Reprinted from:
SYMPOSIA ON THEORETICAL PHYSICS AND MATHEMATICS, Vol. 6
(Plenum Press, 1968)

1968
33B

Recent Developments in Cosmology

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1. INTRODUCTION

Thanks to the progress of observational astronomy, new data of cosmological significance is rapidly becoming available. Under the circumstances it is perhaps not wise to talk about "recent" developments. Some of the observations described in these lectures and the conclusions arrived at may well have become out of date by the time the lectures appear in print. As the symposium is devoted mainly to a discussion of elementary particle physics, for the sake of the noncosmologist in the audience, I will spend a few minutes describing the general background of the subject.

Two important developments early in the present century led to the establishment of cosmology as a branch of science. The first was Einstein's theory of general relativity. This theory of gravitation successfully combines the good points of Newton's law of gravitaion and the special theory of relativity. Newton's law of gravitation involves instantaneous action at a distance and is as such inconsistent with special relativity. Although the law works extremely well in the description of gravitational phenomena on the earth and in the solar system, its validity over the large distances involved in cosmology is very suspect for the above reason. General relativity, being free from this difficulty, provides an adequate tool for a discussion of cosmology. (Newtonian cosmological models have, however, been considered by several authors,[1] but I shall not be describing those here.)

The second event which led to the interest in cosmology was the discovery of nebular red shift by Hubble. Hubble found that the light from distant (extragalactic) sources undergoes an increase in wavelength when it reaches us. The fractional increase in wavelength (called the "red shift") is proportional to the distance of the source. Hubble's observations can be described by the linear relation

$$z = \frac{DH}{c} \tag{1}$$

where z is the red shift, D is the distance of the source, c is the velocity of light, and H is the Hubble constant. The present observations give a value of $H^{-1} \simeq 10^{10}$ years. This relation observed in the case of a large number of distant galaxies, indicated a systematic large scale structure in the universe.

2. RELATIVISTIC COSMOLOGY

Hubble's observations came a few years after general relativity. Theoreticians, therefore, applied general relativity to explain these observations. Relativistic cosmology is the study of the models of the universe obtained by the use of general relativity. The Einstein equations

$$R^{ik} - \frac{1}{2} g^{ik} R = -\frac{8\pi G}{c^4} T^{ik} \tag{2}$$

relate the geometry of space–time (described by the left-hand side) to the distribution of energy momentum (described by T^{ik}). The constant G is Newton's gravitational constant.

The equations (2) are, however, too general, since they are covariant under any coordinate transformation. The equations by themselves are not expected to lead to the picture of the universe as described by Hubble's observations. Two restrictive assumptions are necessary to make any progress at all. These are the Weyl postulate and the cosmological principle.

The Weyl postulate states that the world lines of galaxies form a bundle of nonintersecting timelike geodesics diverging from a point in finite or infinite past. This postulate allows the choice of a special coordinate system. Three spacelike coordinates x^1, x^2, and x^3

describe each geodesic and a timelike coordinate t describes points on any particular geodesic given by $x^\mu = \text{constant}$ ($\mu = 1, 2, 3$). (I shall follow the convention in which the Latin indices take the values 1, 2, 3, 4 and the Greek indices 1, 2, 3.) In terms of these coordinates, the line element becomes

$$ds^2 = c^2 dt^2 + 2g_{\mu 4}\, dt\, dx^\mu + g_{\mu\nu} dx^\mu dx^\nu \tag{3}$$

where $g_{\mu 4}$ are independent of t and t is called the cosmic time.

