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SCALING RELATIONS FOR GRAVITATIONAL CLUSTERING IN TWO DIMENSIONS

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ABSTRACT

It is known that radial collapse around density peaks can explain the key features of the evolution of a correlation function in gravitational clustering in *three* dimensions. The same model also makes specific predictions for *two* dimensions. In this paper we test these predictions in two dimensions with the help of N-body simulations. We find that there is no stable clustering in the extremely nonlinear regime, but a nonlinear scaling relation does exist and can be used to relate the linear and the nonlinear correlation function. In the intermediate regime, the simulations agree with the model.

Subject headings: cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. GRAVITATIONAL CLUSTERING: TWO VERSUS THREE DIMENSIONS

Evolution of density perturbations at scales smaller than the Hubble radius in an expanding universe can be studied in the Newtonian limit in the matter-dominated regime. Linear theory is used to study the growth of small perturbations in density, but a study of nonlinear clustering requires N-body simulations. A number of attempts have been made in recent years to understand the evolution of constructs like the two-point correlation function using certain nonlinear scaling relations (NSRs). (See, for example, Hamilton et al. 1991; Nityananda & Padmanabhan 1994; Padmanabhan 1996a.) These studies have shown that the relation between the nonlinear and the linearly extrapolated correlation functions is reasonably model independent. This relation divides the evolution of correlation function into three parts (Bagla & Padmanabhan 1995): the linear regime, the intermediate regime, and the nonlinear regime. The evolution in the intermediate regime can be understood in terms of radial collapse around density peaks (Padmanabhan 1996a), if it is assumed that the evolution of profiles of density peaks follows the same pattern as an isolated peak. It is customary to invoke the hypothesis of stable clustering (Peebles 1980) to model the nonlinear regime. A large number of studies have examined clustering in this regime, and the general consensus is that the stable clustering limit does not exist (Padmanabhan et al. 1996).

However, the limited dynamic range of currently available three-dimensional N-body simulations poses serious difficulties in investigating this problem in greater detail. It was pointed out (Padmanabhan 1996b) that we can circumvent this problem by simulating a two-dimensional system, wherein a much higher dynamic range can be achieved. For example, since $160^3 \approx 2048^2$, the computational requirements are the same for a two-dimensional simulation with a box size of 2048 and a three-dimensional simulation with box size of 160. Assuming that one can reliably use, say, half of box size as *good* dynamic range, we have a dynamic range of a factor of 1000 in two dimensions as against a factor of about 80 in three dimensions. This allows us to probe higher nonlinearities in two dimensions as compared to three dimensions. As long as we stick to generic features (like the nonlinear scaling relations, investigated here) that are independent of dimension, two dimensions has a definite advantage over three dimensions. Higher dynamic range is the basic motivation for studying gravitational clustering in two dimensions.

When we go from three to two dimensions, we have, in principle, two different ways of modeling the system: (1) We can consider two-dimensional perturbations in a three-dimensional expanding universe. Here we take the force between particles to be $1/r^2$ and assume that all the particles, and their velocities, are confined to a single plane at the initial instant. (2) We can study perturbations that do not depend on one of the three coordinates, i.e., we start with a set of infinitely long straight "needles" all pointing along one axis. The force of interaction falls as $1/r$. The evolution keeps the "needles" pointed in the same direction, and we study the clustering in an orthogonal plane. Particles in the N-body simulation represent the intersection of these "needles" with this plane.

In both of these approaches the universe is three-dimensional and the background is expanding isotropically.

The study of two-dimensional perturbations (like those due to pancakes, for example) in a three-dimensional expanding universe faces an operational problem: To begin with, we do not gain the dynamic range if we stick to three dimensions, even if we consider perturbations in a plane; the force between particles still has to computed by the solution of the Poisson equation in three dimensions. Also, the relevance of the interaction of matter outside the plane with these perturbations makes it, essentially, a three-dimensional problem.

Thus we are left with the second possibility. The two-dimensional system is the intersection of an orthogonal plane and the "needles," and the force between the "particles" in this plane is given by the solution of the Poisson equation in two dimensions. Such a system is somewhat dichotomous with the background universe expanding isotropically. However, convenience is not the only reason for studying this somewhat strange system: relevant results for the evolution of density profiles around peaks in two dimensions have also been computed for this type of system (Filmore & Goldreich 1984).

Generalization of the NSRs to the two-dimensional system was done using relations for cylindrical collapse by

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Padmanabhan (1996b), and we will test these predictions here.

Although the system of infinite needles is appropriate for testing the predictions in the intermediate regime, the same cannot be said for the asymptotic regime. We are dealing with a system that occupies a smaller number of dimensions in the phase space, *and* the interaction of the constituents follows a different force law. Therefore, it is difficult to interpret, or carry over, results regarding stable clustering to the full three-dimensional system.

1.1. *Nonlinear Scaling Relations*

The nonlinear and the linear correlation functions at two different scales can be related by NSRs. The relation between these scales is given by the characteristics of the pair conservation equation (Nityananda & Padmanabhan 1994). For the two-dimensional system of interest, this equation can be written as (Padmanabhan 1996b)

$$\frac{\partial D}{\partial A} - h(A, x)\frac{\partial D}{\partial X} = 2h(A, X) . \quad (1)$$

