

COSMOLOGICAL MODELS IN A CONFORMALLY
INVARIANT GRAVITATIONAL THEORY—II
A NEW MODEL

F. Hoyle and J. V. Narlikar

(Received 1971 July 27)

SUMMARY

It was seen in a previous paper how the mass of a particle can be determined in terms of a mass field $m(X)$. A dimensionless coupling λ constant is introduced, the individual particle mass being $\lambda m(X)$. Since the mass field can be explicitly calculated in terms of cosmic time τ and L^{-3} , the cosmic particle density, the numerical value of λ can be determined by relating $\lambda m(X)$ to empirically known particle masses. For the proton, $\lambda^2 \simeq 5 \cdot 10^{-38}$, which is of the same order as the inverse square root of the number of particles determining the present-day mass field.

In the usual theory it has to be supposed that this circumstance arises from coincidence. If, on the other hand, λ^2 is actually related to the number of particles giving rise to the mass field a new cosmological model is obtained. Since the number of particles giving rise to the mass field is a function of the epoch τ , so must λ be. The model resulting from the dependence of λ on τ therefore involves continuous creation of matter. The situation here is different from the steady-state model, however. With creation occurring in active centres rather than smoothly, the model has radically new astrophysical and geophysical properties. These are examined in some detail.

I. INTRODUCTION

In the previous paper (I) we started with the Friedmann models expressed in Robertson–Walker space. Then because our gravitational theory is conformally invariant we transformed to Minkowski space. We were able to show that the resulting time behaviour of the masses was consistent with our equations provided the Universe is taken to consist of two halves $\tau > 0$, $\tau < 0$ with opposite signs for the mass coupling. This has an effect analogous to the opposite signs of the charges of electrons and protons in electrodynamics. The equation for the mass was

$$m(X) = \sum_a m^{(a)}(X) = \sum_a \int \epsilon_A \tilde{G}(X, A) da, \quad (1)$$

with $\epsilon_A = +1$ for $\tau_A > 0$, $\epsilon_A = -1$ for $\tau_A < 0$.

If instead of starting with the Friedmann models and proving their consistency with (1) we start with (1), can we find any new cosmological models with new astrophysical properties? This is the question we propose to examine in the present paper. The Friedmann models seem to us to be unsatisfactory for two reasons.

(a) They do not meet the perfect absorber requirement in electrodynamics.

(b) In spite of something like 30 years of investigations the link between cosmology and astrophysics is still vague. Attempts to understand the key problem of the origin of galaxies have not met with the success that would be expected from a correct cosmological theory.

The steady state model has hitherto been the only cosmology to give an electro-dynamic arrow of time in the observed sense—that of increasing $\tau > 0$. It is well known (2, 3, 4) that to obtain the correct sense it is necessary for the future half of the light cone taken from any point X , $\tau_X > 0$, to be completely self-absorbing. The free-free absorption coefficient per electron of momentum p in ionized hydrogen of density n atoms per unit volume, with \hbar, c both unity, is

$$\frac{2}{3\sqrt{3}} \frac{e^6}{m_e p \nu^3} \cdot n g, \quad (1)$$

ν being the electromagnetic frequency, m_e the electron mass and g a Gaunt factor. The frequency stays constant in the Minkowski frame, as does p . The Gaunt factor being dependent on ν, p also stays constant.

To see that p can be considered constant for τ large enough we can take the electron-proton gas along the future light cone to be no longer heated by radiation from galaxies—the galaxies eventually burn themselves out. At first sight we might expect that it would be electron energies that would then stay constant, but electrons with random motions are still subject to a mass gradient term in the equations of motion. These terms can be removed and the equations made geodesic by transforming to a frame in which the electron mass is constant. Let the conformal transformation function Ω be such that

$$m_e^* = \Omega^{-1} m_e = \text{constant}. \quad (2)$$

In the cosmological problem—i.e. neglecting local irregularities— Ω is a function of τ only. The metric becomes

$$ds^{*2} = \Omega^2 [d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2)]. \quad (3)$$

With the equations of motion geodesic the random velocities of particles vary with time as Ω^{-1} . But velocities, being dimensionless, are independent of the conformal frame, and therefore vary as Ω^{-1} also in the Minkowski frame. By (2), it therefore follows that momentum is constant in the Minkowski frame.

Hence for any finite electron-proton density the opacity along the future light cone is determined by the behaviour of

$$\int^{\infty} \frac{d\tau}{m_e} \quad (4)$$

at its upper limit. In the Einstein-de Sitter model $m_e \propto \tau^2$ and the opacity is finite, which confirms statement (a) above.

