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Applications of statistical techniques to the angular size–flux density relation for extragalactic radio sources

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Summary. The data on the angular sizes and flux densities of extragalactic radio sources from the 3CR and the Ooty surveys is subjected to two independent statistical tests. The purpose of these tests is to investigate whether the data necessitates the conclusion that the properties of the radio sources evolve with the epoch of the Universe. The first test uses a minimum χ^2 technique to compare the observed (θ, S) plot with a theoretical one. In the second, the so-called median test, the two distributions are compared in a narrow range of flux densities.

It is shown that the present scatter in this data is such that within the usual confidence limits it is not possible to distinguish between a number of evolutionary and non-evolutionary models. In fact, a non-evolutionary model in which there is a reasonable correlation between the linear size and power of radio sources passes both the tests. It is shown how future improved data on (θ, S) may be able to distinguish between the evolutionary and non-evolutionary models.

1 Introduction

There are two distinct ways in which the observational cosmologist is handicapped in settling the cosmological question. On the one hand the insufficient data does not permit him to draw a clear-cut conclusion and a certain amount of guess-work is called for as, for example, in the determination of the redshift of an object with only one spectral line. By contrast, in some situations there are plenty of data points, but in such a scattered form to show up any meaningful trends. In such cases the application of statistical techniques is often useful, if only to lend an objectivity to the conclusions. The data on the angular size (θ) and flux density (S) of extragalactic radio sources is an instance of this type. Fig. 1, adapted from the paper of Swarup (1975, hereafter Paper I), shows a combined plot of θ versus S for the sources from the 3CR and the Ooty survey, and the highly scattered nature of the data points is quite apparent from the figure.

Assuming that the upper limits of values in some of the Ooty sources are the exact values, one can fit regression lines of θ on S, and S on θ, which are shown, for the Ooty sources alone, in Fig. 1. The large angle between these lines ($\sim 72°$) is an indication of the apparent

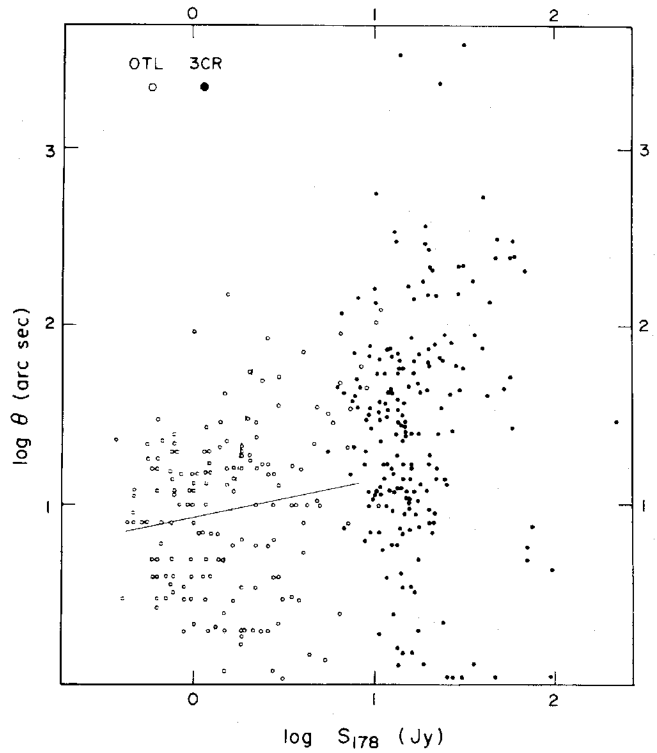

Figure 1. The scatter diagram for θ and S adapted from Fig. 1 of Paper I. The flux levels are normalized to 178 MHz. The two regression lines for the Ooty data are at an angle of 72°.

lack of correlation between θ and S and this is borne out by the correlation coefficient turning out to be ~0.17, statistically consistent with a null hypothesis of zero correlation at 95 per cent confidence level.

At first sight it appears too ambitious to extract any cosmological information from a scattered plot like Fig. 1. But making some allowance for it, if we should be able to understand the *physical* reason for the scatter, it may still be possible to enquire whether this plot contains any significant cosmological information. In the following section we will discuss the nature of the scatter in the θ–S diagram. We find it convenient to compare and contrast this data with the optical redshift–magnitude (z–m) relation. A comparison with this test has been made by Kapahi (1975a, hereafter Paper II) also, but we have adopted a different approach. Having analysed the theoretical nature of the scatter, we wish to subject it to the statistical tests. The main thrust of these tests will be to decide whether the Ooty data warrants the conclusion that the Universe is evolving.

