

Model independent reconstruction of the expansion history of the universe and the properties of dark energy

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ABSTRACT

We have improved upon the method of smoothing supernovae data to reconstruct the expansion history of the universe, $h(z)$, using two latest datasets, Gold and SNLS. The reconstruction process does not employ any parameterisation and is independent of any dark energy model. The reconstructed $h(z)$ is used to derive the distance factor A up to redshift 0.35 and the results are compared with the given value of A from detection of baryon acoustic oscillation peak (BAO). We find very good agreement between supernovae observations and the results from BAO for $\Omega_{0m} \approx 0.276 \pm 0.023$. The estimated values of Ω_{0m} are completely model-independent and are only based on observational data. The derived values of Ω_{0m} are then used to reconstruct the equation of state of dark energy, $w(z)$. Using our smoothing method we can demonstrate that while SNLS data are in very good agreement with Λ CDM, the Gold sample slightly prefers evolving dark energy. We also show that proper estimation of the equation of state of dark energy at the high redshifts would be impossible at the current status of observations.

Key words: cosmology: theory—cosmological parameters

1 INTRODUCTION

Over the last two decades, cosmology has entered the stage of ‘precision’ science, which involve significant improvements in observational techniques, implementation of more powerful statistical and mathematical tools, and, of course, greatly advanced computational facilities. While these advancements have yielded much new insight into the subject, many important questions still remain unanswered. The nature of dark energy has been the subject of much debate over the past decade (Sahni & Starobinsky 2000; Carrol 2001; Peebles & Ratra 2003; Padmanabhan 2003; Sahni 2004; Copeland & Sami & Tsujikawa 2004). Supernovae data, which gave the first indication of the accelerated expansion of the universe, are expected to elucidate this interesting question further, as the quality of the data steadily improves (Riess et al. 1998; Perlmutter et al. 1999; Knop et al. 2003; Tonry et al. 2003; Riess et al. 2005; Astier et al. 2005; Riess et al. 2006). Much attention in recent years has focused on determining the properties of dark energy in a model independent manner. This can be done using either parametric (Starobinsky 1998; Huterer & Turner 1999; Saini et al. 2000; Chiba & Nakamura 2000; Alam et al. 2003; Chevallier & Polarski 2001;

Weller & Albrecht 2002; Gerke & Efstathiou 2002; Maor et al. 2002; Corasaniti & Copeland 2003; Linder 2003; Wang & Mukherjee 2004; Saini, Weller & Bridle 2004; Nesseris & Perivolaropoulos 2004; Roy Choudhury & Padmanabhan 2005; Gong 2005; Linder & Huterer 2005; Huterer & Peiris 2006; Alam, Sahni & Starobinsky 2007; Gong & Wang 2006) or non-parametric methods (Huterer & Starkman 2003; Wang & Lovelace 2001; Saini 2003; Daly & Djorgovsky 2003; Wang & Tegmark 2004, 2005; Huterer & Cooray 2005; Shafieloo et al. 2006; Bonvin, Durrer & Kunz 2006; Fay & Tavakol 2006). A comprehensive recent review has been given by Sahni & Starobinsky (2006). An earlier work by Shafieloo et al. (2006) suggested a non-parametric method based on smoothing the supernova data over redshift in order to reconstruct cosmological quantities, including the expansion rate, $h(z)$, and the equation of state of dark energy, $w(z)$, in a *model-independent* manner. In this approach, the data are dealt with directly, and one does not rely on a parametric functional form for fitting any of the quantities $d_L(z)$, $h(z)$ or $w(z)$. The result obtained by using this approach is, therefore, expected to be model-independent. This method was shown to be successful in discriminating between different models of dark energy if the quality of data is commensurate with that expected from the future SuperNova Acceleration Probe (SNAP). In

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this paper we improve the smoothing method and apply it to two recent sets of supernovae data: Gold (Riess et al. 2006) and SNLS (Astier et al. 2005). We then compare the derived expansion history of universe with the results of baryon acoustic peak observations (Eisenstein et al. 2005). Specifically, we use the improved smoothing method to reconstruct the Hubble parameter, $h(z)$, and then derive the distance factor, A , up to a redshift of 0.35 independently of the assumption of any cosmological model. This derived value, based on supernovae data, is then compared with the distance factor A (which is also claimed to be relatively independent of dark energy model) being determined by the detection of the baryon acoustic oscillation peak. One of the main results of this paper is that there is a good agreement between supernovae data (both Gold and SNLS) and baryon acoustic peak observations for the values $\Omega_{0m} \approx 0.276 \pm 0.023$. The derived value of Ω_{0m} is then used to reconstruct the equation of state of dark energy for both supernovae datasets. We should emphasize here that all the results in this paper are only based on observational data and no theoretical model has been assumed. This is an advantage of this method over the functional fitting methods in which the reconstructed results are biased by an assumed functional form or a theoretical model. The paper is organised as follows. In Section II we briefly explain the smoothing method and we estimate the accuracy of the method based on the quality and the quantity of current datasets. In Section III we apply the smoothing method on the Gold supernovae dataset and by using the results of detection of baryon acoustic oscillation peak, we reconstruct $w(z)$. In Section IV, we carry out a similar treatment on the SNLS dataset. Finally, in Section V we discuss our results along with some concluding remarks.

2 METHOD OF SMOOTHING

The method of smoothing belongs to the category of non-parametric methods of reconstruction which is complementary to the approach of fitting a parametric ansatz to the dark energy density or the equation of state. Most papers using the non-parametric approach exploit a kind of top-hat smoothing in redshift space. Instead, we follow a procedure which is well known and frequently used in the analysis of large-scale structure (Coles & Lucchin 1995; Martinez & Saar 2002); namely, we attempt to smooth noisy data directly using a Gaussian smoothing function. In this method we apply Gaussian smoothing to supernova data (which are of the form $\{\ln d_L(z_i), z_i\}$) in order to extract information about important cosmological parameters such as $H(z)$ and $w(z)$. The smoothing algorithm calculates the luminosity distance at any arbitrary redshift z to be

$$\ln d_L(z, \Delta)^s = \ln d_L(z)^g + N(z) \sum_i [\ln d_L(z_i) - \ln d_L(z_i)^g] \times \exp \left[-\frac{\ln^2 \left(\frac{1+z_i}{1+z} \right)}{2\Delta^2} \right], \quad (1)$$

$$N(z)^{-1} = \sum_i \exp \left[-\frac{\ln^2 \left(\frac{1+z_i}{1+z} \right)}{2\Delta^2} \right].$$