To make (4) divergent at its upper limit we can formulate a new idea in the following general terms. So far as the macroscopic gravitational equations are concerned it is sufficient to consider the energy momentum tensor of a fluid instead of for a set of discrete particles. The critical term in the energy momentum tensor, T_{44} , involves the mass density. From a macroscopic point of view, we can regard the mass density as being made up of a number density $n(\tau)$ of particles each with mass proportional to $\tau^2/n(\tau)$. The variation of n with τ does not affect the mass density and hence does not affect the macroscopic theory. Take $n \propto \tau$, for example. The particle masses then vary only as τ . With $m_e \propto \tau$, and with the additional effect that the number density of particles now increases with τ , the integral (4) becomes divergent as $\tau \rightarrow \infty$, so that the future light cone is perfectly absorbing.

Instead of a constant number of particles, each with mass increasing as τ^2 , we contemplate the possibility of more particles having masses which increase less rapidly than τ^2 . In the following section this idea will be formulated in terms of variable coupling constants for the particles.

In order that the divergence of (4) shall meet the perfect absorber condition it is necessary that the free electron-proton density remain finite at $\tau \rightarrow \infty$. It could be argued that all diffuse gas eventually becomes condensed into stars and other compact objects. We would then need to consider absorption by stars. Since the future light cone eventually intersects a star, the problem is similar in principle to that considered above. The details would be different but we believe the result would be the same. It should make no difference whether matter is spread out smoothly or occurs in clumps so long as there are no gaps as $\tau \rightarrow \infty$ along the light cone.

2. VARIABLE COUPLINGS

We saw in the previous paper how the gravitational equations at a field point X are determined by particles at distances r from X less than τ_X . The summations which appear in the following equations will be regarded as similarly restricted. We also work with the many-particle approximations expressed in (23), (24) of the previous paper. Subject to these approximations the equations determined by

$$\delta S = 0, \quad g_{ik} \rightarrow g_{ik} + \delta g_{ik}, \tag{5}$$

$$g = - \sum_a \sum_b \int \int \epsilon_A \epsilon_B \tilde{G}(A, B) da db, \tag{6}$$

are

$$F(R_{ik} - \frac{1}{2}g_{ik}R) = -3(T_{ik} + \Phi_{ik}) + (g_{ik} \square F - F_{;ik}), \tag{7}$$

where

$$F = \frac{1}{2}m^2(X), \tag{8}$$

$$m(X) = \sum_b \int \epsilon_B \tilde{G}(X, B) db, \tag{9}$$

$$\Phi_{ik} = -[m_{;ik} - \frac{1}{2}g_{ik}m_{;l}{}^l], \tag{10}$$

$$T^{ik}(X) = \sum_a \int \delta_A(X, A)[-g(A)]^{-1/2} \epsilon_A m(A) \frac{da^i}{da} \frac{da^k}{da} da. \tag{11}$$

In macroscopic problems, whether local or cosmological, equations (7) are dealt with by converting the particle summations in (9), (11) to space-time integrations

$$m(X) = \int \tilde{G}(X, B)[\epsilon_B n(B)] d^4b, \tag{12}$$

$$T^{ik}(X) = \int \delta_A(X, A)[-g(A)]^{-1/2} m(A) \frac{da^i}{da} \frac{da^k}{da} [\epsilon_A n(A)] d^4a, \tag{13}$$

where $n(A)$, $n(B)$ are the particle densities at the points A , B where the space-time elements d^4a , d^4b are located. All particles in d^4a in (13) are considered to have the same tangent vector da^i/da .

In the Einstein-de Sitter model, as interpreted in the previous paper, the density function n was taken constant, and with the integrals restricted spatially to

distances $\leq \tau_X$ the coupling factors ϵ_A, ϵ_B were taken as $+1$. The length unit was chosen to make $n = 1$ and $m(X)$ was then found to be $\frac{1}{2}\tau_X^2$, which corresponds in the Minkowski frame to constant mass in the Robertson-Walker frame.

We wish to examine the possibility that the density function n is variable. Evidently so long as the product ϵn remains constant nothing is changed so far as the determination of $m(X)$ is concerned, or with respect to the gravitational equations. The behaviour of an individual particle can be drastically changed, however. The classical motion of a particle in an electromagnetic field depends on an individual term in the summation (11), viz.

$$\int \delta_4(X, A)[-g(A)]^{-1/2} m(A) \frac{da^i}{da} \frac{da^k}{da} \epsilon_A da \quad (14)$$

for particle a . The coupling factor ϵ_A in (14) is no longer accompanied by $n(A)$, so that variability of ϵ_A with A affects the particle. Although cosmology is unchanged so long as the product ϵn remains constant, astrophysical processes depending on the discrete particle structure are affected.

