1982Ap&SS..87..333N

NUCLEOSYNTHESIS IN BOUNCING COSMOLOGIES

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(Received 6 May, 1982)

Abstract. It is shown that certain anomalies connected with the primordial abundances of light nuclei may be resolved if it is assumed that the Universe oscillates between phases of finite densities. Since general relativity does not produce bouncing models of the Universe, such models are obtained through the introduction of a negative energy scalar field of zero rest mass. It is shown that all the relevant parameters of the dynamics of the model and the nucleosynthesis in it are determined by observations and that a self-consistent picture emerges. The model is capable of admitting more than three neutrino flavours without an embarrassingly high primordial helium content. It is also shown that the calculations could be adapted to describe production of light nuclei in compact massive bouncing objects.

1. Introduction

Although it is generally believed that most of the light nuclei found in the Universe were produced in the first few minutes after the big bang, a careful assessment of the presently available data leaves some measure of doubt about the validity of the standard scenario. The fact that all is not well on this front is evidenced by the controversy currently raging amongst the experts (see, for example, Stecker, 1980, 1981; Olive *et al.*, 1981; Rana, 1982).

We will avoid going into the details of this controversy. We shall adopt the view that the standard hot bigh bang model does not satisfactorily reproduce the observed abundances, that the discrepancies between the theoretically computed primordial abundances and the observed abundances are significant and demand a rethinking of the early-universe scenario. In particular, the following discrepancies need to be examined closely:

(a) The canonical value of the ^{4}He mass fraction Y is around 0.25 whereas the primordial component of the observed abundance may be as low as 0.22. (Stecker, 1980, 1981).

(b) With the addition of each neutrino flavour, the value of Y goes up by about 0.02. Thus, three or four (or more according to some grand unified theories) flavours are difficult to accommodate in the standard scenario (Olive *et al.*, 1981).

(c) There is no universally consistent baryon number density which correctly reproduces the primordial abundances of both ^{4}He and ^{2}H (Rana, 1982).

One factor which plays a significant role in the primordial nucleosynthetic calculations is the rate of expansion of the Universe. In the standard big bang scenario, this depends on the effective total number of internal degrees of

Astrophysics and Space Science **87** (1982) 333–346. 0004–640X/82/0872–0333$02.10.
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freedom g of relativistic species of particles, normalized to $g = 2$ for photons. The radiation temperature T depends on g through the relation

$$\frac{\dot{T}^2}{T^2} = \frac{4\pi G}{3c^2} g a T^4, \tag{1}$$

where the dot denotes derivative with respect to the cosmic time. From (1) the usual time-temperature relationship emerges in the form

$$T = \left[\frac{3c^2}{16\pi G a g}\right]^{1/4} t^{-1/2}. \tag{2}$$

It was pointed out by Hawking and Tayler (1966) that in anisotropically expanding cosmologies, the time-temperature relationship would be different. For example, in a Bianchi type I-model with a pancake singularity (2) is changed to

$$T \propto t^{-1/3}. \tag{3}$$

Such a drop in temperature results in the lowering of the value of Y.

In this paper we introduce another device into the conventional picture which preserves the isotropy of the Universe (so necessary for the observed isotropy of the microwave background) but which does away with the big-bang singularity. With this device we have a cosmological model which bounces between states of finite but high and low densities.

In Section 2 we describe the mathematics behind this approach, while in Section 3 we consider how it tends to work in the direction of lowering the value of Y. We will also discuss briefly the production of deuterium. In Section 4 we relate the crucial parameters of the nucleosynthesis problem to the bouncing mechanism. Finally, in Section 5 we briefly discuss a variant of the theory in which the light nuclei are produced in massive oscillators.

2. Oscillating Cosmological Models

It it well known that models of the Universe which oscillate between finite limits of maximum and minimum density are not possible within the framework of the standard general theory of relativity. Cosmological models usually discussed by relativists have an epoch of space-time singularity. This epoch, for lack of any more satisfactory theory, is identified with the 'origin' of the universe. It is assumed (as an article of faith) that extrapolations of present day physics can be pushed arbitrarily close to the singular epoch without actually attaining it.

