

AN ATTEMPT TO EXPLAIN THE SMALLNESS OF THE COSMOLOGICAL CONSTANT

T. P. SINGH, and T. PADMANABHAN

Theoretical Astrophysics Group, TIFR, Homi Bhabha Road, Bombay 400 005, India

Received 9 November 1987

Fields which couple directly to the cosmological constant (Λ) may provide a scenario for explaining the smallness of Λ at the present epoch. In this paper we postulate the existence of a scalar field which couples universally to the trace of energy—momentum tensor of matter. Various possibilities for the explicit form of the coupling function are considered. The field equations in such a theory are derived, and the cosmological models with such a scalar field are analyzed. The proposed coupling makes the effective cosmological constant a dynamically evolving quantity, which can be driven to zero by allowing the scalar field to grow to sufficiently large values. For the case of linear coupling, however, it does not seem to be possible to attain sufficient growth during the age of the universe ($\sim 10^{17}$ s). A quadratic coupling to the trace can evolve Λ to a value consistent with today's observations, but the universe is dominated by the scalar field, rather than by radiation, at late times. The evolution is singular for couplings through a higher power law, in that the scalar field blows up at a finite time. The model is not very sensitive to initial conditions and the problems encountered can be avoided only by a severe fine-tuning of the parameters in the basic theory.

1. Introduction

The observation that the cosmological constant term in Einstein equations has a value smaller than 10^{-120} L_p^2 has not found a satisfactory explanation to date. In other words, it is difficult to understand how the contributions of the various matter fields add up to give a net vacuum energy density as small as $(10^{-12} \text{ GeV})^4$. The various attempts made so far to get rid of the Λ term essentially fall into two categories. The first set consists of those models which establish $\Lambda = 0$ at all stages during the evolution of the universe. The models¹ based on the currently popular superstring theories fall in this category. The second set consists of models² which postulate the existence of a compensatory field which couples to Λ in an appropriate manner. The field dynamically evolves in such a way that it cancels any unacceptably large contributions to the vacuum energy density. Most of the models proposed to date belong to this set. However, a typical problem with these models is that they involve a severe fine-tuning of the relevant coupling constants in the theory.

A proposal for understanding the smallness of Λ was made by Wilczek,³ in analogy to Peccei and Quinn's explanation of the smallness of the ' θ -parameter' in QCD.⁴ Wilczek suggested that the cosmological constant should be looked upon as a dynamical variable (which he names as the cosmion field). In analogy to the axion field

corresponding to the θ parameter, the cosmion field has a coupling, presumably to the trace of the energy-momentum tensor. It is then speculated that, like the axion field, the cosmion field will evolve to the minimum of a potential, and that the minimum will correspond to the vanishing of the effective cosmological constant.

In the present paper we develop a classical model which attempts to explain the smallness of the cosmological constant, based on an idea similar to the cosmion field proposed by Wilczek. We postulate the existence of a scalar field which couples to the trace of the energy-momentum tensor of matter. As we shall see, this automatically makes the Λ term a dynamical variable, which is a function of the scalar field, and which can become very small if the field grows to sufficiently large values. It turns out, however, that the model has problems of its own. For the case of a linear coupling to the trace, it is not possible to get a sufficient growth for the scalar field within the age of the universe. More seriously, the universe does not enter a radiation-dominated phase at late times, but is dominated by the scalar field. It is possible to attain sufficient growth by modifying the coupling to the trace, but the universe still does not evolve into a radiation-dominated phase. We would like to emphasize that this problem is generic to models of this kind, and cannot be done away with by a minor modification of initial conditions. Essentially, the kinetic energy of the scalar field has to be large if rapid growth is to be attained, whereas the kinetic energy density of radiation falls with time.

The plan of the paper is as follows. In Sec. 2, we develop a model for gravity, which includes, besides the usual fields $g_{\mu\nu}$, a massless scalar field coupled to the trace of $T_{\mu\nu}$. The field equations for the model are derived. In Sec. 3, we discuss the solution to these equations for appropriate initial conditions, and study the time-evolution of the cosmological constant. We also consider alternate forms of coupling to the trace, which enhance the growth of the scalar field.

As we shall see, any such dynamical coupling will make the trace of the total $T_{\mu\nu}$ vanish at late times. In other words the late stage evolution of the universe will be radiation dominated. This can also be a potential source of trouble because in such models the growth of fluctuations can be severely inhibited. In this paper, we shall ignore such complications and assume that any matter field which is present is conformally invariant, so that its energy-momentum tensor is traceless. If it is true, as is usually assumed, that particles acquire masses via symmetry-breaking in unified field theories, then the assumption of conformal invariance holds, at least before the relevant phase transitions take place.

