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HEATING OF SOLAR CORONAL LOOPS BY PHASE-MIXING

N.K. Lohani and Lalan Prasad

Department of Physics, M.B. Govt. P.G. College, Haldwani, Nainital - 263141, India

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This paper present an analytical method for the heating of solar coronal loops by phase-mixing. We also discuss herewith the non-linear mode of phase mixing by Alfvén waves. Under typical coronal heating conditions by ohmic dissipation due to phase-mixing can provide magnetic energy on a time scale comparable with the coronal radiative time. For large Lundquist number, it is possible that phase-mixing can attain a hot coronal loop. We introduce two model of loops; i.e., flat symmetric loop model and cylindrical symmetric loop model. The magnetic field assumed to be static and associated with only in inhomogeneities plasma density. The solution under initial boundary condition and the ohmic dissipation have been discussed.
Key words: MHD-Sun: corona, magnetic field.

1. Introduction

In a magnetized medium shear Alfvén waves propagate independentally on each magnetic field line when the dissipative agents such as resistivity and viscosity are absent (Browning 1991; Davila 1987, Nakariakov et al. 1997). When the medium possesses resistivity and viscosity, the waves propagating on neighbouring field line get coupled with medium agents. If there is Alfvén gradient (inhomogeneity in density etc.) the field line come closer at points where the density is smaller and so the larger Alfvén wave speed. Some structures may not be allow to propagate up to infinity. Thus the wave propagating on neighbouring field surfaces becomes more and more out of phase as they propagate onwards due to momentum exchange caused by viscosity and energy dissipation due to resistivity. This process is known as phase-mixing in space and it requires several wavelengths to develop. The waves most likely to dissipate by this mechanisms are the short period waves $\sim 10s$ propagating in magnetic fields $\sim 10G$ in a medium having number density $\sim 10^{11}cm^{-3}$. As a result of such dissipation the turbulence in the medium increase and the effective transport coefficients (resistivity and viscosity) get enhanced. This help in more existing of phase-mixing and the number of wavelengths over which phase-mixing becomes effective depends upon the value of the dissipative coefficients. However, some of the structure do not allow propagation to infinity (or longer extent), such as coronal loops (or open magnetic field lines) with more stratification so that reflection is produced in such cases phase-mixing occurs in time. This phenomena has been intensively studied by Tsiklauri et al. 2003, Botha et al. 2000, Ofman and Aschwanden (2002) analytically. The waves may suffer multiple reflection in such a structure. The time of a phase-mixing state in which the rate of dissipation balances exactly the rate at which the waves are excited which depends upon the values of dissipative coefficients (Hood, et al. 1997a, Heyvaerts and Priest 1983, Priest 1993). The concept of non-zero gyro-radius of the ions were introduced by Voitenko and Goosens (2000) with the creation of short transverse length scales in Alfvén waves. The Alfvén waves become essential in the sense that they have long wavelengths and low energetic along the magnetic field however short-wavelengths across high energy. In this situation the ion polarization drift in the perpendicular direction creates a charge seperation across equilibrium magnetic field $\mathbf{B_0}$, while field aligned electron flows tend to cancel this charge seperation and thus the motions of the ions and electrons decouple from each other. A shear Alfvén wave propagating in a laterally inhomogeneous structure develops strong velocity gradients due to phase-mixing. The strong gradients are subject to ohmic and viscous dissipation so that phase-mixing may greatly enhance the damping of Alfvén waves and thus provide a viable mechanism for coronal heating (Narain and Ulmschneider 1990, Narain et al. 2001).