Here $\ln d_L(z, \Delta)^s$ is the smoothed luminosity distance at any redshift z which depends on luminosity distances of each SNe event with the redshift z_i , and $N(z)$ is a normalisation parameter. The quantity $\ln d_L(z)^g$ represents a guess background model which we subtract from the data before smoothing. This approach allows us to smooth noise only, and not the luminosity distance. After noise smoothing, we add back the guess model to recover the luminosity distance. This procedure is helpful in reducing noise in the results. Since we do not know which background model to subtract, we may take as a reasonable guess that the data should be close to Λ CDM and use $d_L(z)^g = d_L(z)^{\Lambda\text{CDM}}$ as a first approximation and then use a boot-strapping method to find successively better guess models. Having obtained the smoothed luminosity distance, we differentiate it once to obtain the Hubble parameter,

$$H(z) = \left[\frac{d}{dz} \left(\frac{d_L(z)}{1+z} \right) \right]^{-1}, \quad (2)$$

and once again to obtain the equation of state of dark energy $w(z)$,

$$w(z) = \frac{[2(1+z)/3] H'/H - 1}{1 - (H_0/H)^2 \Omega_{0m} (1+z)^3}. \quad (3)$$

In any kind of smoothing scheme for the luminosity distance, some bias is introduced both in d_L and in derived quantities like $H(z)$ and $w(z)$ (see appendix A1 in Shafieloo et al. (2006) to find detailed calculations of the bias). It is important to choose a value of Δ which gives a small value of the bias and also reasonably small errors on derived cosmological parameters. To estimate the value of Δ in (1), we consider the following relation between the reconstructed results, quality and quantity of the data and the smoothing parameters. One can show that the relative error bars on $H(z)$ scale as (Tegmark 2002)

$$\frac{\delta H}{H} \propto \frac{\sigma}{N^{1/2} \Delta^{3/2}}, \quad (4)$$

where N is the total number of supernovae (for approximately uniform distribution of supernovae over the redshift range) and σ is the noise of the data. From the above equation we see that a larger number of supernovae or larger width of smoothing, Δ , will decrease the error bars on the reconstructed H , but as it has been reported earlier (Shafieloo et al. 2006), the bias of the method is approximately related to Δ^2 . This implies that by increasing Δ we will also increase the bias of the results. If we attempt to estimate Δ such that $\frac{\delta H}{H} \propto 3\sigma$, then for $N = 182$ data points (which is the number of data points in the Gold sample), we get $\Delta = 0.084$ for a single iteration of our method. However, with each iteration, the errors on the parameters will increase. Therefore, using this value of Δ when we use an iterative process to find the guess model will result in such large errors on the cosmological parameters as to render the reconstruction exercise meaningless. It has been shown in Shafieloo et al. (2006) that at the M -th iteration, the error on $\ln d_L$ will be approximately $\delta_M(\ln d_L) \simeq \sqrt{M} \delta_0(\ln d_L)$, and the error on $\ln d_L$ scales as $1/\Delta$. We would like the errors after M iterations to be commensurate with the optimum errors obtained for a single iteration, δ_0 , so we require $\Delta_{\text{optimal}} \simeq \sqrt{M} \Delta_0$. Therefore, if we wish to stop the boot-strapping after 50 iterations, then $\Delta_{\text{optimal}} \simeq 0.6$. However,

after this rough estimation of the values of Δ and M , we can still play around these values to find the best combination, by minimising the likelihood of the reconstructed results to the data. In the following, we use $\Delta = 0.6$ and we calculate the χ^2 of the reconstructed distance moduli to the data after each iteration, and we stop the boot-strapping process after reaching the minimum value of χ^2 . This effect, that χ^2 of the reconstructed results goes to a minimum value and increases again with iteration is a reflection of the problem of some iterative reconstruction algorithms which are not error-sensitive. In these cases the noise will be added to the reconstructed results after certain number of iterations and the iterative process should be stopped after reaching the minimum value of χ^2 to get the best result. Similar effect has been reported and studied in the Richardson-Lucy deconvolution algorithm to reconstruct the form of primordial power spectrum from CMB data (Shafieloo & Souradeep 2004).

In appendix A we show that the results are not sensitive to the chosen value of Δ and also to the assumed initial guess model.

3 RESULTS FROM THE GOLD DATASET

The recently released Gold sample (Riess et al. 2006), consist of 182 supernovae type Ia which have been gathered from five different subsets of data, observed during the last 16 years. The range of redshift for these supernovae are between 0.024 and 1.75. In this section we use this dataset to reconstruct $h(z)$, estimate the value of Ω_{0m} , and then reconstruct $w(z)$. We choose a flat Λ CDM model with $\Omega_{0m} = 0.30$ as the initial guess model in our calculation and we fix the value of Δ (width of smoothing) to be 0.6. After each iteration, we compute the χ^2 and we stop the boot-strapping process once χ^2 reaches its minimum value.

The χ^2 at any iteration is calculated from the formula,

$$\chi_{rec,j}^2(H_0) = \sum_i \frac{(\mu_{rec,j}(H_0, z_i) - \mu_{obs}(z_i))^2}{\sigma_i^2} \quad (5)$$

and is followed by marginalising over H_0 . We have marginalised over H_0 by integrating over the probability density $p \propto \exp(-\chi^2/2)$ for all values of H_0 .

In Eq.5, $\mu_{rec,j}(H_0, z_i)$ is the reconstructed result at the j th iteration for the distance moduli at redshift z_i , assuming the value of H_0 , and $\mu_{obs}(z_i)$ is the Gold sample data given by Riess et al. (2006). In Figure 1 we show the χ^2 of the reconstructed results at different iterations, after marginalising over H_0 . As we see, the χ^2 has a minimum around $j = 89$ and after this, χ^2 is slowly increasing. So we stop the boot-strapping process at this iteration and determine $h(z)$. We can also see that for the initial guess Λ CDM model, the $\Delta\chi^2$ of the best recovered result is less than 4 which means that the flat Λ CDM model is in agreement with the Gold sample within 2σ .

By marginalizing over the Hubble parameter, we carry out a similar treatment on the data as it has been done by Riess et al. (2006) to calculate the χ^2 for different cosmological models. As the Gold data are based on a Hubble parameter of 65 km/sec/Mpc, the reconstruction method should be able to recover this value for the Hubble constant. In fact the peak of the probability density of the reconstructed result for different values of the Hubble parameter should

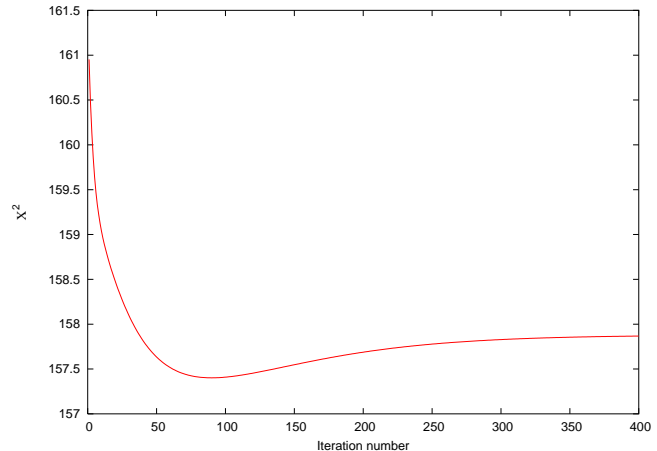


Figure 1. Computed χ^2 for the reconstructed results at each iteration, using Gold sample.