2. Cosmological Model with Scalar Gravity

We consider the system consisting of the gravitational fields $g_{\mu\nu}$, radiation fields, and a new scalar field ϕ which couples to the trace of the energy-momentum tensor of all fields, including itself. The zeroth order action for this system is given by

$$A^{(0)} = A_{\text{grav.}} + A_{\phi}^{(0)} + A_{\text{int.}}^{(0)} + A_{\text{radn.}} \quad (2.1)$$

where

$$A_{\text{grav.}} = (16\pi G)^{-1} \int R\sqrt{-g} d^4x - \int \Lambda\sqrt{-g} d^4x, \quad (2.2)$$

$$A_{\phi}^{(0)} = \frac{1}{2} \int \phi^i \phi_i \sqrt{-g} d^4x, \quad (2.3)$$

and

$$A_{\text{int.}}^{(0)} = \eta \int T f(\phi/\phi_0) \sqrt{-g} d^4x. \quad (2.4)$$

Here, we have explicitly included the cosmological constant term. η is a dimensionless number which ‘switches on’ the interaction. In the zeroth order action, T represents the trace of all fields other than ϕ . Actually, since the $T_{\mu\nu}$ for the radiation fields is traceless, the only zeroth-order contribution to T comes from the Λ term, so that we have

$$T = 4\Lambda. \quad (2.5)$$

The coupling to the trace is through a function f of the scalar field, and we shall consider various possibilities for this function. The constant ϕ_0 converts ϕ to a dimensionless variable, and is also a measure of the strength of the coupling to the trace.

It should be noted that although our primary motive is to couple Λ to the scalar field, the most natural way to do so is through an interaction of ϕ with the trace of $T_{\mu\nu}$ of all fields. This automatically couples Λ to ϕ , and as we shall soon see, the requirement that $\Lambda_{\text{eff.}}$ should decrease with increasing ϕ , emerges in a natural way.

To take into account the back-reaction of the scalar field on itself, we must add to T the contribution T_{ϕ} of the scalar field. Using the definition of the energy-momentum tensor

$$\delta A = -\frac{1}{2} \int T^{ik} \delta g_{ik} \sqrt{-g} d^4x \quad (2.6)$$

in the action A_{ϕ} of the ϕ field, we get

$$T_{\phi}^{ik} = \phi^i \phi^k - \frac{1}{2} g^{ik} \phi^l \phi_l. \quad (2.7)$$

So to T we must add

$$T_{\phi} = -\phi^l \phi_l. \quad (2.8)$$

However, addition of T_ϕ to T in the interaction term $A_{\text{int.}}^{(0)}$ further modifies T_ϕ^{ik} , as per the definition in Eq. (2.6). This again changes T_ϕ . Thus to arrive at the complete action an infinite iteration will have to be performed. Another set of iterations involves the contribution to T_ϕ due to variations of the form

$$\delta \int f(\phi/\phi_0) T \sqrt{-g} d^4x = \frac{1}{2} \int f(\phi/\phi_0) T g^{ik} \delta g_{ik} \sqrt{-g} d^4x, \tag{2.9}$$

with T as in (2.5). Applying the definition (2.6) we see that this gives an additional contribution $-4Tf$ to T_ϕ , which generates further terms $(4f)^2 T$, $-(4f)^3 T$ etc. The complete action is obtained by summing up all the terms. (For a demonstration of this iteration procedure in a similar problem, the reader is referred to a work by J. V. Narlikar, J. J. Rawal, and T. Padmanabhan.⁵) Here we shall obtain the full action by a simpler, consistency argument.

The effect of the iteration is to modify the expressions for A_ϕ and A_Λ . So we consider the following ansatz for the full action:

$$A = (16\pi G)^{-1} \int \dot{R} \sqrt{-g} d^4x - \int \alpha(\phi) \Lambda \sqrt{-g} d^4x + \frac{1}{2} \int \beta(\phi) \phi^i \phi_i \sqrt{-g} d^4x + A_{\text{radn.}} \tag{2.10}$$

Here $\alpha(\phi)$ and $\beta(\phi)$ are functions of ϕ to be determined by the consistency requirement that they represent the iteration of the interaction term. We note that since radiation makes no contribution to T , $A_{\text{radn.}}$ will remain unchanged.

