Mon. Not. R. Astron. Soc. **000**, 000–000 (1994) Printed 14 July 2003 (MN LaTeX style file v1.4)

Small-scale CMB polarization anisotropies due to tangled primordial magnetic fields

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ABSTRACT

Tangled, primordial cosmic magnetic fields create small rotational velocity perturbations on the last scattering surface (LSS) of the cosmic microwave background radiation (CMBR). Such perturbations can contribute significantly to the CMBR temperature and polarization anisotropies at large $l > 1000$ or so, like the excess power detected by the CBI experiment. The magnetic contribution can be distinguished from most conventional signals, as they lead to CMBR polarization dominated by the odd parity, B-type signal. Experiments like DASI and WMAP have detected evidence for CMBR polarization at small l. Many experiments will also probe the large l regime. We therefore calculate the polarization signals due to primordial magnetic fields, for different spectra and different cosmological parameters. A scale-invariant spectrum of tangled fields which redshifts to a present value $B_0 = 3\times10^{-9}$ Gauss, produces B-type polarization anisotropies of $\sim 0.3 - 0.4\mu K$ between $l \sim 1000 - 5000$. Larger signals result if the spectral index of magnetic tangles is steeper, $n > -3$. The peak of the signal shifts to larger l for a lambda-dominated universe, or if the baryon density is larger. The signal will also have non-Gaussian statistics. We also predict the much smaller E-type polarization, and T-E cross correlations for these models.

Key words: magnetic fields-cosmic microwave background-cosmology:theory-large-scale structure of Universe.

1 INTRODUCTION

The origin of large-scale cosmic magnetic fields remains an intriguing question. It is quite likely that magnetic fields in astronomical objects, like galaxies, grew by turbulent dynamo action on small seed magnetic fields (cf. Ruzmaikin, Shukurov & Sokoloff 1988; Beck et al 1996). However, this idea is not without difficulties in view of the constraints implied by helicity conservation and the more rapid growth of small-scale magnetic fields (Cattaneo & Vainshtein 1991; Kulsrud & Anderson 1992; Gruzinov & Diamond 1994; Subramanian 1998, 1999; Blackman & Field 2000; Kleoorin *et al* 2000; Brandenburg 2001; Brandenburg & Subramanian 2000; Brandenburg, Dobler & Subramanian 2002). Magnetic fields with larger coherence scales may also be present in clusters of galaxies (Clarke, Kronberg & Bohringer 2001) and at high redshifts (Oren & Wolfe 1995). Such large-scale coherent fields could present further problems for the dynamo paradigm. Alternatively, galactic or cluster fields could largely be a remnant of a primordial cosmological magnetic field (cf. Kulsrud 1990), although, as yet, there is no entirely compelling mechanism for producing the required field. They could be possibly generated during a period of inflation or at a phase transition, perhaps with an almost scale-invariant spectrum (Turner & Widrow 1988; Ratra 1992; cf. Grasso & Rubenstein 2001 for a review). A primordial field, whose present-day strength is of order 10^{-9} Gauss, and is tangled on galactic scales, can also affect the process of galaxy formation (Rees & Reinhardt 1972; Wasserman 1978; Kim, Olinto & Rosner 1996; Subramanian & Barrow 1998a, SB98a hereafter). It is of considerable interest, therefore, to find different ways of limiting or detecting such primordial fields (see Kronberg 1994 and Grasso & Rubenstein 2001, Widrow 2003 for reviews).