2 The nature of scatter in the θ–S plot

2.1 A COMPARISON BETWEEN THE θ–S AND z–m RELATIONS

It is instructive to compare the θ–S relation with the optical redshift–magnitude relation in order to appreciate the extent of scatter in the former relation. The z–m relation is of the form

$$m = M - 5 + 5 \log [(cz)/H] + f(z), \quad (1)$$

where M is the absolute magnitude of the light emitting source, c speed of light, H Hubble's constant and m is the apparent magnitude. $f(z)$ is a function depending on the cosmological model to be tested. Although $f(z)$ varies considerably from model to model for z as high as 1, these variations are comparable to or swamped by other factors, mainly arising from the uncertainty in M. The K correction, the aperture correction, the possibility of luminosity evolution, the possible presence of intergalactic absorption, the Scott effect are examples of these factors. A detailed discussion (see Gunn & Oke 1975; Chitre & Narlikar 1976) has shown the futility of any attempts to distinguish between the different cosmological models with the present state of the relation.

Compared to this the radio-astronomical θ–S relation is rather more uncertain. Here S plays the part of m, and therefore shares with it many of the uncertainties mentioned above. In fact the optical Hubble relation (1) shows considerably less scatter for galaxies than does the S–z relation for radio galaxies. This was first pointed out by Hoyle & Burbidge (1970) and a later examination of a larger sample from the 3CR catalogue by Burbidge & Narlikar (1976) showed that the enormous scatter still persists. Thus the optical magnitude (even of radio galaxies) appears to be a more reliable distance indicator than the flux density.

The angular size θ also contains two sources of scatter. The first is the projection effect which becomes important because of the linear geometry of the radio sources in most cases. The second factor arises from the fact that the intrinsic linear size (i.e. the separation of the two components of a typical double source) varies considerably from source to source. The ideal θ–z relation was originally derived by Hoyle (1959) for different cosmological models. Most Friedmann models show a definite minimum for θ. The actual relation as reported by Miley (1971) shows a considerable scatter. His plot (Fig. 5, *op. cit.*) is shown in Fig. 2 without the theoretical lines. Although the 'largest angular size' decreases with z, the scatter in the θ values swamps any possible variations due to cosmology.

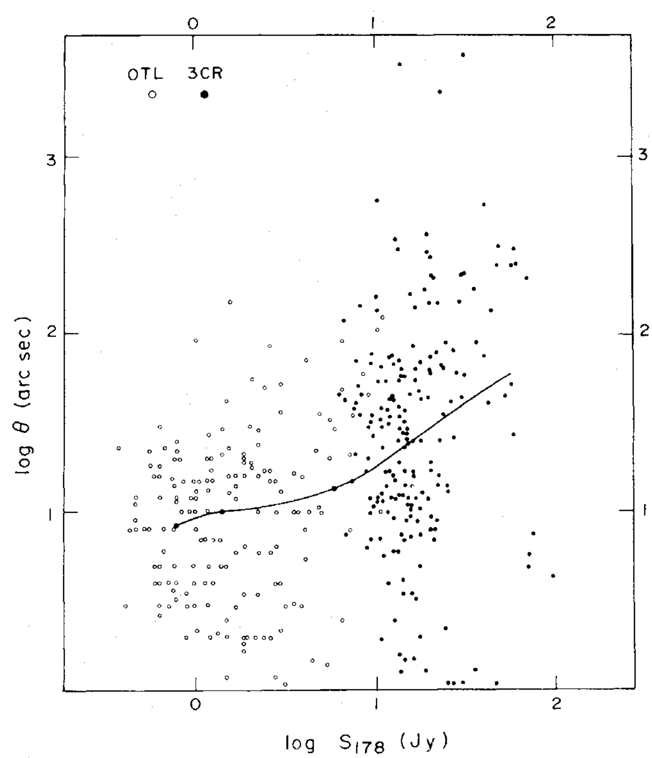

Figure 2. The plot adapted from Fig. 5 of Miley (*op. cit.*) showing the scatter in the angular sizes of radio sources at comparable redshifts. This scatter arises from the projection effects as well as the dispersion in intrinsic linear sizes of the sources.

For example, the ratio of angular sizes of a given source at redshift z in the Einstein–de Sitter and the Steady-State cosmologies is given by

$$r(z) = (z\sqrt{1+z})/[2(\sqrt{1+z} - 1)]. \tag{2}$$

At $z = 0.1$ this is 1.074, while for $z = 1$, $r = 3.414$. The actual linear sizes are known to vary by factors greater than these.

