Perturbed Power Law Parameters

Minu Joy

Alphonsa College, Kerala

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Perturbed Power-Law Parameters from WMAP7

Minu Joy & Tarun Souradeep,

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Introduction

Power spectrum Observations
Cosmological Parameters

Inflation
Gaussian, Adiabatic, nearly Scale Invariant perturbations

Power Law Inflation Models - uniform accelerated approximation

Power Law Inflation

Power law model of inflation $a(t) = t^p$ $p > 1$

$$\epsilon = -\delta = \frac{1}{p}$$

i. uniformly accelerated expansion $-\frac{\dot{H}}{H^2} = const$

ii. $\delta\phi$, massless scalar field perturbations, $m_{eff}^2 = 0$

Scalar and tensor perturbations spectra of identical shape with const n_s and n_t $(n_s = n_t + 1)$

Standard slow-roll inflation

Scale invariant spectrum, $\mathcal{P}(k) \propto k^{(n-1)}$

$n_s = 1 \longrightarrow$ Scale free Harrizon - Zeldovich spectrum

Spectral Index, $n = 1 + \frac{d \ln P(k)}{d \ln k}$
Relative distribution of power on various scales

Running of the Spectral Index $\frac{dn}{d \ln k}$
Deviation of the primordial power specturm from power law

Models with soft departures from scale free spectra

Hubble parameter $H(\phi)$

$$V(\phi) = \frac{3m_{\mathcal{P}}^2}{8\pi} H^2(\phi) \left[1 - \frac{m_{\mathcal{P}}^2}{12\pi} \left(\frac{\partial \ln H}{\partial \phi} \right)^2 \right] \tag{1}$$

The field equation for the modes of the inflaton perturbation, $\delta\phi_k$

$$\delta\phi_k'' + 2\frac{a'}{a}\delta\phi_k' + (k^2 + a^2 m_{eff}^2)\delta\phi_k = 0 \ , \tag{2}$$

$a^2 m_{eff}^2 = \frac{a''}{a} - \frac{z''}{z}$ and $z = a^2 \frac{\phi'}{a'}$.

Power Law Model

Effective mass

$$\frac{m_{eff}^2}{H^2} = -(\epsilon+\delta)(\delta+3) + \frac{\dot{\epsilon}}{H} - \frac{\dot{\delta}}{H} \qquad (3)$$

Slow-roll parameters

$$\epsilon = \frac{m_{\mathcal{P}}^2}{4\pi}\left(\frac{H_\phi}{H}\right)^2 \qquad \text{and} \qquad \delta = -\frac{m_{\mathcal{P}}^2}{4\pi}\frac{H_{\phi\phi}}{H} \qquad (4)$$

$\frac{H_\phi}{H} = (\frac{\partial \ln H}{\partial \phi})$ is a constant for the power law models with uniform acceleration $\Rightarrow \frac{m_{eff}^2}{H^2}$ will be zero

Power Law Model

Scalar and tensor perturbation spectra - parametrized by a common spectral index $\nu = n_s - 1 = n_t$

$$\nu = \frac{-2\epsilon}{1-\epsilon} = \frac{-4\left(\frac{\partial \ln H}{\partial \phi}\right)^2}{\left[1 - 2\left(\frac{\partial \ln H}{\partial \phi}\right)^2\right]} \tag{5}$$

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$$a(\eta) = \left(\frac{\eta}{\eta_*}\right)^{(1/2-\mu)} \qquad \mu = \frac{3}{2} + \frac{\epsilon(\eta)}{1-\epsilon(\eta)}$$

$$H^2(\phi) = \frac{(\mu - 1/2)^2}{\eta^2 a^2(\eta)}\ , \qquad \frac{z''}{z} = \frac{1}{\eta^2}\left[\mu_S^2 - \frac{1}{4}\right] \tag{6}$$

$$\mu_S^2 = \mu^2 - (\mu - 1/2)^2 \frac{m_{eff}^2}{H^2}$$

Perturbed Power Law

Small deviations from uniform acceleration -
$\longrightarrow \partial^2 \ln H / \partial \phi^2$ perturbation

