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Doppler velocities in kink MHD waves.

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IAP P7/08 Charm
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FWO Vlaanderen

1. Transverse and rotational motions

- **Standing transverse MHD waves in coronal loops (Schrijver et al. (1999); Aschwanden et al. (1999)); rare events.**
- **Propagating transverse MHD waves (COMP (Tomczyk et al. 2007), SDO/AIA (McIntosh et al. 2011)); are everywhere.**
- **Transverse waves = kink waves $(m = 1)$ (Nakariakov et al. (1999)).**
- **Doppler signals of torsional motions and swaying motions (De Pontieu et al. (2012))**
- **Torsional motions = signatures of $m = 0$ Alfvén waves.**
- **Axisymmetric $(m = 0)$ Alfvén waves and kink $(m = 1)$ MHD waves.**

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- **Kink MHD waves $\Longleftrightarrow$ transverse + vortical motions**
- **Identify vortical motions $\Longleftrightarrow$ Look at Vorticity Z**
- **Doppler velocities.**

2. Kink MHD waves on flux tubes

- **MHD waves superimposed on an equilibrium magnetic plasma.**
- **Equilibrium model = cylindrical plasma column in static equilibrium.**
- **Equilibrium magnetic field** $\vec{B}_0 = B_0\,\vec{1}_z$**, equilibrium density** $\rho_0(r)$**.**
- $\exp(-i\omega t)$, $\exp(i(m\varphi + k_z z))$ $\quad m, k_z =$ **azimuthal and axial wave numbers.**
- $\vec{\xi} =$ **displacement,** $P' =$ **Eulerian perturbation of total pressure.**
- **Vorticity** $Z = (\nabla \times \vec{\xi}) \cdot \vec{1}_z$

$$Z = (\nabla \times \vec{\xi}) \cdot \vec{1}_z = i\,\frac{m}{r}\,P'\,\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{\underbrace{\rho_0(r)}(\omega^2 - \omega_A^2(r))}\right)$$

- **Local Alfvén frequency** $\omega_A(r)$

$$\omega_A^2(r) \;=\; \frac{(\vec{k}\cdot\vec{B})^2}{\mu\rho_0(r)} \;=\; k_z^2\,v_A^2(r),\quad v_A^2(r) = \frac{B^2}{\mu\,\rho_0(r)}$$

- Z = measure for local rate of rotation.
- $Z \neq 0$ **when?**
- **Uniform plasma of infinite extent:** $Z \neq 0 \iff$ **(bulk) Alfvén waves.**
- **Non-uniform plasma: $\rho_0(r)$ and $\omega_A(r)$ depend on position.**
- **Look back at equation for Z.**
- **Non-uniform plasma:** $Z \neq 0$
- **Non-uniform plasma: Kink MHD waves contain rotational motions!**
- **Details of rotational motions? Where are the transverse motions?**
- $\vec{\xi} = ?$

'Do you realize what this means?' 'Yes, I do, but you say it first!'
Conversation between Bart and Milhouse
The Simpsons.

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3. Kink MHD waves: Piece-wise uniform density.

- **Piece wise constant** $0 \le r \le R : \rho_0 = \rho_i; \;\; r \ge R : \rho_0 = \rho_e; \;\; \rho_i > \rho_e$.

$$\rho_e \; < \; \rho_i, \;\; B_z = \textbf{constant} \;\; \Longrightarrow \;\; \omega_{Ai}^2 = k_z^2 v_{Ai}^2 < \omega_{Ae}^2 = k_z^2 v_{Ae}^2$$

- **Wentzel, 1979; Wilson, 1979, 1980; Edwin and Roberts, 1983.**
- **Bessel functions** $J_m(x)$ $(x = k_i r)$ $(0 \le r \le R)$ **and** $K_m(y)$ $(y = k_e r)$ $(R \le r)$.
- **Continuity of** P' **and** ξ_r **: the dispersion relation.**
- **Popular special case: Thin tube (TT) approximation** $k_z R << 1 = m$.

$$\omega^2 \;\; = \;\; \frac{\rho_i \omega_{Ai}^2 + \rho_e \omega_{Ae}^2}{\rho_i + \rho_e} = \omega_k^2, \quad \omega_{Ai}^2 < \omega_k^2 < \omega_{Ae}^2$$

- **Spatial eigenfunctions**

$$\begin{aligned} \xi_{r,i}(r)/R \;\; &= \;\; C, \quad \xi_{\varphi,i}(r)/R = i\, C, \qquad 0 \le r \le R; \\ \xi_{r,e}(r)/R \;\; &\sim \;\; dK_1(y)/dy, \;\; \xi_{\varphi,e}(r)/R \sim K_1(y) \;\; R \le r < \infty \end{aligned}$$

- **Spatial eigenfunctions, general case: Use $J_1(x)$ and $K_1(y)$**
- ξ_φ **is discontinuous at** $r = R$ **due to change of sign of** $\omega^2 - \omega_A^2$**.**

$$\rho_0(\omega^2 - \omega_A^2)\ \xi_\varphi = \frac{im}{r} P'$$

- **Vorticity** Z

$$Z \ \sim \ \delta(x-1), \quad x = r/R$$

is concentrated at $r = R$ **due to discontinuity in** ξ_φ**.**

- **Rotational motions!**
- **Piece wise uniform density: instructive but pathological.**

The world is full of obvious things which nobody by any chance ever observes.
***Sherlock Holmes*, The Hound of the Baskervilles.**
Sir Arthur Cannon Doyle

4. Kink MHD waves: nonuniform density.

- **Continuous variation of density from ρ_i to ρ_e in $[R - l/2, R + l/2]$.**
- **Density $\rho(r)$ decreases from ρ_i to ρ_e.**
- **Alfvén frequency $\omega_A(r)$ increases from ω_{Ai} to ω_{Ae}.**
- **Recall**

$$\omega_{Ai}^2 < \omega_{kw}^2 \cong \frac{\rho_i \omega_{Ai}^2 + \rho_e \omega_{Ae}^2}{\rho_i + \rho_e} < \omega_{Ae}^2$$

