

# On the wavy mechanics of particles

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Concept of mass is one of the most complicated concepts in physics. Philosophically, the mass can be defined in three different ways and according to these definitions; there are three different kinds of masses. These are named as the active gravitational mass, passive gravitational mass and the inertial mass. Several experiments have been performed to distinguish these masses, and in all these experiments, surprisingly these masses come out to be the same. Therefore, even though conceptually these masses are different but they are considered to be same empirically. Hence, in modern physics, all these different concepts are abolished and people treat all the three masses as the same. However, in this paper the inertial mass is considered to be different from the other ones. It is thought that the inertial mass of a particle is not completely its intrinsic property and it depends on the position of the particle in the universe, or the background of the position where the particle is located. But the background of a particle keeps on fluctuating randomly due to different types of phenomenon occurring in the universe. Therefore, the exact position or the mass of a particle can not be determined at any time, without knowing the exact positions and the states of all the other particles in the universe. Hence, in this paper, the dynamics of a particle has been defined statistically, which leads to the Schrodinger equation. The mathematical formalism presented in this paper can explain the quantum mechanical phenomenon completely classically and thus it gives quantum mechanics a sense of completeness.

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## I. INTRODUCTION

The concept of mass is one of the most fundamental concepts of physics. The definition of mass in use in physics is from the days of Newton. But even after several centuries of work, scientists and philosophers could not agree upon a fixed definition of mass. However to do any physics with the motion of any material particle we need some working concept of mass. In Principia, Newton has described mass as the quantity of matter present in an object. Obviously, this definition of mass is not clear. Therefore, researchers tried to redefine mass in a clearer way. The concepts of mass can be defined in three different ways. The mass can be defined from the inertial properties of matter, and is consequently called the inertial mass. On the application of same amount of forces on two bodies, the inertial masses of the bodies are inversely proportional to the accelerations of the bodies, measured from an inertial reference frame. On the other hand, mass can also be defined from the gravitational properties of matter. From the gravitational properties of matter mass can be defined in two different ways, namely active gravitational mass, which comes from the strength of the gravitational field produced by the mass, and passive gravitational mass, which shows the susceptibility or the response of the body in some gravitational field[1]. Several experiments have been performed to check the equality between inertial mass and passive gravitational mass and all these experiments proved the equality between the two masses up to a surprising accuracy. Therefore, researchers have concluded that these masses are exactly equal and name this as the equivalence principle. However, as the concept of these two different masses evolves from two complete different parts of physics and there is no direct logic to show the equivalence between these two masses, the equality is just an empirical phenomenon. The equivalence principle is discussed in more detail in the next part of this paper.

Now, the inertial masses are responsible for the inertial properties of matter. While determining the inertial masses it is important to define the inertial coordinate system, because based on the motion of different particles in an inertial coordinate frame their inertial masses are determined. However, the problem is how to determine the inertial reference frame i.e. the frame with no acceleration. Philosopher Ernest Mach postulated that the inertial reference could be determined by looking at distant objects in the universe. This implies that the distant objects in the universe actually determine the inertial properties of matter, which is the famous Mach's principle. So, if it is considered that the Mach's principle is correct then if there are some fluctuations in the background of a particle (which is always present due to the motions of different objects in the universe) then the inertial properties of a particle will keep on changing and cannot be calculated deterministically without knowing the position of all the other particles of the universe.

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In this paper, it is shown that, if the background of a particle fluctuates randomly then the dynamics of a particle has to be statistically defined. Our calculations show that the statistical equation, which describes the dynamics of the particle, is same as that of the Schrodinger's equation. Hence, the fluctuations in the background can bring the quantum mechanical properties in a particle.

There have been several attempts to describe Schrodinger's equation[7] from the classical picture. First of such attempts was made by David Bohm in terms of hidden variable theory[5, 6]. This theory is similar to the De Broglie's pilot wave theory and is most popularly known as the Bohmian mechanics. Several versions of the Bohmian mechanics have been developed by researchers through past few decades. Another independent attempt was made by Nelson[10] to explain Schrodinger's equation under the hypothesis that every particle of mass  $m$  is subject to a Brownian motion with diffusion coefficient  $\hbar/2m$  and no friction. Attempts were also made to bring the Schrodinger's equation directly from the uncertainty principle[9, 11]. In some recent works the Schrodinger's equation have been derived from the non-equilibrium thermodynamics[12, 13]. An attempt to bring the Schrodinger's equation from Mach's principle has also been made by Gogberashvili[14]. Gogberashvili's work shows that the Machian model of the universe can provide a nice platform to describe the quantum behavior of nature. The attempt made in the present paper to bring quantum mechanical behaviour from Mach's principle is conceptually different from the Gogberashvili's attempt. In other word, though Mach's principle used in the present paper is similar to Gogberashvili's paper, the mathematical detail of the theory in this paper is completely different from that.

In the following sections, the theory is described conceptually from different thought experiments. Then based on these thought experiments, the mathematical details of the theory are described in the later sections.

## II. EQUIVALENCE PRINCIPLE

The equivalence principle tells about the equality between the passive gravitational mass and the inertial mass of a particle. The logic behind this conclusion can be boldly described as follows. Let us assume that a particle has passive gravitational mass  $m_p$  and inertial mass  $m_i$ . Now if it is placed in a gravitational field, say  $g$ , then the force on the particle will be  $F = m_p g$  and hence the acceleration will be  $a = \left(\frac{m_p}{m_i}\right) g$ . Now, this equation shows that the all the particles, at a given location will fall (in vacuum) with same acceleration or, if released from rest, they will travel through the same distance within the same time, if and only if  $m_p/m_i$  are same for all the bodies. If this is indeed the case, it will be convenient to choose an appropriate unit to get  $m_p = m_i$ . The experiments have been performed with different objects to measure the ratio and they confirm the equality between the two masses. The result of these experiments is most commonly known as the Weak Equivalence Principle (WEP).

However this explanation does not determine if the ratio varies over position and time, i.e.  $\frac{m_p}{m_i}(t, x, y, z)$ . Therefore, in this paper it has been considered that the inertial mass is dependent on the position of the particle in the universe i.e. the background of the particle[2]. Of course, this assumption will not violate any of the experimental results. A more detailed explanation of why the inertial mass will vary from place to place will be discussed later.

The above idea can be explained using the following thought experiment[3]. Suppose there is an observer inside a box and she is conducting some experiment inside the box to check the acceleration of the box. Now, according to the Einstein's Equivalence Principle, there is no experiment using which she could distinguish whether the results, which she is getting from the experiments, are due to some gravitating object outside the box or due to the acceleration of the box, provided the experiment is performed inside a small enough region of space time. This is the standard experiment used to show the equivalence between the acceleration of a particle and the intensity of the gravitational field, which led Einstein to conclude that the gravitational field can be explained by the curvature of space-time.