2. Basic MHD equations

We are expressing the foot-point motion excite linear Alfvén waves in the cavity of coronal loop and we assume to be inhomogeneous only in the x-direction. For

[1]E-mail: lohani@iucaa.ernet.in
[2]E-mail: lprasad@iucaa.ernet.in

the low value of $\beta(= P_{thermal}/P_{magnetic})$ linearized resistive MHD equations are

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\nabla P_1 + \frac{1}{\mu}[(\nabla\times\mathbf{B_1})\times\mathbf{B_0} + (\nabla\times\mathbf{B_0})\times\mathbf{B_1}] + \rho\nu\nabla^2\mathbf{v} \quad (1)$$

$$\frac{\partial \mathbf{B_1}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B_0}) + \eta\nabla^2\mathbf{B_1} \quad (2)$$

$$\frac{\partial \rho_1}{\partial t} + \rho\nabla.\mathbf{v} = 0. \quad (3)$$

Here ρ and $\mathbf{B_0}$ are the equilibrium density and magnetic field, ρ_1, p_1, $\mathbf{v}$ and $\mathbf{B_1}$ are the perturbation of the density, pressure, Alfvén wave velocity and corresponding magnetic field respectively. Here the equilibrium velocity is zero. By expressing the wave frequency space dependent in terms of Alfvén velocity and wave number

$$\omega(x) = v_A(x)k \quad (4)$$

where k = $\frac{2\pi}{L}$, and L is the loop length. For non dissipative plasma the velocity gradient becomes

$$\frac{\partial v}{\partial x} = v.t.\omega'(x). \quad (5)$$

The linear Alfvén wave having its velocity

$$v = exp[i\omega(x)t - ikz]. \quad (6)$$

The velocity gradient corresponding to x-direction equation (5) which increases with time and it represents that the wave are phase-mixed with respect to time (Hood et al. 1997a). Thus the different frequency on each field line means that initially in phase but when wave motion move becomes out of phase with respect to each other which provide phase-mixing.

Here we consider two symmetries for the coronal loops : (i) Flat symmetric loop model (ii) Cylindrically symmetric loop model.

For the flat symmetric loop model, we assume that the density is only a function of the horizontal distance (flat)

$$\rho = \rho(x) \quad (7)$$

and the plasma only moves in the y-direction, the velocity and magnetic field becomes

$$\mathbf{v} = v(x,\ t)sinkz\hat{e_y} \quad (8)$$

$$\mathbf{B_1} = B(x,\ t)coskz\hat{e_y}. \quad (9)$$

Flat symmetry loop model represent that the line tied disturbances that vanish at the photospheric ends of the coronal loops (Hood et al., 1997a). In the cylindrically symmetric model, we assume that the plasma is cylindrically symmetric and it only moves in the θ direction and we get

$$\mathbf{v} = v(r,\ t)sinkz\hat{e_\theta} \quad (10)$$

$$\mathbf{B_1} = B(r,\ t)coskz\hat{e_\theta} \quad (11)$$

by considering equilibrium state so that the equilibrium velocity is zero. We adopt the cylindrical coordinates r, θ and z and we assume that ρ and $\mathbf{B_0}$ depend on r and $\mathbf{B_0}$ is in the z-direction. Thus, the phase-mixing equation can be obtained from equations (1) and (2) in cylindrically symmetric model.

$$\frac{\partial^2\mathbf{B}}{\partial t^2} = -k^2{v_A}^2(r)\mathbf{B} + \eta\nabla^2\frac{\partial \mathbf{B}}{\partial t}. \quad (12)$$

However, for flat symmetric model the phase mixing equation can be written as

$$\frac{\partial^2\mathbf{B}}{\partial t^2} = -k^2{v_A}^2(x)\mathbf{B} + \eta\frac{\partial^2}{\partial x^2}\frac{\partial \mathbf{B}}{\partial t} \quad (13)$$

from equations (12) and (13) it is clear that the two different geometries can be studied by any one of the above equation (12 or 13) is exactly the same by replacing x by r or vice-versa. Both term on right hand side of equation(12) or (13) are responsible for phase-mixing. The second term is of less importance for large horizontal gradient.

