

Creation-field cosmology: A possible solution to singularity, horizon, and flatness problems

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A solution of Einstein's equations which admits radiation and a negative-energy massless scalar creation field as a source is presented. It is shown that the cosmological model based on this solution satisfies all the observational tests and thus is a viable alternative to the standard big-bang model. The present model is free from singularity and particle horizon and provides a natural explanation for the flatness problem. We argue that these features make the creation-field cosmological model theoretically superior to the big-bang model.

I. THE NEED FOR A NEW COSMOLOGY

Almost all investigations dealing with the physical processes in the early universe use a model for the universe, usually called the "big-bang model." This model, described by the Robertson-Walker line element and a matter-energy source that obeys the equation of state $p = \frac{1}{3}\rho$, enjoys considerable popularity among working physicists. Since this paper proposes an alternative scenario, it is probably worthwhile to begin by taking stock of achievements and drawbacks of the big-bang model.

The big-bang model is reasonably successful in the following aspects: (1) It provides an adequate general scenario for the large-scale features of the present-day universe, e.g., expansion rate, overall isotropy, etc. (2) It is consistent with the existence of an isotropic microwave background radiation (MBR) with a temperature ~ 3 K. In the big-bang model it is assumed that all matter (and radiation) "came into existence" at $t=0$. Thus the model does not *explain* the origin of MBR. Rather, it accommodates its existence in a natural fashion. (3) The primordial abundances of light nuclei as calculated for the big-bang model are in reasonable agreement with observation.

The big-bang model is, however, known to have shortcomings as regards the following aspects: (1) The model has a singularity in the past (and possibly one in the future). The singularity signals mathematical inconsistency and physical incompleteness. (2) The conservation of energy—one of the most cherished principles of physics—is violated in the big-bang model. Since the left-hand side of Einstein's equations has zero divergence, it follows that the source on the right-hand side must have zero divergence. On the other hand, the energy density in the big-bang model is positive definite. Thus it is impossible for matter to come into existence without violating energy conservation. It is customary to water down this difficulty by statements like "the laws of physics break down at a singularity;" however the essential truth remains the same. (3) The big-bang models based on reasonable equations of state lead to very small particle horizons in the early epochs of the universe. This fact gives rise to the "horizon problem" in cosmology. (4) No consistent scenario exists—within the framework of the

big-bang model—that explains the origin, evolution, and characteristics of structures in the universe at small scales. (5) Last, one may add to this list the so-called "flatness problem."¹ While the flatness problem is less serious than any of those mentioned above, it certainly has received considerable attention in the recent past—especially because inflationary models seem to solve it.

Thus, it is clear that the big-bang model is observationally supported in the present epoch and sometime in the past. Microwave background radiation can trace the theory up to a red-shift $z \sim 10^3$ while nucleosynthesis probes the theory (at best) up to $z \sim 10^{11}$. We do not have today any observational evidence regarding the validity (or otherwise) of the model at larger red-shifts. (It is to be noted that the standard grand-unified-theory era is at $z \sim 10^{27}$.) On the other hand, the theoretical inconsistencies described above definitely suggest the breakdown of the model at some large but finite z . One is therefore led to the inevitable conclusion that the big-bang scenario can, at best, be an approximation to a more complete theory which must replace it at large z . It is only natural to demand that the extension of the big-bang model should retain the attractive features of the theory while eliminating as many difficulties as possible.

Before one considers a new scenario to replace the simple big-bang model it is relevant to consider the possibility of any conventional improvement which will take care of the difficulties mentioned above. Broadly speaking, the attempts to improve upon the big-bang model may be classified into two sets: (a) quantum cosmological models and (b) inflationary models.

It is an unfortunate fact that we do not have today a complete quantum theory of gravity. All investigations in the field of quantum cosmology resort to many simplifying assumptions in order to produce a tractable mathematical model.² The resulting model can be solved to provide a mathematical solution. However, the physical meaning of such an exercise is far from clear at this stage. What is the meaning of spacetime if light cones are quantum variables? What is the physical meaning of the wave function of the universe when no observers were present? What replaces the singularity in a quantum cosmological model? Until clear answers to these questions are provided, one cannot accept quantum cosmologi-

cal models as providing an answer to the problems of classical cosmology.

The inflationary models (for a review see Ref. 3) are also not without serious drawbacks of their own. All straightforward inflationary scenarios lead to a universe far too inhomogenous to be acceptable. Even assuming that this problem is somehow sorted out inflation does not solve the problem of singularity or "creation." (In the conventional models, the inflationary phase is always preceded by a hot, radiation-dominated epoch which is singular. One can, of course, think of quantum gravitational inflationary models⁴ which may avoid the singularity; the objections raised in the previous paragraph are relevant again.) Thus inflationary models also fall short of the ideal.

