

# Singularity and Matter Creation in Cosmological Models

UNDER "reasonable assumptions" the singularity present in the classical Friedmann cosmological models, and in the gravitational collapse of massive objects, is an unavoidable consequence of general relativity<sup>1,2</sup>. As a rule, singular solutions are unpalatable to theoreticians and suggest something wrong with the theory. Nevertheless some cosmologists make a virtue of this drawback by arguing that the cosmological singularity describes an unusual event, the creation of the Universe, and as such should not be judged by the usual yardstick. Whatever the merit of this argument, it certainly cannot be applied to the case of collapsing massive objects.

Another unsatisfactory feature of the classical cosmological models is the scant attention paid to the question of matter creation. It is assumed that all the present matter (and radiation) in the Universe appeared in its primary form at the time of the "big bang". Subsequent to this event matter as a whole is conserved according to the Einstein equations, although it may change its form as the Universe evolves. So the question "How was the matter created in the first place?" is left unanswered.

I suggest that these two drawbacks in the classical cosmological discussion may be connected and that a proper consideration of matter creation should resolve the problem of singularity. To illustrate the point I consider here two cosmological models in which explicit discussion of matter creation has led to the removal of singularity. The first is the well known steady state model<sup>3,4</sup>; the second is a singularity-free Friedmann model.

The underlying theoretical framework is that first given by M. H. L. Pryce (private communication) and discussed elsewhere<sup>5,6</sup>. It starts from an action principle with the action given by

$$S = \frac{1}{16\pi G} \int R \sqrt{-g} d^4x - \sum_a \int m_a da - \frac{1}{2} \int f C_i C^i \sqrt{-g} d^4x + \sum_a \int C_i da^i \quad (1)$$

In equation (1) the first two terms on the right side represent geometry and inertia, and are the same that lead to the Einstein equation. The last two terms involve creation of matter and are therefore not present in the classical discussion. Here  $C_i$  represents the gradient of a scalar field  $C$  and  $f$  its coupling constant. If the particle is created at world point  $A_1$  and annihilated at world point  $A_2$ , then the integral along its world line is

$$\int C_i da^i = C(A_2) - C(A_1) \quad (2)$$

Thus in a world line variation the last term of equation (1) only contributes at the ends.  $\delta S = 0$  in this case leads to

$$m_a \frac{da^i}{da} \Big|_{A_1} = C^i(A_1), \quad m_a \frac{da^i}{da} \Big|_{A_2} = C^i(A_2) \quad (3)$$

Equation (3) describes energy-momentum conservation when a particle is created or destroyed. The variation of  $C$  leads to its source equation:

$$\square C \equiv C^k_{;k} = f^{-1}n \quad (4)$$

where  $n$  is the number of particles created per unit proper four-volume. (Here annihilations occur with negative signs.) Finally, the variation of the metric tensor gives the field equations

$$R^{ik} - \frac{1}{2}g^{ik}R = -8\pi G [T^{ik}_{(m)} + T^{ik}_{(c)}] \quad (5)$$

Here  $T^{ik}_{(m)}$  is the usual energy momentum tensor of matter and

$T^{ik}_{(c)}$ , the  $C$ -field energy tensor, is given by

$$T^{ik}_{(c)} = -f \{ C^i C^k - \frac{1}{2}g^{ik} C^l C_l \} \quad (6)$$

Thus matter creation is given by

$$T^{ik}_{(m)}_{;k} = -T^{ik}_{(c)}_{;k} = f C^i C^k_{;k} \quad (7)$$

From equations (4) and (7) the law of conservation of matter breaks down at points where  $\square C \neq 0$ . But the combined energy of matter and  $C$ -field is always conserved.