3. The solution under the initial boundary conditions

Let us consider the equilibrium configurations in the form of a magnetic cylinder with coronal loop of length L. At the foot points of the loop determined by $Z = 0$ and $Z = L$, the magnetic field line are anchored in the highly dense electrically conducting photospheric plasma. It is assume to be the plasma at rest at $z = L$, while it moves in the azimuthal direction at $Z = 0$. This leads to the two boundary condition for amplitude function of magnetic field

$$B(x,\ 0) = 1 \quad (14)$$

$$B(0,\ t) = 0 \quad and \quad B(\infty,\ t) = 1. \quad (15)$$

In equation (13) ${v_A}^2$ is the square of Alfvén speed can be expressed in the form of

$${v_A}^2(x) = {v_0}^2 f(x) \quad (16)$$

where v_0 is a typical Alfvén speed in the corona and f(x) is a dimensionless function in the transverse direction equation (16) can be expressed in terms of single parameter equation under the consideration of dimensionless variables (Hood et al. 1997a)

t = $\bar{t}\ \tau_t$ and x = $\bar{x}a$

where a is the typical length-scale for variation of Alfvén speed; $\tau_t = (kv_0)^{-1}$ is the time for the Alfvén wave to propagate along the loop. For the sake of convenience bars are dropped and we get,

$$\frac{\partial^2\mathbf{B}}{\partial t^2} = -f(x)\mathbf{B} + \delta\frac{\partial^2}{\partial x^2}\frac{\partial \mathbf{B}}{\partial t}. \quad (17)$$

and

$$\delta = \frac{\eta}{a^2}\tau_t = \frac{\tau_t}{\tau_d}\frac{L^2}{a^2} \quad (18)$$

where δ is the ratio of the Alfvén travel time (τ_t)to diffusion time (τ_d). We are not taking into the account of viscosity for our further treatment. However, Heyvaerts and Priest (1983) and Ruderman (1999) included a kinematic viscosity of the form $\rho\ \nu\ \nabla^2\ v$ since the flow is incompressible for the Alfvén waves. The dissipation coefficient changed from η to $\eta + \nu$ in equation (12) by the effect of kinematic viscosity. We found that there is no change in the solution of equation (13). Here we also discussed, that the dominant viscosity coefficient is parallel to equilibrium magnetic field. However, for the incompressible Alfvén waves with no velocity component parallel to equilibrium magnetic field does not contribute for the heating of solar corona. Therefore, the remaining terms are small and not taken into the account. Also, viscosity can easily be included by simply changing the dissipation coefficient.

4. Ohmic Dissipation

The presence of resistance in the medium by which the energy of the wave is transfered into heat through ohmic dissipation. The associated current, curl $\mathbf{B} = \frac{4\pi}{c}\ \mathbf{J}$ suffer dissipation through the resistivity of the medium, where $\mathbf{J}$ is the current density. The reflection of wave propagating along field lines also causes momentum exchange between electrons and ions. The time taken to reach a phase-mixed stage, in which the rate of dissipation balances exactly the rate at which the waves are excited depends upon the value of the dissipative coefficients. Under initial condition $x = 0$, $H = Z/s$ the ohmic dissipation become analytically obtained as by using the formulism (Hood et al. 1997b).

$$\frac{j^2}{\sigma} = \eta\frac{(\frac{a^2}{z^2}+1)s^2|\mathbf{B}|}{[1+\delta^2 s^4]^{1/2}} \quad (19)$$

By using approximation $s >> 1$, the magnetic field may be estimated then we have

$$\frac{j^2}{\sigma} \approx \eta s^2 e^{-\delta s^3/3}. \quad (20)$$

For maximum ohmic dissipation

$$s_{max} \approx (2/\delta)^{1/3} \quad (21)$$

and

$$(\frac{j^2}{\sigma})_{max} \approx \eta(\frac{2}{\delta e})^{2/3}. \quad (22)$$