The nearly isotropic nature of the CMBR already places a limit of several nano Gauss on the present strength of any *uniform* (spatially homogeneous) component of the magnetic field (Barrow, Ferreira & Silk 1997). Observations of CMBR anisotropies also provide potentially powerful constraints on tangled magnetic fields (Subramanian & Barrow 1998b (SB98b), 2002 (SB02)). Such fields produce vortical perturbations, which are overdamped in the radiation era and can then survive Silk damping (Silk 1968) on scales much smaller than the compressional modes (Jedamzik, Katalinic & Olinto 1998; SB98a). So their signal, if present, will be particularly evident at small angular scales below the conventional Silk damping scale or at multipoles $l > 1000$ or so (SB98b, SB02). Intriguingly, the CBI experiment identified significant excess power in the CMBR anisotropy spectrum up to $l = 3500$ (with 2 sigma limits of $14 - 31$ μK at $l > 2010$) (Mason *et al* 2002). We argued in SB02 that tangled magnetic fields which redshift to a present-day value of $B_0 = 3 \times 10^{-9}$ Gauss (see below for a definition of B_0), can contribute a non-negligible fraction of this signal. Other possibilities include the Sunyaev-Zeldovich effect (Bond *et al* 2002; Komatsu & Seljak 2002), primordial voids (cf. Griffiths *et al* 2002) or features in the primordial power spectrum (cf. Cooray & Melchiorri 2002). One needs to isolate and test for the possible magnetic field contribution. We point out here that this can be done by looking for the corresponding distinctive polarization signals.

Indeed, Seshadri and Subramanian (2001; Paper I) showed that tangled magnetic fields create distinctive small-scale (large l) polarization anisotropy, dominated by the odd parity B-type signals. Mack *et al.*(2002; MKK02) gave estimates for such signals, but in the low $l < 500$ regime. Recently the DASI experiment detected E-type polarization and both DASI and WMAP have detected the T-E cross correlation, albeit at smaller $l \sim 300$ (Kovac *et al* 2002; Kogut *et al* 2003). Several experiments to probe the polarization anisotropy at the large l regime are also underway or being planned (VSA, ACBAR, ATCA, Polatron and Planck Surveyor). Motivated by these experiments, we compute the B-type polarization signals at larger values of l, for a wider variety of cosmological parameters and spectral indices, than were made in Paper I. In the next section, we first recapitulate the arguments of Paper I. We then present in Section 3, approximate analytic estimates of the signals. The B-type anisotropy for various models, obtained from a detailed numerical integration is given in Section 4. We also calculate there numerically the E-type polarization anisotropy and the T-E cross correlations for these models. Our predictions will allow comparison with future observations and help in detecting or ruling out significant magnetic field contributions to the signal on small angular scales.

2 THE POLARIZATION ANISOTROPY

Polarization of the CMBR arises from the Thomson scattering of radiation from free electrons, and is sourced by the quadrupole component of the CMBR anisotropy. The evolution equations for the moments, Θ_l, E_l and B_l, of the temperature anisotropy ($\Delta T/T$), the electric (E-) type and the odd parity, magnetic (B-) type polarization anisotropies, respectively, for vector perturbations, have been derived in detail by Hu & White (1997a; HW97) (see also Paper I and SB98b). For vector perturbations, the B-type contribution dominates the polarization anisotropy (HW97). We therefore give details of its calculations and summarize the results for the E-type contribution and the T-E cross correlations. The quadrupole anisotropy source term for polarization is given by $P(k,\tau) = [\Theta_2 - \sqrt{6}E_2]/10$. Here k is the co-moving wave number, τ the conformal time. One can analytically estimate P using the tight-coupling approximation, $kL_\gamma(\tau) \ll 1$, where $L_\gamma(\tau)$ is the co-moving, photon mean free path. First, to leading order in this approximation, we have zero quadrupoles, and a dipole $\Theta_1 = v_B$, where $v_B(k,\tau)$ is the magnitude of the rotational component of the fluid velocity v_i^B, in Fourier space. However, to the next order the quadrupole is not zero. It is generated from the dipole at the 'last but one' scattering of the CMBR. From Eq. (60), (63) and (64) of HW97, we get $\Theta_2 = -4E_2/\sqrt{6} = 4kL_\gamma v_B/(3\sqrt{3})$ and hence $P = \Theta_2/4 = kL_\gamma v_B/(3\sqrt{3})$. Using this in Eq. (77) and (56) of HW97 gives an estimate for B_l and the angular power spectra C_l^{BB} due to B-type polarization anisotropy (cf. Paper I),

$$C_l^{BB} = 4\pi\frac{(l-1)(l+2)}{l(l+1)}\int_0^\infty \frac{k^2dk}{2\pi^2}\frac{l(l+1)}{2} \times < | \int_0^{\tau_0} d\tau g(\tau_0,\tau)(\frac{kL_\gamma(\tau)}{3})v_B(k,\tau) \times \frac{j_l(k(\tau_0-\tau))}{k(\tau_0-\tau)}|^2 > . \quad (1)$$