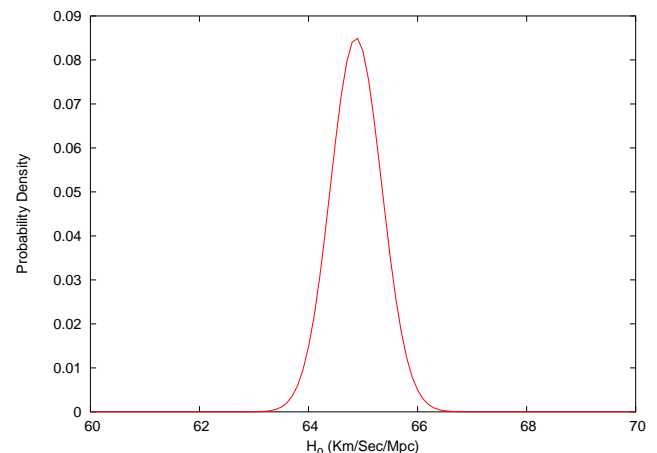


Figure 2. Probability density of the best reconstructed result from Gold data for different values of Hubble parameter.

be close to $H_0 = 65$ km/sec/Mpc. In Figure 2 we show the probability density of the best reconstructed result from Gold data for different values of the Hubble parameter. We see that the probability density has a sharp peak around $H_0 = 65$ km/sec/Mpc.

We should also note here, that the reduced χ^2 of the reconstructed results seems to be consistently below 1 (however it is not trivial to define the degree of freedom in our smoothing method and hence the reduced χ^2 , but we can see that the resultant χ^2 of the reconstructed results is around 25 less than the number of data points). We can also see in Figure 1 that the reduced χ^2 of the first initial guess model, which is a Λ CDM model, is also below 1. It shows that the error-bars of the supernovae data points are quite large and many different reconstructed results may have a reduced χ^2 of less than 1. In this paper we only calculate the χ^2 of the reconstructed results and we compare different results by calculating the $\Delta\chi^2$ to the best result with a minimum χ^2 .

In Figure 3, left panel, we show the reconstructed $h(z)$ for the Gold data set. The red solid line has the highest likelihood and is our best reconstruction. All the other lines are within 1σ away from the best recovered result. These lines are recovered results from our smoothing method by

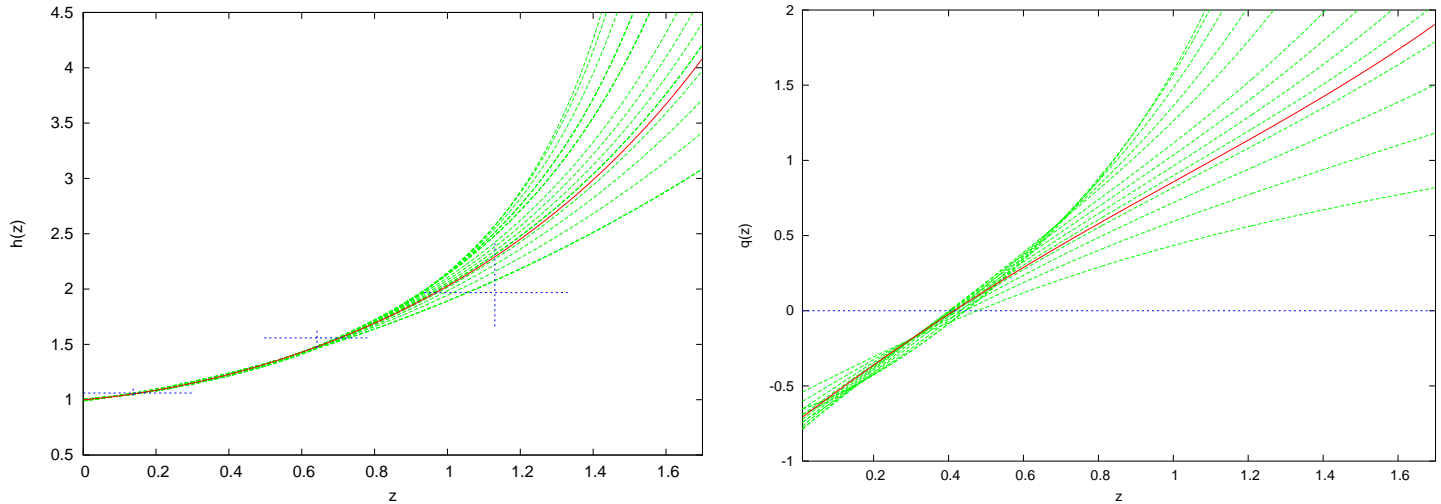


Figure 3. Reconstructed $h(z)$ (left) and $q(z)$ (right) by using Gold dataset. Red solid line is the best recovered result and the green dashed lines are within 1σ away from the best result. Based on our results, the transition between deceleration and acceleration phases of the universe occurs at $0.38 < z < 0.48$ within 1σ error-bar from the best recovered result. In the left panel we can also see 3 uncorrelated and independent measurements of $h(z)$ from the Gold sample (blue dotted crosses from Riess et al. (2006)) for comparison with our reconstructed results.