Individual processes are even affected by multiplying the coupling factor ϵ by a constant, λ say, although equations (7) are not changed. This is because every term in (7) is multiplied by λ^2 which therefore simply cancels so far as gravitation alone is concerned.

We have been concerned so far only with cosmology, but we now see that the cosmological situation alone does not serve to define the theory sufficiently closely from an atomic point of view. To proceed further we need to enquire into individual processes. Laboratory data for such processes, while providing a rich store of information, unfortunately refers entirely to a very small range of τ . Laboratory data can hardly serve therefore to distinguish between ϵ strictly constant and ϵ varying slowly with τ . Yet the issue is of critical importance to astrophysics, since astrophysics is often concerned with problems that involve a considerable range in τ —problems of stellar evolution for example.

However, if laboratory data were to demand $\epsilon = +1$ at present, or ϵ of some simple form— 4π or $1/4\pi$ —this would be strongly suggestive that ϵ should be taken constant for all τ . On the other hand, if we find no such simple result, indeed if we find a distinctly peculiar result, this can at least be taken as providing a *prima facie* case for considering that perhaps ϵ might vary with τ . We examine this question in the following section.

3. LENGTH SCALES AND LENGTH RATIOS IN THE EINSTEIN-DE SITTER MODEL

In the preceding paper (I) equation (77) gave the relation in the Minkowski frame between the red shift z and the spatial distance r or an atomic light source,

$$1 + z = \left(\frac{\tau}{\tau - r} \right)^2, \quad (15)$$

τ being the time of observation. For $r/\tau \ll 1$ we have

$$z \simeq 2r/\tau. \quad (16)$$

Since the Hubble constant H is defined by $z \simeq Hr$,

$$\tau = 2H^{-1}. \quad (17)$$

Empirically, $H^{-1} \simeq 2 \cdot 10^{28}$ cm (the velocity of light being unity). The present epoch is therefore $\sim 4 \cdot 10^{28}$ cm in the Minkowski frame.

For the Einstein–de Sitter model $n(A)$, $n(B)$ in (12), (13) are constants, as also are ϵ_A , ϵ_B . We put

$$n(A) = n(B) = L^{-3}, \quad (18)$$

$$\epsilon_A = \epsilon_B = \lambda, \quad (19)$$

λ being dimensionless. With L as the length unit the particle density is unity. The evaluation of (12) gives

$$m = \frac{1}{2} \lambda \tau^2 L^{-3}, \quad (20)$$

a result already obtained in (1). From (13), together with (18), (19) the mass of a particle is λm . Applied to the proton we have

$$m_p = \lambda m = \frac{1}{2} \lambda^2 \tau^2 L^{-3}. \quad (21)$$

A numerical value for λ^2 can be obtained from the definition

$$8\pi G = \frac{6}{m^2} = \frac{6\lambda^2}{m_p^2}, \quad (22)$$

by using the known values of m_p and of the ‘gravitational constant’ G . Thus

$$\lambda^2 = \frac{4\pi G}{3} m_p^2 \simeq 5 \cdot 10^{-38}, \quad (23)$$

since empirically $G \simeq 10^{-66}$ cm², $m_p \simeq 10^{14}$ cm⁻¹. Putting $\tau \simeq 4 \cdot 10^{28}$ cm in (21), and using (23), we have

$$L^{-3} \simeq 2 \cdot 5 \times 10^{-6} \text{ cm}^{-3}. \quad (24)$$

This is the particle density in the Minkowski frame. In this connection it may be noted that, irrespective of empirical values for m_p , G , τ , equations (17), (21), (22) give

$$m_p L^{-3} = \frac{3H^2}{8\pi G}, \quad (25)$$

which relates the mass density $m_p L^{-3}$ to H , G in the usual way. In fact (25) is nothing more than the definition of G . However, (25) gives a useful result, namely (24), when the empirically known value of G is used.

Multiplying (24) by τ^3 , and putting $\tau \simeq 4 \cdot 10^{28}$ cm, gives a second large dimensionless number

$$L^{-3} \tau^3 \simeq 10^{80}. \quad (26)$$

This is the number of particles with which we are in communication at the present value of τ .