Here $j_l(z)$ is the spherical Bessel function of order l, and τ_0 the present value of τ. The 'visibility function', $g(\tau_0,\tau)$, determines the probability that a photon reaches us at epoch τ_0 if it was last scattered at the epoch τ. We adopt a flat universe throughout, with a total matter density Ω_m and a non-zero cosmological constant density $\Omega_\Lambda = 1 - \Omega_m$ today. We now briefly recall the arguments detailed in Paper I.

Firstly, we approximate the visibility function as a Gaussian: $g(\tau_0,\tau) = (2\pi\sigma^2)^{-1/2}\exp[-(\tau-\tau_*)^2/(2\sigma^2)]$, where τ_* is the conformal epoch of "last scattering" and σ measures the width of the LSS. To estimate these, we use the WMAP results (cf. Spergal *et al* 2003), that the redshift of LSS $z_* = 1089$ and its thickness (FWHM) $\Delta z = 194$. To convert redshift into conformal time we use, $\tau = 6000h^{-1}((a+a_{eq})^{1/2} - a_{eq}^{1/2})/\Omega_m^{1/2}$, valid for a flat universe (cf. Hu & White 1997b). Here, the expansion factor $a = (1+z)^{-1}$ and $a_{eq} = 4.17 \times 10^{-5}(\Omega_m h^2)^{-1}$ (h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}). For the Λ-dominated model suggested by WMAP results, with $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, we get $\tau_* = 201.4h^{-1}$ Mpc and $\sigma = 11.5h^{-1}$ Mpc. For an $\Omega_m = 1$ model, with the same baryon density, (cf. $\Omega_b = 0.0224h^{-2}$), we use the expressions given in Hu and Sugiyama (1995) to estimate $\tau_* = 131.0h^{-1}$ Mpc, and $\sigma = 8.3h^{-1}$ Mpc. We use these numbers in the numerical estimates below.

To evaluate C_l^{BB}, one needs to estimate v_B, the rotational velocity induced by magnetic inhomogeneities. The magnetic field is assumed to be initially a Gaussian random field. On galactic scales and above, the induced velocity is generally so small that it does not lead to any appreciable distortion of the initial field (Jedamzik, Katalinic & Olinto 1998, SB98a). So, the magnetic field simply redshifts away as $\mathbf{B}(\mathbf{x},t) = \mathbf{b}_0(\mathbf{x})/a^2$. The Lorentz force associated with the tangled field is then $\mathbf{F}_L = (\nabla \times \mathbf{b}_0) \times \mathbf{b}_0/(4\pi a^5)$, which pushes the fluid and creates rotational velocity perturbations. These can be estimated as in SB02 or Paper I, by using the Euler equation for the baryons. On scales larger than the photon mean-free-path at decoupling, where the viscous effect due to photons can be treated in the diffusion approximation, this reads (SB02)

$$\left(\frac{4}{3}\rho_\gamma + \rho_b\right)\frac{\partial v_i^B}{\partial t} + \left[\frac{\rho_b}{a}\frac{da}{dt} + \frac{k^2\eta}{a^2}\right]v_i^B = \frac{P_{ij}F_j}{4\pi a^5}. \quad (2)$$