- **Point of resonance $r_A \in]R - l/2, R + l/2[$ where**

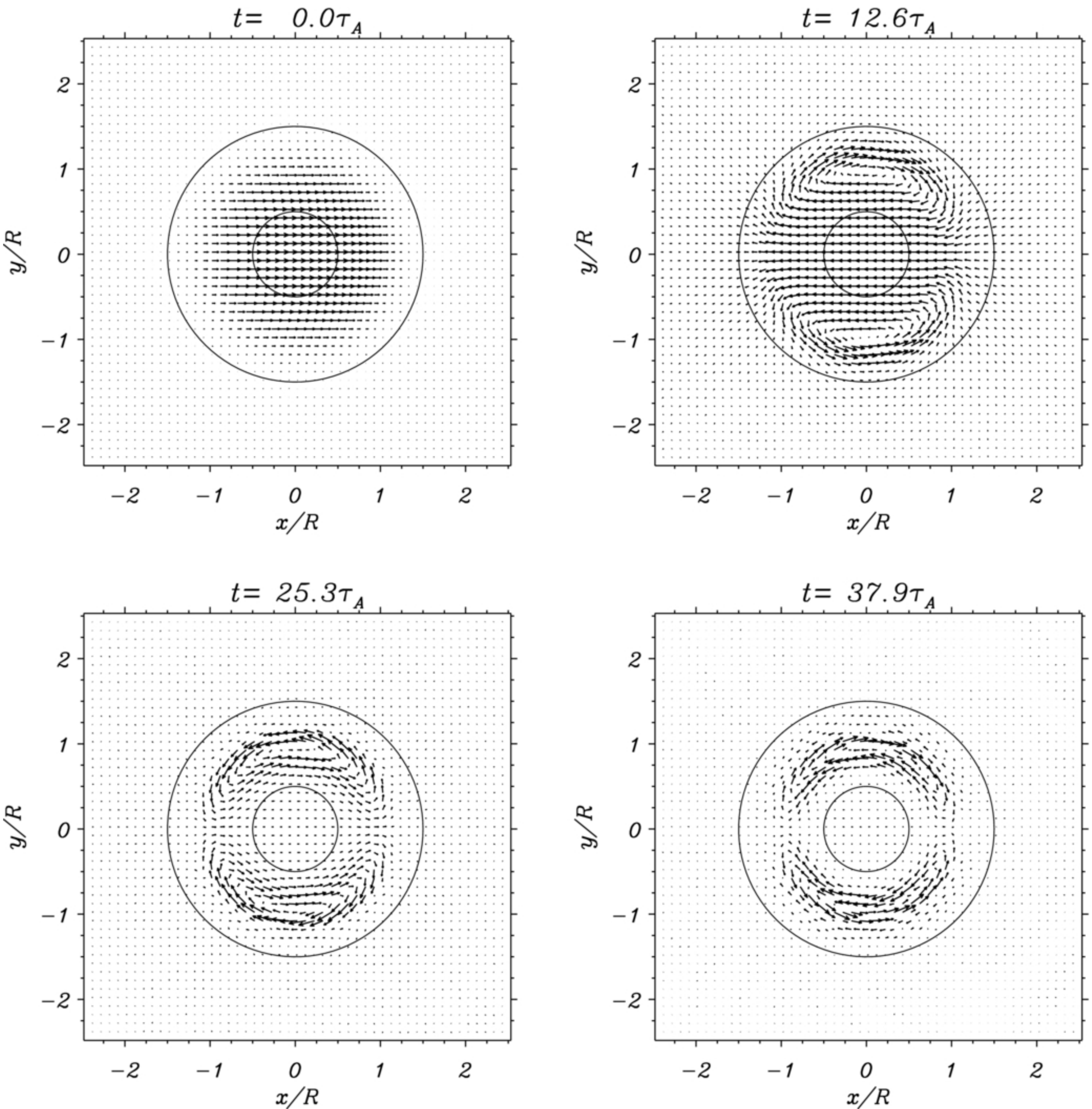
$$\omega_{kw}^2 = \omega_A^2(r_A)$$

- **What about vorticity Z?**

$$Z = (\nabla \times \vec{\xi}) \cdot \vec{1}_z = i\,\frac{m}{r}\,P'\,\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{\rho_0(r)(\omega^2 - \omega_A^2(r))}\right)$$

- **$Z \neq 0$ in non-uniform layer.**
- **Ideal MHD and real frequencies: singular solutions.**
- **$Z \rightarrow \infty$ in ideal MHD for real frequencies ar $r = r_A$.**
- **Damped MHD quasi-modes with complex frequencies: finite solutions (Tirry and Goossens, 1996; Van Doorsselaere et al. 2004; Soler et al. 2013).**
- **Z large but finite for damped kink waves.**
- **At $r = r_A$ kink wave $\approx$ pure local Alfvén wave.**
- **Strong rotational motions in non-uniform plasma in vicinity of $r = r_A$.**
- **Kink wave is Alfvénic in non-uniform plasma.**
- **Time evolution of kink wave in non-uniform plasma (Terradas et al., 2008; talk J. Terradas at BUKS 2009, Leuven).**

"Singularity is invariably a clue."
Sherlock Holmes, The Boscombe Valley Mystery.
A. Cannon Doyle.



t= 0.0τ_A
t= 12.6τ_A
t= 25.3τ_A
t= 37.9τ_A
x/R
y/R
−2
−1
0
1
2

5. Transverse and rotational motions.

- **First attempt: piece wise uniform density and thin tube approximation.**
- **Introduce azimuthal dependency.**
- $$0 \leq r \leq R : \; \frac{\vec{\xi}_h(r,\varphi)}{R} = C \left(\cos\varphi \; \vec{1}_r - \sin\varphi \; \vec{1}_\varphi\right)$$
- **Dependency on z :** $\exp(ik_z z)$
- **Jump in plane $z = z_0$ and define a system of Cartesian coordinates**

$$x = r \;\cos\varphi, \quad y = r \;\sin\varphi$$

$$\vec{1}_r = \vec{1}_x \;\cos\varphi + \vec{1}_y \;\sin\varphi, \quad \vec{1}_\varphi = -\vec{1}_x \;\sin\varphi + \vec{1}_y \;\cos\varphi$$