To discuss the cosmological models mentioned previously I shall apply these equations to the Robertson-Walker line element with  $k=0$ . (The cases  $k = \pm 1$  can be discussed similarly.) This line element is given by

$$ds^2 = dt^2 - Q^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \quad (8)$$

$C$  in this case is a function of cosmic time  $t$  only. The equations (5) then reduce to

$$2\ddot{Q}/Q + \dot{Q}^2/Q^2 = 4\pi G f \dot{C}^2 \quad (9)$$

$$3\dot{Q}^2/Q^2 = 8\pi G \rho - 4\pi G f \dot{C}^2 \quad (10)$$

where  $\rho$  is the density of matter. If  $m$  is the average mass of the particles,  $N(t)$  their number density and  $n(t)$  their rate of creation per unit volume

$$\rho = mN, \quad f \frac{1}{Q^3} \frac{d}{dt} (Q^3 C) = \frac{1}{Q^3} \frac{d}{dt} (NQ^3) = n \quad (11)$$

$n(t)$  essentially determines the nature of the cosmological model. The two cases to be discussed represent two extremes of the behaviour of  $n$ . Reality may lie somewhere in between.

(i) The steady state model has  $n = \text{constant}$  and  $N = \text{constant}$ . Representing these values by  $n_0$  and  $N_0$  respectively,

$$Q(t) = \exp Ht \quad (12)$$

where

$$3H^2 = 4\pi G f m^2, \quad N_0 = f m, \quad n_0 = 3H f m \quad (13)$$

A constant of integration has been absorbed in the scale factor in equation (12).  $Q$  is never zero, that is, a singular state does not exist. Matter is created continually.

(ii) In the non-singular Friedmann model matter is created once and for all in an instant, so that

$$n(t) = N_0 \delta(t) \quad (14)$$

say. To fix ideas I assume the Universe to be empty and flat before  $t=0$ . For  $t > 0$ ,

$$N = N_0 Q_0^3 / Q^3, \quad \dot{C} = N_0 Q_0^3 / f Q^3, \quad Q_0 = Q(0) \quad (15)$$

I also assume  $\dot{Q} = 0$  at  $t=0$ . Then,

$$Q(t) = Q_0 \{ 1 + t^2/t_0^2 \}^{1/2} \quad (16)$$

where

$$t_0 = (12\pi G f m^2)^{-1/2} \quad (17)$$

A non-singular model with the same  $Q(t)$  has recently been obtained by Trautman<sup>7</sup> by adding torsion terms to the usual Einstein equations.

Now  $Q(t)$ , as given by equation (16), is never zero nor is  $N$  as given by equation (15) ever infinite. For  $t \gg t_0$ ,  $Q(t)$  resembles the  $Q(t)$  of the Einstein-de Sitter model. To make the non-singular Friedmann model apply to the real Universe, the coupling constant  $f$  must be several magnitudes higher than if the steady state model represents the Universe.

In both these examples, matter creation is accomplished at the expense of the negative energy  $C$ -field. In general relativity negative energy repels. This causes one of the requirements of Penrose and of Hawking to break down and thus avoids singularity. At this stage I do not want to say that primary matter creation is in fact accomplished by the  $C$ -field, but to emphasize the following general conjecture. If overall energy conservation is to be maintained, primary matter creation

must be accompanied by release of negative energy. The repulsive nature of this negative energy reservoir will be sufficient to prevent the singularity. This conjecture should apply to finite collapsing objects as well as to the Universe.

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## Limit to Pulses of Radiofrequency Emission from the Galactic Centre

THE nucleus of our galaxy, at the position of the Sgr A radio source, has been searched for pulses of radio emission in the frequency range 1,650 to 1,720 MHz. Pulses of short duration might be emitted in association with pulses of gravitational radiation or with other shock wave, collapse, or flare phenomena<sup>1,2</sup>. Previous attempts to detect radio pulses have been made<sup>3-7</sup>: one of these<sup>7</sup> suggested a possible source of pulses near the galactic centre. The work described here was designed to be capable of detecting the relative dispersion delay that the interstellar medium would impose at different frequencies on any signal from the galactic centre; interstellar electrons delay a radio pulse at a frequency  $\nu$  as  $\nu^{-2} \int N_e dl$ , where  $\int N_e dl$  is the integrated electron density or dispersion measure to the source. To accomplish such detection, continuous simultaneous observations at two frequencies were made; the demand for delayed pulses at the lower frequency with respect to the higher provided a discrimination against man-made or terrestrial pulses, which lack dispersion effects.

The 25.6-m radio telescope of Harvard College and Smithsonian Astrophysical Observatories was operated with a wide-band antenna and amplifier system with an antenna efficiency about 40%, beamwidth 29 arc min and noise temperature  $\sim 200$  K; operating frequencies of 1,713 MHz and 1,665 MHz, with bandwidths of 5 MHz and 15 MHz, respectively, were used. A calibration noise temperature of 33 K was injected into the system from time to time. With an effective time constant of 2 ms, typical noise amplitude, while on Sgr A, was about 3 to 5 K peak to peak.