The cosmological principle states that the subspaces $t = \text{constant}$ are homogeneous and isotropic. The subspaces are therefore invariant under the 6-parameter group of translation and rotations in three dimensions. The line element (3) is further simplified to

$$ds^2 = c^2 dt^2 - S^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right] \qquad k = 0, \pm 1 \tag{4}$$

This line element was used by Friedman and was rigorously derived by Robertson[2] and Walker.[3] In equation (4), (r, θ, φ) are constants for any given galaxy and $S(t)$ is called the expansion factor. It can be shown that a light wave emitted from a source at $t = t_1$ and received at $t = t_0$ is red-shifted by

$$z = \frac{S(t_0)}{S(t_1)} - 1 \tag{5}$$

$z > 0$ implies $S(t_0) > S(t_1)$ for $t_0 > t_1$. Thus, red shift is an indication of the "expansion" of the universe. (The proper 3-volume bounded by galaxies expands as S^3.)

By taking the universe to be made up of matter in the form of a smooth fluid with density ρ and pressure p and of radiation with density u, we find that T^{ik} takes the form

$$T^{ik} = \left(p + \rho c^2 + \frac{4}{3} u\right) v^i v^k - \left(p + \frac{u}{3}\right) g^{ik} \tag{6}$$

where $v^i = (0, 0, 0, 1)$ is the flow vector of matter.

The Einstein equations then give

$$3\,\frac{\dot{S}^2 + kc^2}{S^2} = 8\pi G\left(\frac{u}{c^2} + \rho\right) \tag{7}$$

$$2\,\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G}{c^2}\left(p + \frac{u}{3}\right) \tag{8}$$

The present observations indicate that there is very little random motion of galaxies. Although no information is yet available about the intergalactic space we shall assume $p = 0$. Then the divergence of (6) gives

$$\frac{1}{S^3}\frac{d}{dt}(\rho S^3)c^2 + \frac{1}{S^4}\frac{d}{dt}(uS^4) = 0 \tag{9}$$

If the matter and radiation are decoupled or weakly coupled we can assume

$$\rho = \frac{\text{Constant}}{S^3} \qquad u = \frac{\text{Constant}}{S^4} \tag{10}$$

The equation (7) then gives

$$\dot{S}^2 S^2 = -kS^2 + AS + B \tag{11}$$

where A and B are constants.

The radiation term is more important when S is small; as S increases, matter begins to dominate over radiation. In the present epoch of the universe, matter appears to be much more dominant than radiation so that we may neglect B in comparison with AS in (11), thus getting

$$\dot{S}^2 S^2 = -kS^2 + AS \tag{12}$$

The radiation term was explicitly considered by Gamow[4] when he was concerned with the first few seconds after the explosion of a big-bang universe. I shall consider the radiation universe explicitly later. For the purpose of the present discussion (12) is sufficient.

All solutions of (12) require S to become zero at some stage (unless we artificially exclude this possibility). The presence of radiation does not prevent the singularity.

Attempts to avert the singularity by introducing rotation have been made by Heckmann and Schücking,[5] by Raychoudhuri,[6] and others. These attempts regard the universe as homogeneous but anisotropic. Raychoudhuri has shown that rotation inevitably brings in shear and, whereas the former works toward preventing a singulatity, the latter works in the opposite direction, leaving the outcome uncertain. (Indeed, it now appears from investigations of Penrose[7] and Hawking[8] that a singularity is inevitable in relativistic cosmological models.)

Later, I shall describe a somewhat unusual approach which does succeed in preventing the singularity.

3. AGE OF THE UNIVERSE

From (12) it is possible to compute the integral

$$t_0 = \int_0^{S_0} \frac{dS}{\dot{S}} \tag{13}$$

and express the result in terms of the quantities

$$H = \left(\frac{\dot{S}}{S}\right)_{t=t_0} \qquad q = -H^{-2}\left(\frac{\ddot{S}}{S}\right)_{t=t_0} \tag{14}$$

where $S_0 = S(t_0)$, t_0 being the cosmic time for the present epoch, H is the Hubble constant mentioned before, and q is the deceleration parameter. H and q can in principle be determined from observations of distant galaxies. This test will be described later.