One can, however, learn useful lessons from simplified quantum cosmological models as regards the ingredients of the more complete theory. If a model should successfully explain creation of positive-energy matter without violating the conservation of energy, then it is necessary to have some degree of freedom which acts as a negative-energy mode. All quantum gravitational models which describe the creation consistently⁵ use such a "negative-energy mode" arising from the scale degree of freedom of gravity. Thus introducing a negative-energy field, at the classical level itself, may provide a natural way for creating the matter. It is well known that the classical singularity theorems cease to be operational when positivity of energy density is not guaranteed. Thus the introduction of a negative-energy field may solve two of the five difficulties faced by the big-bang model. It is, of course, essential to develop a detailed quantitative model and show that it not only retains the advantages of the big-bang model but that it also solves the major problems outlined above. We shall demonstrate these features with a simple model in the following sections.

II. GRAVITY WITH CREATION FIELD

The simplest possible choice for a negative-energy field is the one with zero mass and zero spin. Such a model was developed and investigated in detail in the sixties.⁶ The generation of a net baryon number as well as the exponentially expanding de Sitter universe arose quite naturally in these scenarios. We shall therefore take our negative-energy field to be the massless scalar field, usually called "creation field" or C -field. The complete action functional describing the C -field, matter, and gravity is

$$\mathcal{A} = \frac{1}{16\pi G} \int R \sqrt{-g} d^4x - \frac{f}{2} \int C_i C^i \sqrt{-g} d^4x + \mathcal{A}_I + \mathcal{A}_{\text{matter}} . \quad (1)$$

Here $C_i = (\partial C / \partial x^i)$ and f is a coupling constant. (Even though f can be absorbed into C , it is convenient to use it in the above form.) The action \mathcal{A}_I in (1) represents the coupling of the C -field to ordinary matter and radiation. In the classical theory, wherein matter and radiation may be represented by particle world lines, \mathcal{A}_I can be taken to be

$$\mathcal{A}_I = \sum_a \int C_i dx_a^i , \quad (2)$$

where the coordinates x_a^i represent the world line of the a th particle. It should be noted that the variation of \mathcal{A}_I produces contributions only at the end points of the world line. [This is because the integral in (2) is independent of the actual path x_a^i and depends only on the end points.] Thus in order to have a completely determined theory, one also has to specify the rate of creation of matter per unit volume.

On varying the action in (1) with respect to g_{ik} we obtain the usual Einstein equations with the addition of C -field energy density:

$$R^i_k - \frac{1}{2} \delta^i_k R = -8\pi G ({}_{(m)}T^i_k + {}_{(C)}T^i_k) . \quad (3)$$

Here, and in what follows, we use the term "matter" to mean also sources other than the C -field. We have

$${}_{(C)}T^i_k = -f(C^i C_k - \frac{1}{2} \delta^i_k C^a C_a) . \quad (4)$$

The variation of the C -field in (1) will give its field equation

$$f \square C = N(x) . \quad (5)$$

In (5) the left-hand side arises from the "kinetic-energy" term for the C -field; the right-hand side $N(x)$ stands for the number of creation events per unit proper volume and arises from the variation of \mathcal{A}_I . In order to solve the field equations one should specify $N(x)$ as a boundary condition. The variation of matter variables will, of course, lead to dynamical equations for matter; however, these equations are not independent of the previous equations but follow from the Bianchi identities.

We next consider the cosmological solutions of these equations.

III. THE COSMOLOGICAL MODEL

The homogeneous and isotropic cosmological model is described by the Robertson-Walker line element:

$$ds^2 = dt^2 - S^2(t) \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] , \quad (6)$$

where $k=0, \pm 1$. We must assume (in order to preserve the maximal symmetry of the spacetime) that $C(\mathbf{x}, t) = C(t)$, and that

$${}_{(m)}T^i_k = \text{diag}(\epsilon, -p, -p, -p) . \quad (7)$$

We have taken $c=1$. We will also take $G=1$ in what follows except where numerical estimates are needed. Under these conditions, the field equations reduce to

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi}{3} (\epsilon - \frac{1}{2} f \dot{C}^2) , \quad (8)$$

$$2 \frac{\ddot{S}}{S} + \frac{\dot{S}^2 + k}{S^2} = -8\pi (p - \frac{1}{2} f \dot{C}^2) . \quad (9)$$