Observations were made between July 30 and August 8, 1972, with the telescope centred on Sgr A. The detected outputs of both radio channels were recorded on an f.m. tape recorder (to facilitate detailed re-examination of any pulses detected) and on a chart recorder with response time  $\sim 0.1$  s. Dispersion measures of from  $5 \times 10^3$  to  $10^6$  electrons  $\text{cm}^{-3}$  pc would have modified any source pulse of duration 0.1 to 20 s, giving a detectable difference in arrival time for the two frequencies; for example, a dispersion measure of  $10^4$  electrons  $\text{cm}^{-3}$  pc to an emitting region would have given a pulse separation between channels of 0.9 s. Electron density to the galactic centre is usually expected to be greater than the value of  $(N_e^2) \sim 10^{-1} \text{ cm}^{-6}$  obtained from radio measurements near the Solar System<sup>8</sup>. Thus, for a distance of 10 kpc, the dispersion measure should be larger than  $3 \times 10^3$  electrons  $\text{cm}^{-3}$  pc.

During 28 h of interference-free operation, no pulses of possible astrophysical origin greater than 5 K over the mean, or 63 f.u., were observed within the duration limit of 0.1 to 20 s. If apparently simultaneous pulses (to within 0.2 s) had been

**Table 1** Radio-frequency Pulse Limits from the Galactic Centre and Andromeda

Direction of source	Pulse duration (s)	Dispersion measure (electrons $\text{cm}^{-3}$ pc)	Flux density ( $\text{W m}^{-2} \text{ Hz}^{-1}$ at 1,665 MHz)	Frequency of occurrence ( $\text{s}^{-1}$ )
Sgr A galactic centre	0.1–20	$5 \times 10^3 - 10^6$	$> 6.3 \times 10^{-25}$	$< \sim 10^{-5}$
M31 Andromeda	0.1–20	$5 \times 10^3 - 10^6$	$> 3.9 \times 10^{-25}$	$< \sim 5 \times 10^{-5}$

found their duration and possible dispersion delay would have been investigated by replaying the tapes with faster recording techniques; the dispersion delay would have had to be less than the equivalent of  $\sim 1,000$  electrons  $\text{cm}^{-3}$  pc and the pulse duration  $< 0.1$  s for any pulse to have been missed in this investigation. In 28 h there should be at least one gravitational pulse in the field of view, if gravitational pulses are generated chiefly at the galactic centre<sup>9</sup>. A lack of electromagnetic pulses associated with gravitational pulses is no evidence against gravitational pulses from this or other regions; however, the limits to pulses, other than gravitational, from this and other work<sup>3-6, 10-13</sup> place interesting constraints on the rate of catastrophic events or emission from them.

The limit implied by this work is shown in Table 1. The flux limit applies for 0.1 s duration pulses and would be lower for longer pulses. For comparison, Weber's gravitational pulses are reported to be of the order of  $10^2 \text{ W m}^{-2} \text{ Hz}^{-1}$ . If the exact times of Weber's pulses during these observations had been available a reduction of a factor of two in sensitivity could be made. This limit does not contradict the results of Hughes and Retallack<sup>7</sup> because the frequency and beamwidth used were different. Some constraints are placed on their pulses; if they are emitted within 0.3 arc deg of Sgr A their frequency spectrum must be steep or their pulse height spectrum must be deficient at small amplitudes. A much shorter period of observation (5 h) on our nearest neighbouring galaxy (M31) allows a similar limit to be estimated.

Finally, on a few occasions outside the 28 interference-free hours used above, groups of interference radar pulses produced "events" on one or both channels. Their nature (as observed on an oscilloscope) and lack of dispersion made their rejection automatic. These observations overlapped the periods of auroral activity following the solar flares of August 2 to 7, 1972, and I believe that the events were caused by incoherent scattering of distant radar transmissions, because the scattering was probably similar to that used in auroral studies<sup>14</sup>. It is conceivable that such effects might also produce spurious coincidences between widely spaced detectors.

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