4. LARGE DIMENSIONLESS NUMBERS

The coupling factor λ^2 must be exceedingly small, $\sim 5 \cdot 10^{-38}$, in order that the dynamical masses of individual particles shall be as small as they are observed to be in atomic processes. How is this small dimensionless number to be explained? In every way of writing cosmologies based on Einstein’s theory this same question arises. It has remained unanswered within the scope of these cosmologies since it

was first noticed some 50 years ago by Einstein himself. A proposal for widening the theoretical basis along lines somewhat similar to the following discussion was made by Dirac (5) in 1937.

Evidently, λ^2 multiplied by the square root of (26) is not unduly different from unity,

$$\lambda^2(\tau^3 L^{-3})^{1/2} = o(1). \quad (27)$$

Can we interpret (27) as a physical relationship? The answer here is negative, since ϵ , L are not epoch dependent, so that (27) is only a coincidence occurring over a limited range of τ . This is the situation in the Einstein-de Sitter model. To make (27) a physical relation we must therefore depart from orthodox cosmology by regarding λ or L , or both, as dependent on τ .

In order that we may continue to use the same macroscopic gravitational results as in the Einstein-de Sitter model it is necessary that $\epsilon_A n(A)$, $\epsilon_B n(B)$ in (12), (13) be independent of points A , B . We require

$$\epsilon_A n(A) = \epsilon_B n(B) = \lambda L^{-3} = \text{constant}. \quad (28)$$

Any dependence of λ on epoch must be compensated by the same dependence in L^3 , L being the interparticle separation. From (27), (28) it is easily seen that

$$\lambda \propto \tau^{-1}, \quad n = L^{-3} \propto \tau. \quad (29)$$

This is the behaviour discussed in the introductory section. Unlike the Einstein-de Sitter model, it admits an electrodynamic arrow of time in the correct sense. The mass field m has the time dependence

$$m = \frac{1}{2} \lambda \tau^2 L^{-3} \propto \tau^2, \quad (30)$$

while

$$m_p = \lambda m \propto \tau, \quad (31)$$

$$G = \frac{3}{4\pi m^2} \propto \tau^{-4}. \quad (32)$$

The constants of proportionality in these relations are fixed by using present-day empirical values, $m_p \simeq 10^{14} \text{ cm}^{-1}$, $G \simeq 10^{-66} \text{ cm}^2$. Care is needed, however, because the numerical value of the present epoch is changed from the above value of $\sim 4 \cdot 10^{28} \text{ cm}$. This is because the relation between z , τ , r is changed from (15) to

$$1+z = \frac{\tau}{\tau-r}, \quad (33)$$

since the masses of individual particles now depend on the epoch linearly instead of quadratically. In place of (16), (17) we have

$$z \simeq \frac{r}{\tau}, \quad \tau = H^{-1}, \quad (34)$$

so that the present epoch is $\tau \simeq 2 \cdot 10^{28} \text{ cm}$. It is easily seen that

$$G \simeq 10^{-66} (\tau/2 \cdot 10^{28})^{-4} \text{ cm}^2, \quad (35)$$

$$m \simeq \left(\frac{3}{4\pi}\right)^{1/2} \cdot 10^{33} (\tau/2 \cdot 10^{28})^2 \text{ cm}^{-1}, \quad (36)$$

$$m_p \simeq 10^{14}(\tau/2 \cdot 10^{28}) \text{ cm}^{-1}, \quad (37)$$

$$L^{-3} \simeq 10^{-5}(\tau/2 \cdot 10^{28}) \text{ cm}^{-3}, \quad (38)$$

$$\lambda \simeq (5 \cdot 10^{-38})^{1/2} (\tau/2 \cdot 10^{28})^{-1}. \quad (39)$$

These relations apply when the epoch T is in centimetres except that they must not be used for $\tau \rightarrow 0$. For $\tau \simeq 10^9$ cm we have $\lambda \simeq 1$. At this epoch $m_p \simeq 10^{-5} \text{ cm}^{-1}$, $L \simeq 10^8$ cm. The Compton wavelength $m_p^{-1} \simeq 10^5$ cm is not greatly less than the interparticle separation. Indeed the Compton wavelength of the electron at $\tau \simeq 10^9$ cm is such that

$$m_e^{-1} \simeq L \simeq 10^8 \text{ cm}, \quad (40)$$

so that classical considerations are quite inapplicable for $\tau < \sim 10^9$ cm.

The length scale in (35) to (39) is the centimetre. It will be noted that the dimensionalities of the various quantities are never the same as the powers in τ . This is because empirical numerical values for G , m_p were used to obtain these relations. Dimensionalities are concealed in the numerical coefficients.