Scalar and Tensor spectra are perturbed from the scale free form
At the leading order maintain a constant difference between their spectral indices

$$\frac{m_{eff}^2}{H^2} \simeq const \text{ (weakly depend on } \eta)$$

PPL models - $n_s \neq n_t + 1$

$$\nu_s(k) = \frac{-2\epsilon(\eta_*)}{1-\epsilon(\eta_*)} + \frac{2\chi(\eta_*)}{1-\epsilon(\eta_*)} + \mathcal{O}(\chi^2) \simeq \frac{-4\left[\left(\frac{\partial \ln H}{\partial \phi}\right)^2 + \left(\frac{\partial^2 \ln H}{\partial \phi^2}\right)\right]}{\left[1 - 2\left(\frac{\partial \ln H}{\partial \phi}\right)^2\right]} \tag{7}$$

$$\chi = \epsilon + \delta = -2\left(\frac{\partial^2 \ln H}{\partial \phi^2}\right) \tag{8}$$

$n_s - 1 = \nu_s \qquad n_t = \nu_t \qquad \nu_{st} = \nu_s - \nu_t$

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$$\chi = \epsilon + \delta = -2\left(\frac{\partial^2 \ln H}{\partial \phi^2}\right) \tag{8}$$

$$\nu_{st}(k) = \frac{2\chi(\eta_*)}{1-\epsilon(\eta_*)} = \frac{-4\left(\frac{\partial^2 \ln H}{\partial \phi^2}\right)}{\left[1 - 2\left(\frac{\partial \ln H}{\partial \phi}\right)^2\right]} \tag{9}$$

Scalar perturbation spectrum

$$\mathcal{P}_s(k) = \frac{\mathcal{A}(\mu_T, \mu_S)}{4\pi} \left(\frac{H_*}{2\pi}\right)^2 \frac{1}{\epsilon(\eta_*)} \tag{10}$$

Tensor perturbation spectrum

$$\mathcal{P}_t(k) = \frac{8\ \mathcal{A}(\mu_T, \mu_T)}{2\pi} \left(\frac{H_*}{2\pi}\right)^2 \tag{11}$$

$\mathcal{A}(x, y) = \frac{4^y \Gamma^2(y)}{(x-1/2)^{2y-1}}$

Perturbed Power Law

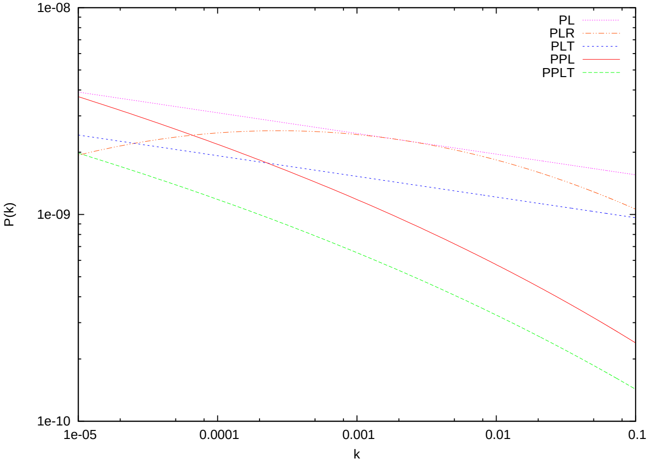
Standard slow-roll approximation $\mu_T = \mu_S = \mu$

PPL approach can be used to study models with comparatively larger ϵ and δ with the condition that $\chi = \epsilon + \delta$ is small (χ is zero for PL case)

$\mathcal{P}_s(k)$ and $\mathcal{P}_t(k)$

Running of their spectral indices also

PPL Spectra



PL spectrum for a model with $n_s = 0.9$, PLR spectrum with $n_s = 0.9$ and $n_{run} = -0.05$ and PLT spectrum with $n_t = -0.1$. The scalar and tensor PPL spectra are plotted with $\nu_s = -0.1$ ($n_s = 0.9$) and $\nu_{st} = -0.15$

- $\mathcal{P}_s(k)$ and $\mathcal{P}_t(k)$

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- ν_s and ν_{st}

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How well one can constrain $H(\phi)$ using CMB observation data ?

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How well one can constrain $H(\phi)$ using CMB observation data ?

T. Souradeep, J. R. Bond and L. Knox *et al*, 1998
Prospects for measuring Inflation parameters with the CMB : Proceedings of COSMO-97 - International workshop on Particle Physics and Early Universe, [arXiv:astro-ph/9802262].