Here, ρ_γ is the photon density, ρ_b the baryon density, and $\eta = (4/15)\rho_\gamma l_\gamma$ the shear viscosity coefficient associated with the damping due to photons, whose mean-free-path is $l_\gamma = (n_e \sigma_T)^{-1} \equiv L_\gamma a(t)$, where n_e is the electron density and σ_T the Thomson cross-section. We have also ignored here a metric perturbation term which is subdominant at large l (cf. Paper I). For $z_* \sim 1089$, we get $L_\gamma(\tau_*) \sim 1.83 f_b^{-1}$ Mpc, where $f_b = (\Omega_b h^2/0.0224)$ (the WMAP value). We have defined the Fourier transforms of the magnetic field, by $\mathbf{b}_0(\mathbf{x}) = \sum_{\mathbf{k}} \mathbf{b}(\mathbf{k}) \exp(i\mathbf{k}.\mathbf{x})$ and $\mathbf{F}(\mathbf{k}) = \sum_{\mathbf{p}} [\mathbf{b}(\mathbf{k}+\mathbf{p}).\mathbf{b}^*(\mathbf{p})]\mathbf{p} - [\mathbf{k}.\mathbf{b}^*(\mathbf{p})]\mathbf{b}(\mathbf{k}+\mathbf{p})$. The projection tensor, $P_{ij}(\mathbf{k}) = [\delta_{ij} - k_i k_j/k^2]$ projects $\mathbf{F}$ onto its transverse components perpendicular to $\mathbf{k}$.

The comoving Silk damping scale at recombination, $L_S = k_S^{-1} \sim 10$ Mpc, separates scales on which the radiative viscosity is important ($kL_S \gg 1$) from those on which it is negligible ($kL_S \ll 1$). For $kL_s \ll 1$, the damping due to the photon viscosity can be neglected compared to the Lorentz force. Assuming negligible initial rotational velocity perturbation, we can integrate the baryon Euler equation to get $v_i^B = G_i D$, where $G_i = 3P_{ij}F_j/[16\pi\rho_0]$ and $D = \tau/(1+S_*)$ (see SB02 or paper I). Here ρ_0 is the present-day value of ρ_γ, and $S_* = (3\rho_b/4\rho_\gamma)(\tau_*) \sim 0.59 f_b$. For $kL_s \gg 1$, we use the terminal-velocity approximation, neglecting the inertial terms in the Euler equation, to balance the Lorentz force by friction. This gives $v_i^B = G_i(\mathbf{k})D$, but with now $D = (5/k^2 L_\gamma)$, on scales where diffusion damping operates. The transition Silk scale can also be estimated by equating v_i^B in the two cases, to give $k_S \sim [5(1+S_*)/(\tau L_\gamma(\tau))]^{1/2}$.

To compute the C_l^{BB}s we need to specify the spectrum of the tangled magnetic field, say $M(k)$. We define, $< b_i(\mathbf{k}) b_j(\mathbf{q}) >= \delta_{\mathbf{k},\mathbf{q}} P_{ij}(\mathbf{k}) M(k)$, where $\delta_{\mathbf{k},\mathbf{q}}$ is the Kronecker delta which is non-zero only for $\mathbf{k} = \mathbf{q}$. This gives $< \mathbf{b}_0^2 >= 2\int (dk/k) \Delta_b^2(k)$, where $\Delta_b^2(k) = k^3 M(k)/(2\pi^2)$ is the power per logarithmic interval in k space residing in magnetic tangles, and we replace the summation over k space by an integration. The ensemble average $< |v_B|^2 >$, and hence the C_l^{BB}s, can be computed in terms of the magnetic spectrum $M(k)$. It is convenient to define a dimensionless spectrum, $m(k) = \Delta_b^2(k)/(B_0^2/2)$, where B_0 is a fiducial constant magnetic field. The Alfvén velocity, V_A, for this fiducial field is,

$$V_A = \frac{B_0}{(16\pi\rho_0/3)^{1/2}} \approx 3.8 \times 10^{-4} B_{-9}, \qquad (3)$$