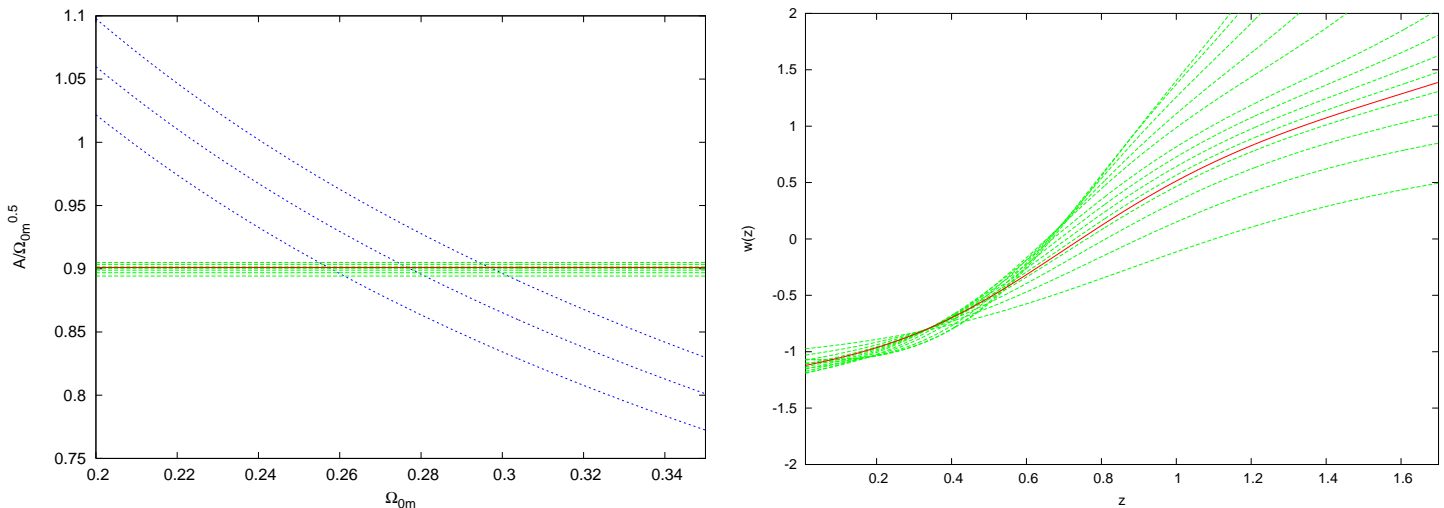


Figure 4. Left panel: The derived value of $A/\sqrt{\Omega_{0m}}$ from supernovae Gold data within its 1σ error-bars (red solid line and green dashed lines) in comparison with its measured value from observation of LRGs within its 1σ error-bars (blue dotted lines) for different values of Ω_{0m} . Right panel: Reconstructed $w(z)$ for the Gold dataset. Red solid line is the best recovered result and the green dashed lines are within 1σ away from the best result. To get these results, we have marginalised over $\Omega_{0m} = 0.277 \pm 0.022$.

using different numbers of iterations in the bootstrapping process. The $\Delta\chi^2$ for all of these lines is less than 1, and so we can consider them to lie within 1σ of the best result. We should note that these green dashed lines in Figure 3 are in fact a non-exhaustive sample of results which are within 1σ away from the best recovered result. As we see in the Figure 3, the reconstructed $h(z)$ at high redshift has a very big degeneracy. This is expected since there is only a single supernova beyond redshift 1.4!

In this figure we can also see 3 uncorrelated and independent measurements of $h(z)$ from the Gold sample (blue

dotted crosses from Riess et al. (2006)) for comparison with our results. We can see that these two results are consistent with each other within their 1σ limits. However we should mention here that in the Wang & Tegmark (2005) method used by Riess et al. (2006) for uncorrelated estimates of the expansion history, there is a slight bias in the reconstruction of $h(z)$ and the higher derivatives of the data. It is mainly because of using the average of the measured quantity $h(z)^{-1}$, which typically is not a straight line. So as it has been mentioned in Wang & Tegmark (2005), the measured average of $h(z)^{-1}$ (and hence $h(z)$) over a redshift bin

will generally lie either slightly above or below the actual curve at the bin center. This can be the reason that why the centres of the crosses in Figure 3, left panel, for uncorrelated estimates of the expansion history, are slightly above or below our reconstructed curve for the $h(z)$.

To reconstruct the Hubble parameter, $h(z)$, we do not need to know the value of Ω_{0m} . Another important cosmological quantity which we can derive from the reconstructed $h(z)$ (independent of the value of Ω_{0m}), is the deceleration parameter, $q(z)$,

$$q(z) = (1+z) \frac{H'(z)}{H(z)} - 1. \quad (6)$$

In Figure 3, right panel, we show reconstructed $q(z)$. For the Gold data our method shows that the transition between deceleration and acceleration occurred at $0.38 < z < 0.48$ (at 1σ). The best reconstruction shows the redshift of transition to be $z_a \simeq 0.42$. This is in agreement with results obtained using parametric methods (Alam, Sahni & Starobinsky 2007; Gong & Wang 2006).

To derive the equation of state of dark energy $w(z)$, one needs to know the value of Ω_{0m} , as we see in Eq.3. To estimate the value of Ω_{0m} , without using any parameterisation and in a model-independent way, we can use the results of the detection of the baryon acoustic oscillation peak (Linder 2003; Eisenstein et al. 2005). The distance factor A up to redshift 0.35, measured by observation of luminous red galaxies in detection of baryon acoustic oscillation peak (which have been claimed to be relatively independent of the model of dark energy), can be derived directly for different values of Ω_{0m} by using the reconstructed $h(z)$,

$$A = \frac{\sqrt{\Omega_{0m}}}{h(z_1)^{1/3}} \left[\frac{1}{z_1} \int_0^{z_1} \frac{dz}{h(z)} \right]^{2/3}, \quad (7)$$

where the measured value of A is $A = 0.469 \left(\frac{n}{0.98} \right)^{-0.35} \pm 0.017$ at $z_1 = 0.35$. The 3-year WMAP results, when combined with the results of baryon acoustic oscillations, yield $n = 0.951$ for the spectral index of the primordial power spectrum (Spergel et al. 2006; LAMBDA web site). By using the best reconstructed results for $h(z)$, we get $A/\sqrt{\Omega_{0m}} = 0.901$. In Figure 4, left panel, we see the derived value of $A/\sqrt{\Omega_{0m}}$ from supernovae data in comparison with its measured value from observation of LRGs for different values of Ω_{0m} . It is clear that these two independent observations which are completely different by nature, are very much in agreement if $0.255 < \Omega_{0m} < 0.299$. This derived value of Ω_{0m} is completely independent of any dark energy model assumption (within the framework of standard general relativity) and is in very close agreement with the results from large scale structure measurements from 2dF (Tegmark 2004) and SDSS (Cole et al. 2005). This derived value of Ω_{0m} is also in good agreement with the results from Fay & Tavakol (2006), where a different model-independent method of reconstruction has been used.