It is preferable to work with the energy conservation

equation, rather than Eq. (9). This allows us to use, instead of (8) and (9), the following set:

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi}{3}(\epsilon - \frac{1}{2}f\dot{C}^2); \quad (10)$$

$$\frac{d}{dS}(\epsilon S^3) + 3pS^2 = f\dot{C}(s) \frac{d}{dS}[S^3\dot{C}(s)]. \quad (11)$$

We are thus left with four unknown functions of time $C(t)$, $S(t)$, $\epsilon(t)$, $p(t)$ related by three equations: Eqs. (10), (11), and one equation of state between ϵ and p . The missing fourth equation should specify the interaction between C -field and matter. As we discussed before, one should specify the rate of creation of matter energy (at the expense of the negative energy of the C -field) in order to obtain a unique solution. We shall make the natural assumption that at any epoch the rate of creation of matter energy density is proportional to the strength of the existing C -field energy density. That is, the rate of creation of matter energy density per unit proper volume is given by

$$\frac{d}{dS}(\epsilon S^3) + 3pS^2 = \alpha^2 \dot{C}^2 \equiv \alpha^2 g^2(S), \quad (12)$$

where α is a constant and we have defined $\dot{C}(S) \equiv g(S)$. Thus our complete set of equations now reads as (12) above, together with

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi}{3}(\epsilon - \frac{1}{2}fg^2), \quad (13)$$

$$\frac{d}{dS}(\epsilon S^3) + 3pS^2 = fg(S) \frac{d}{dS}(S^3g). \quad (14)$$

In addition, in the radiation-dominated era the equation of state is $\epsilon = 3p$.

Though we have introduced (12) as an extra assumption, our results do not critically depend on it. We discuss this aspect in the Appendix. For the sake of simplicity we shall also take $k=0$ in the solution obtained below. (Generalizations to other cases are discussed in the next section.)

Combining (12) and (14) we get

$$g(S) \frac{d}{dS}(gS^3) = \frac{\alpha^2}{f} g^2(S), \quad (15)$$

which integrates to

$$g(S) = \frac{A}{S^3} \exp\left[-\frac{\alpha^2}{2fS^2}\right], \quad (16)$$

where A is an arbitrary constant. Substituting (16) into (12) and using the equation of state $\epsilon = 3p$, we get

$$\frac{d}{dS}(\epsilon S^3) + \epsilon S^2 = \frac{\alpha^2 A^2}{S^6} \exp\left[-\frac{\alpha^2}{fS^2}\right], \quad (17)$$

which can be integrated to give

$$\epsilon(S) = \frac{A^2 f^2}{2\alpha^2 S^4} \left[1 + \frac{\alpha^2}{fS^2}\right] \exp\left[-\frac{\alpha^2}{fS^2}\right]. \quad (18)$$

Substituting (18) and (16) into (13), we get the equation for $S(t)$ to be ($k=0$)

$$\frac{dx}{dt} = B \exp\left[-\frac{\alpha^2}{2fx}\right] \quad (19)$$

with

$$x \equiv S^2(t), \quad B = \left[\frac{\pi}{3}\right]^{1/2} \frac{Af}{\alpha}. \quad (20)$$

The solution to (19) is indicated in Fig. 1. Though (19) cannot be integrated in closed form it is easy to obtain the general characteristics of $S(t)$. The expansion factor $S(t)$ is nonzero at all finite times. It starts at vanishingly small value at the asymptotic past ($t \rightarrow -\infty$) and grows steadily. In other words, we have achieved a nonsingular model which expands steadily from the infinite past to infinite future.

The nonsingularity of the epoch $t \rightarrow -\infty$, $S \rightarrow 0$ deserves some comment. In a naive interpretation of what constitutes lack of singularity in a Robertson-Walker model it might be argued that (i) the fundamental observers whose world lines are orthogonal to the hypersurfaces ($t = \text{constant}$) of homogeneity and isotropy are past and future complete and (ii) the curvature invariants do not become unbounded at any epoch. It can be shown that both these criteria are satisfied in our model.

However, even if these criteria are met, it is possible for Robertson-Walker spacetimes to be considered singular from the standpoint of null-geodesic incompleteness. Physically speaking, this criterion implies that the affine parameter length for a past- (or future-) directed null geodesic is finite. For an observer at epoch t_0 and the Robertson-Walker line element (6) this condition becomes, for past-directed null geodesics,

$$\int_{t_p}^{t_0} S(t) dt < \infty,$$

where t_p is the lower bound on the permitted time axis of the model. It is interesting to note that this condition is satisfied by the steady-state model which could therefore be considered singular. In our model, however, the integral on the left-hand side becomes infinite for $t_p = -\infty$. Thus the present model emerges as nonsingular under the more stringent criterion of null-geodesic completeness.