2.2 THE POWER–SIZE FUNCTION

From this discussion it is clear that the power–size function must inevitably enter any discussion of the θ–S relation. Suppose we know this function very well. Let $F(l, P, z)\, dl\, dP$ denote the number of sources per unit proper volume with the maximum linear size l in the range $(l, l + dl)$ and the power per steradian P, in the range $(P, P + dP)$ at the epoch of redshift z. Knowing this function we can calculate in a given cosmology the average number of sources $n(S, \theta)\, dS\, d\theta$ in the flux range $(S, S + dS)$ and angular size range $(\theta, \theta + d\theta)$ for a survey covering a *known* area of the sky. Here the word 'average' refers to averaging with respect to the projection effects. A comparison can then be made between theory and observation.

In Papers I and II this approach was followed, but only partially. Instead of comparing the entire distributions of data points, Swarup and Kapahi concentrated on the variation of the median value θ_m of θ in a given S range $(S, S + dS)$ with S. The median would be a reliable statistic for the overall distribution provided the scatter about it were small. In Fig. 3 the observed (θ_m, S) curve is superposed on the data points shown in Fig. 1. Clearly the scatter is large enough to warrant the examination of the entire (θ, S) plot rather than the median curve only.

In this paper we will examine this plot in two different ways. First we will assess an overall comparison between theory and observation with a variant of the familiar χ^2-test. We will then apply the so-called median test which takes into account the statistical fluctuations of the median. The purpose of both the tests is to look at the plausibility of any assumed cosmological hypothesis, evolutionary or otherwise.

3 Theoretical inputs

We will be concerned here with three types of models which are briefly described below.

Type A. These are evolutionary models with the assumption that there is no correlation between l and P. In this case we may write

$$F(l, P, z) = \lambda(l, z)\, \rho(P, z), \tag{3}$$

where λ and ρ are specified functions. The appearance of z in the arguments of these functions is an indication of cosmological evolution in the properties of radio sources. We will give numerical results for the model considered by Kapahi (*cf.* Paper II) with the following details

$$\lambda(l, z) = \frac{2}{l_0(z)} \left\{1 - \frac{l}{l_0(z)}\right\},$$

$$l_0(z) = l_0(1+z)^{-n},$$

$$\rho(P) = \begin{cases} KP^{-2.1}, & P_L < P \leqslant P_m, \\ K(1+z)^{\beta} P^{-2.1}, & P_m < P \leqslant P_u, \end{cases} \tag{4}$$

where K and l_0 are constants and $P_L = 10^{23}\,\mathrm{W\,Hz^{-1}\,ster^{-1}}$, $P_m = 10^{26}\,\mathrm{W\,Hz^{-1}\,ster^{-1}}$, $P_u = 2 \times 10^{28}\,\mathrm{W\,Hz^{-1}\,ster^{-1}}$. The size evolution parameter n is estimated by Kapahi to be in the range 1–1.5, while the density evolution of strong sources is taken in Paper II to be given by $\beta = 5.5$. We shall refer to this specific type of model as the Kapahi model.

Type B. These are *non*-evolutionary models with no correlation between l and P. In this case we have

$$F(l, P) = \lambda(l)\,\rho(P). \tag{5}$$

The absence of z dependence in the above functions is taken to imply that the radio source properties do not depend on the cosmological epoch. We will continue to take $\rho(P)$ as a suitable combination of $P^{-\gamma}$-type functions.

Type C. These are *non*-evolutionary models *with* a possible correlation between l and P. We shall discuss a typical case in detail in a later section.

For any model of the three types mentioned above it needs a straightforward calculation to generate the quantities $n(S, \theta)$. We have calculated these numbers using the electronic computer DEC 10 and have chosen the grid of S and θ as follows. Divide the range $\theta = 1$–100 arcsec into logarithmic subranges $\Delta \log \theta = 0.2$. In addition, a separate calculation is required for the range $\theta = 0$–1 arcsec. The S range 0.1–100 Jy is similarly divided into equal divisions of $\Delta \log S = 0.2$. The theoretical numbers $n(S, \theta)\,\Delta S\,\Delta\theta$ are to be scaled suitably for comparison with the observed numbers at 178 MHz, taken from the 3CR sample used in Paper I and the Ooty sample from Kapahi (1975b). The spectral index was taken uniformly as $\alpha = 0.75$ as in Paper II.

4 The minimum chi-square test

4.1 THE LIMITATIONS OF THE STANDARD CHI-SQUARE TEST

Having obtained the theoretical distribution of points on the plane we could consider comparing it with the observed distribution by using the familiar χ^2 test. However, one difficulty presents itself when we try to do this for the 3CR + Ooty data.