If taken seriously, the singular epoch itself denotes a stage when the conservation law of matter and energy is violated. It was noted by one of us (Narlikar, 1973) that the breakdown of this conservation law and the appearance of singularity occur at the same stage and that both reflect an incompleteness of the relativistic framework. An augmented theory may 'cure' both these 'ills'. In

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this connection it was shown (Narlikar, 1973 and 1974) that a field-theoretic discussion of matter creation removes both the space-time singularity and the violation of the conservation of matter and energy. In this modified framework the big bang epoch of infinite density was replaced by an epoch of very high but *finite* density. This is the framework we adopt for our discussion of nucleosynthesis in this paper.

In the action principle formulation we have to add two extra terms to the Hilbert action which gives Einstein's equations. The extra terms consist of

$$f \int_{\mathcal{V}} C_i C^i \sqrt{-g}\, \mathrm{d}^4x - \sum_a \int_{\Gamma_a} C_i\, \mathrm{d}a^i . \tag{4}$$

To fix the notation, we use here a Riemannian space-time manifold described by coordinates x^i, metric tensor g_{ik} $(i, k = 1, 2, 3, 4)$, signature $(---+)$ and the metric determinant g. C is a scalar field of zero rest-mass and $C_i \equiv \partial C/\partial x^i$. f (>0) is a coupling constant between the C-field and matter represented by particles $a, b, c \ldots$. The volume integral in (4) is over the same space-time region $\mathcal{V}$ over which the Hilbert action is defined. The line integral of (4) is along the world line Γ_a of the typical particle a, $\mathrm{d}a^i$ denoting the coordinate differentials along Γ_a. Note that the line integral makes a trivial contribution to the variation of the action *unless* Γ_a has ends which fall inside $\mathcal{V}$. Physically this result means that the C-field couples with matter only at the time of its creation (or annihilation).

This formulation is based on the work of the late M. H. L. Pryce (private communication in 1960) and was used extensively by Hoyle and Narlikar (1963, 1964, 1966a–c) for describing the nonconservation of baryons in the steady-state cosmology. Baryon-non-conservation typified by world lines Γ_a with ends was anathema to particle physicists in the 1960s. It is no longer so now, and is being actively discussed by those interested in the 'early Universe' phase of the big bang cosmology. For, even in the big bang theory the baryon to photon ratio in the range 10^{-8}–10^{-10} needs to be explained. We will describe here the application of the C-field to an evolving rather than the steady-state model.

2.1. Cosmological solutions

The field equations are given in the usual notation by

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -\frac{8\pi G}{c^4}[T_{ik} - f\{C_i C_k - \tfrac{1}{2} g_{ik} C^l C_l\}] ; \tag{5}$$

while the source equation for the C-field is

$$C^i_{;i} = f^{-1} n , \tag{6}$$

where n = number of particles created per unit proper 4-volume.

The steady-state cosmology of Hoyle (1948) follows from the above equations for n = constant. To get the analogue of the big bang (Narlikar, 1974) n has to

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have a delta-function behaviour with respect to time. Thus the created matter is accompanied by an equal amount of C-field energy of *negative* sign. (The term containing the C-field energy tensor on the right-hand side of (5) has the negative coefficient $-f$.) Hence, even at the creation epoch energy is conserved.

In Section 4 we will consider the integration of (6) with a delta function source. For the time being we consider the developments after the once-for-all creation. To proceed further we use the cosmological principle and the Robertson-Walker line element

$$ds^2 = c^2\,dt^2 - S^2(t)\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \tag{7}$$

Thus we have chosen the closed model in anticipation of an oscillating solution. From the cosmological principle it also follows that C is a function of t only.