Now by marginalising over $\Omega_{0m} = 0.277 \pm 0.022$, which is the range of agreement between the two observations, we can reconstruct $w(z)$ from our previously reconstructed $h(z)$. In Figure 4, right panel, we show the reconstructed $w(z)$, marginalised over Ω_{0m} for the Gold dataset. We see that the data prefer evolving dark energy to the cosmological constant. The degeneracy for the equation of state of

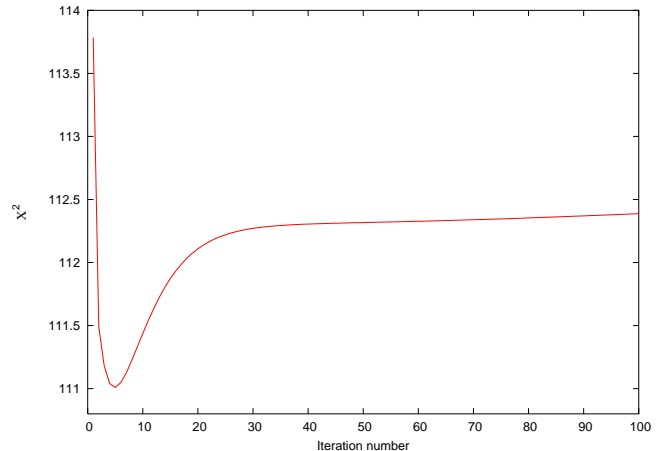


Figure 5. Computed χ^2 for the reconstructed results at each iteration for the SNLS dataset.

dark energy at high redshifts is very large and it is almost impossible to say much about $w(z)$ at high redshifts.

4 RESULTS FROM THE SNLS DATASET

In this section we use the same procedure as we used in the previous section to deal with SNLS supernovae data. The SNLS dataset contains 115 data points in the range of $0.1 < z < 1.0$. We use this dataset, first to reconstruct the Hubble parameter, $h(z)$, and the deceleration factor, $q(z)$, up to redshift 1. Then by using the results of detection of baryon acoustic oscillation peak we derive the value of Ω_{0m} , following which we recover the form of $w(z)$. We use the distance modulus of the supernovae available in Tables 8 and 9 in Astier et al. (2005) as our dataset in this section.

In Figure 5 we see the computed χ^2 for the reconstructed results using smoothing method at each iteration. As we see, the χ^2 diverges to its minimum value very fast at just the 5th iteration. In Figure 6 we show the reconstructed $h(z)$ (left panel) and $q(z)$ (right panel) for the SNLS dataset. The red solid line has the best likelihood, which is our best reconstructed result. All the other lines are within 1σ away from the best recovered result. We should like to emphasise here that these results (green dashed lines) are not representative of all the possibilities which give the likelihood within 1σ of the best recovered result. However they can show the overall behaviour of the quantities which we have studied. Our results for SNLS data show that the transition from deceleration to acceleration phase of the universe occurs at redshifts higher than 0.7. The fact that we cannot put an upper limit to the redshift of the commence of acceleration is due to the absence of supernovae data at $z > 1$ in SNLS dataset.

As we have discussed earlier in the previous section, we use the results of detection of baryon acoustic oscillation peak to determine the value of Ω_{0m} . Then by marginalising over the recovered value of Ω_{0m} , we derive the dynamics of $w(z)$. In Figure 7 we see the derived value of Ω_{0m} and the reconstructed form of $w(z)$. We see that the Λ CDM model is in much better agreement with SNLS data than with Gold data.

By comparing the recovered results from the SNLS and

Gold datasets, we can clearly see an inconsistency between these two supernovae datasets. This inconsistency is obvious by looking at the reconstructed $q(z)$ and $w(z)$ in the middle and high redshift ranges. Gold data suggest the redshift of the commencement of acceleration at $z_a \simeq 0.42$ while SNLS data suggest $z_a \simeq 0.80$. The reconstructed $w(z)$ from these two datasets also shows a very different behavior in the middle and high redshift ranges. The discord between Gold and SNLS supernovae datasets has been reported and studied earlier by Nesseris & Perivolaropoulos (2006), and a similar discord between Gold supernovae data and other cosmological observations like Cosmic Microwave Background observations from WMAP and observations of cluster abundance, has also been reported earlier by Jassal, Bagla & Padmanabhan (2006). However SNLS supernovae data seem to be in good agreement with the other cosmological observations. Based on all these results and analyses, we may conclude that some significant systematics in the Gold data (or in a part of the data) might be the reasons for these inconsistencies.

Interestingly, the recovered values of Ω_{0m} from Gold and SNLS data (by using the results of detection of baryon acoustic oscillation peak), are in very close agreement. In both cases the derived value of Ω_{0m} is around 0.276 ± 0.022 . We should note here that the two data sets rely on pretty much the same nearby supernovae samples and that is why the results are similar in this range. It is something which we logically expect to get. But in fact it shows one of the advantages of this method over the functional fitting methods. By using a functional fitting method, the recovered results at any redshift would be equally dependent on the data set in the whole redshift range. But here, by using our smoothing method, we can clearly see that despite the significant differences between the reconstructed results from Gold and SNLS datasets in the middle and high redshift regions, the reconstructed results for the expansion history at low redshifts (which we use to estimate the value of matter density), are not affected by the big differences between the two datasets at the higher redshifts.

5 DISCUSSION AND CONCLUSION

In this paper we have shown that by improving the efficiency of the smoothing method (Shafieloo et al. 2006), we can reconstruct the expansion history of the universe in a model-independent way, using current supernovae data. We have used the smoothing method to reconstruct the expansion history of the universe, $h(z)$, the deceleration parameter, $q(z)$, the value of Ω_{0m} and the equation of state of dark energy, $w(z)$, independently of any assumption of the theoretical model of the universe, within the framework of standard general relativity. This is an advantage of this method over the functional fitting methods where the results are usually biased by the form of the functional fitting or the assumed theoretical model. We dealt with two recent datasets, Gold and SNLS in our analysis. In determining the value of Ω_{0m} , we found excellent agreement between Gold and SNLS datasets. This determination is directly related to the supernovae data points at redshifts lower than $z = 0.35$. We have got $\Omega_{0m} \approx 0.276 \pm 0.023$ for both Gold and SNLS datasets, which is in good agreement with results of SDSS

and 2dF large scale structure observations, and also with results of recent Chandra X-ray observations of the relaxed galaxy clusters (Allen et al. 2007).

This derived value of Ω_{0m} also agrees with the recent WMAP 3 years CMB data, if we assume the broken scale invariant spectrum for the form of the primordial spectrum (Shafieloo & Souradeep 2007). In the derivation of $q(z)$ and the stage of transition from deceleration to acceleration in the dynamics of the universe, we found disagreement between Gold and SNLS datasets. Gold data suggest the redshift of the commencement of acceleration at $z_a \simeq 0.42$ while SNLS data suggest $z_a \simeq 0.80$.

After marginalizing over the derived value of Ω_{0m} , we have reconstructed $w(z)$. The inconsistency between Gold and SNLS supernovae datasets is also obvious by looking at the reconstructed $w(z)$ from these two datasets. The derived form of $w(z)$ from SNLS dataset, is in good concordance with Λ CDM model, while Gold dataset prefers an evolving form of dark energy (however Λ CDM is still in agreement with the Gold dataset to within 2σ). This discrepancy between Gold and SNLS datasets has been reported earlier by other groups (Nesseris & Perivolaropoulos (2006); Alam, Sahni & Starobinsky (2007)). As the Gold sample is also relatively in disagreement with the other cosmological observations like CMB and observations of cluster abundance (Jassal, Bagla & Padmanabhan (2006)), we may conclude that the effect of systematics in the Gold dataset (or at least in a part of the data) is significant.