Following Swarup and Kapahi (*cf.* Papers I and II) we may choose a complete sample of 199 3CR sources with $\delta > +10°$, $|b| > 10°$, $S_{178} > 9$ Jy in the 4.25 ster of the sky (Longair & Pooley 1969; Mackay 1973). But the Ooty data is not a complete sample in this sense. As pointed out by Kapahi (1975b) the Ooty survey progressively covers smaller and smaller areas of the sky at lower S values. The areas covered in a given flux range cannot be estimated accurately by geometrical methods since the Ooty survey uses the lunar occultation method and no systematic effort has been made to calculate the region covered by the Moon during the period of the survey. For this reason the expected numbers in the Ooty part of the survey cannot be properly scaled for comparison with the observed distribution.

Nevertheless we can apply a variant of the χ^2 test which is based on the following assumption.

Assumption (E). The area of the sky covered in a narrow flux range $(S, S + \Delta S)$ in the Ooty survey is independent of the source angular size θ.

This assumption is strongly supported by Swarup & Kapahi (private communication), and though it is not explicitly stated it forms an important basis for their conclusions. If this assumption were not true and for example if the areas of the sky covered are smaller for large θ than for small θ, then the observed value of θ_m for sources in this flux range will be underestimated. Thus assumption (E) will be taken as the basis of the proposed statistical test.

It is worth pointing out that a similar assumption does not hold for S and θ interchanged. Thus not equal areas of the sky are covered for different S values in a given narrow θ range $(\theta, \theta + \Delta\theta)$. For this reason it is not possible to define a median value S_m of S in such a θ range. The use of S_m in Paper I is therefore misleading since there it refers to the median value of S for *all* sources in the flux range $(S, S + \Delta S)$.

Suppose we now divide the S scale into k narrow ranges. On the basis of assumption (E), we will associate an undetermined area α_i with the ith flux range on the S scale, and calculate the χ^2 in terms of the α_i. We will then minimize χ^2 with respect to all α_i and test the significance of the minimum value. If $\chi^2_{\min}$, the minimum value of χ^2 turns out to be significant then we can argue that the theoretical model does not represent the observed data. For, in this case the *actual* value of χ^2 (if all α_i's were known) would certainly be greater and more significant than $\chi^2_{\min}$. If $\chi^2_{\min}$ is not significant the case for agreement remains unproven. The levels of significance may be set at the usual 5 or 1 per cent probabilities.

4.2 THE THEORY OF MINIMUM CHI-SQUARES

Suppose we divide the ith flux density range into j_i sub-ranges of θ so that altogether we have

$$\sum_{i=1}^{k} j_i = r \tag{6}$$

different 'rectangular' subdivisions of the (θ, S) plane. Let O_{il} denote the observed number of data points in the lth sub-range of the ith range: $1 \leqslant l \leqslant j_i$, $1 \leqslant i \leqslant k$. It is not necessary that j_i be independent of i, nor is it essential that all rectangles in $\log \theta$ and $\log S$ have equal areas.

If unit angular area of the sky were surveyed, the theoretical model will predict a certain number of sources, f_{il} (say) in the above (θ, S) rectangle. However, by the assumption (E), the number to be compared with O_{il} is not f_{il} but $\alpha_i f_{il}$. Writing this expected number as

$$e_{il} = \alpha_i f_{il}, \tag{7}$$

we get χ^2 for the distribution as

$$\chi^2(\alpha_1, \ldots, \alpha_k) = \sum_{i=1}^{k} \sum_{l=1}^{j_i} \frac{(O_{il} - e_{il})^2}{e_{il}}. \tag{8}$$

The e_{il} have to satisfy the condition that their sum equals the total number of observed points:

$$\sum_{i=1}^{k} \sum_{l=1}^{j_i} e_{il} = N. \tag{9}$$

The minimum value of χ^2 is easily obtained from (8) by writing

$$n_i = \sum_{l=1}^{j_i} \frac{O_{il}^2}{e_{il}}; \quad \sum_{l=1}^{j_i} e_{il} = E_i. \tag{10}$$

Using Lagrange's method of undetermined multipliers we see that the minimum occurs for

$$\alpha_i = \sqrt{n_i/(\lambda E_i)} = N\sqrt{n_i/E_i}\left(\sum_{i=1}^{k} \sqrt{n_i E_i}\right)^{-1}, \tag{11}$$

and is equal to

$$\chi^2 = (1/N)\left(\sum_{i=1}^{k} \sqrt{n_i E_i}\right)^2 - N. \tag{12}$$

The appropriate number of degrees of freedom is $(r - 1)$, since we are arguing that in the 'best' possible case from the point of view of agreement between theory and observations, the actual α_i happen to be those given by (11).

4.3 THE APPLICATION TO THE KAPAHI MODEL

The median curve (see Fig. 3) shows a fall in the value of θ_m between the 3CR and the Ooty survey. In his discussion of the various cosmological models Kapahi (see Paper II) has argued that this change from high- to low-flux sources is a 'distance' effect. Taking the interpretation of low-flux sources as distant sources and hence sources observed at earlier epochs, he fitted an evolutionary model of type A to the observed median curve. The details of this model are given in Section 3. However, taking into account the necessity of fitting the entire distribution of points in the (θ, S) diagram, we now subject this model to the above statistical test, for the case $n = 1.5$.