where $B_{-9} \equiv (B_0/10^{-9}\text{Gauss})$. We will also consider as in SB02, power-law magnetic spectra, $M(k) = Ak^n$ cut-off at $k = k_c$, where k_c is the Alfvén-wave damping length-scale (Jedamzik, Katalinic & Olinto, SB98a). We fix A by demanding that the smoothed field strength over a "galactic" scale, $k_G = 1h\text{Mpc}^{-1}$, (using a sharp k-space filter) is B_0, giving a dimensionless spectrum for $n > -3$ of

$$m(k) = (n+3)(k/k_G)^{3+n}. \qquad (4)$$

3 ANALYTIC ESTIMATES

The dominant contributions to the integral over τ in Eq. (1) come from a range σ around the epoch $\tau = \tau_*$. Furthermore, $j_l(k(\tau_0 - \tau))$ picks out (k, τ) values in the integrand which have $k(\tau_0 - \tau) \sim l$. Thus, following the arguments detailed in Paper I and SB02, for $k\sigma \ll 1$ we get the analytical estimate, $l(l+1)C_l^{BB}/(2\pi) \approx (kL_\gamma(\tau*)/3)^2(\pi/4)\Delta_v^2(k,\tau_*)|_{k=l/R_*}$. Here, $\Delta_v^2 = k^3 < |v_B(k,\tau_*)|^2 > /(2\pi^2)$ is the power per unit logarithmic interval of k, residing in the *net* rotational velocity perturbation, and $R_* = \tau_0 - \tau_*$. In the opposite limit, $k\sigma \gg 1$, we get $l(l+1)C_l^{BB}/(2\pi) \approx (kL_\gamma(\tau*)/3)^2(\sqrt{\pi}/4)(\Delta_v^2(k,\tau_*)/(k\sigma)|_{k=l/R_*}$. At small wavelengths, C_l^{BB} is suppressed by a $1/k\sigma$ factor due to the finite thickness of the LSS. Further in both cases, the polarization anisotropy, $\Delta T_P^{BB}(l) \approx (kL_\gamma(\tau_*)/3) \times \Delta T(l)$, where, $\Delta T(l)$ is the temperature anisotropy computed in SB02. We can now put together the above results to derive approximate analytic estimates for the CMBR polarization anisotropy induced by tangled magnetic fields. As a measure of the anisotropy we define the quantity $\Delta T_P^{BB}(l) \equiv [l(l+1)C_l^{BB}/2\pi]^{1/2} T_0$, where $T_0 = 2.728$ K is the CMBR temperature. On large scales, such that $kL_s < 1$ and $k\sigma < 1$, the resulting CMBR anisotropy is (see Paper I)

$$\Delta T_P^{BB}(l) = T_0 (\frac{\pi}{32})^{1/2} I(k) \frac{k^2 L_\gamma(\tau_*) V_A^2 \tau_*}{3(1+S_*)}$$
$$\approx 0.4\mu K \left(\frac{B_{-9}}{3}\right)^2 \left(\frac{l}{1000}\right)^2 I(\frac{l}{R_*}). \qquad (5)$$

Here, $l = kR_*$ and we adopted the Λ-dominated model favored by WMAP, with $\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$, $\Omega_b h^2 = 0.0224$ and $h = 0.71$ (in Paper I, we used a purely matter-dominated $\Omega_m = 1$ model). We also use the fit given by Hu & White (1997b) to calculate $\tau_0 = 6000h^{-1}((1+a_{eq})^{1/2} - a_{eq}^{1/2})(1 - 0.0841 \ln(\Omega_m))/\Omega_m^{1/2}$, valid for flat universe. On scales where $kL_S > 1$ and $k\sigma > 1$, but $kL_\gamma(\tau_*) < 1$, we get

$$\Delta T_P^{BB}(l) = T_0 \frac{\pi^{1/4}}{\sqrt{32}} I(k) \frac{5V_A^2}{3(k\sigma)^{1/2}}$$
$$\approx 1.2\mu K \left(\frac{B_{-9}}{3}\right)^2 \left(\frac{l}{2000}\right)^{-1/2} I(\frac{l}{R_*}). \qquad (6)$$