Now this experiment can be modified a bit. We can consider a similar thought experiment, but this time instead of a single box, two boxes are kept at two different parts of space time (see Fig.1). There are two observers inside the two boxes, say  $A$  and  $B$ , and there is a third observer, say  $S$ , who can observe both the boxes from outside. Suppose that the boxes are exactly identical to each other and there are exactly identical gravitating objects near the boxes. Consider that all the other background objects are far away from the systems and the backgrounds are almost symmetric around the masses. This is just to ensure that the gravitational intensity on a box is only due to the gravitating body kept near the box. Suppose the backgrounds at the place  $A$  and place  $B$  are not similar.

Let us consider the case when both the boxes are freely falling due to the gravitational field. The observers inside the boxes will not feel any types of forces. Therefore, if they carry out similar experiments then they will find exactly identical results. Now suppose the gravitating objects are stationary in the reference frame of the outside observer  $S$ . So the outside observer can measure the acceleration of both the boxes. She might find out that at the freely falling condition the box  $A$  has an acceleration  $a_A$ , where as for box  $B$  the acceleration is  $a_B$ , where  $a_A \neq a_B$ . If she has to explain this phenomenon then the only possible explanation she has is the background, because everything else in  $A$  and  $B$  coordinate system are identical. Her possible explanations can be

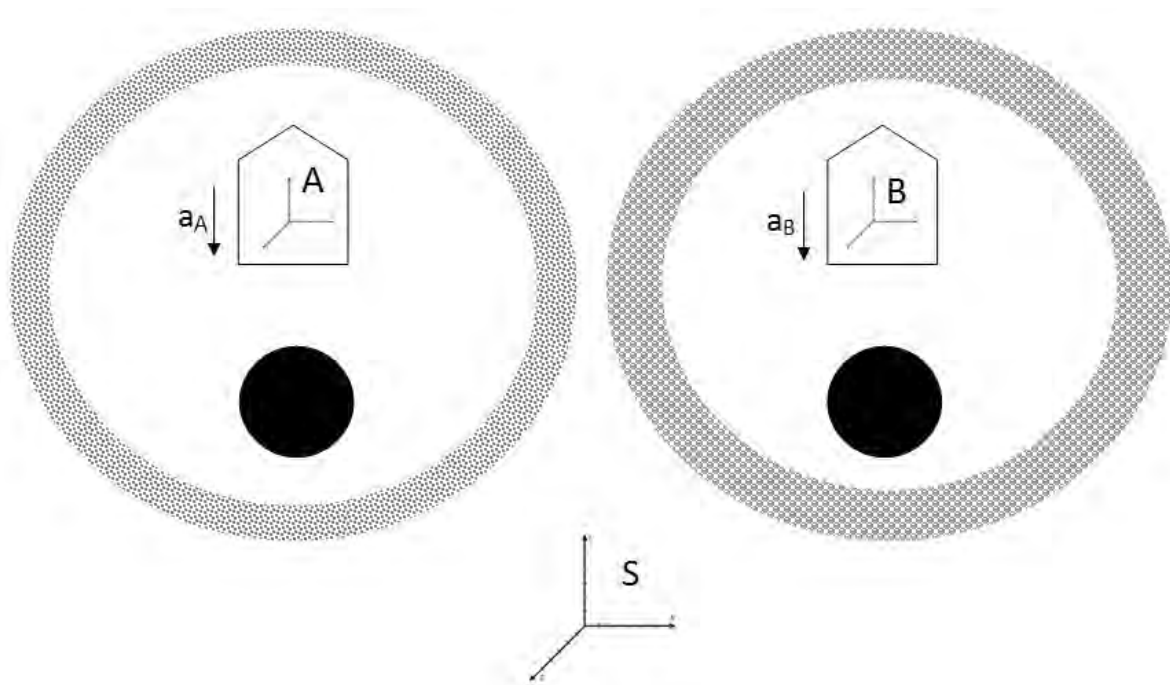


FIG. 1: Two identical systems are kept at two different places. The outer sphere represents the background, which is different at the two different places.

1. The gravitational constant changes due to the background
2. The gravitational mass changes due to the background (may be active gravitational mass of the gravitating object or the passive gravitational mass of the box)
3. The inertial mass of the box changes due to the background

In this paper, the last option has been explored. The logic behind this will be discussed in the later sections.

Also there is one more problem in this experiment. The above experiment by no means contradicts the Einstein's Equivalence Principle (EEP), i.e. "The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in space-time." However, the conclusion that was drawn previously from this experiment that the gravity could be described only by the curvature of space-time, will not be valid anymore. The experiment shows that the background will also take a significant role in determining gravity. Therefore it is important to fix the  $m_i/m_p$  ratio at all points of space time before applying General theory of relativity and hence the General theory of relativity will not work correctly. In this paper, only the effect of fluctuating background on the inertial mass and the dynamics of a particle have been considered. The details of gravitational field will be discussed in in the next paper.

### III. INERTIAL MASS

From the physics point of view, mass was first described by Newton in his Principia, as the amount of matter in a body and the amount of mass can be measured by multiplying its volume with the density. Now it is quite obvious that the density can only be defined by the amount of mass per unit volume. Hence, the Newton's definition of inertial mass is circular. To avoid the circularity of the statement Mach had defined the inertial mass in a new way. If there are two bodies  $A$  and  $B$ , then they will apply force on each other. That will induce opposite accelerations on the two bodies along their line of junction. While looking from an inertial frame if  $a_{A/B}$  be the acceleration of the body  $A$  towards  $B$  and  $a_{B/A}$  the opposite then according to the Mach's definition of inertial mass, the quantity  $-\left(\frac{a_{A/B}}{a_{B/A}}\right)$  will be a positive quantity which will give the ratio of the inertial masses of the two bodies, i.e.  $m_{A/B} = \frac{m_A}{m_B}$ .

Then by introducing a third body  $C$  which is interacting with  $A$  and  $B$  he had shown that the mass ratios satisfy transitive relations, i.e.  $m_{A/B} = m_{A/C}m_{C/B}$ . Therefore, he had shown that the mass ratio of two bodies would

remain a positive quantity. Therefore, if we take one body as the standard, then we can define the mass of the other bodies.

However, Mach's definition of inertial mass does not work well for a system consisting of  $n$  bodies. Pendse gave an argument showing that the Mach's calculation fails to determine the unique mass for all the bodies in a system when we have more than 4 bodies in the system[1, 4]. In brief, Pendse's statement was that if a system has  $n$  bodies, the observable induced acceleration of the  $k^{th}$  body is given by  $a_k$  and  $u_{ik}$  is the unit vector from the  $i^{th}$  body to the  $k^{th}$  body, then the accelerations of the bodies can be written as

$$a_k = \sum_{j=1}^n \alpha_{kj} u_{kj} \quad \dots k = 1(1)3n. \quad (1)$$

Here  $\alpha_{ik}$  ( $\alpha_{ik} \neq 0$ ) are  $n(n-1)$  unknown numerical coefficients for  $3n$  equations. These numerical coefficients can be used for determining the mass ratios. However these numerical coefficients are uniquely determined only if number of unknowns does not exceed the number of equations i.e.  $n(n-1) \leq 3n$ , i.e.  $n \leq 4$ . Therefore if there is a universe with only 4 or less numbers of particles then only the inertial masses of the particles can be determined uniquely. Otherwise, it is not possible to determine the mass ratios of any two particles in the universe uniquely because the ratio will depend on the position of all other particles in the universe. Therefore, the inertial mass ratios of any two particles in the universe depend on the background of those two masses.