Now, t_0 can be looked upon as the age of a big-bang universe. Its maximum value turns out to be $H^{-1} \sim 10^{10}$ years. Although this is larger than the age of the earth and of the solar system, it is *less* than the estimated age of the galaxy, which is put at around 1.3–1.5 $\times 10^{10}$ years.[9] The discrepancy becomes more marked when we notice that (1) galaxies older than our own appear to exist and (2) the value H^{-1} is an upper limit on t_0; the actual value of t_0 may be even less.

Unless future observations require an increase in H^{-1} or the calculations of the stellar and galactic ages (which depend on nuclear physics) are modified in the right direction, the big-bang models appear to be inconsistent with the galactic ages.

4. THE STEADY-STATE MODEL

This model was put forward by Bondi and Gold[10] and by Hoyle[11] in 1948, with a view to avoiding the singular state and the age difficulty of the big-bang models. Although they arrived at the same model their approaches were different.

Bondi and Gold started with the perfect cosmological principle. This states that the universe presents the same large-scale view over long intervals of time. Thus, it is a stage further than the cosmological principle described before in that homogeniety along the t-axis is also introduced. Such a principle guarantees that the universe is in a steady state. From an observational point of view this has the advantage that we can confidently assert that the physics we know here and now also operates elsewhere at other times, and therefore we can interpret observations of distant objects confidently.

To reconcile the expansion of the universe with a constant density of matter (required by the steady state) Bondi and Gold had to introduce the concept of the creation of matter. The matter must appear continually to make up for the depletion caused by the expansion. Continuous creation is therefore a consequence of the perfect cosmological principle of Bondi and Gold.

In the steady-state theory H must be independent of epochs. Also the spatial curvature kS^{-1} is an observable quantity and can be constant only if $k = 0$. This leads to a line element of the form

$$ds^2 = c^2 dt^2 - e^{2Ht}[dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2)] \tag{15}$$

The same line element can be arrived at by more rigorous group theoretic arguments.

Notice that a definite line element is obtained without solving any field equations. This is an example of the deductive power of the perfect cosmological principle. Bondi and Gold do not follow the Einstein equations but deduce what they can from the above principle.

Hoyle adopted a different standpoint. He argued that in the big-bang models creation of matter takes place all at once at the time of the bang. This event is usually excluded from considerations of physics. Why not look upon creation of matter as a phenomenon that can be studied by physics? He proceeded to do so by introducing additional terms on the right-hand side of the Einstein equations. The right-hand side *taken as a whole* was conserved, but the T^{ik} for matter alone was not. The effect of the additional terms was to produce the line element (15) in the homogeneous isotropic case. Thus the steady-state solution, according to Hoyle, was a consequence of the phenomenon of creation of matter. (Notice the reversal of cause and effect with respect to Bondi and Gold.)

A few years ago M.H.L. Pryce showed that Hoyle's ideas could be described in a covariant manner, starting from an action principle. The total action including the Einstein term, inertial term, and the extra creation terms is given by

$$J = \frac{c^4}{16\pi G} \int R\sqrt{-g}\, d^4x - \Sigma \int mcds + \frac{1}{2} f \int C_i C^i \sqrt{-g}\, d^4x - \Sigma \int C_i dx^i \tag{16}$$

where C is a scalar field and $C_i = \partial C/\partial x^i$. The resulting field equations obtained by variation of C and g_{ik} are

$$fC^i_{;i} = n$$
$$R^{ik} - \frac{1}{2} g^{ik} R = -\frac{8\pi G}{c^4}\left[T^{ik} - f\left(C^i C^k - \frac{1}{2} g^{ik} C^l C_l\right)\right] \tag{17}$$

where n =number of particles created per unit volume per unit time. Divergence of the gravitational equations gives

$$T^{ik}_{;k} = fC^i C^k_{;k} \tag{18}$$

Thus matter alone is not conserved. It can be verified that (15) is a solution of (17), with matter density

$$\rho = \frac{3H^2}{4\pi G} \tag{19}$$

In the Bondi–Gold theory no such expression could be obtained for ρ because of the absence of any field equations. From present estimates of H and G, $\rho \simeq 3.10^{-29}$ gms cm^{-3} sec^{-1}.