The function $I^2(k)$ in the Eqs.(5) and (6) is a dimensionless mode-coupling integral given in our previous papers (cf. Eq. (7) of Paper I). Analytic approximations to $I(k)$ exist for power-law spectra and for $k \ll k_c$ (as generally relevant even at high l; cf. Paper I and MKK02) For $n > -3/2$,

$$I^2(k) = \frac{28}{15} \frac{(n+3)^2}{(3+2n)} (\frac{k}{k_G})^3 (\frac{k_c}{k_G})^{3+2n}, \qquad (7)$$

dominated by the cut-off scale k_c. For $n < -3/2$ (SB02),

$$I^2(k) = \frac{8}{3}(n+3)(\frac{k}{k_G})^{6+2n} \qquad (8)$$

independent of k_c, where we neglect a subdominant term of order $(k_c/k)^{3+2n} \ll 1$; note that $k \ll k_c$. A nearly scale-invariant spectrum, say with $n = -2.9$, then gives $\Delta T_P^{BB}(l) \sim 0.16\mu K (l/1000)^{2.1}$ for scales larger than the Silk scale, and $\Delta T_P^{BB}(l) \sim 0.51\mu K (l/2000)^{-0.4}$, for scales smaller than L_S but larger than L_γ. Larger signals result for steeper spectra, $n > -2.9$ at the higher l end.

4 NUMERICAL RESULTS

We now compute $\Delta T_P^{BB}(l)$ for the above spectra, by evaluating the τ and k integrals in Eq.(1) numerically. For this we first express Eq. (1) explicitely in terms of the magnetic correlation function. We have for $l \gg 1$, $(\Delta T_P^{BB}(l)/T_0)^2 = \int_0^\infty (dk/k)(l^2 k V_A^2 I(k) U(k)/\sqrt{8})^2$, where

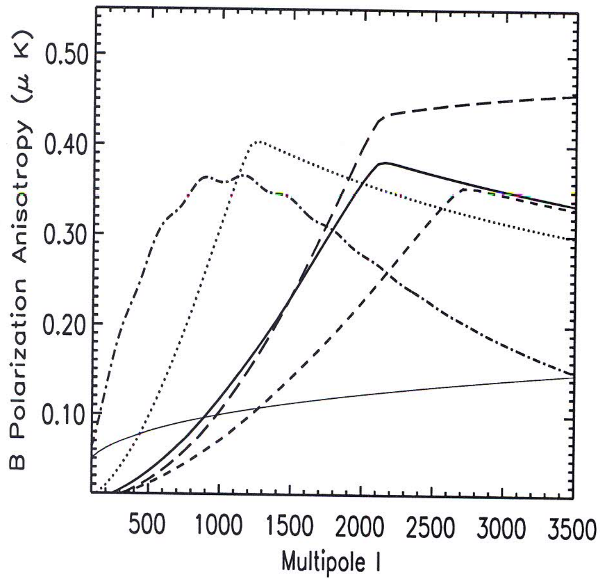

Figure 1. ΔT_P^{BB} versus l predictions for different cosmological models and magnetic power spectrum $M(k) \propto k^n$, for $B_{-9} = 3$. The bold solid line is for a standard flat, Λ-dominated model, with $\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$, $\Omega_b h^2 = 0.0224$, $h = 0.71$ and almost scale invariant spectrum $n = -2.9$. The long dashed curve obtains when one changes to $n = -2.5$, while the short dashed curve is for a larger baryon density $\Omega_b h^2 = 0.03$. The dotted curve gives results for a $\Omega_m = 1$ and $\Omega_\Lambda = 0$ model, with $n = -2.9$. These curves show the build up of power in B-type polarization, due to vortical perturbations from tangled magnetic fields which survive Silk damping at high $l \sim 1000 - 3500$. The eventual flattening or slow decline is due to the damping by photon viscosity, although this is only a mild effect as the magnetically sourced vortical mode is overdamped. We also show for qualitative comparison (dashed-dotted curve), the B-type polarization anisotropy due to gravitational lensing, in the canonical Λ-CDM model, computed using CMBFAST (Seljak & Zaldarriaga 1996; Zaldarriaga & Seljak 1998). The signal due to magnetic tangles dominate for l larger than about 1000. Finally, the thin solid line gives the expected galactic foreground contribution estimated by Prunet *et al* (1998), which is also smaller than the predicted signals.