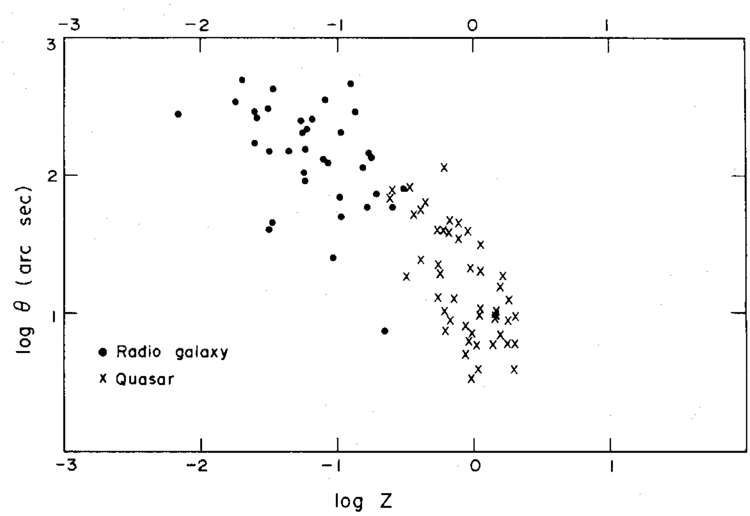

Figure 3. The scatter diagram of Fig. 1 is reproduced with the median curve $\theta_m(S)$ drawn through the observed points,

In Fig. 4 is given the observed and expected number of points in suitable rectangles in the $\log \theta$, $\log S$ plane. The sizes of the rectangles have been determined by the requirement that the expected number of points in each does not fall below 5. This makes the χ^2 relatively stable and hence reliable. It must be pointed out that for the χ^2_{min} test it is necessary that there be at least two rectangles in each flux range. Otherwise the contribution to χ^2_{min} from that rectangle is trivially zero. The flux ranges extend between 1 and 100 Jy at 178 MHz. The reason for limiting the Ooty part to fluxes greater than 1 Jy is observational and will be discussed in the following subsections – see point (4) Section 4.4. There are in all 298 sources involved in the above test.

The value of χ^2_{min} is 33.50 for $r = 16$. At 15 degrees of freedom the 1 per cent level of χ^2 is 30.58. Thus the difference between the Kapahi model and the observed data is significant. In other words the evolutionary model chosen in Paper II does not adequately describe the observations. Also, this high value of χ^2 is stable against merging or redistribution of neighbouring rectangles provided the expected numbers do not fall below 5.

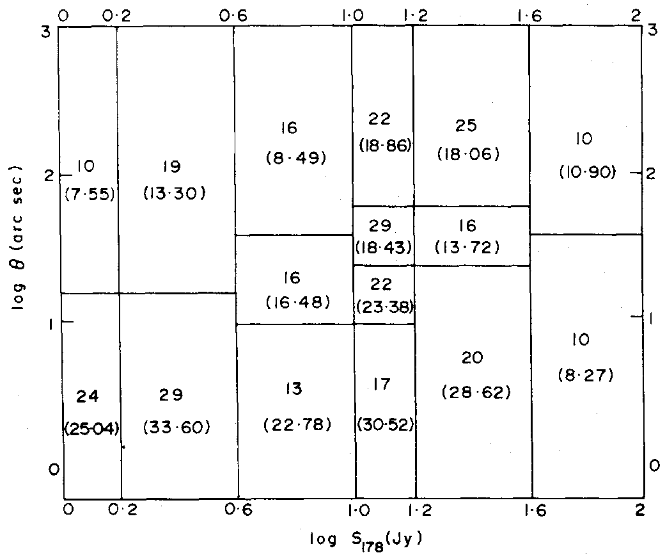

Figure 4. The χ^2_{min} table showing the observed and the expected number of sources in the Kapahi model for the different rectangles in the (log θ, log S) plane. The expected numbers are indicated in the parentheses.

4.4 DISCUSSION

In the following section we will discuss the application of the minimum χ^2 test to the non-evolutionary models. Here we briefly discuss the implications of the above negative result for the Kapahi model. The following points and likely objections to our approach are worth mentioning.