The steady-state model is free from singularity. Also the age difficulty does not arise in an *acute* form. There is some difficulty, however. The age distribution $P(\tau)$ of galaxies is of the form

$$P(\tau) d\tau \, \alpha \, e^{-3H\tau} d\tau \qquad \tau \geq 0 \tag{20}$$

where τ is the age. The average age is $\frac{1}{3} H^{-1} \simeq 3 \cdot 10^9$ years. Thus our galaxy and the nearby galaxies are much older than the average. This may be the case if galaxies condense in large age-correlated groups and we happen to live in an old group. (The origin of life may be possible only in older galaxies). Also, young galaxies may be invisible because of being less bright. Clearly, whether an age difficulty also exists in the steady-state theory can be decided only when we have a better understanding of galactic evolution.

5. TESTS OF COSMOLOGICAL MODELS

As in any other branch of science, the predictions of the cosmological theories are open to verification. However, the verification proceeds through observation only since no experiments are possible on the large scale structure of the universe. If the observations corrected for all possible errors point against the predictions of a theory, the theory must be discarded. Although a large number of observations are available today, they are fraught with uncertainty, and no definitive conclusion can yet be drawn. The observation regarding the ages of galaxies described above is an example. During the rest of this talk I shall enumerate other observational tests.

However, before considering tests which depend on observational techniques available now, I wish to describe another criterion which is of a theoretical nature and which, in my opinion, appears to be strongly linked with cosmology. This is the question of the arrow of time.

6. THE ARROW OF TIME

The question is often raised "Why is there an arrow of time when all microscopic laws of physics are time symmetric?" Broadly speaking the arrow of time is detected through time asymmetric phenomena which occur in thermodynamics, electrodynamics, and cosmology. Is there any connection between the three types of time-asymmetric phenomena? Some physicists believe there is only a weak connection, if at all. Others suspect a strong connection. I am more in sympathy with the latter view and will now describe some work done in the last few years to support this point of view.

The asymmetry in electrodynamics arises from the choice of retarded solutions of Maxwell's equations. Thus an oscillating electric charge radiates (rather than receives) energy because retarded potentials are used to describe the electromagnetic field around the charge. The asymmetry in cosmology comes from the choice of space-time which corresponds to an *expanding* universe. To establish a strong connection between electrodynamics and cosmology, it is necessary to show that the retarded solutions *alone* are consistent in an expanding universe. This work was undertaken by Hogarth[12] and later by Hoyle and Narlikar[13] and will be described below.

The starting point is the basically time-symmetric Fokker action describing electrodynamics:

$$J = -\sum_a m_a \int da - \sum_{a<b}\sum e_a e_b \iint \delta(S^2_{AB})\eta_{lm}\, da^l\, db^m \tag{21}$$

where $a, b, \ldots$ are charges, S^2_{AB} is the square of the interval between typical points A, B on the world lines of charges a, b, and η_{lm} is the flat-space metric. This action does away with the idea of fields as independent entities. *The fields are defined in terms of particle world lines.* Thus the 4-potential arising from the world line of charge a is

$$A^{(a)}_i(x) = e_a \int \eta_{il}\, \delta(S^2_{AB})\, da^l \tag{22}$$

Since $\delta(S^2_{AB})$ occurs in the expressions both advanced and retarded fields exist on equal footing. Thus instead of the usual retarded field $F^{(a)}_{\rm ret}$ the field is

$$\tfrac{1}{2} F^{(a)}_{\rm ret} + \tfrac{1}{2} F^{(a)}_{\rm adv} \tag{23}$$

Wheeler and Feynman[14] demontrated that the explicit presence of advanced fields is no embarrassment if the universe responds suitably. They found that in a static Euclidean universe with a uniform distribution of charges the reaction from the universe is

$$\tfrac{1}{2} F^{(a)}_{\rm ret} - \tfrac{1}{2} F^{(a)}_{\rm adv} \tag{24}$$

This is the radiative reaction assumed by Dirac.[15] Equations (23) and (24) add up to give $F^{(a)}_{\rm ret}$.