$$U(k) = \int_0^{\tau_0} d\tau \; g(\tau_0, \tau) \frac{kL_\gamma(\tau)}{3} D(k, \tau) \frac{j_l(k(\tau_0 - \tau))}{k(\tau_0 - \tau)}. \qquad (9)$$

In doing the above integral numerically, we retain the analytic approximations to $I(k)$ and D with a transition between the limiting forms of D, at wavenumber k_S. (The function j_l is treated as in codes like CMBFAST). The numerically evaluated results are shown in Figures 1 and 2.

We see that for $B_0 \sim 3 \times 10^{-9} G$, this leads to a predicted RMS B-type polarization anisotropy in the CMBR of order $0.3 - 0.4\mu K$ for $1000 < l < 5000$, for a nearly scale-invariant power law spectra with $n = -2.9$. Larger signals result, at the high l end, even for a moderately steeper spectral index, with $n = -2.5$, which has more power on small-scales (compare the long dashed and solid curves). Much larger signals result from even steeper spectra with $n > -2.5$. However, these cases are probably ruled out for $B_{-9} \sim 3$, as they lead to an over production of gravitational waves (Caprini & Durrer 2002). Hence we do not display the results for such spectra explicitely. The peak of the polarization signal shifts to larger values of l with increasing Λ (compare the solid and dotted curves), due to an increase in R_*. This also happens for larger baryon density (compare the solid and short dashed curves), due to the decrease in L_γ and hence the damping effects of radiative viscosity. The l-dependence of $\Delta T_P^{BB}(l)$ got by analytic approximation matches very well with that obtained by numerical integration. The amplitude is, however, somewhat overestimated by the analytic treatment; by a factor of about 1.2 and 1.6 respectively, for the regimes above and below the Silk scale. So one can indeed get a reasonable idea of the signals from the analytics, but for more accurate amplitudes, one needs the numerical integration done above.

We have also numerically computed the E-type polarization (C_l^{EE}) as well as the T-E cross correlation (C_l^{TE}) for the above models. For this we use Eqns. (77), (79) and (56) given in HW97 and evaluate the k and τ integrals numerically, adopting the same analytic approximations for $I(k)$ and D given above. In calculating the T-E signal, we have included a small polarization contribution to T as well, which contributes negligibly to T, but significantly to C_l^{TE}. The cross correlation due to the vortical modes induced by magnetic tangles has a negative sign and so we define the corresponding effective 'temperature' after taking a modulus of C_l^{TE}. We show in Figure 2 the resulting E and T-E anisotropies, for the standard Λ-dominated model. For comparison, we have also give the temperature (T) and B-type polarization anisotropies (for T we have corrected a normalisation error made in SB02). The E-type polarization has a peak value of $\sim 0.1\mu$ K and so is much smaller that the B-type signal, as expected for vector perturbations (cf. HW97). The T-E cross correlation is $\sim 0.1 - 0.2\mu K$, for $l > 1000$. However, both E and T-E power are subdominant to that produced by the standard scalar perturbations.