The Eq.(1) shows that if the inertial mass is defined in the way Mach has proposed, then there will be more unknowns than the number of equations. Hence, it is not possible to get any definite solution for the system of equations, i.e. there are an infinite number of solutions for the equations. There will be no way to determine which of those set of solutions will describe the actual masses of the particles in the universe. Also, in such a universe if the particles move from one place to another in a completely random way then the mass of each particle will vary from location to location, and at each location of a particle there will be an infinite number of possible solutions for the inertial mass of the particle. In addition, if any of the random solution from this set of infinite solutions is chosen then there is no guarantee that the mass ratios determined by this relation will follow the positivity conditions or the transitive relations.

The above approach also creates another problem. Suppose two identical particles are kept at same (or nearby) position in space-time. Then the above equation can allow different masses for the two particles. However if both the particles are identical and the background is also the same therefore we should expect the same mass for those particles. Therefore, this approach can not define the inertial masses correctly.

The problem in this approach is that no two particles are talking amongst themselves for determining their masses. So the above problem can be solved by producing some kind of interaction between the particles. Suppose initially all the particles in the universe have some masses. Now it can be considered that whenever some particle is moving through space-time, it is sending some signal (may be or may not be instantaneous) through space-time, which carries information about the mass. When that signal reach other masses, they change their inertial masses accordingly. So in this way each mass will be connected to all the other masses in the universe. Hence, each mass will have some definite value of the inertial mass and the above problems will get sorted out. This will make the inertial masses deterministic and still dependent on the background.

Therefore, we see that the inertial mass is not an intrinsic property of the body and has its origin in the background provided by the rest of the universe. In a more mathematical sense, it can be written as follows. Let us consider that  $A$  and  $B$  are two different locations in space. Also consider that  $a, b$  are the particles in the universe and the inertial mass of the particle  $a$  is given by  $m_{ia}$ . Now, as  $m_i$  is not an intrinsic property, we can write

$$m_{ia}(A) = \sum_{b \neq a} m_i^{(b)}(A). \quad (2)$$

Here  $m_i^{(b)}(A)$  is some scalar contribution to the inertial mass of the particle  $a$ , which is located at  $A$ , from the particle  $b$  which is located at the position  $B$ .

Therefore, the inertial mass of a particle is a background dependent quantity. If it is considered that the universe is almost homogeneous and isotropic and several small-scale random fluctuations keep on occurring in different parts of the universe, then the inertial mass or the inertial properties of an object will keep on changing continuously. Therefore, only the probabilistic behaviour of the position or momentum of the particles can be calculated.

### A. Mach's principle and energy conservation

In the previous sections, detailed description of the equivalence principle and the logic behind background dependent inertial mass have been discussed. This section is primarily concerned about the mathematical derivation of the previous logic and energy conservation under Mach's background dependent inertial mass hypothesis.

Suppose there are two identical particles, which are kept at two different places in the universe such that the backgrounds at the two different positions are different. Now, in accordance with the previous discussions, inertial properties of the two particles are different at those two locations. If the special theory of relativity is valid then if those masses are converted to energy completely at those two different places then the amount of energy released will be different. This will clearly violate the energy conservation principle. Therefore, some changes are needed to save energy conservation principle, or some technique is needed which can relate the masses at the two places.

The following thought experiment can throw some light on this context. Suppose there is an object at some point, say  $A$ , in the universe. Assume that all the particles in the background are far apart to show any type of gravitational attraction. It can also be thought that all the gravitational forces are switched off for the time being. According to Mach's hypothesis all the particles in the background have some contribution to the mass of the particle kept at  $A$ . Suppose in this configuration the inertial mass of the particle is  $m_1$ . Now suppose all the particles in the background slowly move apart and give the background a new configuration. Thus previous discussion shows that for this new configuration of the background, the inertial mass of the particle at  $A$  will change. Suppose this new mass is  $m_2$ . As we have switched off the gravitational attraction between the particles for the moment therefore while changing the configuration of the background, it can be assumed that, the particle at  $A$  has not felt any force. Hence, it does not receive any energy in the process.

If it is considered that the total energy of the particle for the first configuration of the background be  $E_1$  and that of for the  $2^{nd}$  case be  $E_2$ , then according to the special theory of relativity the energy can be related to the inertial mass as

$$E_1^2 = m_1^2 c^4 + p_1^2 c^2, \quad (3)$$

and for the  $2^{nd}$  case

$$E_2^2 = m_2^2 c^4 + p_2^2 c^2. \quad (4)$$

Here  $p_1$  and  $p_2$  are the momentums of the particles at these two configurations of the background and in the above thought experiment we had considered that  $p_1$  be zero. As the particle faces no force,  $p_2$  should also be 0. The conservation of energy demands that  $E_1 = E_2$ . However, here  $v = 0$  and  $c$  is constant in both the equations where as  $m_1 \neq m_2$ . This implies  $E_1 \neq E_2$ , and hence the energy conservation principle is violated. This is not expected as all the standard interactions are switched off here.

To save the energy conservation principle, some extra terms should be added to the energy equations. Let the equation for energy be

$$E^2 = m^2 c^4 + p^2 c^2 + \epsilon E_m^2. \quad (5)$$

Here  $E_m$  can be thought of as the energy coming from the background. Eq.(3) and Eq.(4) show that the energy conservation principle can be regained both by adding or by subtracting some extra term in the energy conservation principle. Therefore  $\epsilon$  can take values  $+1$  or  $-1$ . This term is kept there to determine the sign of the energy contribution from the background.