The calculations of Wheeler and Feynman were done in a universe which is time symmetric. Pure advanced solutions are also therefore consistent. To resolve the issue, Wheeler and Feynman resorted to thermodynamics and suggested that retarded solutions would be consistent on thermodynamical grounds.

Hogarth pointed out that the issue does not arise at all in expanding cosmological models. The retarded interactions are red-shifted and the avanced ones blue-shifted. Also the matter density in the universe may be different in the future and in the past. Thus advanced and retarded interactions are not on the same footing.

Detailed calculations show that retarded (not advanced) interactions are consistent in the steady-state model and advanced (not retarded) interactions are consistent in the big-bang models. The case of the oscillating models is not clear cut. If this point of view is accepted, it provides an elegant way of distinguishing between cosmological models. (However, it must be remembered that this calculation is based on the Fokker action principle which may not be accepted by every physicist as a starting point for electrodynamics.)

A connection between electrodynamics and cosmology may imply the arrow of time in thermodynamics. The expanding universe provides a sink and the retarded interactions guarantee that the tendency of local thermodynamics *will* be toward filling the sink. The question "why an arrow of time?" is then resolved. For, if instead of an expanding universe we choose a contracting universe, all arrows will be reversed and the physically observable effects still remain the same. In other words, we grow old because the universe expands!

7. THE RED SHIFT-DISTANCE RELATION

The proper distance of a distant galaxy with radial coordinate r, at the present epoch is

$$D = rS(t_0) \tag{25}$$

Its red shift is given by (5). The relation between z and D can be worked out for any specified $S(t)$. For small z and D, power series expansion of the following form is obtained:

$$z = \frac{DH}{c} - \frac{1}{2}(1+q)\frac{D^2H^2}{c^2} + O(D^3) \tag{26}$$

Thus all cosmological models give the same first term, which was observed by Hubble for not too distant galaxies. The second order term involves the parameter q which is different for different models. For the steady state model, $q = -1$; for the Einstein–de Sitter model (big bang with $k = 0$), $q = \frac{1}{2}$; while for big-bang models with $k = 1$, $q > \frac{1}{2}$. The observations at Mt. Palomar, by Sandage[16] show that $q = 1$ gives the best fit. However, Sandage points out that there is enough scatter in the observed diagram to rule out a definite conclusion. The scatter can be reduced by selecting only the brightest member of a cluster of galaxies. It is usually found that a cluster of galaxies is dominated by a massive elliptical and that the intrinsic brightness of the dominating elliptical is more or less constant from cluster to cluster. Observations are in progress to improve the test in this direction.

At large red shifts, different models predict widely different values of D. So the test may be applied more fruitfully to quasi stellar objects which have large red shifts (the largest known is $z = 2.1$ at the time of writing). However, at present, there is some doubt whether the red shift of quasi stellars is due to expansion of the universe.[17] Also, it is not known how big is the scatter in the intrinsic brightness of these objects. The application of the test must await further observations of the properties of quasi stellar objects.

8. COUNTING OF RADIO SOURCES

Consider a Euclidean universe with a uniform distribution of bright objects. Then, if objects are counted up to different radial distances r, the number of objects within a sphere of radius r and the apparent brightness of the faintest object vary, respectively, as

$$N \propto r^3 \qquad S \propto r^{-2}$$

i.e.,

$$\frac{d(\log N)}{d(\log S)} = -1.5 \tag{27}$$

What is the actual relation in the universe? Hubble attempted to apply this test to the counting of galaxies, but their large number rendered it an impossible task. Later this was applied to radio sources (which are not so numerous). The latest Cambridge survey (Ryle and Clark[18]) gives a slope of the $\log N/\log S$ curve to be -1.8.