- **From polar to Cartesian coordinates**

$$\vec{\xi} = \begin{bmatrix} \xi_x \\ \xi_y \end{bmatrix} = C \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \vec{\xi}_{TR}$$

- **Uniform motion of the entire internal plasma along the x-axis.**

- **How did we get the uniform motion along the x-axis**
- **Necessary conditions: uniform, thin tube, piece-wise constant density.**
- $0 \le r \le R$. **What happens outside tube $r \ge R$?**

- **Remember strong rotational motions in non-uniform tube .**
- ξ_r **and** ξ_φ **have unequal amplitudes .**
- $$0 \le r \le R : \ \frac{\vec{\xi}_h(r,\varphi)}{R} \ = \ C_{TR} \ \cos\varphi \ \vec{1}_r - \ C_{AZ} \sin\varphi \ \vec{1}_\varphi, \ \ C_{TR} \ne C_{AZ}$$
- **Again from polar to Cartesian coordinates**

$$\vec{\xi} = \begin{bmatrix} \xi_x \\ \xi_y \end{bmatrix} = C_{TR} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + C_{ROT} \ \sin\varphi \begin{bmatrix} \sin\varphi \\ -\cos\varphi \end{bmatrix}$$

$$C_{ROT} = C_{AZ} - C_{TR}$$

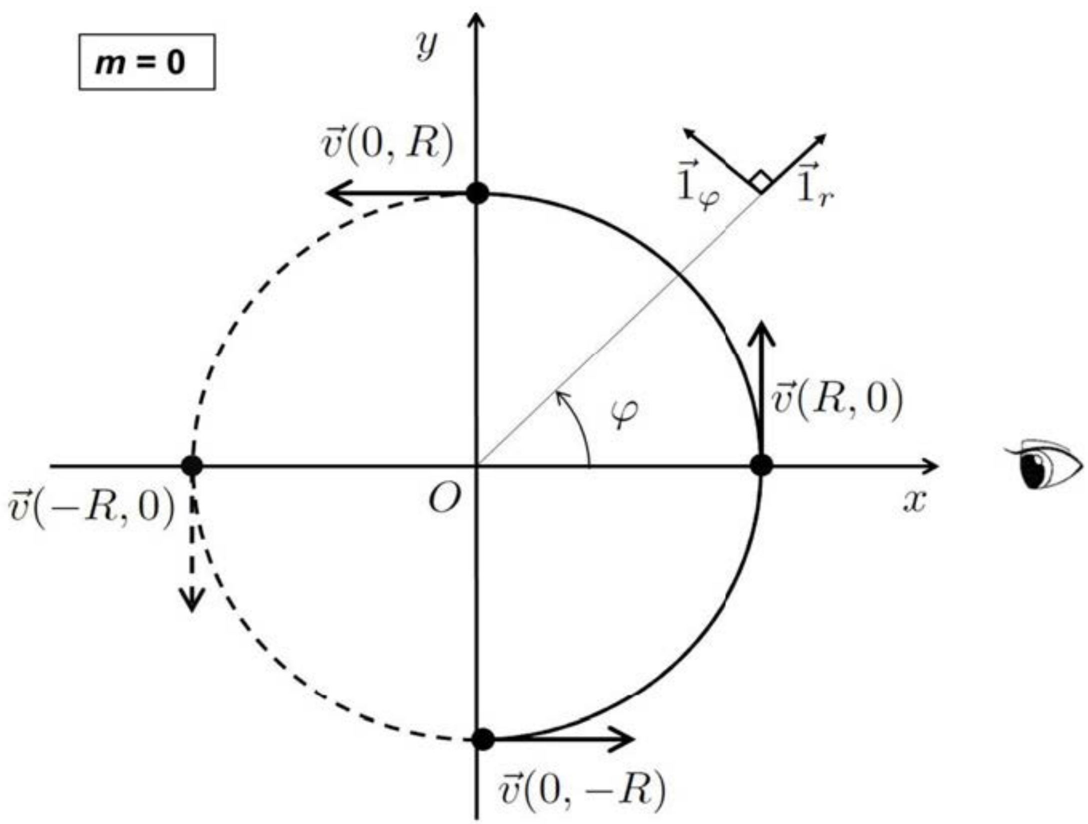
- ξ_x = **constant part + variable part** φ**.**