### B. Defining the coordinate system

The discussion in the previous section shows that Mach's principle demands an extra term in the energy equation and gives the new energy equation as

$$E^2 = m^2 c^4 + p^2 c^2 + \epsilon E_m^2. \quad (6)$$

The previous discussions show that in Eq.(6) the last term does not dependent on the space-time but on the geometry of the background. Therefore, it cannot be determined by the space and time, and needs a completely independent dimension which can measure the properties of the background. Dividing the Eq.(6) by  $m^2 c^2$  we obtain

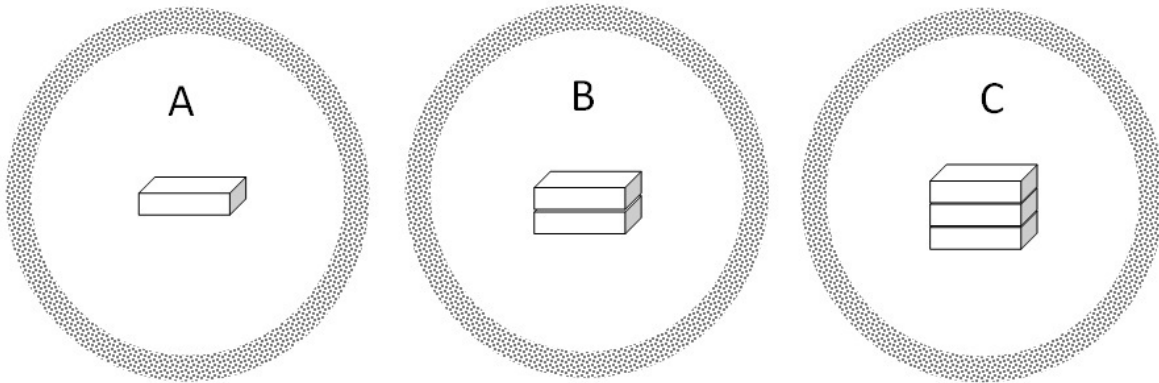


FIG. 2: The background of A, B and C are identical. A has one, B has 2 and C has 3 identical blocks. So one can expect that the inertial mass of B is twice and C is thrice that of A.

$$c^2 \frac{dt^2}{ds^2} = 1 + \frac{dx^2}{ds^2} + \frac{dy^2}{ds^2} + \frac{dz^2}{ds^2} + \epsilon K^2 \frac{d\zeta^2}{ds^2}. \quad (7)$$

Here  $\zeta$  is the new dimension which is measuring the effect from the background and  $K^2 \frac{d\zeta^2}{ds^2} = \frac{E_m^2}{m^2 c^2}$ . Of course  $\zeta$  need not have the dimension of space because it is just coming from the background matter distribution and the inertial mass of the particle. Therefore, a constant  $K$  is multiplied to make the equations dimensionally correct. In the previous section, it was argued that the inertial properties of matter could be quantified by properties of the background. Also the Pendse's principle, as describe before gives an impression that the extra dimension should contain some term which is inversely proportional to the inertial mass or momentum of the particle (as both of them measures the inertial properties of the particle). Therefore, it will be convenient to choose the dimension of this to be  $[M^{-1}]$  or  $[M^{-1}L^{-1}T]$  or something similar which carries dimension of  $[M^{-1}]$ . The value of the constant  $K$  will be calculated in later sections.  $\zeta$  is the dimension which carries the information of the background of the test particle. Knowing the value of  $\zeta$  at some location of space-time, the inertial properties of the particle can be determined.

According to Eq.(2) the inertial mass is not totally an intrinsic property of a particle. It gains its inertial properties from the objects in the background, and gets a definite inertial mass. Now suppose we have taken a block of mass, and the amount of material in it is say 1 unit. Suppose we keep this block of material at some point of space time and it gains an inertial mass  $m_i$ . Now if we keep another similar slab at the same place then that block will also gain a similar inertial mass  $m_i$ . Here we have considered that there are infinite number of objects in the background and therefore the two masses, which are kept on top of each other, will not affect the masses of one another. This is shown in the *B* part of figure 2 . Therefore the total inertial mass of the object kept at *B* will be  $2m_i$ . Now, going on in this way, if we keep  $n$  number of objects on top of each other and make an object then the mass of that object will be  $nm_i$ . It can be seen that the mass of an object is some multiple of the amount of material coming from its intrinsic property, and the multiplicative factor is provided by the background. Therefore the background dimension i.e.  $\zeta$  provides some kind of multiplication factor in the mass equations. It can also be assumed that if  $\zeta \propto \frac{1}{m_i}$  (provided  $m_i \neq 0$ ) then that will be a very good approximation for the background measurement.

Eq.(7) gives the line element as

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 - \epsilon K^2 d\zeta^2, \quad (8)$$

whereas the line element from the special theory of relativity is  $ds_c$  say, where  $ds_c$  is given by

$$ds_c^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \quad (9)$$

$$ds^2 = ds_c^2 - \epsilon K^2 d\zeta^2. \quad (10)$$

The above equations also show that the extra dimension takes care of all the background contribution on the inertial mass. Therefore, all the energy imbalance and the other changes due to the change of the background will

get balanced by the change of the background coordinate and the line element  $ds^2$  and not  $ds_c^2$  will remain invariant under any coordinate transformation.

### C. Calculating the action

This section shows which kind of variation will be there in the action due to the introduction of the new coordinate dimension in the line element and what will be the consequence of this change in the action. According to the classical mechanics the action can be defined as

$$S = -mc \int ds, \quad (11)$$

where  $ds$  is the line element and  $m$  is the mass of the object, a constant. As the background dependency of inertial mass is already been taken care of by defining  $\zeta$ , here we should not put a background dependent mass. Also, as we demand that this line element will remain invariant under the coordinate transform therefore the action should be some constant multiplied with the line element.

In the non-relativistic limit, the standard classical mechanics line element  $ds_c$  can be written as

$$ds_c = cdt \left( 1 - \frac{1}{2} \frac{v^2}{c^2} \right). \quad (12)$$

However, if the background contribution is taken into account for the calculations then the new line element becomes

$$ds = cdt \left( 1 - \frac{1}{2} \frac{v^2}{c^2} - \frac{1}{2} \frac{\epsilon K^2}{c^2} \frac{d\zeta^2}{dt^2} \right). \quad (13)$$

Here it is considered that the time variation of  $\zeta$  is very small. Though if  $\frac{K^2}{c^2} \frac{d\zeta^2}{dt^2}$  becomes very large then the above assumption may not be correct. In a later section of this paper it is shown that this assumption is in-fact a valid assumption.

Now according to the line element given in Eq.(13), the actual action of the particle will be the action from the standard classical mechanics plus a small quantity

$$S = S_c + \delta S, \quad (14)$$

where  $\delta S = \frac{1}{2} m c \epsilon K^2 \int dt \left( \frac{d\zeta}{dt} \right)^2$ .

Therefore, the actual action from the Machian model is not same as the action from standard classical mechanics. If we calculate the equation of motion from standard classical mechanics then we cannot get the actual value of momentum and the energy of a particle because there is one more dimension yet to be fixed, and the actual energy and the momentum will depend on that particular dimension also. Suppose a particle is going from point  $A$  to point  $B$ . By minimizing the action, the path of the particle from one place to another can be found out. If it is considered that the background dimension fluctuates randomly due to random events in the universe, then if a particle goes from  $A$  to  $B$  for  $n$  number of times then it will go through  $n$  different paths. Therefore, this will cause a quantum mechanics like behaviour for the particle motion. The next section is about this statistical behaviour of the particles motion.