The behavior of the radiation density and the C -field

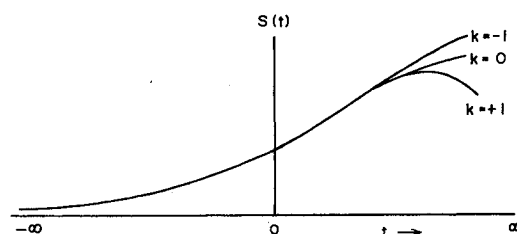


FIG. 1. The expansion factor $S(t)$ as a function of time. Note that $S(t)$ is positive for all finite values of t .

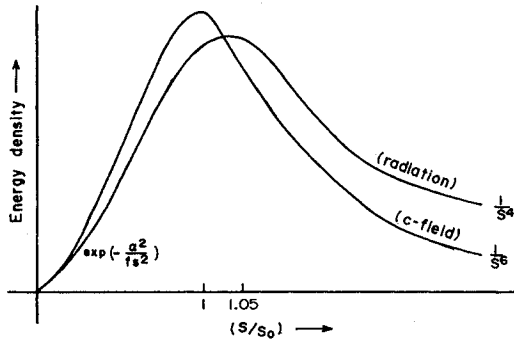


FIG. 2. The behavior of energy densities of the C -field and radiation as functions of the expansion factor. (The absolute value of the C -field energy density is plotted.)

energy density g^2 are plotted against S in Fig. 2 and against the cosmic time t in Fig. 3. The radiation density starts with a zero value in the infinite past and grows to a maximum value of

$$\epsilon_{\max} = \left[1 + \frac{\sqrt{3}}{2} \right] \exp[-(\sqrt{3}+1)] \left[\frac{A^2 f^4}{\alpha^6} \right] \cong 0.12 \frac{A^2 f^4}{\alpha^6} \quad (21)$$

at the epoch

$$S_{\max} = \left[\frac{3}{1+\sqrt{3}} \right]^{1/2} \frac{\alpha}{\sqrt{3}f} \cong 1.05 \frac{\alpha}{\sqrt{3}f} \equiv 1.05 S_0. \quad (22)$$

For $S > S_m$, $\epsilon(s)$ decreases with S and for $S \gg S_m$ the radiation density obeys the usual Friedmann relation

$$\epsilon(S) \propto \frac{1}{S^4}. \quad (23)$$

The C -field energy density behaves similarly. It starts from zero in the infinite past and grows to the maximum value of

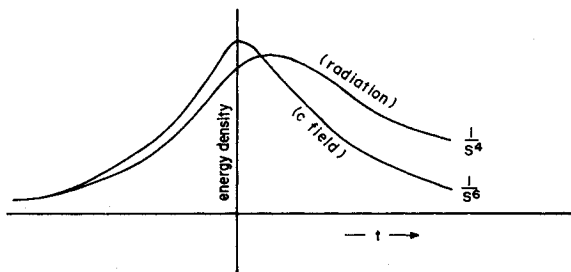


FIG. 3. The behavior of energy densities as functions of cosmic time t . Note that both matter and C -field densities start at zero value in the infinite past.

$$|\epsilon_{\max}(C\text{-field})| \equiv \left[\frac{f}{2} g^2 \right]_{\max} = \frac{27}{2e^3} \frac{A^2 f^4}{\alpha^6} \cong 0.67 \frac{A^2 f^4}{\alpha^6} \quad (24)$$

at the epoch

$$S_{\max}(C\text{-field}) = \frac{\alpha}{\sqrt{3}f} = S_0. \quad (25)$$

In the $S \gg S_0$ phase, the magnitude of C -field energy density decreases and for $S \gg S_0$, this energy density falls as S^{-6} .

It is clear that both the radiation and the C -field start at zero value in the infinite past and grow to a maximum value around the same epoch S_0 . We identify this nonsingular epoch with the conventional big bang. The explosive, singular creation of the big bang is replaced by steady creation of matter and (compensating) negative-energy C -field from $t = -\infty$ to $t = 0$. (We choose the time coordinate such that $S = S_0$ at $t = 0$.) For epochs $S \gg S_0$ the conventional big-bang model takes over with $S \propto t^{1/2}$ and $\epsilon \propto S^{-4}$. Clearly, the existence of the universe for infinite time in the past of "big-bang epoch" $t = 0$, allows all species of matter to come into thermal equilibrium through normal interactions. We shall now show that our model is horizon-free, allowing the interactions to homogenize the whole universe.