6. Doppler velocities due to axi-symmetric Alfvén waves.

- **Velocity in axi-symmetric $(m = 0)$ Alfvén wave**

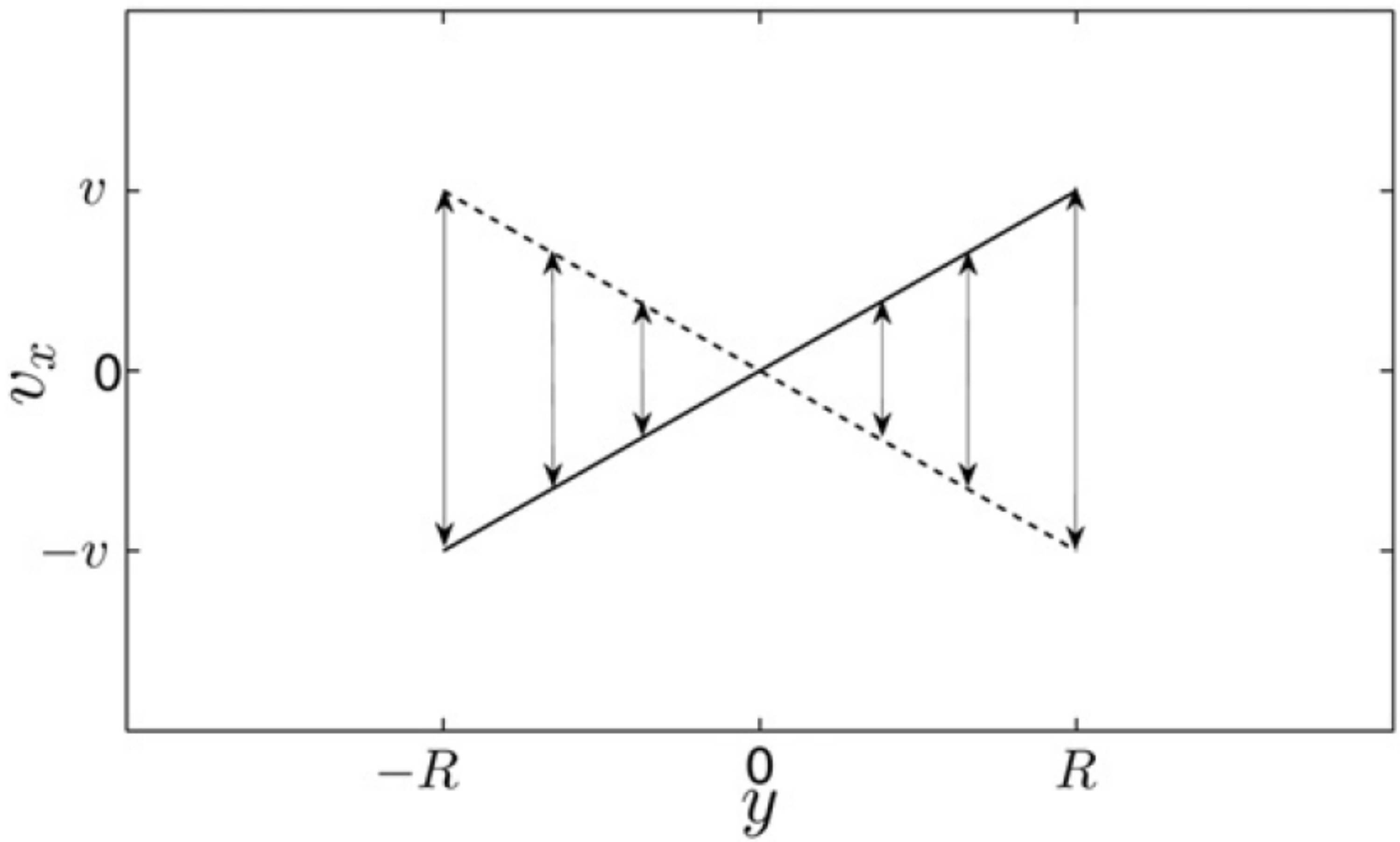
$$\vec{v} = v\,\vec{1}_\varphi$$

- **Take x-axis in the direction of observer.**
- **y-axis is across the flux tube**



- **Component of velocity in the direction of the observer, i.e. along x-axis**

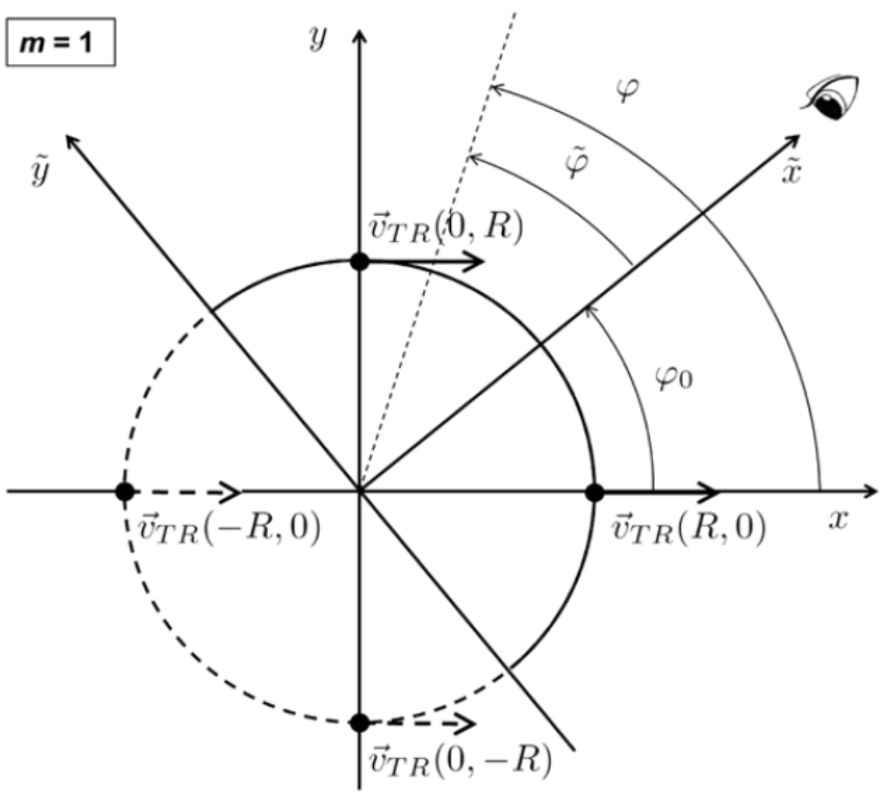
$$v_x = -v\frac{y}{R}$$



- **Linear variation.**

7. Doppler displacements due to kink motions across loop.

- Remember translational motion along x-axis, $\varphi = 0$.
- Line of sight: perpendicular to z-axis and makes angle φ_0 with x-axis.
- Rotate old system of Cartesian coordinates (x, y) over angle φ_0
- New system of Cartesian coordinates $(\tilde{x}, \tilde{y})$