(1) It might be argued that many of the Ooty angular sizes are given as upper limits only. It is therefore not correct to treat these as exact values. In principle this is a valid objection. In practice, however, its implications for the χ^2_{min} are unimportant, if not slightly damaging for the Kapahi model. This is seen by examining the changes in Fig. 4 if the relevant source points were shifted downward in θ. Since the changes occur at lower flux levels there are not many such cases at $S > 1$ Jy. Also, the agreement in this part of the plane is so good that any shift of observed points will, if at all, increase the value of χ^2_{min}.

(2) Fig. 4 shows that the agreement between the observed and expected values is not good at high flux levels. It might be argued in general that the high-flux part covers a smaller volume and hence it should be given less importance. This argument would be valid for a complete sample which is certainly not true in the present context. If the source density were uniform, an equal number of sources in a Euclidean universe occupy the same proper volume regardless of the flux density. In the Kapahi model the source density was considerably higher in the past so that the Ooty sources (~150 in number) occupy a smaller volume than the 3CR sources (~150 in number) in the above analysis. Thus the above analysis, if at all, leans marginally in favour of the Ooty sample rather than the 3CR one.

(3) The large value of χ^2_{min} could be due to the fact that two different samples have been taken together. This may well be the case and calibration errors may be responsible to some extent for this. However, if a comparison of the two samples is to be rendered invalid for this reason then it removes the very basis of the cosmological conclusions arrived at in Papers I and II, where the evolutionary effect was concluded on the basis of such a comparison.

(4) We feel that a valid criticism of the above test can be made on the grounds that the angular-size measurements may not be accurate enough. This criticism is more likely to be relevant in the Ooty range than in the 3CR range. In the latter case many of the single-

component sources are scanned in several directions so that the 'largest' angular size (the so-called $\omega_{\|}$) can be estimated with some confidence. In the former case the measurements are not based on many scans so that the θ values, if at all, may have been underestimated. This effect has been discussed in Paper I. We have restricted our analysis to $S > 1$ Jy because we feel that the effect may be larger at lower values and may introduce more uncertainties.

(5) Finally the entire analysis breaks down if the assumption (E) of equal area surveyed for all θ values is not valid. In other words, the Ooty or the 3CR survey may not be complete in *any* narrow flux-range. In view of the extensive work done on the survey we feel reasonably confident that the assumption (E) applies to the measurements at high flux densities ($S > 10$ Jy). Future work on low-flux density sources from Ooty may throw light on whether this assumption applies for $S < 10$ Jy.

5 Non-evolutionary models

We now consider the application of the minimum χ^2 test to non-evolutionary models, and to fix ideas we use the steady state model for the detailed analysis.

5.1 TYPE B MODELS

In this case there is no correlation between size l and power P of the source. Assuming a size-distribution function as given by (4) and a power-distribution function of the form

$$\rho(P) \propto P^{-\gamma}, \tag{13}$$

where γ is a constant which may be varied over different ranges of P, as for example in (4), we generated a number of theoretical cases. In all the cases considered χ^2 turns out to be significant at 1 per cent level. Thus all non-evolutionary models of type B appear to be ruled out.

5.2 TYPE C MODELS

Suppose we divide the range of P into a number of sub-ranges, $[P_{i+1}, P_i]$, $i = 1, 2, \ldots, (n-1)$ say. In the ith subrange we have

$$F(l, P) = \lambda_i(l)\,\rho_i(P), \tag{14}$$

where

$$\lambda_i(l) = (2/l_i)[1 - (l/l_i)], \quad \rho_i(P) = K_i P_i^{-\gamma_i}. \tag{15}$$

This is an example where power and size are not independent. We can use this to represent the result that 'larger radio sources are more powerful', or the opposite of it. To what extent will such a modified $F(l, P)$ affect the θ–S plot? We found that the numbers $n(S, \theta)$ are very sensitive to such modifications, and used this result to generate a model which is not rejected by the $\chi^2_{\min}$ test. This model has only two sub-ranges of power and three of size. The details are as follows. For P in units of 10^{26} W Hz^{-1} ster^{-1},

$$\begin{aligned} &1 \leqslant P \leqslant 200, && \rho_1(P) \propto P^{-2.1}, && l_1 = 750\,\text{kpc}, \\ &0.1 \leqslant P \leqslant 1, && \rho_2(P) \propto P^{-1.9}, && l_2 = 600\,\text{kpc}, \\ &0.001 \leqslant P \leqslant 0.1, && \rho_3(P) \propto P^{-1.9}, && l_3 = 500\,\text{kpc}. \end{aligned} \tag{16}$$

The constant of proportionality is the same in all four cases in the units chosen. The (θ, S)