Above approximation is must valid for $\delta s_{max}^2 << 1$. Under usual condition fig(1) represent the ohmic dissipation as a function of height for $\delta = 10^-4$ and η as unity. However Hood et al 1999(b) having similar result for $\delta = 10^-8$. However, there is also the magnitude of the wave amplitude still to be included. The maximum ohmic dissipation scales H_s under the consideration of $\delta = \eta/(a^2\omega)$ as

$$H_s = \eta^{1/3}\omega^{2/3}a^{4/3} \quad (23)$$

where a is the length-scale of the plasma. This equation represents that the amount of coronal heating by phase-mixing are depends on the frequency, phase-mixing length and resistivity. By integrating equation (20), we get the total amount of ohmic heating under considerable limits

$$\int \frac{j^2}{\sigma}dz \approx \frac{n}{\delta} = a^2\omega. \quad (24)$$

Thus, under the approximation the total ohmic dissipation is depends on frequency and independent of resistivity. This technique physically represents that the ohmic dissipation. Fig(2) shows that the height of the maximum ohmic heating as a function of δ.

4.0.1. Figures

5. Discussion and conclusions

The Lundquist number S followed by Hood et al. (1997b). In terms of the size of the coronal loop region A

$$S = \frac{A^2\omega}{\eta}. \quad (25)$$

Numerically by taking the loop length L ranging from 10^6 to 10^8 m, $L/a \approx 10$ and $v_{A0} \approx 2 \times 10^6$ m/s (Karpen et al. 1994, Ofman et al. 1995) then the time of maximum ohmic dissipation is obtained as

$$t_{max} \approx s^{1/3} \times 10^{-2} \quad (26)$$

in seconds. For high Lundquist number $S = 10^{12}$, we get 10^2 to 10^4 seconds of maximum phase-mixing ohmic dissipation (Woo, 1996 and Malara et al. 2001). Phase-mixing is the main mechanism which is responsible for keeping hot coronal loops, provided that the disturbances (pulses) are repeated severally. According to Narain et al. (2001) a shear Alfvén wave propagating in a laterally inhomogeneous structure develops strong velocity gradients due to phase-mixing. From fig(1) it is clear that the location of ohmic dissipation and their corresponding maximum value can be estimated. However fig(2) represents that the hight of maximum dissipation depends on the value of δ and therefore on the value of a , η and ω. The dissipation height decreases as δ increases. A highly appreciable work has been done by Hood et al. in this field. The strong gradients are subjected to ohmic and viscous dissipation so

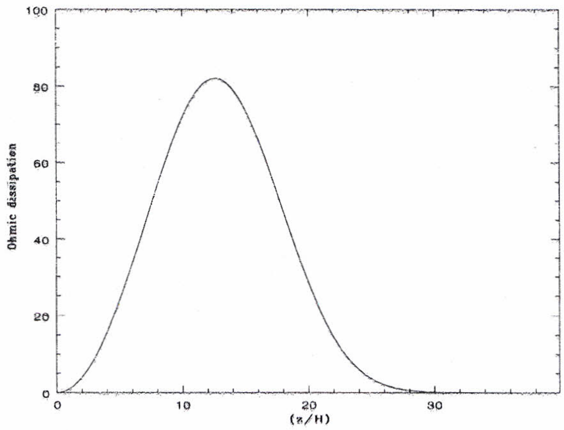

Figure 1: The ohmic dissipation as a function of height.

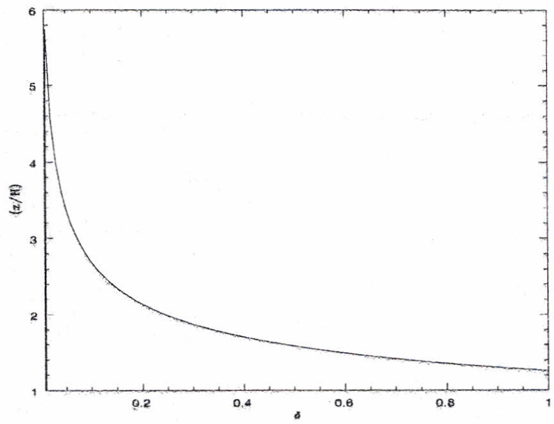

Figure 2: The variation of height with respect to δ for maximum dissipation.

that phase-mixing may greatly enhanced the damping of Alfvén waves and this provide mechanism for coronal heating. It is concluded that the solar coronal heating by Phase-mixing is the dominant process.

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