Parameter Estimation with PPL

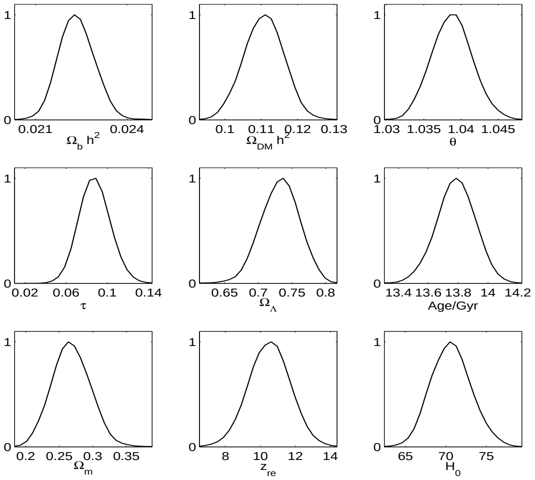
Markov Chain Monte Carlo sampling of the parameter space

Constraints on the inflationary parameters and the various background cosmological parameters

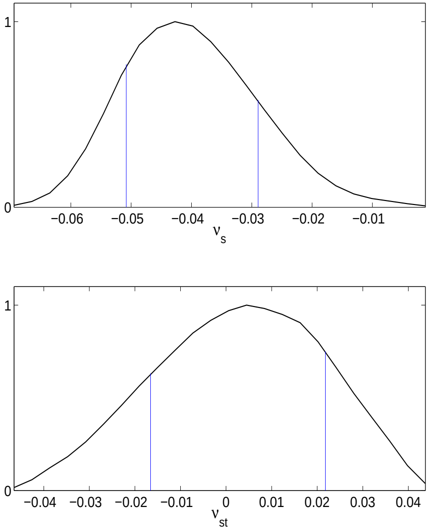
Parameter	Lower Limit	Upper Limit
$\Omega_b h^2$	0.005	0.1
$\Omega_c h^2$	0.01	0.99
θ	0.5	10
τ	0.01	0.8
ν_s	-0.15	0.0
ν_{st}	-0.06	0.06
$log[10^{10} A_s]$	2.0	4.2

Priors for the background cosmological parameters and the PPL inflationary parameters

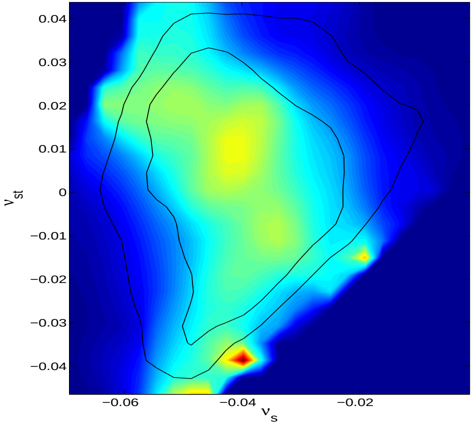
Background Cosmological Parameters



1D marginalized posterior distribution of the background cosmological parameters obtained by PPL fitting to WMAP7 data

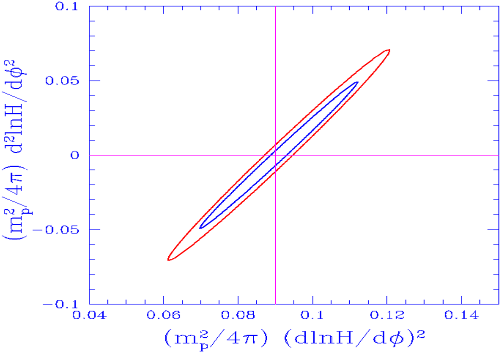


1D marginalized posterior distribution of the PPL inflationary parameters ν_s and ν_{st} obtained by fitting to WMAP7 data. The 95% marginalised limit on the parameters are shown by the vertical lines.