The large error-bars at the high redshifts for the reconstructed results, reflect the significant lack of data points. This effect may not be seen if we use some of the parametric methods of analysis, but as we deal with the data directly here, we notice that the lack of data points at high redshifts limits our ability to say much about the behavior of the Universe at the early stages at high redshifts. This is another important feature of our smoothing method in which the reconstructed results at any redshift rely mostly on the supernovae data points at the same redshift range.

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REFERENCES

- U. Alam, V. Sahni, T. D. Saini, A. A. Starobinsky, 2003, *Mon. Not. Roy. Ast. Soc.* **344**, 1057.
- U. Alam, V. Sahni, A. A. Starobinsky, 2007, *J. Cosmo. Astropart. Phys.* **0702**, 002.
- S. W. Allen et al., 2007, (astro-ph/0706.0033).
- P. Astier et al., 2005, *Astron. Astroph.* **447**, 31.
- C. Bonvin, R. Durrer, M. Kunz, 2006, *Phys. Rev. Lett.* **96**, 191302.
- S. M. Carroll, 2001, *Living Rev. Rel.* **4**, 1.
- M. Chevallier and D. Polarski, 2001, *Int. J. Mod. Phys. D* **10**, 213.

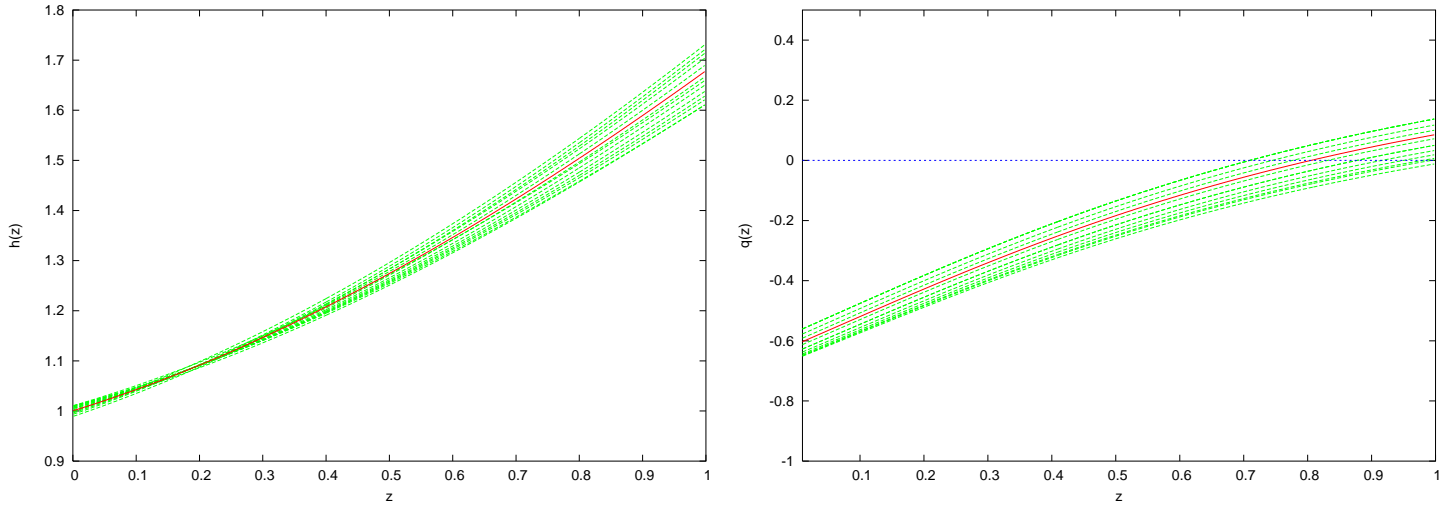


Figure 6. Reconstructed $h(z)$ (left) and $q(z)$ (right) by using SNLS dataset. Red solid line is the best recovered result and the green dashed lines are within 1σ away from the best result. Based on our results, the transition between deceleration and acceleration phases of the universe occurs at $z > 0.70$ within 1σ error-bar from the best recovered result.

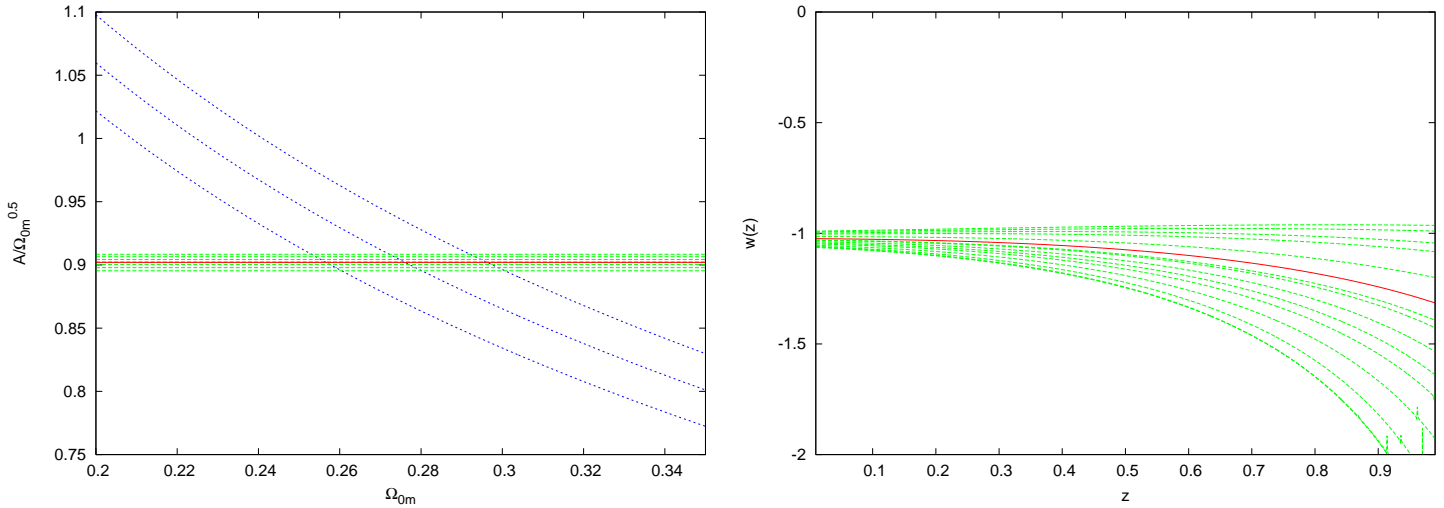


Figure 7. Left panel: The derived value of $A/\sqrt{\Omega_{0m}}$ from supernovae SNLS data within its 1σ error-bars (red solid line and green dashed lines) in comparison with its measured value from observation of LRGs within its 1σ error-bars (blue dotted lines) for different values of Ω_{0m} . The consistency of this result with the result from the Gold sample is obvious. Right panel: Reconstructed $w(z)$ for the SNLS dataset. Red solid line is the best recovered result and the green dashed lines are within 1σ away from the best result. To get these results, we have marginalised over $\Omega_{0m} = 0.276 \pm 0.023$.