## IV. STATISTICS OF THE PARTICLE MOTION

According to the discussion of the previous section, the inertial mass of a particle depends on the background of particle's position. Therefore, without knowing the positions and the states of all the particles in the universe, the inertial mass of a test particle cannot be determined. If it is considered that the universe is perfectly isotropic and homogeneous then two identical particles at two different locations of the universe should have the same inertial mass because the background of both the particles are identical. However, if it is considered that the universe is not perfectly homogeneous and isotropic and different kinds of fluctuations are occurring in the space, then, if a particle

is kept at any point of space-time, the mass of the particle will keep on varying since the background of the particle will keep on changing.

It is known that the universe is almost homogeneous and isotropic. The time dependent fluctuations due to the motions of different particles are small. For the time being, we will also consider the universe to be nonexpanding for calculating the statistics of the particle motion. The reason behind considering a nonexpanding universe is that we do not want to compare the particles between two large redshifts. In that case the background of a particle will change continuously. Those effects are discussed in [15]. This paper mainly focuses on a random change of  $\zeta$ .

Now as we can see from the previous discussion that in the universe the time varying fluctuations in the background at some point is very small, time variation in  $\zeta$  is also small. Therefore, for classical particle even if we neglect the change in  $\zeta$ , we can come to a really good approximation for the line element. Therefore, for classical particles special theory of relativity will be valid. However, if we want to take the effect of  $\zeta$  in our calculations, then its not practically possible to calculate the effect deterministically, because the  $\zeta$  will continuously vary due to position of all the other particles in the universe. Therefore, in the following section the equation of motion of a particle has been determined statistically.

### A. Calculating the probability density function

This section describes the detail calculation of the probability density function. Discussions in the previous sections show that to incorporate the Mach's principle in the theory, it is important to consider a five-dimensional space-time-background coordinate system instead of a standard four dimensional space-time. As in this paper, we are interested in the calculation in a nonrelativistic limit, it can be considered that the four-dimensional line element from the space and the background will remain invariant. Therefore the line element that will remain invariant is  $ds_{sb}^2 = g_{\Theta\Phi} dx^\Theta dx^\Phi$ . Here  $\Theta$  and  $\Phi$  varies from 1 to 4, that is the time dimension is not included here. If a particle moves freely then the particle will go through a straight line or the geodesic line. Though  $g_{\Theta\Phi}$  is not flat, at any place we can define a local flat coordinate system. So any point we can write the line element in the form  $ds_{sb}^2 = \delta_{\Theta\Phi} dx^\Theta dx^\Phi$  locally. Therefore, at any point of the particle's path  $\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2} + \frac{dz^2}{dt^2} + K^2 \frac{d\zeta^2}{dt^2}$  or rather  $p_x^2 + p_y^2 + p_z^2 + K^2 p_\zeta^2$  will remain invariant. Now,  $p_\zeta$  depends completely on the background. Therefore, if  $p_\zeta$  changes then all the other momentums of the particle will also change.

Now in the standard classical mechanics, it is considered that the particle is moving in three-dimensional space (in nonrelativistic limit). There we never consider the motion of the particle along the background dimension. As the particle is actually moving in a four-dimensional space and background, if someone measures the motion of the particle in three-dimension then to her the energy from  $p_\zeta$  term will act as an extra potential energy in the energy conservation equation. The concept can be explained as follows. Suppose a ball is moving in a 'rolling ball sculpture'. If we track the shadow of the ball on a wall then it moves slowly when the ball is moving faster in the perpendicular direction to the wall (As to total energy remain constant therefore more momentum in one dimension implies less momentum in other dimensions.). Therefore, if someone wants to define the laws of motion of the particle just by looking its motion on the two-dimensional wall then the energy from the motion of the particle along the perpendicular dimension of the wall will behave as a potential energy acting on the shadow. As in our theory, the particle is actually moving in a five dimensional space-time-background, when we will analyse the motion of the particle from the four-dimensional space-time we need to add extra potential energy on the Hamiltonian, which will be equal to the kinetic energy of the particle in the background dimension.

Suppose a particle at time  $t$  was at  $A=(x, y, z, \zeta)$  and its momentum along  $\zeta$  direction was  $m \frac{d\zeta}{dt}$ . So when we will do our calculations in four dimension then this quantity will behave as a field which can provide extra momentum along different directions. So if the particle moves to  $B=(x + dx, y + dy, z + dz, \zeta + d\zeta)$  after some time  $dt$  through the path  $s$ , then the line integral  $I_{AB} = \int_A^B m \left| \frac{d\zeta}{dt} \right| ds$  can be related to the probability of finding the particle at  $B$ , i.e.  $P_{AB}$  at  $t + dt$  given that the particle was at  $A=(x, y, z, \zeta)$  at time  $t$ . Now suppose the particle goes from  $A$  to  $C$  via  $B$ . Then the line integral goes as summation i.e.  $I_{ABC} = \int_A^B m \left| \frac{d\zeta}{dt} \right| ds + \int_B^C m \left| \frac{d\zeta}{dt} \right| ds = I_{AB} + I_{BC}$ . However, the probability will go as multiplication i.e.  $P_{ABC} = P_{AB} P_{BC}$ . Therefore the best way to relate these two quantities is  $I_{AB} = -\log P_{AB}$ . We can always absorb all the multiplication constants in the definition of  $\zeta$ . This will lead to  $m \left| \frac{d\zeta}{dt} \right| = \left| \frac{\nabla P}{P} \right|$ .

Here the modulus is taken because  $\nabla P$  is the rate of change of the probability. To know the direction of  $\nabla P$  we need to know the actual shape of the geodesic at a particular point, which cannot be known without knowing the position and the states of all the particles of the universe at that instant.

This will give the three dimensional Hamiltonian a new form

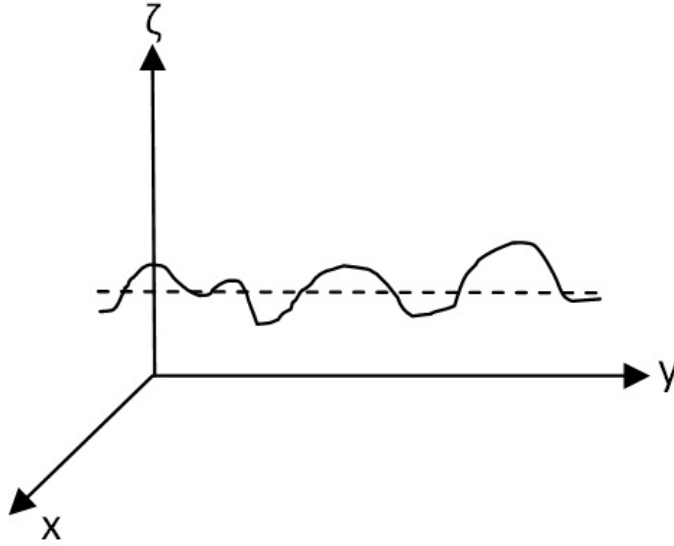


FIG. 3: The continuous dark line is the geodesics path through which the particle is moving. But in standard classical mechanics it is thought that the particle is moving through the dotted line. The motion of the particle in the background dimension is not considered the standard classical mechanics.

$$H = \frac{1}{2m} \left( p_x^2 + p_y^2 + p_z^2 + K^2 \left( \frac{\nabla P}{P} \right)^2 \right) + V. \quad (15)$$

### B. Calculating the wave function

In this section, quantum mechanical wave function for a non-relativistic particle has been derived from the fluctuating background model. Hamiltonian mechanics has been used to derive the wave function.