5. OBSERVATIONAL CONSIDERATIONS

Astrophysics is largely concerned with the behaviour of individual particles. To establish connections with known results it is convenient to apply the conformal transformation

$$\Omega = \frac{\tau}{2 \cdot 10^{28}}, \quad \tau \neq 0. \quad (41)$$

Then for $\tau \neq 0$ the relations (35) to (39) transform to

$$G^* \simeq 10^{-66}(\tau/2 \cdot 10^{28})^{-2} \text{ cm}^2, \quad (42)$$

$$m^* \simeq \left(\frac{3}{4\pi}\right)^{1/2} \cdot 10^{33}(\tau/2 \cdot 10^{28}) \text{ cm}^{-1}, \quad (43)$$

$$m_p^* \simeq 10^{14} \text{ cm}^{-1}, \quad (44)$$

$$L^{*-3} \simeq 10^{-5}(\tau/2 \cdot 10^{28})^{-2} \text{ cm}^{-3}, \quad (45)$$

$$\lambda^* = \lambda \simeq (5 \cdot 10^{-38})^{1/2} (\tau/2 \cdot 10^{28})^{-1}, \quad (46)$$

and the metric is

$$ds^{*2} = \left(\frac{\tau}{2 \cdot 10^{28}}\right)^2 [d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2)]. \quad (47)$$

Writing

$$2t = \left(\frac{\tau}{2 \cdot 10^{28}}\right)^2, \quad (48)$$

we have

$$ds^{*2} = dt^2 - 2t[dr^2 + r^2(d\theta^2 - \sin^2 \theta d\phi^2)]. \quad (49)$$

This line element is the same as that of a radiation-dominated Friedmann model. However the proportionality $G^* \propto \tau^{-2}$ differs from such a model. It is seen from (44) that the conformal transformation (41) makes individual particle masses independent of τ .

The astrophysical consequences of the present model are drastically different from the Einstein-de Sitter model from which we started. This again is due to $G^* \propto \tau^{-2}$. Perhaps the most telling argument against the model in its present form lies in the past brilliance of the Sun. Taking opacity by electron scattering the luminosity of a star when particle masses are constant is proportional to the fourth power of the 'gravitational constant'. With $G^* \propto \tau^{-2}$ this gives $L_{\odot} \propto \tau^{-8}$. It follows that the Sun was brighter in the past than it is now by a factor $\sim (1 - \Delta H^{-1})^{-8}$, where Δ is the age of the solar system. Taking $\Delta/H \simeq \frac{1}{3}$ this is a factor of ~ 3 .

While the past history of the Earth may preclude an increase in the solar constant by a factor of this order it is worth noting that temperatures at the Earth's surface are not as sensitive to the solar constant as might at first sight be supposed. Because of the albedo of the cloud cover the Earth possesses a self-adjusting process whereby even large increases in the solar constant produce only comparatively small variations of temperature. Thus an increase of cloud cover from the present-day value of about 50 per cent to 90 per cent would be equivalent at the Earth's surface to a fivefold reduction of the solar constant.

It is also worth recalling the difficulty of reconciling measurements by Davis (6) of the neutrino flux from the Sun with theoretical expectations based on conventional astrophysics. Every effort has been made in the theoretical calculations to force down the expected flux. Yet the observed value is significantly lower than the calculated value. The discrepancy could imply that the past history of the Sun has been very different from the usual picture. The present model would require the Sun to start with considerably less helium than is usually assumed.

Because all stars would be much brighter in the past, distant galaxies would be brighter. This has the effect of causing the Hubble red shift magnitude relation for galaxies to turn upwards at large z , leading to a significantly positive value being inferred for the parameter usually denoted by q_0 . While this expectation of the present model is in qualitative agreement with observation, quantitative calculation suggests that the effects would be more drastic than observation seems to permit. On the other hand the high past brilliance of stars could resolve a difficulty that has never been overcome in the usual model. Stars in galaxies formed at early epochs would be subject to extremely rapid evolution. The initial main-sequence would soon be eaten away to considerably smaller masses than we normally take to be the case. As τ increases towards the present-day epoch the luminosities of the remaining unevolved stars would fall to low values. Consequently such systems would appear at the present epoch as galaxies of very high mass-to-light ratios. Elliptical galaxies do indeed have very high mass-to-light ratios, a circumstance that causes severe difficulty in the usual theory.