The energy-momentum tensor for ϕ and Λ is now given by

$$T^{ik} = \alpha(\phi) \Lambda g^{ik} + \beta(\phi) \{ \phi^i \phi^k - \frac{1}{2} g^{ik} \phi^a \phi_a \}, \tag{2.11}$$

so that the total trace is

$$T_{\text{tot.}} = 4\alpha(\phi) \Lambda - \beta(\phi) \phi^i \phi_i. \tag{2.12}$$

$\alpha(\phi)$ and $\beta(\phi)$ are determined by the consistency requirement

$$\begin{aligned} & - \int \alpha(\phi) \Lambda \sqrt{-g} d^4x + \frac{1}{2} \int \beta(\phi) \phi^i \phi_i \sqrt{-g} d^4x \\ & = - \int \Lambda \sqrt{-g} d^4x + \frac{1}{2} \int \phi^i \phi_i \sqrt{-g} d^4x + \eta \int T_{\text{tot.}} f(\phi/\phi_0) \sqrt{-g} d^4x. \end{aligned} \tag{2.13}$$

Using $T_{\text{tot.}}$ from (2.12) and by comparing terms in the above equation we find that

$$\alpha(\phi) = [1 + 4\eta f]^{-1}, \beta(\phi) = [1 + 2\eta f]^{-1}. \tag{2.14}$$

Finally, the complete action can be written as

$$A = (16\pi G)^{-1} \int R \sqrt{-g} d^4x - \int \frac{\Lambda}{1 + 4\eta f} \sqrt{-g} d^4x + \frac{1}{2} \int \frac{\phi^i \phi_i}{1 + 2\eta f} \sqrt{-g} d^4x + A_{\text{radn.}}. \quad (2.15)$$

The same action would be obtained if one uses the iteration procedure. We also note that setting η as zero switches off the interaction, and reproduces the original action.

The action in (2.15) leads to the following field equations,

$$R_{ik} - \frac{1}{2} g_{ik} R = -8\pi G \left\{ \beta(\phi) \left(\phi^i \phi^k - \frac{1}{2} g^{ik} \phi^a \phi_a \right) + \frac{\Lambda}{8\pi G} \alpha(\phi) g_{ik} \right\}, \quad (2.16)$$

$$\square \phi + \frac{1}{2} \frac{\beta'(\phi)}{\beta(\phi)} \phi^i \phi_i + \frac{\Lambda}{8\pi G} \frac{\alpha'(\phi)}{\beta(\phi)} = 0. \quad (2.17)$$

Here, \square stands for a covariant d'Lambertian, and a prime denotes differentiation with respect to ϕ .

We shall assume that the scalar field evolves in an isotropic and homogeneous space-time, which is described by the Robertson-Walker metric

$$ds^2 = dt^2 - S^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\psi^2) \right]. \quad (2.18)$$

We shall also assume that the scalar field ϕ is a function of time alone. Then the Einstein equations corresponding to action (2.15) and the metric (2.18) are

$$\ddot{\phi} + \frac{3\dot{S}}{S} \dot{\phi} = \eta \dot{\phi}^2 \frac{f'}{1 + 2\eta f} + \eta \frac{\Lambda}{2\pi G} \frac{f'(1 + 2\eta f)}{(1 + 4\eta f)^2}, \quad (2.19)$$

$$\frac{\dot{S}^2 + K}{S^2} = \frac{8\pi G}{3} \left\{ \frac{1}{2} \frac{\dot{\phi}^2}{1 + 2\eta f} + \frac{\Lambda}{8\pi G} \frac{1}{(1 + 4\eta f)} + \frac{\rho_0}{S^4} \right\}. \quad (2.20)$$

Here (2.19) is the equation of motion for ϕ , which reduces to a force-free equation when η is set as zero. The terms on the right-hand side of (2.19) should thus be looked upon as the forcing function, which arises because of the coupling we have introduced. The second equation determines the time-evolution of the scale-factor for a given density. The $\dot{\phi}^2$ term is the kinetic energy for the scalar field. (ρ_0/s^4) represents the net contribution of radiation, and ρ_0 is the energy-density of radiation at an appropriate initial time. We note that setting η as zero in (2.20) reproduces the conventional field equation for the evolution of $S(t)$ in a universe filled with radiation and the scalar field.