The coordinate distance to the horizon $r_H(t)$ is the maximum distance a null ray could have traveled at time t starting from the infinite past, i.e.,

$$r_H(t) = \int_{-\infty}^t \frac{dt}{S(t)}. \quad (26)$$

Notice that we could extend the proper time to $(-\infty)$ in the past because of the nonsingular nature of the spacetime. Using (19), we find that

$$r_H = \int_{-\infty}^t \frac{dt}{S} = \int_0^{S(t)} \frac{dS}{SS} = \int_0^{S(t)} \frac{2}{B} \exp \left[\frac{\alpha^2}{2fy} \right] dy. \quad (27)$$

The last integral diverges at the lower limit showing that the model is free of horizons (in other words, signals could have traveled to infinite distance at any t , because the universe "existed" since $t = -\infty$).

In order to get a feeling for the numbers involved we shall rewrite (16), (18), etc., in terms of the parameters

$$\epsilon_0 \equiv \frac{A^2 f^4}{\alpha^6}, \quad S_0^2 \equiv \frac{\alpha^2}{3f}, \quad q \equiv \frac{S}{S_0}, \quad (28)$$

obtaining

$$\epsilon(\text{radiation}) = \frac{9}{2} \frac{1}{q^4} \left[1 + \frac{3}{q^2} \right] e^{-3/q^2} \epsilon_0, \quad (29)$$

$$|\epsilon(C\text{-field})| = \frac{27}{2} \frac{1}{q^6} e^{-3/q^2} \epsilon_0. \quad (30)$$

To assign some numerical values, we should know the

value of q at the present epoch. We see from (28) that $q_P \equiv S_P/S_0$ measures the factor by which the universe has expanded since the "big bang" (i.e., the epoch of maximum density). In order to accommodate nucleosynthesis we certainly need $q_P \geq 10^{11}$. If one has to accommodate grand unified theories and baryosynthesis, one should have $q_P \geq 10^{27}$. We shall take, as a fiducial value $q_P = 10^{30}$. Then we get

$$\frac{|\epsilon(C\text{-field})|}{\epsilon(\text{radiation})} \Bigg|_{\text{epoch}}^{\text{present}} \lesssim \frac{3}{q_P^2} \sim 3 \times 10^{-60}. \quad (31)$$

Quite clearly the C -field exerts negligible influence on the cosmological evolution in the "recent" past. Further, using $\epsilon_P(\text{radiation}) \approx 10^{-33} \text{ g cm}^{-3}$ and $q_P \approx 10^{30}$, we can estimate ϵ_0 from (29). We get

$$\epsilon_0 \approx \frac{2}{9} \epsilon_P q_P^4 \approx 10^{86} \text{ g cm}^{-3}. \quad (32)$$

The second parameter in the equations, S_0 , can be determined once the present value of S is given. Assuming that $S(\text{present epoch}) \equiv S_P \geq 10^{28} \text{ cm}$ we get

$$S_0 = \frac{S_P}{q_P} \geq 10^{-2} \text{ cm}. \quad (33)$$

The so-called flatness problem in the conventional big-bang model arises from the smallness of the parameter

$$|\Omega - 1| = \frac{|\rho - \rho_c|}{\rho} = \frac{1}{S^2} \frac{1}{(8\pi/3)\rho}. \quad (34)$$

For example, at the red-shift of $\sim 10^{30}$ (corresponding to an energy scale of 10^{17} GeV) the value of $|\Omega - 1|$ in the conventional big-bang model had to be smaller than 10^{-50} . In our C -field model this small value is explained in a natural fashion. Ever since $t = -\infty$, the scale factor was increasing and hence the curvature term (k/S^2) in the Einstein's equation was decreasing. On the other hand, the creation of matter which dominates the "pre-big-bang" epoch causes the $(8\pi/3)\rho$ term on the right-hand side of Einstein's equation to steadily increase. Thus around $t=0$, the curvature term (k/S^2) is negligible compared to the other terms in Einstein's equation. The effect can be numerically estimated by reintroducing G and c :

$$\begin{aligned} |\Omega - 1|_{S_0} &= \left[\left[\frac{1}{S^2} \right] \left[\frac{8\pi G}{3c^2} (\epsilon - \frac{1}{2} f g^2) \right]^{-1} \right]_{S=S_0} \\ &\approx \left[\frac{12\pi G \epsilon_0 S_0^2}{c^2} \right]^{-1}. \end{aligned} \quad (35)$$

Taking $\epsilon_0 \sim 10^{86} \text{ g cm}^{-3}$, $S_0 \sim 10^{-2} \text{ cm}$ [see Eqs. (32) and (33)] we get

$$|\Omega - 1|_{S_0} \approx 10^{-55}. \quad (36)$$

Thus the model naturally leads to the flatness observed in the universe.