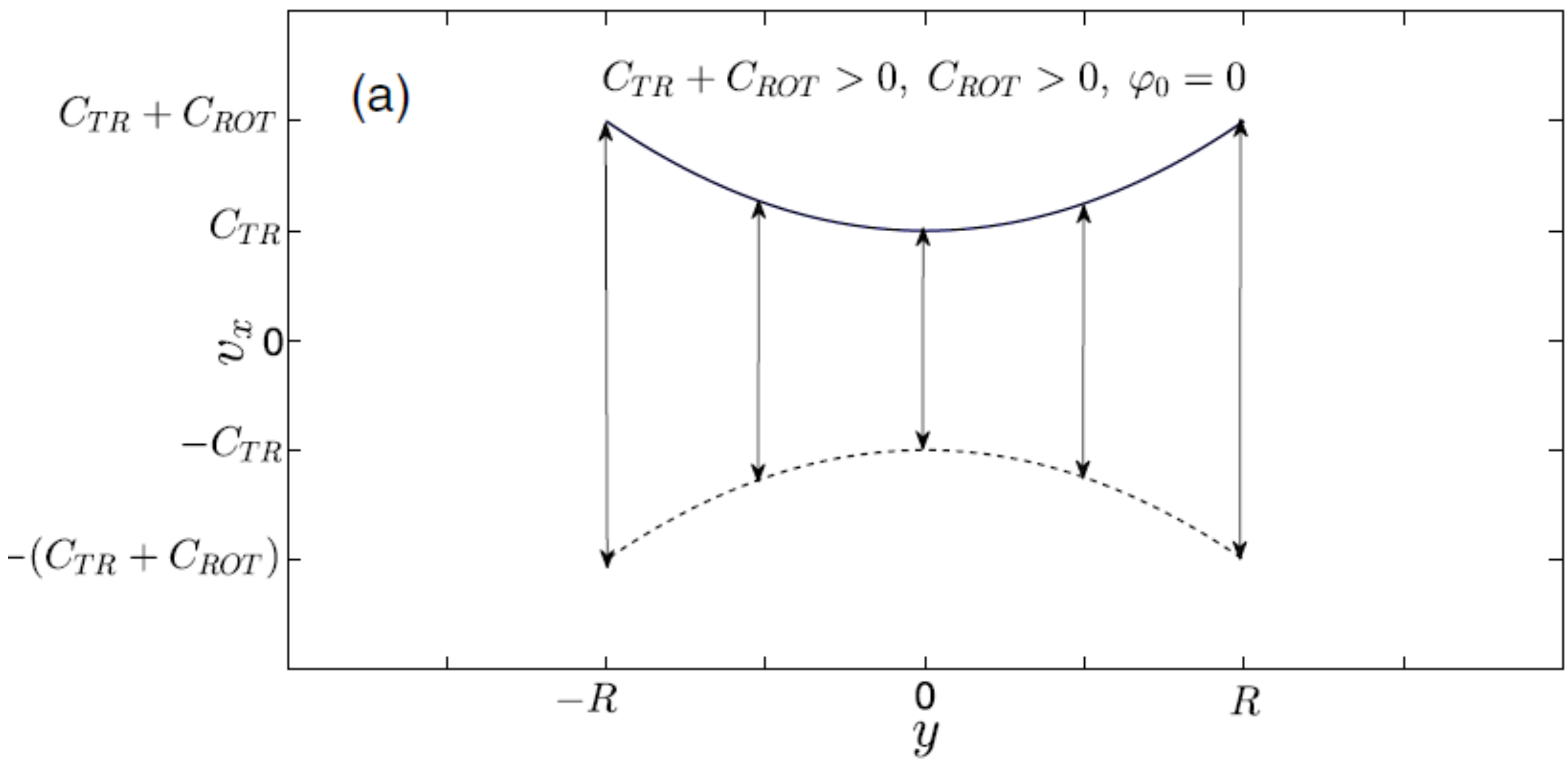
- **$\tilde{x}$-axis is towards the observer, $\tilde{y}$-axis is across the flux tube.**
- **Measure velocity in unit** ωR
- **Purely transverse motion:** $C_{AZ} = C_{TR}, \quad C_{ROT} = 0$
- **Component of velocity in the direction of the observer, i.e. along $\tilde{x}$-axis**

$$v_{\tilde{x}} \;=\; C_{TR}\, \cos\varphi_0$$

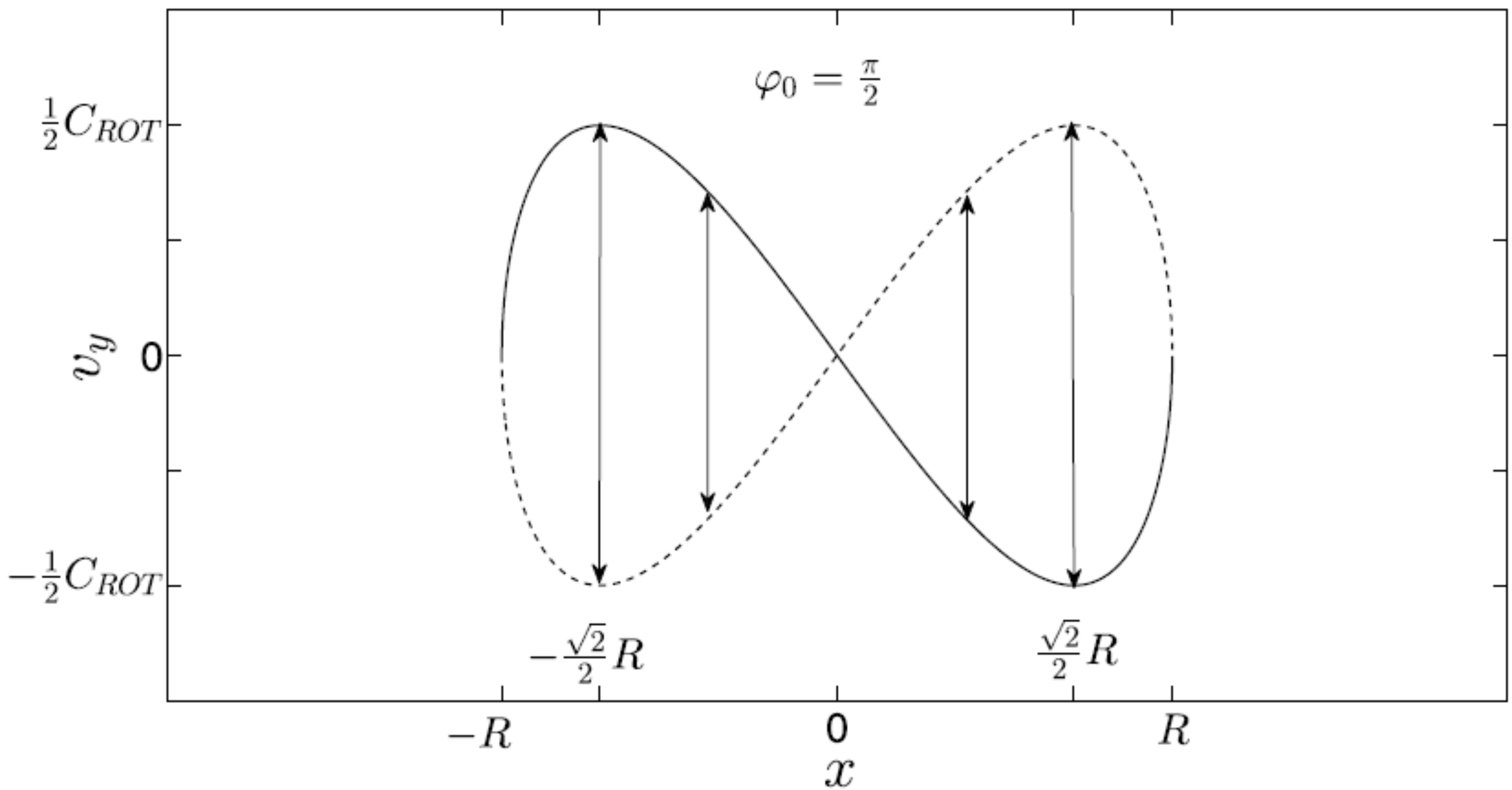
- **Constant Doppler velocity across flux tube.**
- **Role of** φ_0
- **Case** $C_{AZ} \neq C_{TR}, \quad C_{ROT} \neq 0$
- **Component of velocity in the direction of the observer, i.e. along $\tilde{x}$-axis**