The interplay which we have just described between difficulties and attractive features of the model suggests the point of view that while the model possesses a vein of truth the position has been oversimplified. The point of departure from the usual model lies in our interpretation of (27) as a relation valid at all τ . This can be achieved, however, without requiring λ, L to vary smoothly with τ . So long as (28) remains satisfied it would be adequate if λ, L were to change in discontinuous jumps, so long as the jumps were not too large. Since the interparticle distance L decreases according to (38) as $\tau^{-1/3}$, particle creation is implied. The condition (28) is a form of conservation equation. Decreasing λ with τ can be interpreted as a decrease of the individual particle masses, which is then made good through the

appearance of more particles. It is as if existing particles were to subdivide into more particles. If such an idea is to be related to observation it would be much preferable to suppose that new particles emerge from active centres rather than have new particles appearing homogeneously throughout space. And since λ is associated by (28) with the appearance of new particles this suggests that λ may not change homogeneously. The possibility has to be considered that λ may decrease more rapidly, possibly discontinuously, in active centres than it does in comparatively undisturbed circumstances.

In accordance with what has just been said we regard (29) as applying over a fair span of τ and as a spatial average rather than as a relation which applies in detail in all localities. In a particular locality we might have

$$\lambda \propto \tau^{-q}, \quad n = L^{-3} \propto \tau^q. \quad (50)$$

In such a locality we have the further proportionalities

$$G \propto \tau^{-4}, \quad m \propto \tau^2, \quad m_p = \lambda m \propto \tau^{2-q}. \quad (51)$$

Individual particle masses can be transformed to constants through a conformal transformation which is locally of the form $\Omega \propto \tau^{2-q}$. This gives

$$G^* \propto \tau^{-2q}. \quad (52)$$

For $q < 1$ local astrophysical problems are affected less drastically than they were in the uniform model.

However (50), with the possibility that q may be different in different localities, raises a further drastic feature which must now be considered. Light passing between two localities with different values of q will show a red shift effect, in the sense that light from the region of higher q will appear red shifted to an observer in the region of lower q .

Anomalous red shift effects of this kind have hitherto been dismissed by most astronomers even though there is now a considerable body of evidence to show that such effects exist (7, 8, 9, 10, 11, 12).

A striking feature of the data is that blue shifts are never observed. This requires q to be smaller in our own locality than it is in the anomalous regions, suggesting that the galaxy is in a general field of uniform q , whereas the anomalous regions are active centres, or have recently been active centres, of the kind mentioned above. Indeed the classical red shift-magnitude relation for galaxies implies the existence of such a uniform field. The two extreme possibilities for the field are.

- (i) $q = 1$, the model discussed above.
- (ii) $q = 0$ over limited ranges of τ , but with λ decreasing in jumps from time to time, so that q averages to unity when taken over a large range of τ .

There can of course be intermediate possibilities. It is attractive in relation to (ii) to suppose that active centres manage to anticipate the next downward transition of λ . This idea requires further investigation before it can be decided whether it can be incorporated into a quantitative theory.

6. THE STRUCTURE OF THE EARTH

Of all the effects of a model with the parameter q in (50) non-zero the effect on the structure of the Earth is the most immediate to our environment. It should be possible to decide from geophysical data whether a model with $q \neq 0$ is ruled out or whether indeed such a model is demanded by the data. Because in principle geophysics could be of decisive importance to cosmology we shall discuss the problem of the structure of the Earth at some length in this final section.

From the outset we choose a conformal frame in which the masses of individual particles are constant. In this frame $G^* \propto \tau^{-2q}$, as in (52). We have to study the effect on the Earth of such a changing 'gravitational constant'.

For our present purpose we can treat the Earth in terms of a 3-zone model, a fluid core of radius about 3500 km, a non-liquid mantle extending to about 6300 km and a thin crust of a few tens of kilometres. In this structure only the crust need be thought of as solid in the ordinary sense. The mantle as well as the core satisfies the equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G^* M_r}{r^2} \rho, \quad (53)$$

where r is the radial coordinate, P the pressure, ρ the density, and M_r is given by

$$M_r = 4\pi \int_0^r \rho x^2 dx. \quad (54)$$

It is sufficient to use the instantaneous value of G^* in (53).

Eliminating M_r between (53), (54) gives the following second order differential equation

$$\frac{1}{\rho r^2} \frac{d}{dr} \left(r^2 \frac{dP}{dr} \right) = -4\pi G^*. \quad (55)$$

In solving (55) account must be taken of the jump of density between core and mantle. The relevant boundary condition is that P , M_r be continuous.