The post-creation wave equation satisfied by C is $C^{i}_{;i} = 0$ which in our special case becomes

$$\frac{d}{dt}(\dot{C}S^3) = 0\,.$$

The integral of this equation is of the form

$$\dot{C} = \frac{A}{S^3}, \tag{8}$$

where the constant A will be related in Section 4 to the delta function source. The $\binom{4}{4}$ component of the field Equations (5) gives us

$$3\frac{\dot{S}^2 + c^2}{S^2} = 8\pi G(\rho - \tfrac{1}{2}f\dot{C}^2)\ . \tag{9}$$

Following the standard Friedmann cosmology, we distinguish between two phases of the Universe.

(i) *Matter-dominated phase.* In this case the energy tensor T^{ik} of matter has the form suitable for dust. In (9) ρ denotes the density of dust. $T^{4k}_{;k} = 0$ gives

$$\rho S^3 = \text{constant} = \rho_0 S_0^3 \tag{10}$$

where ρ_0 and S_0 denote the values of ρ, S at the present epoch t_0.

Since $\dot{C}^2$ decreases as S^{-6}, for large S we anticipate that our model will simulate the behaviour of the standard closed Friedmann model. Accordingly we use the parameters H_0 and q_0 to characterize the model (cf. Weinberg, 1972), for $S \geq S_0$. Then we get

$$S_0 = \frac{1}{(2q_0 - 1)^{1/2}}\left(\frac{c}{H_0}\right), \tag{11}$$

$$S_{\max} = \frac{2q_0}{(2q_0 - 1)^{3/2}}\left(\frac{c}{H_0}\right); \tag{12}$$

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S_{max} being the maximum value attained by S during the expanding phase. For numerical calculations we will write

$$H_0 = 100\, h_0 \text{ km s}^{-1} (\text{Mpc})^{-1}\,; \tag{13}$$

with h_0 a scaling parameter. Observations suggest that $0.5 \leqslant h_0 \leqslant 1$.

The present density ρ_0 and the minimum density ρ_{min} are given by

$$\rho_0 = \frac{3H_0^2}{4\pi G} q_0 \simeq 4 \times 10^{-29}\, h_0^2 q_0 g \text{ cm}^{-3}\,, \tag{14}$$

$$\rho_{min} = \frac{3H_0^2}{32\pi G}\,\frac{(2q_0-1)^3}{q_0^2} \simeq 5 \times 10^{-30}\, h_0^2 \frac{(2q_0-1)^3}{q_0^2} \text{ g cm}^{-3}. \tag{15}$$

We also have

$$\frac{8\pi G\rho S^3}{3} = \frac{2c^3}{H_0}\,\frac{q_0}{(2q_0-1)^{3/2}} = Kc^2 \text{ (say)}\,. \tag{16}$$

(ii) *Radiation-dominated phase.* Taking the present temperature of the microwave background radiation as $T_0 \simeq 3$ K, we find that the radiation dominated expansion (or contraction) holds for $S \leqslant S_R$ where

$$aT_0^4 \left(\frac{S_0}{S_R}\right)^4 = \rho_0 c^2 \left(\frac{S_0}{S_R}\right)^3,$$

i.e.,

$$S_R = S_0 \frac{aT_0^4}{\rho_0 c^2} \cong 1.7 \times 10^{-5}\, S_0 h_0^{-2} q_0^{-1}$$

$$\cong 1.7 \times 10^{23} (q_0 h_0^3)^{-1} (2q_0 - 1)^{-1/2} \text{ cm}\,.$$

Notice, however, that we now have a third phase to consider, the phase during which the C-field term in (9) becomes comparable to the radiation term. For, if we want to have a 'hot' phase during the course of oscillations, the bounce must occur at a value of S substantially *lower* than S_R. Clearly, the minimum value of S, S_{min}, depends on the magnitude of the C-field strength, i.e., on the constant A.

Earlier work (Narlikar, 1974) has shown that the constant A can be determined from the primary creation event. We will return to the determination of A in Section 4 of this paper. For the time being we will keep it as a free parameter to be determined from empirical considerations.