We have, therefore, obtained a model which is practically indistinguishable from the big-bang model at moderate red-shifts and thus satisfies all the observational tests of big bang. However, this model does better. It solves the problems of singularity, horizon, creation of

matter without violating energy conservation, and flatness. Having demonstrated this feature, we shall discuss some more details of the model in the next section.

IV. FURTHER GENERALIZATIONS AND DETAILS

In order to present the basic picture in the clearest possible manner, we have made some simplifying assumptions in the previous section. The general scenario remains valid even when these assumptions are relaxed.

In Eq. (12) we set the rate of creation to be proportional to the strength of the C -field energy density. That the results remain valid under more general conditions is shown in the Appendix.

In Eq. (19) we have put $k=0$, for discussing the evolution of $S(t)$. When $k=-1$ the qualitative behavior of the universe is the same as for $k=0$. When $k=+1$, the expansion factor oscillates between two values S_1 and S_2 . The behavior of $S(t)$, is different at large t for $k=+1, -1$ as indicated in Fig. 1. Again the main conclusions remain unchanged. It should be emphasized that the expressions for $\epsilon(S)$ and $g(S)$ [Eqs. (18) and (16)] are independent of the value for k . Thus the increase of energy density during $S \lesssim S_0$ and decrease during $S \gtrsim S_0$ occurs in all models.

Finally, we have taken the equation of state for matter to be $\epsilon = 3p$. Clearly such an extreme relativistic limit is not valid throughout the epoch. With matter treated as dust (with $p=0$), the equations can be integrated in terms of incomplete Γ functions. Once again, no new features emerge.

It is possible to discuss the process of particle creation by the C -field by considering the individual world lines of particles. (The details can be found in Ref. 7.) The end-point contribution of the world line of a group of created particles leads to the creation condition,

$$\sum p^i(\text{created particles}) = C^i. \quad (37)$$

In the homogeneous and isotropic case, particles can be created in pairs with equal and opposite three-momentum ($p, -p$). Then (37) reduces to the energy equation,

$$\dot{C} = 2(p^2 + m^2)^{1/2} \equiv 2E. \quad (38)$$

The pressure and density of the cosmological fluid can be determined (at any epoch with the scale factor S) from the momentum distribution of particles. Let $N(p, S) dp$ be the number of particles in the momentum range $(p, p + dp)$. Then

$$\text{pressure} \equiv P(S) = \frac{1}{3} \int_0^\infty \frac{p^2}{E} N(p, S) dp, \quad (39)$$

$$\text{energy} \equiv \epsilon(S) = \int_0^\infty (p^2 + m^2)^{1/2} N(p, S) dp. \quad (40)$$

The created particles, in the absence of collisions will move along geodesics, leading to the momentum dependence

$$p \propto \frac{1}{S}. \quad (41)$$

When the collisions are not frequent enough to change the above rule, the function $N(p, S)$ must satisfy the phase-

space conservation equation

$$\frac{\partial N}{\partial S} - \frac{p}{S} \frac{\partial N}{\partial p} + \frac{2N}{S} = R(p, S), \quad (42)$$

where $R(p, S)S^3 dS dp$ is the number of particles created between the epochs $(S, S+dS)$ in the momentum range $(p, p+dp)$ per unit coordinate volume. Using the Green's-function techniques, it is easy to solve (42). We get

$$N(p, S) = \frac{1}{S^2} \int_{S_0}^S x^2 R \left[\frac{pS}{x}, x \right] dx, \quad (43)$$

where S_0 is an arbitrary constant to be determined by initial conditions. From (38) it follows that $R(p, S)$ must have the form