If a series of homologous models with respect to G^* are to exist, it is immediately clear from (55) that dP/dr must scale with respect to

$$r \rightarrow \alpha r, \quad \rho \rightarrow \alpha^{-3} \rho, \quad (56)$$

according to some power of α ,

$$\frac{dP}{dr} \rightarrow \alpha^m \frac{dP}{dr}. \quad (57)$$

Then (55) requires

$$\alpha^{5+m} \propto G^*. \quad (58)$$

Lyttleton (13) has discussed Earth models analysed by Bullard (14). By taking seismically known values of the bulk modulus $k = dP/d \ln \rho$ he arrives at the conclusion that in the core

$$P = \frac{a_1}{b} \left[\left(\frac{\rho}{\rho_1} \right)^b - 1 \right], \quad (59)$$

and that in the mantle

$$P = \frac{a_2}{b} \left[\left(\frac{\rho}{\rho_2} \right)^b - 1 \right]. \quad (60)$$

Lyttleton emphasizes the remarkable point that to within the accuracy of the data the constant b is the same in the core and mantle. He finds

$$\begin{aligned} a_1 &= 1.3417 \times 10^{12} \text{ dyne cm}^{-2}, & \rho_1 &= 6.107 \text{ g cm}^{-3}, \\ a_2 &= 2.1505 \times 10^{12} \text{ dyne cm}^{-2}, & \rho_2 &= 4.057 \text{ g cm}^{-3}, \\ b &= 3.5. \end{aligned} \quad (61)$$

Since (59), (60) represent the seismic data with remarkable accuracy over a range of P of about a factor 10, it is reasonable to suppose they will continue to be satisfied for the smaller pressure variation produced by the changing value of G^* . According to (59), (60), dP/dr behaves with respect to a homologous change like

$$\frac{dP}{dr} \rightarrow \frac{1}{\alpha^{3b+1}} \frac{dP}{dr}, \quad (62)$$

and hence it happens, whether by chance or due to some physical adjustment which has taken place inside the Earth, that changing G^* can be followed by a sequence of homologous models. For $b = 3.5$ we have

$$\alpha \propto G^{*-1/3b-4} = G^{*-1/6.5}. \quad (63)$$

It will be recalled that we are working in the conformal frame in which particle masses are constant. In this frame $G^* \propto \tau^{-2q}$ so that—neglecting the thin solid crust—the radius r_s of the Earth is proportional to $\tau^{q/3.25}$. We have

$$\frac{d \ln r_s}{d\tau} = \frac{q}{3.25\tau} \simeq \frac{1}{2} q H, \quad (64)$$

where H is the Hubble constant. Empirically, H^{-1} is about 1.8×10^{10} years. Inserting this value in (64), together with $r_s \simeq 6300$ km, and measuring τ in units of 10^8 years gives

$$\frac{dr_s}{d\tau} \simeq 10 q \text{ km } (10^8 \text{ yr})^{-1}, \quad (65)$$

$$8\pi r_s \frac{dr_s}{d\tau} \simeq 2.10^6 q \text{ km}^2 (10^8 \text{ yr})^{-1}, \quad (66)$$

for the present rate of increase of the Earth's radius and for the rate of increase of the surface area.

As Lyttleton points out (13) the forms (59), (60) lead to polytropic solutions of (55). Because b is the same for core and mantle the Emden equation is the same. In fact the Emden equation of index 0.4 applies throughout the Earth, except for the thin crust. Now the Emden equation, being of second order, has a twofold infinity of solutions. The core and mantle follow different members of this family, the change from one member to the other arising because dP/dr is discontinuous at the core-mantle interface. A homologous sequence of the kind considered above requires that the same two members of the Emden family shall apply at all values of G^* . Now what determines the two members? Each needs two boundary conditions and two members together require four boundary conditions. They are

$$\begin{aligned} P &\rightarrow 0 \quad \text{as } r \rightarrow r_s, \\ \frac{dP}{dr} &\rightarrow 0 \quad \text{as } r \rightarrow 0, \end{aligned} \quad (67)$$

P continuous, M_r continuous at the core-mantle interface. Each of these conditions can be expressed in terms of the dimensionless variables used in the Emden equation. The length scales appropriate to the core and mantle are fixed by knowing the masses of the core and mantle, and by knowing G^* . It is implicit in (67) that M_r is independent of time at the core-mantle interface. Over the intervals of interest here ($\sim 10^8$ years) and for an iron core this can be accepted as a good approximation.