2.2. The bounce temperature

The universe as described above bounces at a low value of S since even in the radiation dominated phase the C-field term grows faster than the radiation term. Assuming that the number of degrees of freedom of relativistic species of particles is g, we write

$$L = \frac{4\pi G a T_0^4 S_0^4}{3c^4} g\,, \qquad B = \frac{4\pi G f A^2}{3}\,, \tag{17}$$

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and get the following differential equation for the dynamical expansion of the early Universe:

$$\dot{S}^2 = c^2\left(\frac{L}{S^2} - 1\right) - \frac{B}{S^4}. \tag{18}$$

We will assume this relation to hold for $S < S_R$. For $S > S_R$ (the matter-dominated phase) the corresponding equation is

$$S^2 = c^2\left(\frac{K}{S} - 1\right) - \frac{B}{S^4}, \tag{19}$$

with $L = KS_R$.

The bounce is expected to occur at a very small value of S where the curvature term of (18) may be neglected. Then the bounce value S_B of S is given by

$$S_B = \left(\frac{B}{Lc^2}\right)^{1/2}. \tag{20}$$

The neglect of the curvature term is justified, provided that

$$Lc^2 \gg B. \tag{21}$$

The bounce temperature may be determined on the assumption that black-body radiation at temperature T_B has essentially cooled down adiabatically to the present temperature $T_0 \sim 3$ K of the microwave background as S increased from S_B to S. Thus

$$T_B = \frac{S_0 T_0}{S_B}. \tag{22}$$

Anticipating that $S_0 \gg S_R \gg S_B$, we expect $T_B \gg T_0$. Further, if we are discussing the 'early Universe' – i.e., the Universe close to $S = S_B$ – we may approximate (18) by

$$\dot{S}^2 \cong \frac{c^2 L}{S^2} - \frac{B}{S^4}. \tag{23}$$

In the relativistic regime we therefore have $S \propto T^{-1}$, and

$$\frac{\dot{T}^2}{T^2} = \frac{4\pi G g a}{3c^2} T^4 \left(1 - \frac{T^2}{T_B^2}\right). \tag{24}$$

The last factor on the right-hand side arises from the introduction of the C-field. Notice that we may express (24) in a form similar to (1) for the standard hot big bang by writing

$$\frac{\dot{T}^2}{T^2} = \frac{4\pi G a}{3c^2} g(T) T^4 \tag{25}$$

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with

$$g(T) = g\left(1 - \frac{T^2}{T_B^2}\right). \tag{26}$$

Since $T < T_B$, the effective g-factor in this cosmology is always *less* than that in standard cosmology, for the same species of the primordial brew. This result has a bearing on the problem of helium production if it occurs soon after the bounce.

3. The Production of Light Nuclei

3.1. Differences from standard cosmology

The oscillating model presented here differs from standard hot big bang in the following important respects where nucleosynthesis is concerned.

In the hot big bang, the early Universe calculations begin from the end of the quantum era at the age of the so called Planck-time

$$t_p = \sqrt{\frac{G\hbar}{c^5}}. \tag{27}$$

Much of the present work on the early Universe is concerned with the evolution of the material content of the universe from the Planck-time and up to the hadron era. The work on primordial nucleosynthesis begins from the epoch when the rate of nucleon-neutrino interaction has become about equal to the rate of expansion of the Universe. This is the last epoch t_d when we expect the ratio of the number densities of neutrons to protons to be equal to that given by thermodynamic equilibrium. Denoting the number fractions of neutrons and protons by X_n and X_p respectively, it is customary to write the above relation as

$$\frac{X_n}{X_p} = \exp\left\{-\frac{(m_n - m_p)c^2}{kT_d}\right\}, \tag{28}$$

where m_n and m_p are the masses of these two particles and T_d is the temperature of the Universe. In standard cosmology, the nucleon neutrino reaction rate goes as (cf. Weinberg, 1972)

$$r = \mu T^4, \quad \mu = \text{constant}; \tag{29}$$

while the Hubble constant goes as

$$H = pT^2, \quad p = \text{constant}. \tag{30}$$

Thus the last epoch at which (28) can hold is given by

$$T_d = \left(\frac{p}{\mu}\right)^{1/2}. \tag{31}$$

In the present cosmology, we anticipate that the bounce temperature will be lower than that at the hadron era so that all of the standard work on grand