In the standard non-relativistic classical mechanics, the kinetic energy for a moving particle is given by

$$K.E. = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}. \quad (16)$$

If the particle is moving in a potential  $V(\vec{x})$ , then the Hamiltonian for the particle can be written as

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + V(\vec{x}). \quad (17)$$

However in this simple Hamiltonian the effect of the 5<sup>th</sup> dimension i.e. the contribution from the background dimension  $\zeta$  has not been considered. So introducing the contribution of  $\zeta$  in the Hamiltonian equation we can obtain

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + V(\vec{x}) + \frac{K^2}{2m} \left( \left( \frac{\nabla P}{P} \right)^2 \right). \quad (18)$$

Here  $\epsilon$  is taken as +1 because in [15] it is shown that  $\epsilon = -1$  can not produce the correct cosmological model. This Hamiltonian is a random Hamiltonian, because  $\left| \frac{\nabla P}{P} \right| = m \left| \frac{d\zeta}{dt} \right|$ , is a random variable, with a given probability distribution. Now if a particle is taken with a given action from standard classical mechanics (i.e. without considering the background contribution) then the probability that

$$\begin{aligned}
P \left( \mathbf{H} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + V(\vec{x}) + \frac{K^2}{2m} \left( \left( \frac{\nabla P}{P} \right)^2 \right) \right) \\
= P \left( \left( \frac{d\zeta}{dt} \right)^2 = \left( \frac{d\zeta}{dt} \right)^2 \right) \dots \dots \text{[for the given state the other parts will be fixed]} \\
= P.
\end{aligned} \tag{19}$$

In the second line the quantity in the left hand side is the variable and in the right hand side is its value. Once the probability density function of the Hamiltonian to occur for a fixed classical action is determined, the expectation value of the Hamiltonian can be calculated as

$$\begin{aligned}
\bar{H} &= \int P(\vec{x}) \left( \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + V(\vec{x}) + \frac{K^2}{2m} \left( \left( \frac{\nabla P}{P} \right)^2 \right) \right) dx^3 \\
&= \int P(\vec{x}) \left( \frac{(\nabla S_c)^2}{2m} + V(\vec{x}) + \frac{K^2}{2m} \left( \left( \frac{\nabla P}{P} \right)^2 \right) \right) dx^3.
\end{aligned} \tag{20}$$

Here  $S_c$  is the standard classical mechanics action from the 4 dimensional theory as discussed in previous section and  $V(x)$  is the potential as discussed before.

Now for a moment we can treat  $P$  as some field and  $S_c$  to be its conjugate momentum[5, 6]. Such an assumption is a valid assumption and that can be seen from the equations derived from this. The equations can be derived from the conservation equation also, but this way of deriving these equations will be easier. Hamiltonian mechanics gives

$$\dot{P} = \frac{\delta \bar{H}}{\delta S_c} = -\frac{1}{m} \nabla(P \nabla S_c). \tag{21}$$

The expression can be obtained by taking ‘integration by parts’ of Eq.(20) and then differentiating with respect to  $S_c$ . Similarly, the time derivative of  $S_c$  can be obtained by

$$\dot{S}_c = -\frac{\delta \bar{H}}{\delta P} = -\left[ \frac{1}{2m} (\nabla S_c)^2 + V(x) - \frac{K^2}{m} \left( \frac{\nabla^2 P}{P} - \frac{1}{2} \left( \frac{\nabla P}{P} \right)^2 \right) \right]. \tag{22}$$

Therefore, the equations become

$$\dot{P} = -\frac{1}{m} \nabla(P \nabla S_c), \tag{23}$$

$$\dot{S}_c = -\left[ \frac{1}{2m} (\nabla S_c)^2 + V(x) - \frac{K^2}{m} \left( \frac{\nabla^2 P}{P} - \frac{1}{2} \left( \frac{\nabla P}{P} \right)^2 \right) \right]. \tag{24}$$

If we look at the Eq.(23) and Eq.(24) then we can physically interpret the first equation as the conservation equation and the second equation as the Hamilton Jacobi equation. Intuitively, these equations are same as the equations for the irrotational barotropic flow with density equal to  $P$  and the internal energy equal to  $P \left( V(\vec{x}) + \frac{K^2}{2m} \left( \left( \frac{\nabla P}{P} \right)^2 \right) \right)$ . Under such assumption, the above equations will match with the equations of motions of fluid dynamics equations for a irrotational barotropic flow.

Now the above equations i.e. Eq.(23) and Eq.(24) can be rearranged and combined together to give a single equation of the form

$$i \frac{\partial \Psi}{\partial t} = -\frac{2K^2}{m} \nabla^2 \Psi + V \Psi. \tag{25}$$

where  $\Psi = P^{\frac{1}{2}} \exp(iS_c/2K)$ . The Eq.(25) is nothing but the Schrodinger equation provided we take the constant  $K = \frac{\hbar}{2}$ . From the above explanation, it is clear that the Schrodinger’s wave equation can be derived in a completely classical way from the Mach’s principle.

In case, the logic discussed in the previous section for adding the extra potential equivalent to the kinetic energy in the  $\zeta$  direction is not convincing, the concept can be rethought as follows. Suppose there is an ensemble of particle trajectories following the equation of motions Eq.(23) and Eq.(24). If we consider a completely standard classical picture then the kinetic energy in the  $\zeta$  dimension will vanish, i.e. the extra potential term in the Hamiltonian (Eq.(18)) will vanish. This can be done by putting  $\hbar = 0$  in the above equations. The trajectories are now behaving as the standard classical mechanics and hence they will be normal to any constant  $S_c$  hyperspace and at any point  $(x, y, z)$  of that hyperspace,  $\frac{\nabla S_c}{m}$  will give the velocities vector of the particles. The Eq.(23) gives the standard continuity equation. Therefore, Schrodinger equation in the standard classical limit approximation is just a composition of the two equations.

Now this interpretation can be extended further. When we are looking at the dynamics of the particles then the particles are not actually moving through the four dimensional space-time but in a five dimensional space-time-background. But when we calculate the dynamics of a particle then the motion in the background dimension is never considered. Therefore, the kinetic energy of the particle along the background dimension projects itself as a quantum mechanical potential acting on each particle along with the classical potential  $V$ . The trajectories of all the particles can not be calculated independently without knowing the details of all the particles in the background and hence the dynamics of the particles has to be computed statistically. This gives a new interpretation to the Schrodinger's equation.