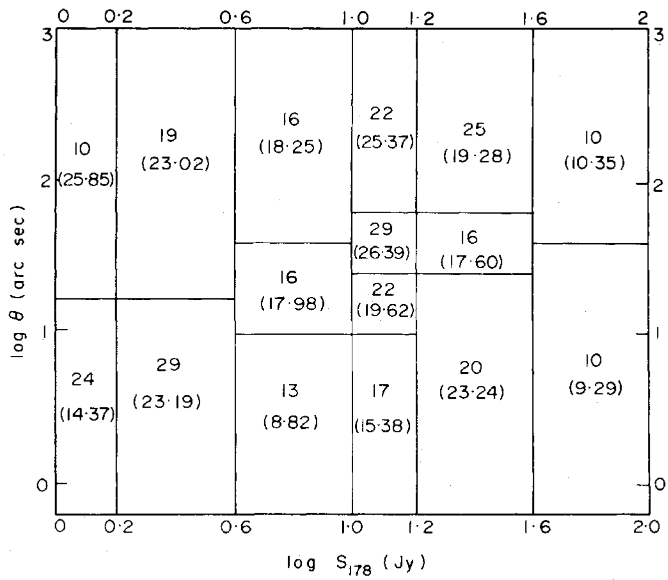

Figure 5. The $\chi^2_{\min}$ table showing the observed and the expected number of sources in a type C model with a mild positive correlation between source size and source power. The expected numbers are indicated in the parentheses.

distribution is shown in Fig. 5. The $\chi^2_{\min}$ at 15 degrees of freedom is 24.85. The 5 and 1 per cent significance levels of χ^2 at 15 degrees of freedom are 25.00 and 30.58 respectively.

We are not proposing this model as *the* model for this data. Rather we wish to demonstrate that a better agreement with the overall data is possible *without* recourse to evolution even with a mild correlation between l and P. No doubt models of type C with better performance than this can be constructed as a parameter fitting exercise. However, any such models including the one above must be consistent with certain other observational checks which we discuss below.

5.3 DISCUSSION

The following remarks may be made in connection with the above result for type C models.

(1) The comparison made here between the observed and calculated numbers $n(S, \theta)$ is between relative rather than absolute numbers. It is generally recognized (*cf.* for example von Hoerner 1973) that non-evolutionary models do not fit the source count data at high as well as low fluxes. For this reason the present $\log N$–$\log S$ data of complete surveys would rule out the models of type C discussed above. However, as emphasized by Swarup and Kapahi, we wish to look at the θ–S relation *as an independent cosmological test.* The Ooty survey does not provide independent estimates of the areas of the sky covered in the different flux density ranges. Hence a comparison of absolute numbers is not possible at present. It will be interesting to repeat this analysis in future, if and when such area estimates do become available. It will then be possible to see whether the comparison of absolute numbers does rule out the type C model. An independent estimate of areas covered by the Ooty survey and its comparison with complete surveys in the same flux density range will also be useful to check whether the survey is complete in the sense of assumption (E).

(2) An additional check is provided by the N–θ relation discussed in Paper II. This refers to the 3CR part of the data where Kapahi finds that a plot of $\log N$ against $\log \theta$ has a straight-line slope of -1.1 ± 0.25 in the range 100–1000 arcsec for θ. If we lower γ_i and ρ_i

for weaker sources, in order to obtain a better fit to the $\theta_m(S)$ relation, the $\log N$–$\log \theta$ curve tends to steepen. For the type C model described above the slope is -1.34, i.e. within 1σ range.

(3) Is the correlation postulated for l and P in this model in conflict with the usual claims of no correlation? In Paper II reference is made to the sample of 3CR sources examined by Mackay (1973) who finds for the 64 sources in question no obvious correlation for l and P. We have examined the plot for a larger sample of 64 *galaxies* (whose redshifts are known) + 18 galaxies (whose redshifts are estimated from their magnitudes) from the latest compilation by Spinrad & Smith (1975 December, private communication) of the optical properties of 3CR sources, and we find a correlation coefficient of ~ 0.33 between $\log l$ and $\log P$.

For comparison we generated a sample from type C model of 86 sources with $S > 10$ Jy. The correlation coefficient for this sample, between $\log l$ and $\log P$, was ~ 0.1. In other words a high-flux sample of the 3CR type cannot rule out the mild correlation of the type C models.

6 The median test

This is a rank test which tells us whether the median values of two samples differ significantly in view of their intrinsic scatters. For details of the theory behind it, see Hajek & Sidak (1967).

6.1 THE OUTLINE OF THE MEDIAN TEST

Consider two numerical sequences $\{T_1\}$, $\{T_2\}$ each consisting of n members arranged in ascending order:

$$\{T_1\}: x_1, \ldots, x_n,$$
$$\{T_2\}: y_1, \ldots, y_n. \tag{17}$$

In (18) there are nx's and ny's. Let there be p of the x's less than the median of the combers.) Let $\tilde{x}$ and $\tilde{y}$ be the medians of the above two distributions. Is the difference $|\tilde{x} - \tilde{y}|$ statistically significant?