The clue to the solution of the cosmological constant problem is now seen in Eq. (2.20). If an appropriate physical function f can be found, so that f becomes very large as ϕ evolves, then the contribution to the Λ term can be made small, effectively killing the cosmological constant. We shall now proceed to discuss the solution of the coupled Eqs. (2.19) and (2.20).

3. Cosmological Solutions of the Model

The kind of solution we are looking for is the following. It is desirable that in Eq. (2.19), $\phi(t)$ grows sufficiently rapid from suitable initial conditions. In Eq. (2.20) this should have the effect of bringing the Λ term below the observed bound today. Moreover, the kinetic energy term for the scalar field should be small, compared to the radiation term, so that the universe is radiation-dominated at late times.

We also note from Eq. (2.19) that for ϕ such that $(1 + 4\eta f(\phi/\phi_0)) = 0$, the forcing term is singular. This suggests that around such an initial condition, growth will be rapid. Besides, the right-hand side in (2.19) is always positive, again suggesting that a growing solution will exist. However, since the equations themselves are sufficiently complicated, we have resorted to numerical analysis.

Before going on to consideration of the various choices for the function f , we would like to discuss the question of initial conditions. We shall assume, as a first approximation, that the equations are valid up to Planck-time, and so fix the initial values of various quantities at this time. We also have in our equations, five free parameters — η , Λ , ρ_0 , ϕ_0 and the curvature parameter K . Numerical analysis shows that our results are not significantly affected by the value of K . So we shall set K as zero. The dimensionless variable η which switches on the coupling, is set as one. Λ is the value of the cosmological constant corresponding to $\eta = 0$, and it is desirable that the evolution should ‘kill’ even a high initial value of Λ . So we assume that at Planck-time t_p , $\Lambda = L_p^{-2}$, in Planck units. Again, our results are not very sensitive to the initial value of Λ . The radiation density at t_p is conveniently taken to be m_p^4 , by choosing $\rho_0 = m_p^4$, and $S(t_p) = 1$. A value for ρ_0 higher than this appears unnatural, and a lower value does not improve the results. The coupling constant ϕ_0 is also chosen in Planck units, as L_p^{-1} . A fine-tuning of ϕ_0 can change the evolution drastically, but makes the model inelegant. Thus we shall not undertake any fine-tuning of the free parameters, and study the evolution for the choice made above. We shall also assume that at t_p the field ϕ has a value of L_p^{-1} , and we will consider two choices for the initial field velocity $\dot{\phi}(t_p)$ —zero and L_p^{-2} (no initial push, and finite initial push, respectively). The evolution is not significantly different in these two choices. We would like to remark once again that although we have initialized a large number of parameters, the results to follow are not strictly a consequence of this particular choice. Rather, they are fairly general. So long as an extremely severe fine tuning of the parameters is not resorted to, the problems encountered will continue to persist.

It is also useful, for the purpose of numerical computation, to scale the time-coordinate in units of Planck-time, and ϕ in terms of ϕ_0 . So we define

$$\tau = \frac{1}{\sqrt{2\pi}}(t/t_p), \quad x = \phi/\phi_0. \quad (3.1)$$

Noting that in the units $\hbar = c = 1$, we have $G = L_p^2$, Eqs. (2.19) and (2.20) can be written as

$$x_{\tau\tau} + \frac{3S_\tau}{S}x_\tau = \frac{f_x x_\tau^2}{1+2f} + \frac{f_x(1+2f)}{(1+4f)^2}, \quad (3.2)$$

and

$$\frac{S_\tau^2}{S^2} = \frac{2\pi}{3} \left\{ \frac{2x_\tau^2}{1+2f} + \frac{1}{1+4f} + \frac{8\pi}{S^4} \right\}. \quad (3.3)$$

(f_x stands for df/dx , and subscript τ denotes differentiation with respect to τ). We shall now consider various choices for the function f . We solve the pair of Eqs. (3.2) and (3.3) numerically, for the case of linear coupling ($f(x) = x$), and quadratic coupling ($f(x) = x^2$). We also consider some couplings with a higher power, and the case of exponential coupling. None of these models provides a satisfactory explanation for the smallness of Λ . This leads us to consider the evolution of the scalar field when a potential $V(\phi)$ is included.