## V. RELATIVISTIC EFFECTS OF THE MOTION OF THE PARTICLES

According to the discussions in the previous sections the line element can be taken as

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 - \frac{\hbar^2}{4} \epsilon d\zeta^2. \quad (26)$$

Suppose an observer is looking at a particle of mass  $m$  moving along the  $x$  axis. Let the observer's reference frame be  $K$  and the reference frame which is moving with particle be  $K'$ . Also consider that the  $x$  axes of both the coordinate system are aligned along the same direction. Let the velocity of the particle in  $K$  reference frame be  $v$  and the velocity along  $\zeta$  direction be  $\frac{d\zeta}{dt}$ . Of course, in the  $K'$  reference frame all these velocities will be 0.

So the line element in both these coordinates can be related by

$$d\tau^2 = dt^2 - \frac{dx^2}{c^2} - \frac{\hbar^2}{4c^2} \epsilon d\zeta^2, \quad (27)$$

here  $\tau$  is the proper time of the particle i.e time in  $K'$  frame and the quantities in the right hand side are the quantities in the  $K$  frame. So the equation shows that the coordinate time and the proper time of a particle can be related by the expression

$$d\tau = dt \sqrt{\left(1 - \frac{1}{c^2} \frac{dx^2}{dt^2} - \epsilon \frac{\hbar^2}{4c^2} \frac{d\zeta^2}{dt^2}\right)}, \quad (28)$$

which implies

$$dt = \frac{d\tau}{\sqrt{\left(1 - \epsilon \frac{\hbar^2}{4c^2} \frac{d\zeta^2}{dt^2} - \frac{1}{c^2} \frac{dx^2}{dt^2}\right)}}, \quad (29)$$

provided  $\epsilon = +1$ , the velocity of a particle in a 5 dimensional system can't reach  $c$  at any stage unless  $\frac{d\zeta}{dt} = 0$ . If  $\epsilon = -1$ , the particle can have a velocity higher than  $c$ . But in [15] it is shown that  $\epsilon$  will in fact be  $+1$ . Therefore there is not such problem of superluminal velocity.

## VI. EXAMPLE

In the previous section, the details of the theory are described and it is shown that the Schrodinger's equation can be derived completely classically. In this section some problems are discussed which are usually solved by quantum mechanics and have no solution in the classical mechanics. Using this Machian 5 dimensional model, it can be shown that all those problems can be solved in a completely classical way.

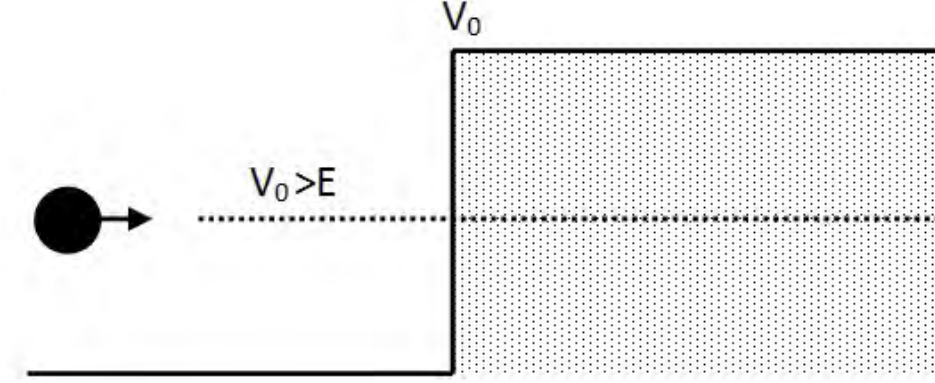


FIG. 4: A potential wall with finite barrier of energy  $V_0$ .

### A. Tunneling through a potential wall

Tunneling phenomenon is one of the many quantum mechanical problems that have no classical explanation. Therefore, in this section, it is described how this quantum mechanical phenomenon can be explained. The potential wall is shown here in the figure 4. The potential is 0 for negative  $x$  and there is a constant potential  $+V_0$  for positive  $x$ .

$$V = \begin{cases} 0 & x < 0 \\ V_0 & x \geq 0 \end{cases} \quad (30)$$

Now from the quantum mechanics, it is known that when the particle will travel in the positive  $x$  direction, the wave function will be of the form

$$\Psi(x) = \exp\left(-\frac{\sqrt{2m(V_0 - E_c)}}{\hbar}x\right). \quad (31)$$

Here  $E_c$  is the total energy of the particle calculated from the 4 dimensional motion of the particle and differ from the actual energy of the particle by the energy contribution from the 5<sup>th</sup> dimension.

According to quantum mechanics, the kinetic energy of the particle when it is at a distance  $x$  from the origin will be given by

$$K.E. = -\frac{1}{\Psi(x)} \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x) \quad (32)$$

$$= -(V_0 - E_c). \quad (33)$$

Therefore, we have to consider that the kinetic energy of the particle is getting negative. However, if the background dimension gives some contribution to the kinetic energy then the kinetic energy can be made positive and everything will behave completely classically.

The wave function shows that the probability of finding the particle at  $x$  is given by

$$P = \Psi^*(x)\Psi(x) \quad (34)$$

$$= \exp\left(-\frac{2\sqrt{2m(V_0 - E_c)}}{\hbar}x\right). \quad (35)$$

Hence we will have

$$\frac{\nabla P}{P} = -\frac{2\sqrt{2m(V_0 - E_c)}}{\hbar}. \quad (36)$$

Now according to the relation between  $\frac{\nabla P}{P}$  and  $m\frac{d\zeta}{dt}$

$$\frac{1}{2}m\left(\frac{d\zeta}{dt}\right)^2 = (V_0 - E_c)\frac{4}{\hbar^2}. \quad (37)$$

Now according to the theory discussed in this paper, the total energy of a particle can be written as

$$E = \frac{1}{2}\frac{p^2}{m} + V_0 + \frac{1}{2}m\frac{\hbar^2}{4}\left(\frac{d\zeta}{dt}\right)^2. \quad (38)$$

Therefore, there is no need to consider that the total kinetic energy is getting negative or the particle is having some imaginary momentum.

An alternate explanation to the phenomenon can be that the particle will stop when the kinetic energy and the potential energy of the particle will be the same. Therefore, suppose the particle stops at a distance  $x$  and at  $x$ ,  $E = V_0$

Also 4 dimensional model total energy of the particle is

$$E_c = \frac{1}{2}\frac{p^2}{m} + V_0, \quad (39)$$

which gives Eq.(37), and doing the calculation in the backward direction we can find the probability of a particle that will stop at a distance  $x$  is

$$P = \exp\left(-\frac{2\sqrt{2m(V_0 - E_c)}}{\hbar}x\right). \quad (40)$$

Therefore, it will give a classical mechanics explanation of quantum tunneling.