$$\begin{aligned} v_{\tilde{x}} \;&=\; \underbrace{\left\{C_{TR} + C_{ROT}\,(\tilde{y}/R)^2\right\}}_{I} \cos\varphi_0 \\ &+\; \underbrace{C_{ROT}\,(\tilde{y}/R)\,\left(1 - (\tilde{y}/R)^2\right)^{1/2}}_{II} \sin\varphi_0 \end{aligned}$$

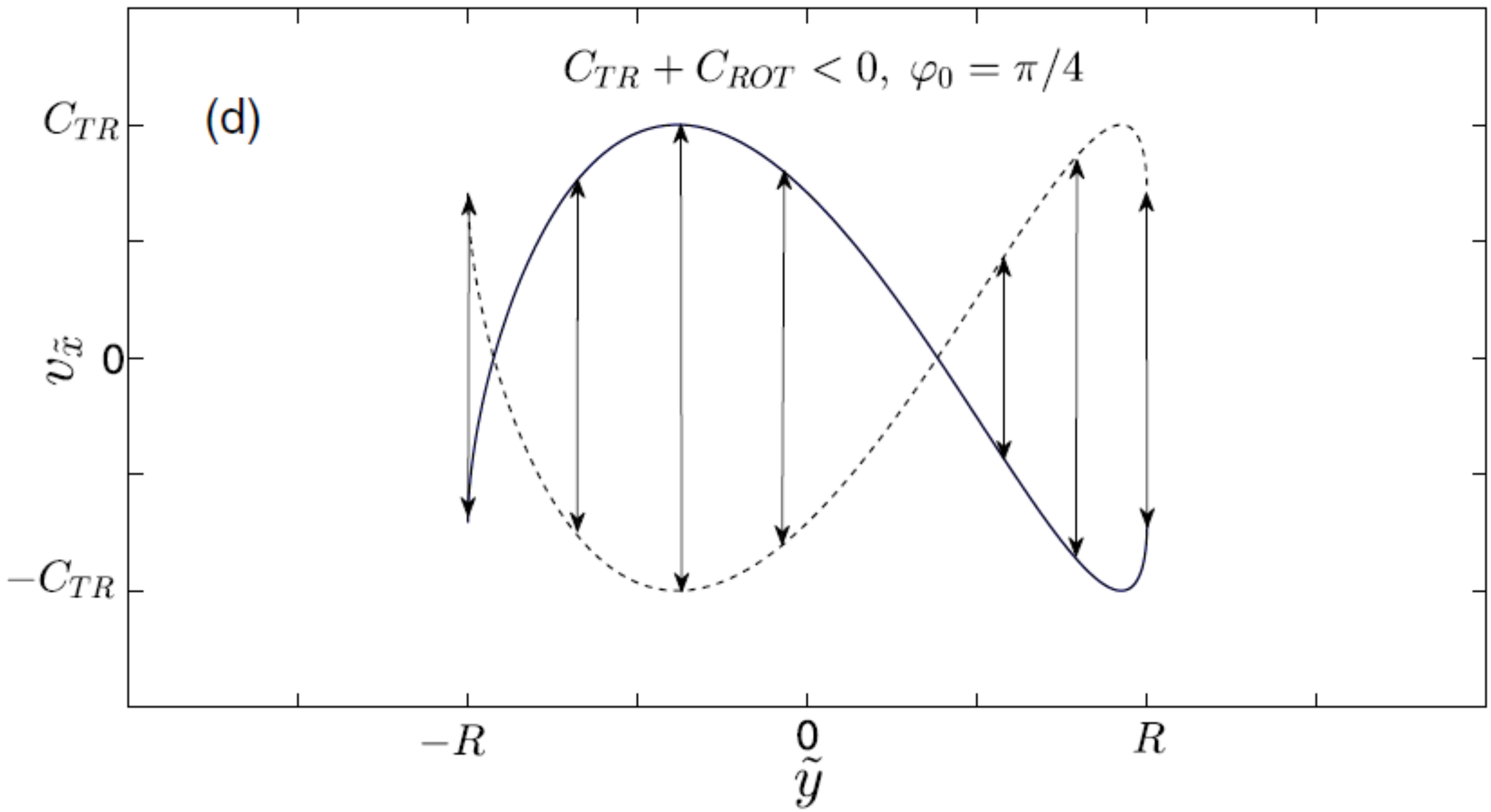
- $I\ \varphi_0 = 0$: **Parabolic variation (GV : horseshoes.)**



