of this analysis in the curved space-time. For example, a detector at rest in a black hole space-time is accelerated with respect to local inertial frames, so that an analysis similar to that of Part III can be performed. One also expects the results in Part II to be carried over to curved space-times. Already, it has been shown (Candelas, 1980) that in the Schwarzschild metric the vacuum fluctuation pattern observed by an Unruh-type detector is different from the result expected, based on the Hawking process. These points are under investigation.

Putting these together, one might conclude that the results for accelerated detectors are not physically realizable.

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