## B. Stationary states

Stationary states are the states where the probability of finding a particle does not change with time. Stationary states only occur when the particle is bounded inside some infinite potential wall. The conditions for the stationary states are

1. The particle's energy is a constant of motion and hence independent of time.
2. The probability density of finding the particle at some point will remain stationary, i.e.  $\dot{P} = 0$ .

Therefore, the equations for finding the stationary states of a particle will be

$$0 = \nabla(P\nabla S_c), \quad (41)$$

$$E = \frac{1}{2m}(\nabla S_c)^2 + V(x) - \frac{\hbar^2}{m}\left(\frac{\nabla^2 P}{P} - \frac{1}{2}\left(\frac{\nabla P}{P}\right)^2\right). \quad (42)$$

Also, to solve the equations we need the boundary conditions, which can be found as follows. Suppose, there is a infinite potential energy boundary at a position  $x = 0$ . (check the figure 5). Let us consider a point  $Q$  at a distance  $\delta$ , which is very close to the wall. Now if the particle has to move to that point then the particle needs an  $\infty$  energy contribution from the background, and hence it needs  $\frac{d\zeta}{dt}$  to be infinity at that place. As  $\nabla \ln P = -m\frac{d\zeta}{dt} = -\infty$ , we should get  $P = 0$  inside the potential wall. Considering the probability distribution of a finding a particle at some point in space to be continuous we can have  $P(x = 0) = 0$ .

As the wave function  $\Psi$  of a particle is related with the probability  $P$  with the equation  $\Psi = P^{1/2}\exp(iS_c/\hbar)$ , therefore  $\Psi$  will also become 0 at position  $x = 0$ .

The above equations can be solved using the boundary conditions to get all the energy levels of a particle, though the calculations are done in the framework of classical mechanics.

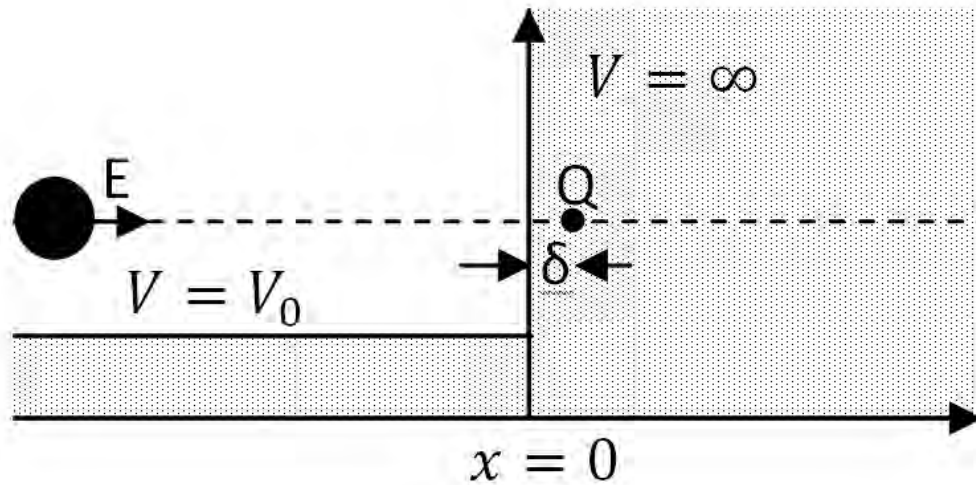


FIG. 5: A potential wall with infinitely high energy at one side and a finite energy of  $V_0$  at the other side.

## VII. CONCLUSION

Mach's principle has been studied in detail in this paper and a new mathematical formalism has been derived. Logically, it has been shown that the inertial mass of a particle depends on the background of the position of the particle. Random fluctuation of the background can give the particle quantum mechanical behaviour. It has also been shown that the quantum mechanical behaviours of a particle can have a complete classical description. Therefore, the quantum mechanical behaviours of a particle can be described logically without assuming the Schrodinger's equation. It is shown that even in the quantum level there exist some definable variable that gives the actual behaviour of a system and the system in the quantum level does not behave in a merely probabilistic sense. Therefore, this mathematical formalism can provide quantum mechanics a sense of completeness. It shows that the radical departure from classical mechanics, which took place due to the introduction of quantum mechanics, is unnecessary, and correct formalism of classical mechanics can explain the entire quantum mechanical phenomenon.

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- [1] **Max Jammer** Concepts of mass in Contemporary Physics and Philosophy. 1999, *Princeton University Press*
  - [2] **Hoyle, F.** and **Narlikar, J.V.**, A new theory of gravitation. *Proc. R. Soc. Lond. A* 1964 282, 191-207
  - [3] **Einstein, A.** The meaning of relativity, 1954, *Princeton University Press*
  - [4] **Pendse, C. G.** A Note on the Defination and Determination of Mass in Newtonian Mechanics. 1937, *Philos Mag* 24, 1012-1022
  - [5] **Bohm, D.** A suggested interpretation of the quantum theory in terms of 'Hidden' variables, I. 1952, *Phys Rev*, Vol 85, Num 2, 166-179
  - [6] **Bohm, D.** A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden Variables', II. 1952. *Phys Rev* 85: 180-193
  - [7] **Schrodinger, E.** An undulatory theory of the mechanics of atoms and molecules. 1926, *Physical Review*, Vol 28, No. 6, 1049-1070
  - [8] **Yang, C.D.** Quantum Hamiltonion mechanics: Hamilton equations of quantum motion, origin of quantum operators, and proof of quantization axiom. 2006, *Annals of physics* 321 (2006) 2876-2926
  - [9] **Hall, M.J.** and **Reginatto, M.** (2002) Quantum Mechanics from a Heisenberg-type Equality. *Fortschritte der Physik*, 50: 646-651. doi: 10.1002/1521-3978(200205)50:5/7 (646::AID-PROP646) 3.0.CO;2-7
  - [10] **Nelson, E.** Derivation of the Schrodinger Equation from Newtonian Mechanics. 1966, *Physical Review*, Vol 150, Num 4

- [11] **Hall, M.J.W.** and **Reginatto, M.** Schrodinger equation from an exact uncertainty principle. 2002, *J. Phys. A: Math., Gen.* 35 3289 doi:10.1088/0305-4470/35/14/310
- [12] **Grossing, G.** On the thermodynamic origin of the quantum potential. 2009, *Physica A: Statistical Mechanics and its Applications*, Volume 388, Issue 6, Pages 811–823
- [13] **Grossing, G.** The vacuum fluctuation theorem: Exact Schrodinger equation via nonequilibrium thermodynamics. 2008, *Physics Letters A*, Volume 372, Issue 25, Pages 4556–4563
- [14] **Gogberashvil, M.** Thermodynamic Gravity and the Schrodinger Equation. 2011, *Int J Theor Phys* 50:2391–2402
- [15] **Das, S.** Cosmological solution of Machian gravity. arXiv:1205.4055v1 [gr-qc]