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unified theories (GUTs) and the early Universe will not be applicable here. Furthermore, although (29) holds, (30) is modified to

$$H = pT^2\left(1-\frac{T^2}{T_B^2}\right)^{1/2}, \tag{32}$$

as implied by (24). Equating (29) to (32) now gives for T_d the relation

$$T_d^2\left(1-\frac{T_d^2}{T_B^2}\right)^{-1/2} = \frac{p}{\mu}. \tag{33}$$

Evidently the solution of (33) gives a value for T_d, which is *lower* than $(p/\mu)^{1/2}$. The result is that the X_n/X_p ratio is *lower* in the present theory when compared to standard cosmology.

The second source of difference comes from the time-temperature relation. In the detailed calculation of the X_n/X_p ratio subsequent to t_d, we have to take into account the reaction rates of various processes which convert neutrons to protons and vice-versa. The rates depend on ambient temperature which in turn depends on the age t of the Universe. Comparing (24) with (1) we note that

$$\frac{\dot{T}_{\text{present model}}}{\dot{T}_{\text{standard model}}} = \left(1-\frac{T^2}{T_B^2}\right)^{1/2}. \tag{34}$$

Since the temperature falls with time more *slowly* in the present model, compared to the standard model, the (n, p) reactions can go on longer. The effect of these reactions is to reduce X_n and increase X_p. Therefore, we expect the X_n/X_p ratio to be lower in the present model compared to the standard model and, hence, detailed computations should give a smaller value of Y, the helium mass fraction.

These expectations are borne out by the numerical computations described below.

3.2. Primordial abundance of helium

It is not our purpose here to discuss the problem of primordial nucleosynthesis in all its gory details of reaction rates of participating nuclei. Instead we wish to demonstrate the difference that is likely to emerge in such a calculation if we take into account the bounce of the universe at very high temperature. A lower value of Y will establish a *prima facie* case for undertaking a detailed calculation along the lines of Wagoner *et al.* (1967, WFH in brief). For this purpose we will compare our results with the classic work of Hoyle and Tayler (1964) which adopted a somewhat simplified approach to primordial nucleosynthesis without losing any of its essential features.

Starting their computations at $t = t_d$, Hoyle and Tayler wrote down a differential equation giving the rate of change of the neutron to baryon number ratio as a

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function of temperature. In our notation this differential equation becomes

$$\frac{dX_n}{dt} = 0.142\, T_{10}^3(1+0.476q\, T_{10})^2 \times \times \left[\left\{1+\exp\left(-\frac{1.506}{T_{10}}\right)\right\}X_n - \exp\left(-\frac{1.506}{T_{10}}\right)\right], \tag{35}$$

where T_{10} is the temperature expressed in units of 10^{10} K and q is a slowly varying quantity which makes $mc^2 + qkT$ equal to the average electron energy at temperature T, m being the electron mass. So long as we are considering a population of free protons and neutrons, $X_n + X_p = 1$.

The numbers appearing in (35) are those given by Hoyle and Tayler on the basis of the experimental data available in 1964. More recent data and a more elaborate approach would no doubt alter these numbers. However, as mentioned before we wish to show how the above differential equation gives a different answer in a bouncing cosmology. Thus whereas Hoyle and Tayler used (2) for converting (35) into a differential equation with respect to T_{10} as the independent variable, we will use (24) in our integration.

Thus instead of Equation (12) of Hoyle and Tayler (1964), we get

$$\frac{dX_n}{dT_{10}} = 0.308(1+0.476q\, T_{10})^2\left(1-\frac{T_{10}^2}{T_{10B}^2}\right)^{-1/2} \times \times \left[\left\{1+\exp\left(-\frac{1.506}{T_{10}}\right)\right\}X_n - \exp\left(-\frac{1.506}{T_{10}}\right)\right]. \tag{36}$$

The extra factor is that containing T_{10B}, the bounce temperature in units of 10^{10} K. Since it is always greater than unity, X_n decreases at a faster rate than in standard cosmology, as the Universe cools. Further, the starting value of X_n is determined from the fact that (28) holds at $t = t_d$, i.e., the right-hand side of (36) vanishes at $T = T_d$.