$$R(p, S) = \delta(E - \frac{1}{2}g(S))L(p), \quad (44)$$

where $L(p)$ is arbitrary. Under this condition (43) can be evaluated to give

$$N(p, S) = \frac{1}{S^2} F(pS), \quad (45)$$

where

$$F(pS) = x^2 L \left[\frac{pS}{x} \right], \quad (46)$$

and x is the (real, positive) root of the equation

$$(p^2 S^2 + m^2 x^2)^{1/2} = \frac{1}{2} x g(x). \quad (47)$$

When two solutions exist the contributions are added together in (46). When no real positive root of (47) exists F is zero by definition. Mathematically, given an evolution function for C -field $g(S)$, Eqs. (47), (46), and (45) determine the collisionless particle spectrum created by this C -field. Physically, a particle of momentum p_1 created at S_1 will have an energy

$$E_1 = (p_1^2 + m^2)^{1/2} = \frac{1}{2} g(S_1). \quad (48)$$

Subsequent to creation the momentum of the particle red-shifts as

$$p = \frac{p_1 S_1}{S} = \frac{1}{S} \left[\frac{1}{4} S_1^2 g(S_1)^2 - m^2 S_1^2 \right]^{1/2}. \quad (49)$$

We can work out the particle spectrum (in the absence of collisions) for the evolution discussed in the previous section. Taking $m=0$, at any epoch $S_1 > S_0$, the relation (47) implies that

$$\frac{1}{2} x g(x) = p S_1. \quad (50)$$

There is no solution for x if $p S_1$ exceeds the maximum value of $x g(x)/2$; i.e., for

$$p > \frac{2A}{3S_0^2 S_1} \frac{1}{e} \equiv p_{\max} \quad (51)$$

there is no solution. For $p < p_{\max}$ but $p > \frac{1}{2} g(S_1)$, the equation (47) has two roots corresponding to, say, S_- and S_+ . Here $S_- < S_0$ and $S_+ > S_0$, and we get

$$N(p, S_1) = \left[\frac{S_+}{S_1} \right]^2 N_+ + \left[\frac{S_-}{S_1} \right]^2 N_-, \quad (52)$$

where N_+ and N_- are related to the creation rates at S_+ and S_- . If $p < \frac{1}{2} g(S_1)$ then the only contribution is from S_- .

We have presented the above calculation in order to show that the C -field model is capable of predicting the spectrum of the created particles. As it stands, the calculation does not take into account the effects of collisions and consequent thermalization. In a model with infinite age, thermalization will be dominant and cannot be ignored. Nevertheless, the model may produce some *deviations* from the thermal spectrum which may be used to test the theory. This aspect is currently under investigation.

V. CONCLUSIONS

Let us compare the successes and failures of the C -field model with those of the big-bang model.

The C -field model satisfies all the observational tests satisfied by the big-bang model. (Incidentally, this fact shows that the observational tests for the big bang are far from "crucial.") The evolution of the C -field model at moderate red-shifts is indistinguishable from that of the big-bang model. However, as we emphasized before, the C -field model successfully tackles many other conceptual problems unanswered by big-bang cosmology. First of all, it provides a well-defined mechanism for creation of positive-energy matter. Second, it eliminates the problems of singularity, horizon, and flatness.

What possible objections could there be to the C -field model? Since any theory which does not disagree with observations cannot be questioned in an objective fashion, the C -field cosmology can only be questioned on what may be called "aesthetic" grounds. We now discuss some of the possible objections that may be raised against this model.

Objection 1. The model is based on the existence of a negative-energy C -field.

As we have argued earlier, the negative-energy field is the most natural choice for (a) providing a mechanism which will create matter without violating energy-momentum conservation and (b) escaping the consequences of singularity theorems. In fact, one may argue that singularity theorems added to the inadmissibility of singularities in physics *imply* the existence of negative-energy modes. It should also be noted that quantum theory, by itself, does not violate the law of conservation of energy. Unless certain degrees of freedom "act like" carriers of negative energy, quantization of a classical system cannot solve this problem. Thus the negative-energy field may be a physical necessity.

Objection 2. The creation condition assumed in this paper is *ad hoc*.

While the specific creation condition used in this paper is an assumption, the main results of the paper do not depend on this assumption. We show in the Appendix that the basic features of the model remain true under a wide class of creation conditions.

Objection 3. The theory does not make *new*, verifiable predictions.

In the context of the present paper, this objection remains valid. However, we feel that this objection is partially answered in this paper on two counts. (i) We have shown that the theory does solve some of the theoretical problems of the big-bang model. The initial acceptance of grand unification or supersymmetry, for example, was based on conceptual considerations rather than on detailed observational characteristics. We feel that the analysis in this paper does establish a *prima facie* case for the *C*-field cosmology. (ii) We have also shown that the model is capable of describing the spectrum of created particles. It is likely that thermalization does not wipe out this signature altogether. Thus one may be led to the prediction of deviations from the thermal nature of the spectrum of microwave radiation.