We now ask: are all of (67) the same irrespective of the value of G^* ? An affirmative answer to this question was implicit in our use of the homologous sequence and hence affects the results (65), (66). Only $P \rightarrow 0$ as $r \rightarrow r_s$ needs consideration, since the other three conditions are rigorously satisfied at all G^* . It is here that the thin outer crust needs consideration. The presence of the crust requires the polytropic solution for the mantle to be terminated not at $P = 0$ but at some non-zero P . However, the maximum pressure that a crust of a few tens of kilometres can withstand from below is small compared to the interior pressures, so $P \rightarrow 0$ is a good approximation to the outer boundary condition of the Earth. The answer to our question is therefore affirmative to an adequate degree of approximation.

Although nothing new comes out of the question of the previous paragraph so far as the actual situation for the Earth is concerned, it is instructive to consider a drastically different case. Suppose the outer crust had infinite strength. Then the Earth could not expand. As G^* decreased, as the interior was released from gravitational compression, expansion would be resisted by the crust. A pressure would build up at the base of the crust, a pressure sufficient to force the interior solutions for the mantle and core on to the drastically different members of the Emden family demanded by a boundary condition very different from the first of (67). The new boundary condition would be that the volume of the Earth should stay fixed. As G^* became appreciably reduced the pressure on the crust would become comparable with interior values. The actual crust is of course far too flimsy to withstand any such enormous force. What this argument shows for the actual crust, however, is that it must all the time be at the limit of its strength.

The crust is under tension and it must be burst apart to provide for the increased surface area required by (66). Material must well up from below to fill the gaps between portions of the fragmented crust. Material fluid in the ordinary sense—fluid at low pressure—will come up most readily into the gaps. Because it happens that the sub-crustal part of the mantle is subject to significant radioactive heating such magmatic material exists in considerable quantity. Fluid can transmit pressure from below to the immediate sub-surface of the Earth, so that horizontal pressure gradients can arise. The necessary condition for significant horizontal pressures to arise in this way is that the fluid material be a somewhat lower (uncompressed) density than that of the overlying rocks. For fluid rising from a depth ~ 300 km as little as 1 per cent difference of density is sufficient to give rise to pressures of $\sim 10^9$ dyne cm^{-2} near the surface, and this may be sufficient to move portions of the crust horizontally.

The present picture would seem to fit the data in a satisfactory qualitative way. It provides, we believe for the first time, an escape from Jeffreys' argument against continental drift. From discussions we have had with geophysicists the situation is that recent evidence, for example sea-floor spreading, has carried widespread conviction that drift is actually taking place. Hence it is argued that Jeffreys must be wrong 'somewhere'. But where? Jeffreys, for his part, continues to point out

that the assumptions he made concerning the viscosity of crustal material have if anything proved conservative. Hence the forces required to move the continents must be at least as great as they were calculated to be. But was it correct to argue that these calculated forces are impossibly large? The only way to reconcile the data with the calculations is to suppose that the actual forces are very much larger than was thought plausible originally. Accepting drift as being demanded by the data, and accepting reasonable viscosity values for the crustal material, it seems clear that the drift forces must be exceedingly large. How else could a crack between Africa and S. America be opened up on so large a scale? The critical issue is whether, within the scope of conventional theory, there is a sensible way of explaining the magnitude of the required forces. If there is no sensible way—and the lack of a coherent explanation to date would suggest that this is so—then the theory discussed above receives powerful support.

Institute of Theoretical Astronomy, Madingley Road, Cambridge

REFERENCES

- (1) Hoyle, F. & Narlikar, J. V., 1971. *Mon. Not. R. astr. Soc.*, **155**, 305.
- (2) Hoyle, F. & Narlikar, J. V., 1963. *Proc. R. Soc. A*, **277**, 1.
- (3) Hogarth, J. E., 1962. *Proc. R. Soc. A*, **267**, 365.
- (4) Wheeler, J. A. & Feynman, R. P., 1945. *Rev. mod. Phys.*, **17**, 156.
- (5) Dirac, P. A. M., 1937. *Nature*, **139**, 323.
- (6) Davis, R., 1971. *Abstract. Am. Phys. Soc.*
- (7) Zwicky, F., 1956. *Naturwissenschaften*, **29**, 344.
- (8) Sargent, W. L. W., 1968. *Astrophys. J.*, **153**, L 135.
- (9) Sargent, W. L. W., 1970. *Vatican Conference*.
- (10) Arp, H. C., 1971. *Astrophys. Lett.*, **7**, 221.
- (11) Burbidge, E. M., Burbidge, G. R., Solomon, P. M. & Strittmatter, P. A., 1971. Preprint
- (12) Arp, H. C., 1971. Preprint
- (13) Lyttleton, R. A., 1963. *Proc. R. Soc.*, **A 275**, 1.
- (14) Bullard, E. C., 1954. *The Earth as a Planet*, University of Chicago Press.