- $II\ \varphi_0 = \pi/2$

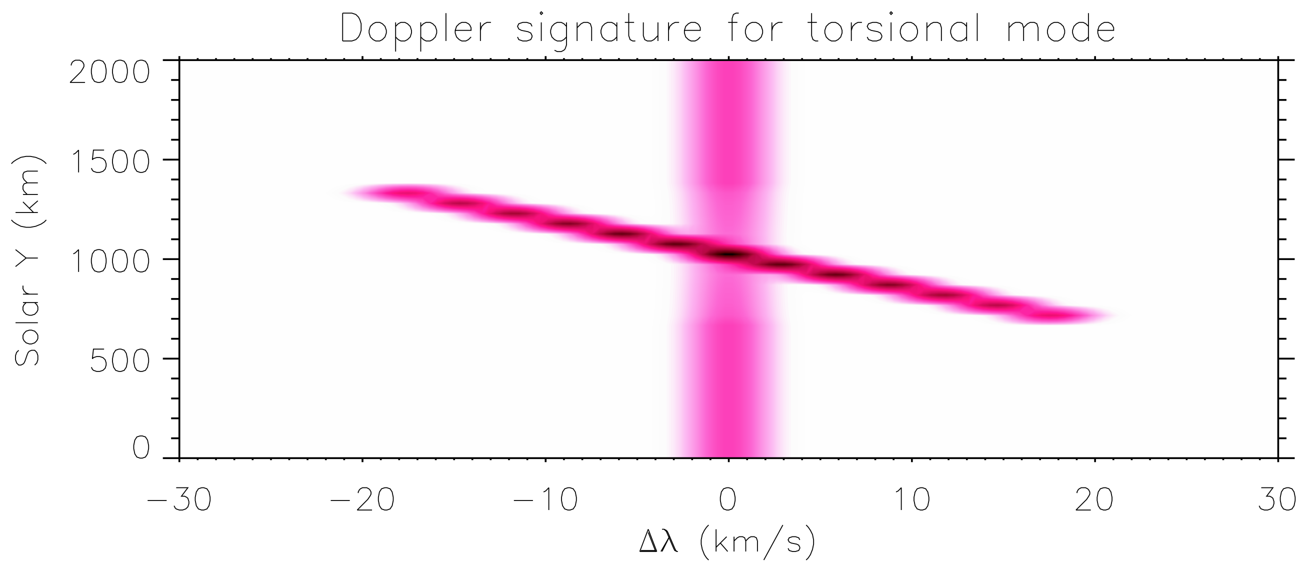


• *III* **General** φ_0

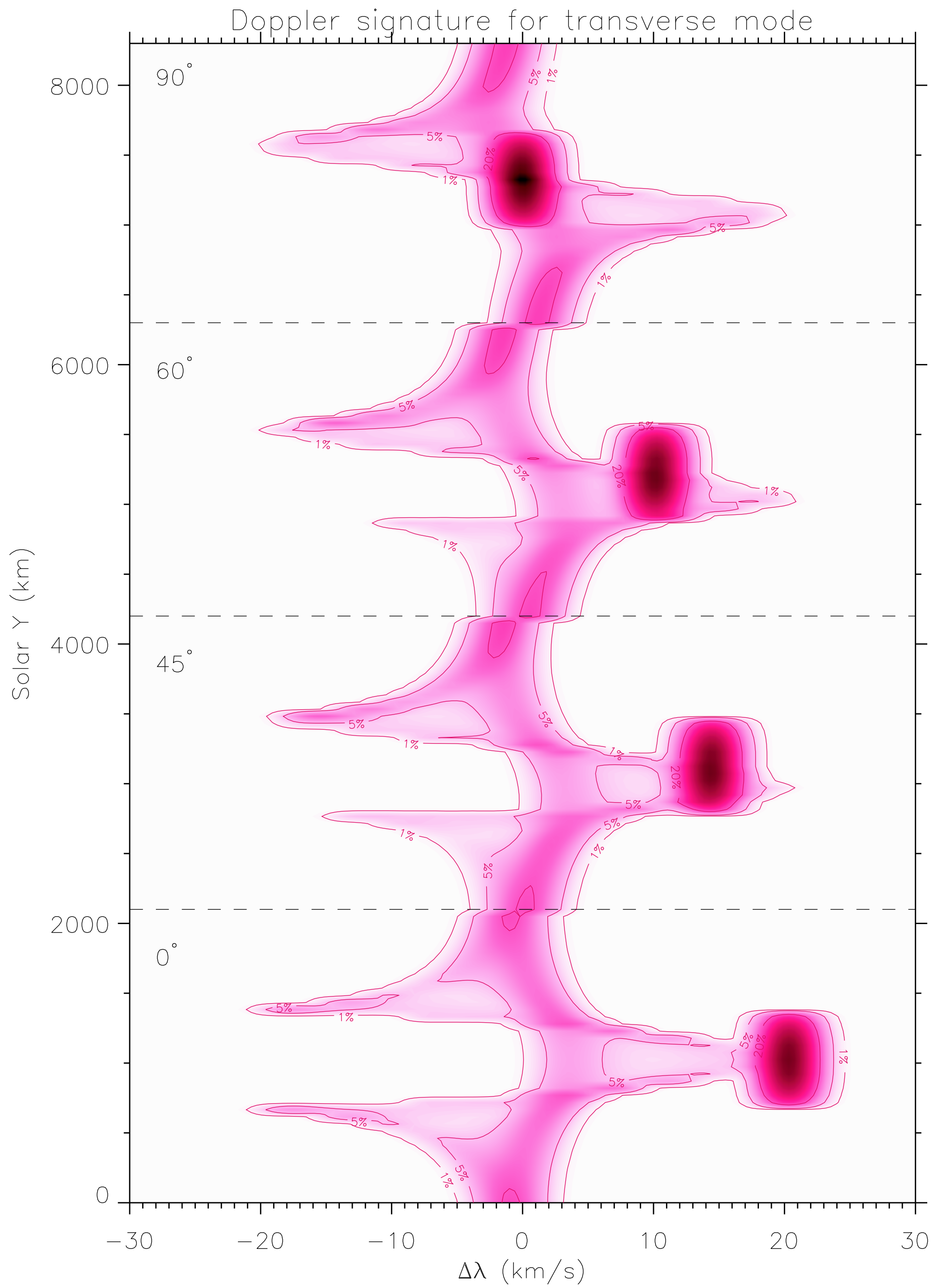


8. Integrated intensities for axi-symmetric Alfvén waves and kink waves.

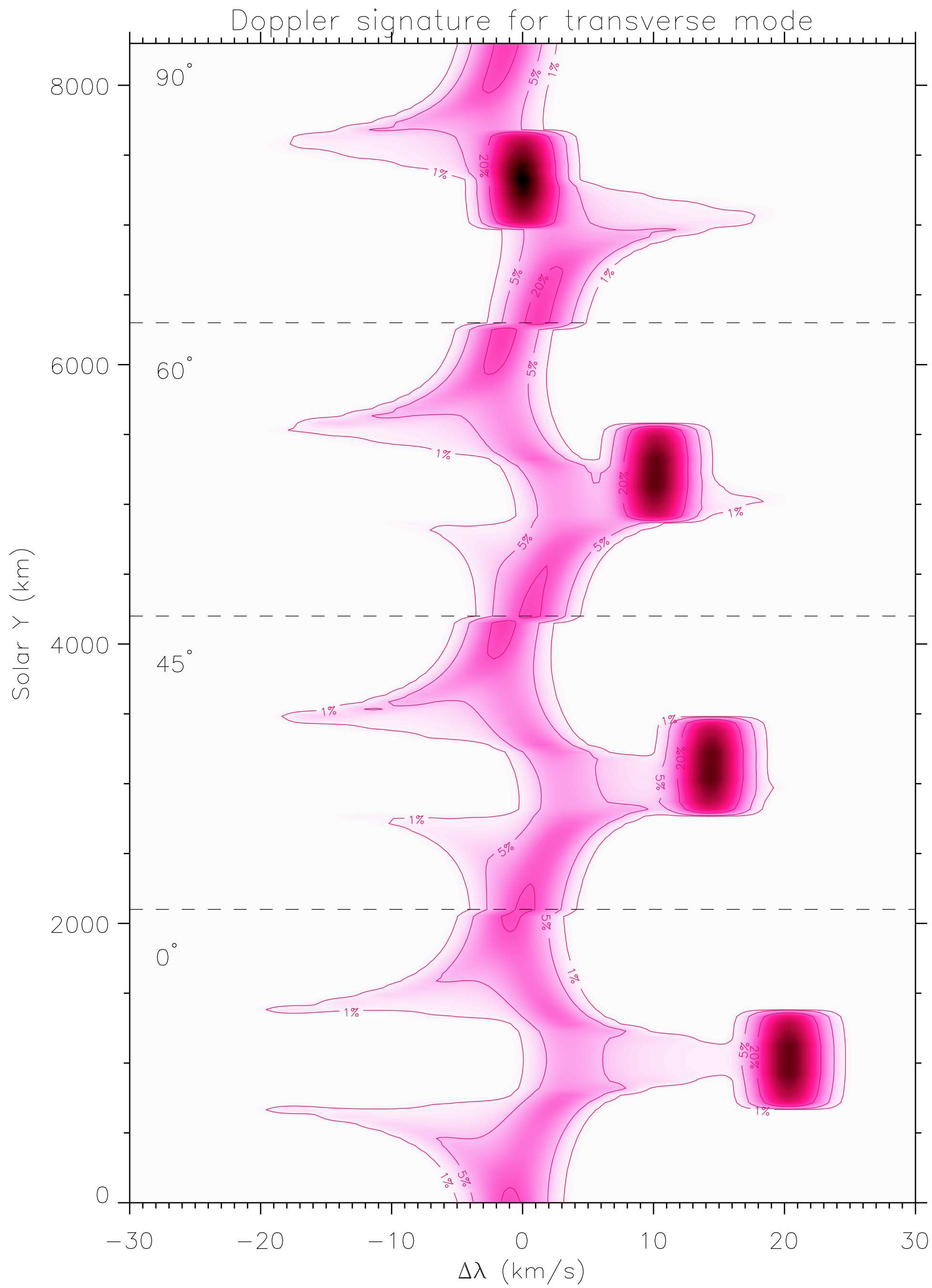
- **Details: consult TVD.**
- **Integrated intensity for the axi-symmetric Alfvén wave.**



- **Result for AW is independent of angle** φ_0
- **Next 2 figures: Integrated intensity for the kink waves.**
- **Piece wise constant density** $\rho_i/\rho_e = 10$
- **Smooth density profile** $l/R = 1$



Doppler signature for transverse mode
90°
60°
45°
0°
Solar Y (km)
Δλ (km/s)
0
2000
4000
6000
8000
−30
−20
−10
0
10
20
30
1%
5%
20%



Doppler signature for transverse mode
Solar Y (km)
Δλ (km/s)
0
2000
4000
6000
8000
−30
−20
−10
0
10
20
30
90°
60°
45°
0°
1%
5%
20%

9. Conclusions

“Has anything escaped me?” I asked with some self-importance.
“I trust that there is nothing of consequence which I have overlooked?”
“I am afraid my dear Watson, that most of your conclusions were erroneous.”
The Hound of the Baskervilles.
A. Conan Doyle.

- Kink waves in a non-uniform plasma: $Z \neq 0$.
- Motion of plasma = translational motion + rotation.
- Variation of $v_{\tilde{x}}$ across loop = linear for axi-symmetric Alfvén wave.
- $v_{\tilde{x}}$ = is constant for special case of uniform translation in kink wave.
- Variation of $v_{\tilde{x}}$ across loop = horseshoe + compressed sinus.
- Integrated intensities for axi-symmetric Alfvén wave and kink waves.

References

- Goossens, M., Soler, R., Terradas, J., M., Van Doorsselaere, T., & Verth, G. 2014, ApJ, 788:9