To answer this question we rearrange the combined sequence $\{T_1 \cup T_2\}$ in an ascending order:

$$z_1, z_2, \ldots, z_{2n}. \tag{18}$$

In (18) there are nx's and ny's. Let there be p of the x's less than the median of the combined distribution, which lies anywhere between z_n and z_{n+1}. Then the quantity

$$Q = (n/2) - p \tag{19}$$

for large n, is a normal variate with zero mean and the standard deviation given by

$$\sigma = n/(2\sqrt{2n} - 1). \tag{20}$$

Thus if we find that $|Q| > 2.52\sigma$ we may reject at 1 per cent level the null hypothesis that the two samples are drawn from the same population.

6.2 THE APPLICATION TO THE RADIO DATA

Consider a narrow range of flux density, say with $\Delta \log S = 0.2$. Let the observed θ values in this range form the sequence $\{T_1\}$ in the above example. For a given (evolutionary or non-evolutionary) model generate a theoretical sequence $\{T_2\}$ of the same number of sources. This is a straightforward procedure once the numbers $n(S, \theta)$ are obtained numerically. From these construct the composite sequence $\{T_1 \cup T_2\}$ and obtain the value of Q. If $|Q| > 2.52\sigma$, we may reject the theoretical model at 1 per cent level. If $|Q| > 1.95\sigma$, we may reject the model with somewhat less confidence, i.e. at the 5 per cent level.

The Kapahi model, not surprisingly, lies well within these limits except at high flux densities. But even here it lies within the 2σ limits. The steady-state model of type B lies well within these limits at high flux levels ($S \lesssim 10$ Jy), barely within these limits between $S \sim 1$ and 10 Jy and outside these limits in $S < 1$ Jy. At low flux densities of this type the data is less reliable but even here the value of Q does not exceed 3.5σ. The non-evolutionary model of type C lies well within these limits throughout the S range.

7 Conclusions

Having applied the χ^2_{min} and the median tests to the angular diameter–flux density relation we find that with its present large scatter the data on θ–S is not able to clearly distinguish between the evolutionary and the non-evolutionary models – especially those models where there exists a correlation between size and power of radio sources. In the case of the χ^2_{min} test it may be possible to make this distinction in future if the areas covered by the Ooty survey at different flux ranges can be estimated. The importance of this has been emphasized in Section 5.3.

Despite two decades of theoretical inputs it has not yet been possible to construct viable models of extragalactic radio sources. Even ignoring the notorious redshift controversy for QSOs, the problems of radio source structure present a number of difficulties of interpretation. Should one distinguish at the outset between quasars, single component radio sources and double component radio sources? The detailed analysis of the only complete survey the 3CR, has yielded valuable information. But as discussed here, the 3CR may turn out to be the tip of the iceberg. A power-size correlation, of even a mild nature, can seriously affect the θ_m–S curve at low flux levels, although it may not be detected at high fluxes. Whether such a correlation would emerge from a viable theoretical model of radio sources is an open question. We also find that variation of the size function $\lambda(l)$ can also affect the $\theta_m(S)$ curve significantly. Hence we feel that until a better understanding of radio source structure is reached, the θ–S measurements should be looked upon as providing important inputs to determine $F(l, P)$ rather than settling the long-standing cosmological problem.

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References

Burbidge, G. R. & Narlikar, J. V., 1976. *Astrophys. J.*, **205**, 329.
Chitre, S. M. & Narlikar, J. V., 1976. *Astrophys. Space Sci.*, **44**, 101.
Gunn, J. E. & Oke, J. B., 1975. *Astrophys. J.*, **195**, 255.
Hajek, J. & Sidak, Z., 1967. *Theory of rank tests*, Academic Press, New York.

Hoerner, S. von, 1973. *Astrophys. J.*, **186**, 741.

Hoyle, F., 1959. *Paris Symposium on Radio Astronomy*, p. 529, ed. R. Bracewell, Stanford University Press.

Hoyle, F. & Burbidge, G. R., 1970. *Nature*, **227**, 359.

Kapahi, V. K., 1975a. *Mon. Not. R. astr. Soc.*, **172**, 513.

Kapahi, V. K., 1975b. *PhD thesis.*

Longair, M. S. & Pooley, G. G., 1969. *Mon. Not. R. astr. Soc.*, **145**, 121.

Mackay, C. D., 1973. *Mon. Not. R. astr. Soc.*, **162**, 1.

Miley, G. K., 1971. *Mon. Not. R. astr. Soc.*, **152**, 477.

Swarup, G., 1975. *Mon. Not. R. astr. Soc.*, **172**, 501.