In the steady-state model, the slope starts off at -1.5 and diminishes in magnitude. So the Cambridge survey appears to disprove the steady-state model. It indicates that there were more radio sources in the past than there are now.

Recently, Veron[19] at Mt. Palomar has further analyzed the Cambridge data. He finds that the source count for radio galaxies has a slope -1.56 and for quasi stellar radio sources has the slope -2.2. Radio galaxies are not very distant and therefore their slope -1.56 would be consistent with any cosmological theory within the observational errors. If the quasi stellars are really distant objects, the slope -2.2 would indicate strong evidence against the steady-state theory.

All these conclusions are subject to the assumption that the distances of radio sources have been correctly estimated. No direct measurement of red shift or distance of a radio source is yet possible. Some of them have been optically identified and their red shifts measured. A vast majority are still to be dealt with this way. Any conclusion from this test must await until the "distance" issue is satisfactorily resolved.

9. BACKGROUND RADIATION

The radiation in the universe can be classified into two categories. The first category consists of radiation from specific sources such as galaxies, radio sources, etc. The second categrory of radiation cannot be attributed to any source. It is isotropic in character and is called the background radiation.

Recently, Penzies and Wilson[20] at the Bell Telephone Laboratories announced the detection of background radiation at 7.3 cm, with intensity corresponding to a temperature of 3.1°K. Similar observations have been reported by the group at Princeton[20]; they find a background radiation temperature of 2.5°K at 3.2 cm. Such an appreciable amount of radiation cannot be explained within the present framework of the steady-state theory.

In the big-bang cosmologies, this radiation can be accounted for in terms of the radiation left over from an earlier highly dense phase. As was pointed out in Section 2, near $S = 0$, radiation terms are the most important, and equations (7) and (8) lead to

$$\frac{\dot{S}^2}{S^2} = \frac{8\pi G u}{3} \qquad u \propto \frac{1}{S^4} \tag{28}$$

Models of the radiation universe have been considered in detail by Dicke,[21] Peebles,[21] and others. The present observations can be qualitatively accounted for in terms of the big-bang models, but not quantitatively. Dicke suggests that a variation of gravitational constant G with epoch may resolve the discrepancy. The problem of the radiation universe is also related to another observation which will now be described.

10. HELIUM/HYDROGEN RATIO

Observations in our galaxy show that the ratio of the number of atoms of helium to hydrogen ranges from 0.09 to 0.19. Such observations, coming from widely different objects (e.g., Orion nebula, B stars, planetary nebulae, solar cosmic rays, solar evolution) indicate that the ratio has a universal value. Less precise measurements from other nearby galaxies also support this.

Stellar evolution alone does not appear to provide this ratio. Assuming that our galaxy has been radiating at the present rate of

4×10^{43} ergs/sec all its life (3×10^{17} sec) the amount of energy radiated is 1.2×10^{61} ergs. Now one gram of hydrogen when converted to helium in stellar evolution releases 6×10^{19} ergs. Hence, some 2×10^{42} gms of hydrogen have been converted to helium. This may be increased by a factor of three to include radiation in the infrared and ultraviolet. The mass of the galaxy is 10^{11} solar masses. Hence, the ratio of He to H by mass is ~ 0.03 and by number ~ 0.01. This is too low by an order of magnitde—unless our Galaxy was more luminous in the past. There is no evidence for this.

The conclusion is then the galaxy started its life with some helium already in it. Where did this come from? In the big-bang radiation universe temperatures are high enough to provide synthesis of helium.