We have assumed $T_B \geqslant T_d$, i.e., we have supposed that the Universe bounces at a temperature not lower than that at which neutrons and protons cease to be in strict thermal equilibrium. According to the present day calculations in standard cosmology, $T_d \leqslant 10^{10}$ K, although Hoyle and Tayler took $T_d \sim 2.5 \times 10^{10}$ K. In our work, therefore, we expect T_d to be somewhat lower. Also the most dramatic differences occur from the standard theory when $T_B \simeq T_d$. In Table I we have given the results of our numerical integration for this case.

Hoyle and Tayler had integrated their differential equation until the temperature dropped to $T_{10} \simeq 0.5$, at which stage they found $X_n = 0.18$, with

$$Y = 2X_n = 0.36\,. \tag{37}$$

For comparison, we have given in Table I two cases where the final temperature $T_{\min} = 3 \times 10^9$ K as well as $T_{\min} = 5 \times 10^9$ K. The Hoyle–Tayler value corresponds to $T_B = \infty$ in Table I.

We suggest that a bounce temperature T_B in the neighbourhood of 10^{10} K will be sufficient to lower the value of Y appreciably without contradicting observations.

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TABLE I

Neutron fraction in bouncing models[a]

T_B	X_n	
	$T_{min} = 0.3$	$T_{min} = 0.5$
∞	0.180	0.180
2.50	0.152	0.173
1.50	0.144	0.162
1.20	0.135	0.154
1.16	0.132	0.150
1.10	0.129	0.147
1.00	0.121	0.139
0.90	0.112	0.128

[a] All temperatures are in units of 10^{10} K.

Notice that there is a significant drop from the standard value given by (37) for $T_B \leqslant 2.5 \times 10^{10}$ K. Qualitatively this is to be expected because the factor which makes the difference – viz.

$$\left(1 - \frac{T^2}{T_B^2}\right)^{1/2},$$

is significantly different from unity provided T_B is close to the relevant range of integration. From Table I it is apparent that the helium mass-fraction Y ($= 2X_n$ at the *end* of integration) may be reduced by as much as 20% from its standard value.

The standard primordial estimate of Y from more recent calculations (Olive *et al.*, 1981) comes close to 0.25 for three flavours of neutrinos. The above arguments suggest that similar calculations in a bouncing cosmology may lead to $Y \leqslant 0.18$ for two flavours of neutrinos and to $Y \simeq 0.20$ for three favours.

3.3. The break-up of nuclei during contracting phase

It is necessary to clarify another important question which arises only in the oscillating models. The question may be posed thus: Do successive oscillations steadily increase the value of Y? If the answer is 'yes', then clearly it will be an embarrassment to the present theory.

Bouncing massive objects were discussed earlier in a different context by WFH as alternative agents for nucleosynthesis at high temperatures. The scenario presented here differs from the massive bouncers in the important respect that the bouncing mechanism is different and the resulting dynamics of the model is also different. Moreover, the baryon number density is much lower than that necessary in bouncing massive objects. We cannot therefore try to answer the question by using the above results on massive objects.

The following argument, however, shows that during the collapse phase, in the

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presence of a high density of photons the ^{4}He nucleus may well break into lighter nuclei by photodisintegration, before the bounce occurs. The dominant processes for photodisintegration are

$$^4\mathrm{He} + \gamma \rightarrow p + {}^3\mathrm{H}\,, \qquad (-19.81\ \mathrm{MeV})\,;$$

$$^4\mathrm{He} + \gamma \rightarrow n + {}^3\mathrm{He}\,, \qquad (-20.58\ \mathrm{MeV})\,;$$

$$^4\mathrm{He} + \gamma \rightarrow D + D\,, \qquad (-23.85\ \mathrm{MeV})\,.$$

In the bracket against each reaction is given the energy needed to achieve the breakup (all reactions are endothermic). The reaction rate depends on the number density of photons, i.e., on the ambient temperature T. From Table II of WFH we find that the reaction rates λ_γ are comparable for the three modes. Thus using these three modes we may write

$$\frac{dY}{dt} \simeq -[\lambda_\gamma(^4\mathrm{He})_p + \lambda_\gamma(^4\mathrm{He})_n + \lambda_\gamma(^4\mathrm{He})_D]\,, \tag{38}$$

i.e.,

$$\left|\frac{dY}{dt}\right| > \lambda_\gamma(^4\mathrm{He})_n\,, \text{ say}\,. \tag{39}$$