T. Chiba and T. Nakamura, 2000, Phys. Rev. D, **62**, 121301(R).
 S. Cole, et al., 2005, Mon. Not. Roy. Ast. Soc. **362**, 505.
 P. Coles and F. Lucchin, 1995, “Cosmology, the origin and evolution of large scale structure”, John Wiley and sons.
 E. J. Copeland, M. Sami and S. Tsujikawa, 2006, (hep-th/0603057).
 P. S. Corasaniti and E. J. Copeland, 2003, Phys. Rev. D, **67**, 063521.
 R. A. Daly and S. G. Djorgovsky, 2003, Astroph. J. **597**, 9.
 D. J. Eisenstein, et al., 2005, Astroph. J. **633**, 560.

S. Fay and R. Tavakol, 2006, Phys. Rev. D, **74**, 083513.
 B. Gerke and G. Efstathiou, 2002, Mon. Not. Roy. Ast. Soc. **335**, 33.
 Y. Gong, 2005, Int. J. Mod. Phys. D **14**, 599.
 Y. Gong and A. Wang, 2006, (astro-ph/0612196).
 D. Huterer and M. S. Turner, 1999, Phys. Rev. D, **60**, 081301.
 D. Huterer and G. Starkman, 2003, Phys. Rev. Lett. **90**, 031301.
 D. Huterer and A. Cooray, 2005, Phys. Rev. D **71**, 023506.
 D. Huterer and H. V. Peiris, 2006, (astro-ph/0610427).
 H. K. Jassal, J.S.Bagla, T.Padmanabhan, 2005, Phys. Rev.

- D **72**, 103503
- R. A. Knop et al., 2003, *Astroph. J.* **598**, 102(k).
Legacy Archive for Microwave Background Data, web site:
<http://lambda.gsfc.nasa.gov/>
- E. V. Linder, 2003, *Phys. Rev. Lett.* **90**, 091301.
- E. V. Linder, 2003, *Phys. Rev. D*, **68**, 083504.
- E. V. Linder, D. Huterer, 2005, *Phys. Rev. D*, **72**, 043509.
- I. Maor, R. Brustein, J. McMahon and P. J. Steinhardt, 2002, *Phys. Rev. D*, **65**, 123003.
- V. J. Martinez and E. Saar, 2002, “Statistics of Galaxy Distribution”, Chapman and Hall.
- S. Nesseris and L. Perivolaropoulos, 2004, *Phys. Rev. D*, **70**, 043531.
- S. Nesseris, L. Perivolaropoulos, 2006, (astro-ph/0612653).
- T. Padmanabhan, 2003, *Phys. Rep.* **380**, 235.
- P. J. Peebles and B. Ratra, 2003, *Rev.Mod.Phys.* **75**, 559.
- S. J. Perlmutter et al., 1999, *Astroph. J.* **517**, 565.
- A. G. Riess et al., 1998, *Astron. J.* **116**, 1009.
- A. G. Riess et al., 2005, *Astroph. J.* **607**, 665(R).
- A. G. Riess et al., 2006, (astro-ph/0611572).
- T. Roy Choudhury and T. Padmanabhan, 2005, *Astron. Astrophys.* **429**, 807.
- V. Sahni and A. A. Starobinsky, 2000, *Int. J. Mod. Phys. D* **9**, 373.
- V. Sahni, 2004, *Lect. Notes Phys.* **653**, 141.
- V. Sahni and A. A. Starobinsky, 2006, (astro-ph/0610026).
- T. D. Saini, S. Raychaudhury, V. Sahni, A. A. Starobinsky, 2000, *Phys. Rev. Lett.* **85**, 1162.
- T. D. Saini, 2003, *Mon. Not. Roy. Ast. Soc.* **344**, 129.
- T. D. Saini, J. Weller and S. L. Bridle, 2004, *Mon. Not. Roy. Ast. Soc.* **348**, 603.
- A. Shafieloo and T. Souradeep, 2004, *Phys. Rev. D*, **70**, 043523.
- A. Shafieloo, U. Alam, V. Sahni, A. A. Starobinsky, 2006, *Mon. Not. Roy. Ast. Soc.* **366**, 1081.
- A. Shafieloo and T. Souradeep, 2007, *In preparation*.
- D. Spergel et al., 2006, (astro-ph/0603449).
- A. A. Starobinsky, 1998, *JETP Lett.* **68**, 757.
- M. Tegmark, 2002, *Phys. Rev. D*, **66**, 103507.
- M. Tegmark, et al., 2004, *Astroph. J.* **606**, 702.
- J. L. Tonry et al., 2003, *Astroph. J.* **594**, 1.
- Y. Wang and G. Lovelace, 2001, *Astroph. J.* **562**, L115.
- Y. Wang and P. Mukherjee, 2004, *Astroph. J.* **606**, 654.
- Y. Wang and M. Tegmark, 2004, *Phys. Rev. Lett.* **92**, 241302.
- Y. Wang and M. Tegmark, 2005, *Phys. Rev. D*, **71**, 103513.
- J. Weller and A. Albresht, 2002, *Phys. Rev. D*, **65**, 103512.

$\Omega_{0m} = 0.25$ after 91 iteration, and $\chi^2 = 157.39$ for a flat quiescence model with $\Omega_{0m} = 0.30$ and $w(z) = -0.8$ after 104 iteration. In figure A1 we can see the reconstructed $h(z)$ and $w(z)$ for the Gold data set by assuming these three different initial guess models. As we see, the robustness of the method for the choice of the initial guess model is obvious.

We have also used different values of Δ (width of smoothing in Eq.1), in our reconstruction process to check the reliability and stability of our results against the changes in the value of Δ . We have used three values of Δ equal to 0.30, 0.60 and 0.90 in our smoothing method and we have applied it on the Gold dataset. By using $\Delta = 0.30$ we have got $\chi^2 = 157.38$ after 9 iteration, while we have got $\chi^2 = 157.40$ by using $\Delta = 0.60$ after 89 iteration, and $\chi^2 = 157.41$ by using $\Delta = 0.90$ after 407 iteration. In figure A2 we can see the reconstructed $h(z)$ and $w(z)$ for the Gold dataset by using these three values of Δ . We can clearly see that the results are not sensitive to the given value of Δ . These two examinations confirm the overall robustness of the method for different initial assumptions.