According to Gamow's original calculations, the solution of equation (28) in terms of radiaton temperature can be written as

$$T_{10} = 1.52 \times t^{-1/2} \tag{29}$$

where T_{10} is temperature in 10^{10} °K and t is in seconds. If the universe started with matter in the form of neutrons, some neutrons will decay in 10^3 sec into protons. If the matter density is too high, all protons will combine with neutrons to form He. If the matter density is too low, no He can be formed. Only a precise adjustment of temperature and density can lead to the observed He/H ratio.

Gamow had ignored pair creation in the radiation field. When pairs and neutrinos are included (29) is changed to

$$T_{10} = 1.04 \times t^{-1/2} \tag{30}$$

Calculations have been performed by Hayashi[22] and later by Hoyle and Tayler,[23] and the latter obtain the He/H ratio ~ 0.14 which lies well within the observed range.

While this is very satisfactory from the point of view of the radiation universe, it must be noted that a lower value of He/H than 0.14 is an embarrassment. Such a value is found in the sun and careful observations must be done to check this.

In the steady-state universe, such a ratio is explained if there is a large number of massive objects in gravitational collapse. The conditions inside such an object are similar to those in a radiation universe at a time of high density. In either cosmology, indications of this ratio are that matter in the universe passed through a high-temperature phase.

11. FORMATION OF GALAXIES

One of the difficulties of cosmological theories has been to provide an explanation of how galaxies were formed. Attempts to condense galaxies out of the cosmological substratum have come to nothing, largely because the expansion of the universe acts against condensation.

Recently a reverse process was put forward by Hoyle.[24] He argued that a cloud of gas expanding with the universe can be *restrained* by a concentrated mass in the center. The cloud then forms a galaxy. Such a process, instead of condensing from low to high density, envisages expansion from high to low density. The distribution of light in the galaxy so formed closely resembles that of elliptical galaxies. A mass of the order of 10^9 solar masses can control the expansion of a cloud of mass as high as 10^{12} solar masses. The nuclei of elliptical galaxies are highly concentrated objects of masses 10^8–10^9 solar masses. One prediction of this theory is that ellipticals would not show significant rotation. Observations should be made to test this point. The elliptical shapes arise in this theory due to shear in the motion of the cloud and/or the motion of the restraining mass relative to the cosmological substratum.

The spirals may arise as a result of condensation in the field of ellipticals. This shows up in the form of a cluster of spirals with a dominating elliptical in the center. This observational fact has been described before.

12. AN OSCILLATING UNIVERSE?

The evidence presented in Sections 7–11 indicates that the universe was denser in the past. (This statement must be taken with reservation because of the "ifs" attached to all the observational tests). A model of the universe which could account for these and which would also be free from singularity would be an oscillating universe which has finite maximum and minimum densities. However, as mentioned in Section 2, classical general relativity does not provide such a model.

It would be possible to produce such a model with the help of the C-field. If there is no creation of matter,

$$\dot{C} = \frac{\text{Constant}}{S^3} \tag{31}$$

in the homogeneous isotropic case. Thus instead of (12) we get for $k = +1$,

$$S^2\dot{S}^2 = -S^2 + AS - \frac{\alpha^2}{S^2} \tag{32}$$

where α is a real constant, related to that in (31). This model oscillates between finite radii. This would be consistent with considerations of Sections 7–11, although its verdict on the arrow of time is not clear.

There is a difficulty with this model—or any finite oscillating model. Dissipative processes acting in it would tend to damp the oscillations, and it would eventually become static. If the universe is infinitely old, why is it not already static? This difficulty has not yet been resolved.

This difficulty does not arise if the oscillations are not universal, but occur relatively locally. Recently Hoyle and Narlikar[25] put forward a model in which the universe is in a very dense steady state (with large f). It is kept in that state by creation in the neighborhood of massive objects. It is possible for large finite regions of the universe (but not for the universe as a whole) to develop instability in this process. Creation in such regions is switched off and these regions behave like the oscillating model described by (32). Although these oscillations may be subsequently damped out, the universe as a whole continues to expand. Also in such a universe the arrow of time would point the right way!

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