3.1. The case of linear coupling

The most natural choice for the coupling to the trace is linear coupling: $f(\phi/\phi_0) = \phi/\phi_0$. For this case, we carried out the numerical integration up to a stage from where the equations can be integrated analytically. The results of the numerical integration are:

$$\text{at } \tau = \tau_0 = 10^4, \quad x = 154, \quad x_\tau = 5 \times 10^{-3}, \quad x_{\tau\tau} \cong 10^{-6}, \quad \frac{8\pi}{S^4} < 10^{-294}, \quad \frac{3S_\tau}{S} = 0.18. \quad (3.4)$$

With the help of these numbers, Eqs. (3.2) and (3.3) can be approximated to

$$\left(\frac{3S_\tau}{S} \right) x_\tau = 1/8x, \quad S_\tau/S = \sqrt{\pi/6}x^{-1/2} \quad (3.5)$$

with the solution

$$x = [2 \times 10^3 + 0.1(\tau - \tau_0)]^{2/3},$$

$$\frac{S_\tau}{S} = [5.3 \times 10^3 + 2.3(\tau - \tau_0)]^{-1/3}, \quad (3.6)$$

where τ_0 is as in (3.4). If we take the current age of the universe as 10^{17} s ($\cong 10^{60}t_p$),

we find that the scalar-field ϕ grows to a value $10^{40}L_p^{-1}$ in this time. As is evident from Eq. (3.3), this growth is insufficient to bring the Λ term below the observed value of 10^{-120} (in Planck units). Moreover, the kinetic energy term for the scalar field dominates over that for radiation.

For these reasons, the linear coupling to the trace cannot be considered satisfactory, and leads us to investigate alternate forms of the coupling function f .

3.2. Power law coupling to the trace

The failure of the linear coupling and the form of the Λ term in (3.3)— $(1 + 4f)^{-1}$, suggest that we consider other power law forms for $f(x)$.

For the coupling $f(x) = x^2$, and the same initial conditions as before, numerical integration gives the following results:

$$\text{at } \tau = 10^{60}, x = 10^{59}, x_\tau = 0.1, x_{\tau\tau} \cong 10^{-62}, \quad \frac{8\pi}{S^4} < 10^{-294}, \frac{S_\tau}{S} \sim 10^{-60}. \quad (3.7)$$

With this value of x , and with $f(x) = x^2$, the Λ term equals $\sim 10^{-119}$, which is just near the observed bound.

However, there is still the problem that the radiation density is negligible compared to the kinetic energy of the scalar field. Namely, the kinetic energy term for radiation is less than 10^{-294} (in the scaling that we have chosen), as contrasted to that for the scalar field, which is $\sim 10^{-120}$. This is the major problem with all forms of the coupling function f , and the reason for it is that the scalar field always has a nonzero velocity. The only plausible way to kill the kinetic energy term of the scalar field is to introduce a potential in the theory, which has a minimum at a nonzero finite value of the field. The scalar field should then evolve to the minimum, where it comes to rest because the oscillations about the minimum are damped by say, conversion of the scalar field to radiation fields. But then we have to fine-tune the parameters of the potential in such a way that the minimum is arranged at a sufficiently high value. To see this, consider a Higgs potential of the form $V(\phi) = (\phi^2 - \phi_m^2) + \Lambda$. If ϕ is to come to rest at ϕ_m , then the smallness of $\Lambda(\phi)$ at the current epoch implies (as may be seen from the modified field equation (3.11)), that $f(\phi_m/L_p^{-1}) \sim 10^{120}$. For $f(x) = x^n$ this means that $\phi_m = 10^{120/n}L_p^{-1}$, which is an absurdly high value for ϕ_m . In the next section we shall discuss in some details, the possibility of including a power law potential for the scalar field in the action.

What about a power law coupling of the form $f(x) = x^n$, $n > 2$? From numerical integration for the case $n = 3, 4$, we find that in these cases, the equation of motion (3.2) for the scalar field can be approximated, at a few Planck-times, to

$$\ddot{x} = \frac{n\dot{x}^2}{2x}, \quad n = 3, 4. \quad (3.8)$$

It can be easily shown that for $n > 2$, this equation has a singular solution (i.e. $x \rightarrow \infty$), and from numerical integration one finds that the singularity is reached very early

during the evolution—at only a few Planck times! For the case of the coupling $f(x) = \exp(x)$ also, similar singular behavior is observed. These results strongly suggest that the evolution is singular for all $n > 2$. This then rules out the model for all power law couplings to the trace.

3.3. Does including a potential help?

As argued above, the inclusion of a potential which has a minimum for a nonzero value of the field entails fine-tuning of the minimum. Thus a Higgs potential of the form $(\phi^2 - \phi_m^2)^2$ will not do. But it is instructive to investigate how a power law potential with a minimum at $\phi = 0$ effects the evolution.