5 DISCUSSION

We have re-examined the small angular scale polarization anisotropy induced by tangled magnetic fields, for different cosmological parameters and spectral indices. A major motivation arises from ongoing and future experiments, which probe this large l regime. Tangled magnetic fields generate vortical perturbations, which lead to a distinctive polarization anisotropy dominated by a B-type contribution. Vortical perturbations survive Silk damping on much smaller scales than do compressional modes and their damping due to the finite thickness of the LSS is also milder. By contrast, in the standard non-magnetic models the polarization anisotropy is dominated by E-type contributions. A scale-invariant spectrum of tangled fields which redshifts to a present value of $B_0 = 3 \times 10^{-9}$ Gauss, produces B-type polarization anisotropies of order $\sim 0.3 - 0.4\mu K$ between $l \sim 1000 - 5000$. Larger signals are produced for steeper spectra with $n > -3$. These signals are also larger than the small B-type polarization induced by gravitational lensing at $l > 1000$ or the expected galactic foreground contribution. The peak of the signal shifts to larger l, in a Λ-dominated universe, or in a universe with larger Ω_b. Our results complement the small l ($l < 500$), and purely analytical estimates of MKK02. The polarization anisotropy peaks or troughs

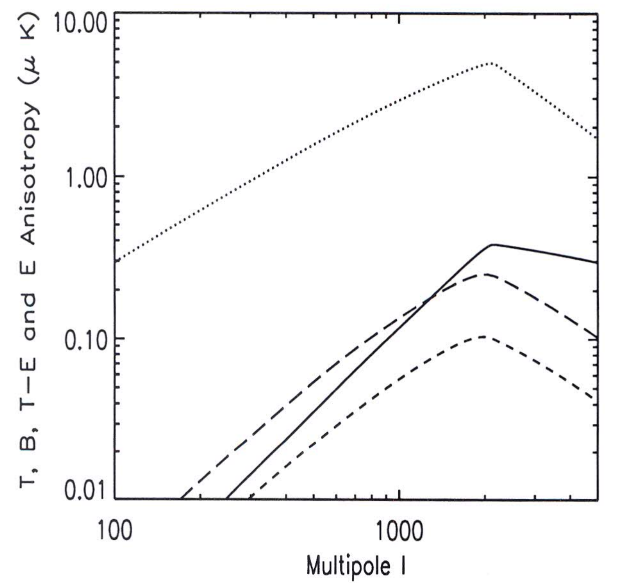

Figure 2. The predicted anisotropy in temperature (dotted line), B-type polarization (solid line), E-type polarization (short dashed line) and T-E cross correlation (long dashed line) up to large $l \sim 5000$ for the standard Λ-CDM model, due to magnetic tangles with a nearly scale invariant spectrum.

could be much larger, because the non-linear dependence of C_l^{BB} on $M(k)$, implies non-Gaussian statistics for the anisotropies. Clearly, with the sub-micro-Kelvin sensitivities expected from experiments like the Planck Surveyor, these signals can be detected.

An important contributor to small scale anisotropies, like that seen the CBI experiment, would be the Sunyaev-Zeldovich effect. This signal can be isolated by its frequency dependence. Also, scattering in clusters produces a much smaller statistical polarization anisotropy compared to the magnetically-induced signals (even individual clusters produce maximum signals of only $\sim 0.1\mu$K (cf. Sazonov & Sunyaev, 2000)). The polarization induced by the SZ effect, primordial voids, or features in the power spectrum are all expected to be predominantly E-type, in contrast to predominantly B-type signals predicted here. Primordial magnetic fields can also lead to depolarization due to differential Faraday rotation. This effect is only important at frequencies lower than about $16.4\mathrm{GHz}(B_{-9}/3)^{1/2}$ (Kosowsky & Loeb 1996; Harari, Hayward & Zaldariaga 1997). Additional B-type polarization can arise from tensor modes generated during inflation or those induced by the primordial magnetic fields (cf. Durrer, Ferreira & Kahniashvili 2000; MKK02; SB02); but these are expected to be important only at $l < 100$ or so. Helicity in the magnetic spectrum can also leave interesting signatures on the CMBR (Pogosian, Vachaspati & Winitzki 2002).

In summary, sensitive observational searches for B-type polarization anisotropies at large l will allow us to detect or constrain primordial, tangled magnetic fields. They will also tell us whether such fields are significant contributors to the excess power at large l detected by the CBI experiment and help probe possible new physics in the early universe.

ACKNOWLEDGMENTS

We would like to thank Pedro Ferreira for discussions.

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