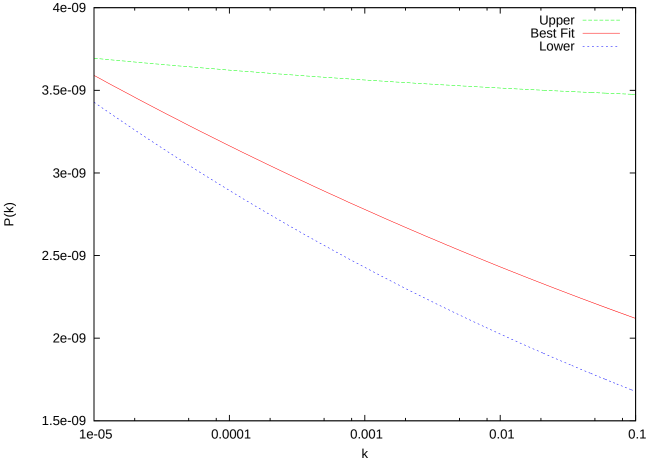


The joint 2D marginalized posterior distribution of ν_s and ν_{st}

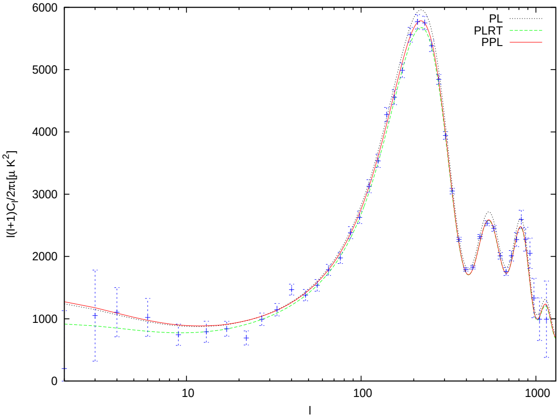
Forcast



The expected 1-σ error ellipses in the $(\partial \ln H/\partial\phi)^2$ - $(\partial^2 \ln H/\partial\phi^2)$



Ranges of possible deviation from PL spectrum with in 95% CL. Best fit PPL $\mathcal{P}(k)$ (red, solid) and the PPL spectra with upper ($\nu_s = -0.03$ and $\nu_{st} = 0.02$) and with lower ($\nu_s = -0.05$ and $\nu_{st} = -0.02$) limit of 95% CL of ν_s and ν_{st}



Best fit $C_\ell s$ for power-law (PL) (black, dotted), power law with tensor and running of scalar spectral index (PLRT) (green, dashed), power law with perturbative method (PPL) (red, solid)

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- Perturbative approach for models with small deviations from uniform acceleration
- PL consider the scalar and tensor spectral indices n_s, n_t and the running of scalar spectral index n_{run} as separate independent parameters.
- With PPL just the two parameters ν_s and ν_{st} will capture all these.
- PPL spectra takes care of the running of tensor spectral index also
- In contrast to the slow roll expansion method, for which ϵ and δ have to be small, PPL can look also at models with comparatively larger ϵ and δ with the condition that $(\epsilon + \delta)$ is small.

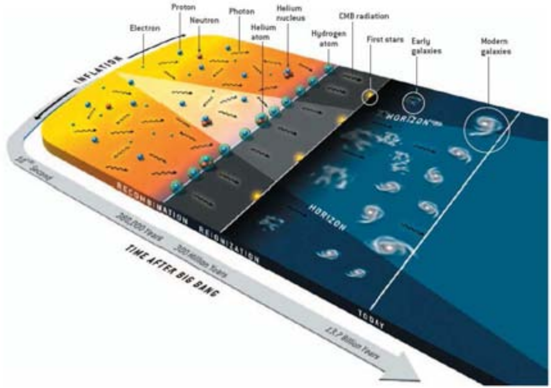
Perturbed Power Law

How the general models of inflation are best studied as departures from the power law model.

The probable values of the **deviation parameter** ν_{st}

Contained between $-0.04 < \nu_{st} < 0.04$.

Thank You !



Proton
Photon
Helium nucleus
Electron
Neutron
Helium atom
CMB radiation
Hydrogen atom
First stars
Early galaxies
Modern galaxies
INFLATION
HORIZON
RECOMBINATION
REIONIZATION
HORIZON
TODAY
TIME AFTER BIG BANG
300 Million Years
13.7 Billion Years

Model	No. of extra Parameters	χ^2_{eff}
PPL	1	7474.96
PL	0	7475.14
PLT	2	7475.00
PLTC	1	7475.07
PLR	1	7473.74
PLRT	3	7473.58
PLRTC	2	7473.61

The best fit likelihood χ^2_{eff} values for the perturbed power law (PPL), power law (PL), power law with tensor (PLT), power law with tensor obeying the consistency relation $n_t = -r/8$ (PLTC), power law with running of scalar spectral index (PLR), power law with running and tensor (PLRT), power law with running and tensor obeying the consistency relation (PLRTC) models.