When a potential $V(\phi)$ for the scalar field is added, the Λ term can be included in the potential, with the understanding that Λ now has a contribution from the scalar field also. (Thus $V(0) = \Lambda$.) The modified action reads

$$A = (16\pi G)^{-1} \int R\sqrt{-g} d^4x + \frac{1}{2} \int \phi^i \phi_i \sqrt{-g} d^4x - (8\pi G)^{-1} \int V(\phi)\sqrt{-g} d^4x + \eta \int Tf(\phi/\phi_0)\sqrt{-g} d^4x. \quad (3.9)$$

Taking account of the back reaction of ϕ in the same way as before, now leads to the following field equations,

$$x_{\tau\tau} + \frac{3S_\tau}{S} x_\tau = x_\tau^2 \frac{f'}{1+2f} + 4V(x) \frac{(1+2f)}{(1+4f)^2} f' - V'(x) \frac{(1+2f)}{1+4f}, \quad (3.10)$$

$$\frac{S_\tau^2}{S^2} = \frac{8\pi}{3} \left\{ \frac{x_\tau^2}{2(1+2f)} + \frac{V(x)}{1+4f} + \frac{1}{S^4} \right\}. \quad (3.11)$$

Here, all scalings have been done as before. We can arrive at some general conclusions by a simple analysis. Let us consider a potential of the form

$$V(x) = ax^\alpha + \Lambda L_p^2, \quad (3.12)$$

and a coupling to the trace of the form $f(x) = bx^n$. If the term $V(x)/(1+4f)$ is to fall as the field grows, we must have $\alpha < n$. Let us now look at the effect of $V(x)$ on the equation of motion (3.10) for the scalar field. For large x , the terms involving the potential have the form

$$\frac{1}{2} V(x) \left[\frac{f'(x)}{f(x)} - \frac{V'(x)}{V(x)} \right] = \frac{1}{2} a(n-\alpha)x^{\alpha-1} + n\Lambda L_p^2/2x. \quad (3.13)$$

This expression is positive for $n > \alpha$. This implies that in Eq. (3.10), the inclusion of a potential hastens the growth of the field when $n > \alpha$. Thus in those cases when the

evolution is singular, the potential does not prevent the singularity. This conclusion is supported by numerical computation. For the cases $n = 1$ and $n = 2$, a viable choice of the potential cannot be made, because of the requirement $\alpha < n$.

This completes our investigation into the cosmological solutions of the model. We find that, owing to some rather general reasons, the model does not satisfactorily explain the smallness of the cosmological constant. It appears difficult to lower Λ by a mechanism consistent with cosmology and without resorting to a fine-tuning of initial conditions or coupling constants.

4. Concluding Remarks

In this paper we have examined the possibility of making the cosmological constant a dynamical quantity by coupling it to a scalar field. The nature of the interaction is such that as the scalar field grows, Λ_{eff} progressively becomes smaller. It is possible to attain adequate growth for the scalar field, but the scalar field kinetic energy dominates over that for radiation. This feature is generic to all models of this kind, essentially because there are two competing factors. Rapid growth would require a high field velocity, which implies a high kinetic energy for the scalar field. Such a scalar-field dominated universe is not consistent with cosmology.

Acknowledgment

We thank Mr. T. R. Seshadri for useful discussions and for collaboration in the earlier stages of the investigation.

References

1. D. Arnaudon, C. P. Bachas, V. Rivasseau and P. Ve'greville, *Phys. Lett.* **B195** (1987) 167.
2. A. D. Dolgov, in *The Very Early Universe*, eds. G. W. Gibbons, S. W. Hawking and S. T. C. Siklos (Cambridge University Press, Cambridge, 1983); L. F. Abott, *Phys. Lett.* **150B** (1986) 427; L. H. Ford, *Phys. Rev.* **D35** (1987) 2339; S. M. Barr, BNL preprint BNL 38423 (1986); R. D. Peccei, J. Solà and C. Wetterich, *Phys. Lett.* **B195** (1987) 183.
3. F. Wilczek, in *The Very Early Universe*, *op. cit.*
4. R. Peccei and H. Quinn, *Phys. Rev. Lett.* **38** (1977) 1440; *Phys. Rev.* **D16** (1977) 179.
5. J. J. Rawal and J. V. Narlikar, *J. Astrophys. Astron.* **3** (1982) 393; J. V. Narlikar and T. Padmanabhan, *J. Astrophys. Astron.* **6** (1985) 171.