APPENDIX A: EXAMINING THE ROBUSTNESS OF THE METHOD

In this section we show that the results of the smoothing method are robust against the choice of the initial guess model and also to the chosen value of Δ .

We assumed three different cosmological models as our initial guess model and we applied our smoothing method on the Gold dataset. The final results by using these three different initial guess models are almost identical with $\Delta\chi^2 < 0.01$. We have got $\chi^2 = 157.40$ by using a flat Λ CDM model with $\Omega_{0m} = 0.30$ as the initial guess model after 89 iteration, while we have got $\chi^2 = 157.40$ for a flat Λ CDM model with

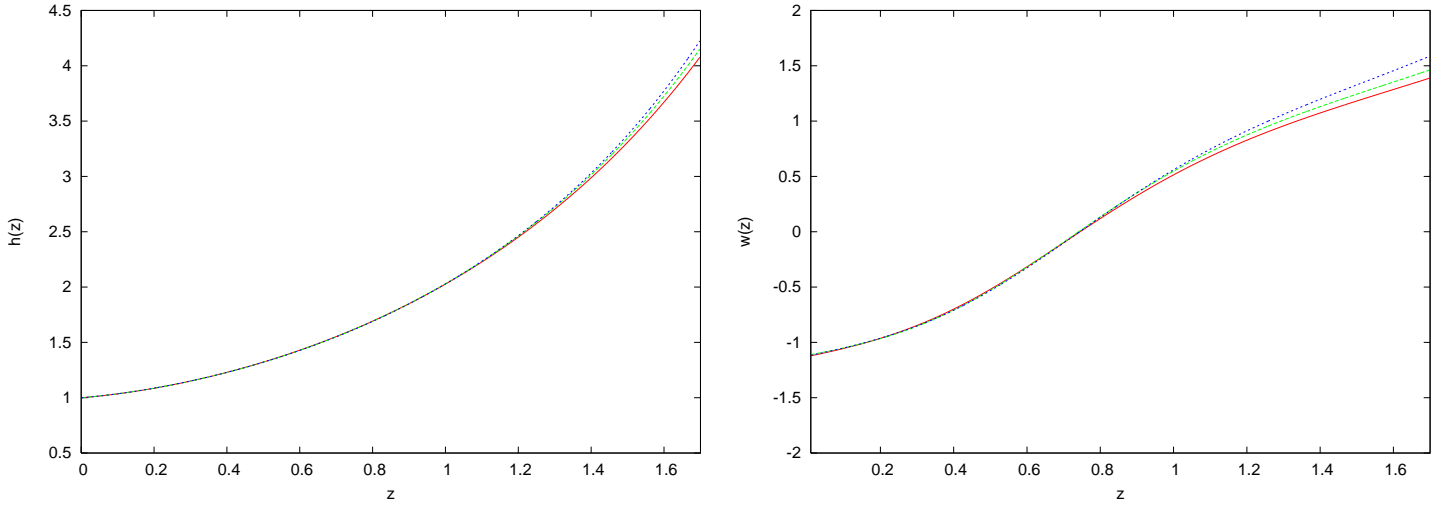


Figure A1. Reconstructed $h(z)$ (left panel) and $w(z)$ (right panel) for the Gold dataset by assuming three different initial guess models. The red solid line is the reconstructed result by using a flat Λ CDM model with $\Omega_{0m} = 0.30$ as the initial guess model. The green dashed line is the reconstructed results by using a flat Λ CDM model with $\Omega_{0m} = 0.25$, and the blue dotted line is the reconstructed result by using a flat quiescence model with $w(z) = -0.8$ and $\Omega_{0m} = 0.30$ as the initial guess models. We can clearly see that the results are almost identical which shows the robustness of the method for the different choices of the initial guess model.

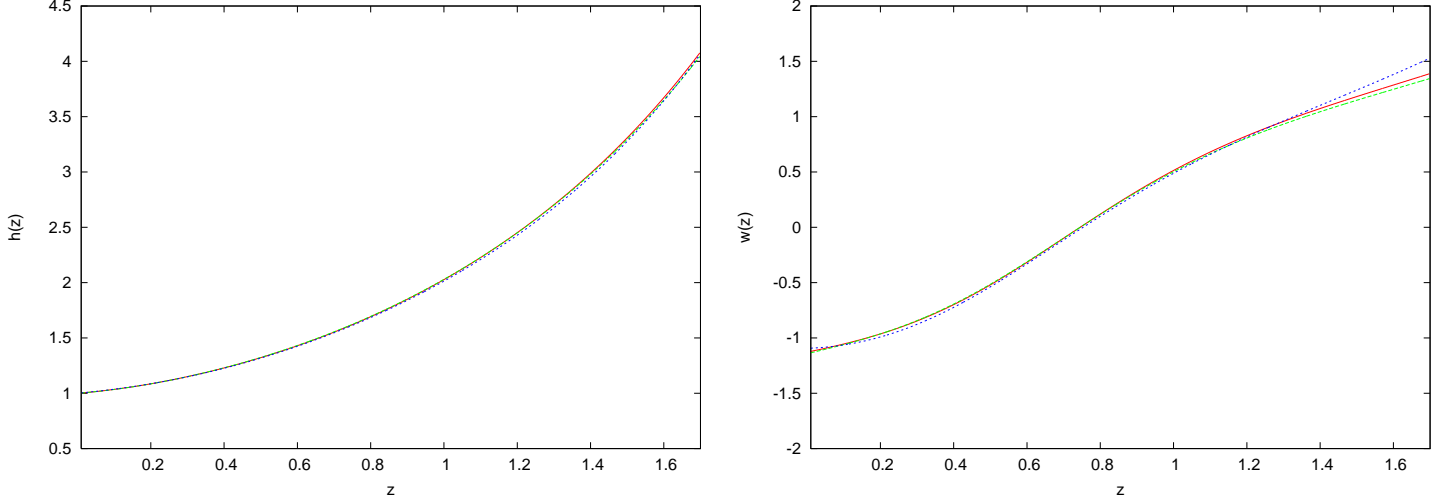


Figure A2. Reconstructed $h(z)$ (left panel) and $w(z)$ (right panel) for the Gold dataset by using three different values of Δ (width of smoothing). The red solid line is the reconstructed result by using $\Delta = 0.60$. The green dashed line is the reconstructed results by using $\Delta = 0.90$, and the blue dotted line is the reconstructed result by using $\Delta = 0.30$. In all these cases we have stopped the bootstrapping process after reaching to the minimum χ^2 . We can see that the method is robust against the variation of Δ in a wide range.