Following WFH, Table II we get

$$\lambda_\gamma(^4\mathrm{He})_n \cong 4.93 \times 10^{16}\, T_{10}^{5/2} \exp\left(-\frac{23.88}{T_{10}}\right) \mathrm{s}^{-1}\,. \tag{40}$$

Using (24) with $g = 9$, in conjunction with (39), we obtain

$$\xi \equiv \left|\frac{d \ln Y}{dT_{10}}\right| \geq 1.08 \times 10^{17}\, T_{10}^{-1/2} \exp\left(-\frac{23.88}{T_{10}}\right)\left(1 - \frac{T_{10}^2}{T_{10B}^2}\right)^{-1/2} \mathrm{K}^{-1}\,. \tag{41}$$

Table II gives values of ξ for a range of values of T_{10}. The last factor is insensitive to the values for $T_{10B} \geq 2$ since it is the exponential term which dominates in the relevant range. It has been ignored in Table II. If $T_{10B} \leq 2$, ξ will be higher at the same temperature than what is given in Table II.

TABLE II

Lower limit on $\xi \equiv (d \ln Y/d \ln T_{10})$ in collapsing phase of the Universe

T_{10}	ξ
0.5	2.56×10^{-4}
0.6	0.67
0.7	1.83×10^{2}
0.8	1.22×10^{4}
0.9	3.16×10^{5}
1.0	4.25×10^{6}

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It is evident from Table II that a significant drop in Y begins at $T_{10} \simeq 0.6$ and that most of ^{4}He is broken up into lighter constituents by $T_{10} \simeq 0.7$. Thus by the time the bounce occurs at $T_{10} \simeq 1$ we are back to bare nucleons and a 'new deal' begins with the next expansion stage.

3.3. The abundance of deuterium

Only detailed calculations of nucleosynthesis will tell us what is the eventual mass fraction of deuterium produced in the hot era. We have not undertaken this calculation and our arguments here are based on qualitative comparisons with the corresponding calculations of the hot big bang.

First we note that since we are considering closed models in the present framework, we need the present density of matter to exceed the closure density. Therefore, we would have the same deuterium problem (i.e., underabundance of primordial deuterium) as the closed Friedmann models if we assume that most of the matter in the Universe is in baryonic form. The way out of the difficulty is provided by massive neutrinos which may easily make up 90% of the total density of matter.

Given that baryonic density is sufficiently low we find that for the same neutrino flavours the present model produces slightly more deuterium than the standard model of the same baryonic density. This follows from the inverse correlation of deuterium and helium abundances found in the usual calculations (Schramm and Wagoner, 1977; see also Figure 4 in their article). The increased deuterium abundance given by the present model does not, however, pose a problem because of the large uncertainty in spallation subsequent to the synthesis.

4. Primary Creation

One of us (Narlikar, 1974) had discussed in an earlier paper how the constant A can be determined from explosive creation of matter. This work was concerned with a cold big bang, i.e., a big bang in which the matter created was essentially pressure-free. The created universe would, in this case, immediately follow the dust solution given by (19). Here we adapt the calculation to discuss the hot big bang in which the particles created are moving relativistically and in random directions. Let us denote the creation epoch by t_c and the value of S at t_c by S_c.