The evolution of perturbations in the new cosmological model is under investigation. The existence of a pre-big-bang epoch in which the energy density is increasing with time helps the growth of inhomogeneities and can solve the problem of galaxy formation. The details of this process are under investigation.

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APPENDIX

In the text we have assumed that the rate of creation is proportional to the *C*-field energy density. Actually, the main results of the paper continue to be valid even if Eq. (14) is replaced by a more general assumption,

$$f \frac{d}{dS}(gS^3) = \mu g^{1+m}, \quad (\text{A1})$$

where μ is a constant and $m > 0$. (In the text we have considered the $m=0$ case.) We shall briefly discuss the case with $m > 0$ here.

We consider the case $k=0$, with (14) in the text replaced by (A1). When (14) is replaced by (A1), $g(S)$ becomes

$$g(S) = \left[\left[\frac{\mu}{f} \right] \left[\frac{m}{3m+2} \right] \frac{1}{S^2} + cS^{3m} \right]^{-1/m}, \quad (\text{A2})$$

where c is the integration constant. For large and small S this has the limiting forms

$$\begin{aligned} g(S) &\simeq \left[\frac{1}{c} \right]^{1/m} \frac{1}{S^3} \quad (\text{large } S) \\ &\simeq \left[\frac{1}{A} \right]^{1/m} S^{2/m} \quad (\text{small } S). \end{aligned} \quad (\text{A3})$$

Thus $g(S)$ dies down for both large and small S and reaches a maximum in between. The qualitative features are very similar to that of (16). Using this form for $g(S)$ we can find $\epsilon(S)$. Direct calculation shows that

$$\frac{1}{S} \frac{d}{dS}(\epsilon S^4) = \mu \left[\frac{\mu}{f} \left[\frac{m}{3m+2} \right] \frac{1}{S^2} + cS^{3m} \right]^{-(m+2)/m}. \quad (\text{A4})$$

Therefore,

$$\begin{aligned} \epsilon(S) &\simeq \frac{B}{S^4} \quad (\text{large } S) \\ &\simeq B'S^{4/m} \quad (\text{small } S), \end{aligned} \quad (\text{A5})$$

where B, B' are constants. Again we have the same qualitative features as in (18); ϵ vanishes at large and small S with one maximum in between. Using these equations, we can write the evolution equation for $S(t)$ as

$$\dot{S}^2 = \frac{\Lambda^2}{S^2} \int_0^S du u^3 g^2(u), \quad \Lambda^2 = \frac{8\pi Gf}{3}. \quad (\text{A6})$$

This equation integrates to give

$$\begin{aligned} S^{2/m} &\simeq -\frac{(\text{const})}{t} \quad (\text{small } S; S \simeq 0) \\ &\simeq (\text{const})t^{1/m} \quad (\text{large } S; S \simeq \infty). \end{aligned} \quad (\text{A7})$$

It follows that S goes to zero only in the infinite past of time. Thus the spacetime is nonsingular and goes over to the radiation model ($t^{1/2}$) for large t . The main conclusions of the paper are therefore independent of the value chosen for m .

¹R. H. Dicke and P. J. E. Peebles, in *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979).

²For a review see, e.g., J. B. Hartle, in *The Very Early Universe*, edited by G. Gibbons, S. W. Hawking, and S. Siklos (Cambridge University Press, Cambridge, 1983); J. V. Narlikar and T. Padmanabhan, *Phys. Rep.* **100**, 153 (1983); T. Padmanabhan, *Phys. Rev. D* **28**, 756 (1983).

³A. D. Linde, *Rep. Prog. Phys.* **47**, 925 (1984).

⁴A. Vilenkin, *Phys. Rev. D* **27**, 2848 (1983); T. Padmanabhan,

Phys. Lett. **104A**, 196 (1984).

⁵R. Brout *et al.*, *Ann. Phys. (N.Y.)* **115**, 78 (1978); D. Atkatz and H. Pagels, *Phys. Rev. D* **25**, 2065 (1982); T. Padmanabhan, *Phys. Lett.* **93A**, 116 (1983).

⁶M. H. L. Pryce (private communication); F. Hoyle and J. V. Narlikar, *Proc. R. Soc. London* **A273**, 1 (1963); **A278**, 465 (1964); **A282**, 178 (1964); **A290**, 143 (1966); **A290**, 166 (1966); **A294**, 138 (1966).

⁷J. V. Narlikar, *J. Astrophys. Astron.* **5**, 67 (1984).