At $t = t_c$, the action principle gives

$$\langle \dot{C} \rangle_{t_c} = \langle E \rangle . \tag{42}$$

The brackets $\langle\ \rangle$ denote the averaging over a number of particles created at neighbouring space-time points in a given small volume. Denoting by u_c the energy density and by N_c the number density of the particles created, we write for $\langle E \rangle$ the average energy per particle as

$$\langle E \rangle = \frac{u_c}{N_c} . \tag{43}$$

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The explosive creation may be approximated by

$$n_c = N_c \dot{} \delta(t - t_c) \tag{44}$$

at t_c. Thus, (6) can be solved to give for $t > t_c$

$$f\dot{C}S^3 = cN_cS_0^3 . \tag{45}$$

From (42)–(45) we get

$$u_c = f\dot{C}^2|_{t_c} . \tag{46}$$

For $t > t_c$, $u \propto S^{-4}$ while $f\dot{C}^2 \propto S^{-6}$. A comparison with (18) then gives

$$S_c^2 = \frac{2B}{Lc^2} = 2S_B^2 . \tag{47}$$

Thus the Universe is created explosively with $\dot{S} > 0$; it attains a maximum value for S and then collapses to S_B ($< S_c$) and subsequently oscillates. We can express all the unknown parameters of the problem in terms of the bounce temperature T_B. Thus we get

$$u_c = \frac{1}{4}u_B = \frac{1}{8}gaT_B^4 , \tag{48}$$

$$N_c = \frac{1}{2\sqrt{2}}N_B \cong \frac{1.2\tilde{g}}{\sqrt{2}\pi^2}\left(\frac{kT_B}{\hbar c}\right)^3 , \tag{49}$$

$$\dot{C}|_{t_c} \cong \frac{\sqrt{2}\pi^4}{144}\frac{g}{\tilde{g}}\frac{kT_B}{c} , \tag{50}$$

$$f \cong \frac{1.728\tilde{g}^2c^2}{2\pi^6 g}(kT_B)^2(\hbar c)^{-3} ; \tag{51}$$

where $g = g_b + \frac{7}{8}g_f$, $\tilde{g} = g_b + \frac{3}{4}g_f$, g_b and g_f being the spin states of bosons and fermions.

From these considerations we see that at the moment of creation the C-field is strong enough to produce particles of energy ~ 1 MeV. While this energy limit can accommodate electrons and the neutrinos, it falls far short of the energy needed for creating baryons. To get around this difficulty we suggest the following possible scenarios.

(a) To produce baryons, local inhomogeneities are needed which collapse and bounce and in the process increase the value of $\dot{C}$ above the cosmological value (Hoyle and Narlikar, 1964, 1966; Apparao and Narlikar, 1982). This line of argument suggests that baryons may be much less common than leptons and photons and that the baryon to photon ratio may not have a universal value after all.

(b) The Universe may have been created with a much higher value of $\dot{C}$. Thus in (50) if the original value of T_B was $\geqslant 10^{13}$ K, the Universe would be created predominantly with baryons. In subsequent oscillations we may imagine a

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systematic steady decrease in the magnitude of C-field energy, leading to a decrease in T_B. This may occur through dissipation or destruction of baryons. In the latter case the baryon number will be lowered.

A value of $T_B \gg 10^{10}$ K will produce essentially the same quantity of helium as the standard model. At each oscillation the helium produced is broken up and remade. Thus Y decreases as T_B decreases. If T_B falls below $\sim 5 \times 10^9$ K, there will be no break-up or reprocessing and the Universe will settle down to a fixed Y. This value of Y may be lower than ~ 0.2.

5. Massive Oscillators

It is also possible to argue that the universe did not have a hot phase at all, but that it has processed (and is probably still processing) light nuclei in massive objects which oscillate between maximum and minimum density-phases. Such massive oscillators were considered by us (*op. cit.*) as sources of high energy in the universe. In the present context these oscillators would serve the same purpose as the bouncing supermassive objects of WFH.

These objects can produce light nuclei and the microwave background although in order to have the isotropy of the latter, we may need them to be of the size of large scale superclusters which merge into each other. This possibility was advocated by Hoyle as a means of reinstating the steady-state theory. We have not investigated all the implications and constraints of this point of view.

Acknowledgement

We thank Mr N. C. Rana for helpful comments and discussions.

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