Here $D = \log(1 + \bar{\xi})$, $h = -v_p/Hr$ is the scaled pair velocity, $\bar{\xi}(x) = 2x^{-2}\int^x r\xi(r)dr$ is the mean correlation function (ξ is the correlation function), H is Hubble's constant, $X = \log x$ and $A = \log a$. The characteristics of this equation are $x^2[1 + \bar{\xi}(x, a)] = l^2$, where x and l are the two scales used in NSRs. The self-similar models due to Filmore & Goldreich (1984) imply that for collapse of cylindrical perturbations the turnaround radius and the initial density contrast inside that shell are related as $x_{\rm ta} \propto l/\bar{\delta}_i \propto l/\bar{\xi}_L(l)$. (Here $\bar{\xi}_L$ is the linearly extrapolated mean correlation function.) Noting that in two dimensions $M \propto x^2$, we find $\bar{\xi}(x) \propto [\bar{\xi}_L(l)]^2$ in the regime dominated by infall. Stable clustering limit implies $\bar{\xi}_{\rm NL}(a, x) \propto \bar{\xi}_L(a, l)$ (Padmanabhan 1996b). Thus in two dimensions the scaling relations are

$$\bar{\xi}(a, x) \propto \begin{cases} \bar{\xi}_L(a, l) & \text{(linear)} , \\ \bar{\xi}_L(a, l)^2 & \text{(radial infall)} , \\ \bar{\xi}_L(a, l) & \text{(stable clustering)} . \end{cases} \quad (2)$$

A more general assumption compared to stable clustering involves taking $h =$ constant asymptotically. In a system reaching steady state with both virialization and mergers contributing to the evolution, one may reach a constant value for h, though it will not be unity if mergers are a dominant phenomenon. (This assumption has been discussed in, for example, Padmanabhan 1996a, 1996b, 1997.) It also allows a larger parameter space to compare simulation results. If $h =$ constant asymptotically, then $\bar{\xi}(x) \propto \bar{\xi}_L^h(l)$ in this limit. Note that in three dimensions the indices for three regimes are 1, 3, and $3h/2$, respectively.

All features of clustering in three dimensions are present here as well. In particular:

1. If the asymptotic value of h scales with n such that $h(n + 2) =$ constant, then the final slope of the nonlinear correlation function will be independent of the initial slope.
2. If a NSR exists, then it will predict a *specific* index in the intermediate and asymptotic regimes that will depend on the initial power spectrum. In other words, the existence of a NSR implies that gravitational clustering does not erase the memory of initial conditions.
3. It is, however, possible that spectra which are not scale free acquire universal critical indices at which the correlation functions grow in a "shape-invariant" manner. This comes about because the growth rate of the correlation function varies with the local index, and for an index that is not globally constant the correlation functions may "straighten out" by this process.
4. In three-dimensional clustering, $n = -1$ in the intermediate regime and $n = -2$ in the asymptotic regime (Bagla & Padmanabhan 1997a) are the critical indices. These are the same for clustering in two dimensions.

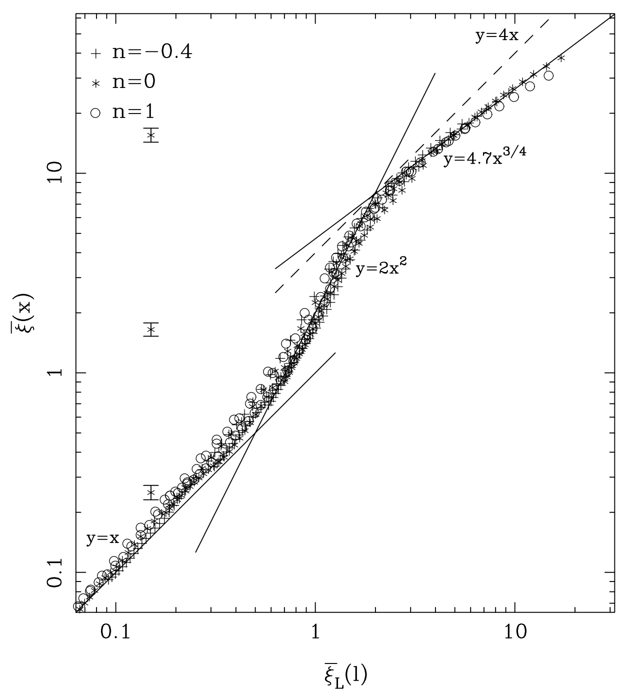

FIG. 1.—Nonlinear correlation function $\bar{\xi}(x)$ as a function of the linearly extrapolated correlation function $\bar{\xi}_L(l)$. Here the scales x and l are related by $x^2(1 + \bar{\xi}) = l^2$. Data for $n = 1$ is represented by circles, those for $n = 0$ by stars, and Plus signs mark the points for $n = -0.4$. For each of these models we have plotted data for the three epochs mentioned in the text. The estimated 2 σ error bars are shown as vertical lines at three representative values of $\bar{\xi}$, viz., $\bar{\xi} = 15.582$, 1.65, and 0.25, covering the nonlinear, intermediate, and linear regimes. The error bars are shown away from the NSR plot for the sake of visibility. It is clear from this figure that there are no systematic differences between the three models, and they trace out a simple curve with three distinct slopes. The slope of the curve in the intermediate regime is the same as that predicted by the radial infall model. The stable clustering limit is shown as the dashed line, and it is clear that the data points deviate from this curve.

2. SIMULATIONS AND RESULTS

We carried out a series of numerical experiments to test the ideas outlined above. We used a particle mesh code (Bagla & Padmanabhan 1997b) to simulate power-law models. The simulations were done with 1024^2 or 2048^2 particles in order to ensure that we had sufficient dynamic range to study all three regimes in the evolution of nonlinear clustering. In particular, it is necessary to use larger simulations for power-law spectra with a negative index. Here we will present results for three models: $n = 1$, $n = 0$, and $n = -0.4$.

All the models are normalized by requiring the linearly extrapolated rms fluctuations in density, computed using a Gaussian filter, to be unity at a scale of 10 grid points at $a = 1.0$. The results we present are for $a = 1$, 2, and 5 for $n = 0$ and $n = 1$, and for $a = 1, 2$, and 3 for $n = -0.4$.

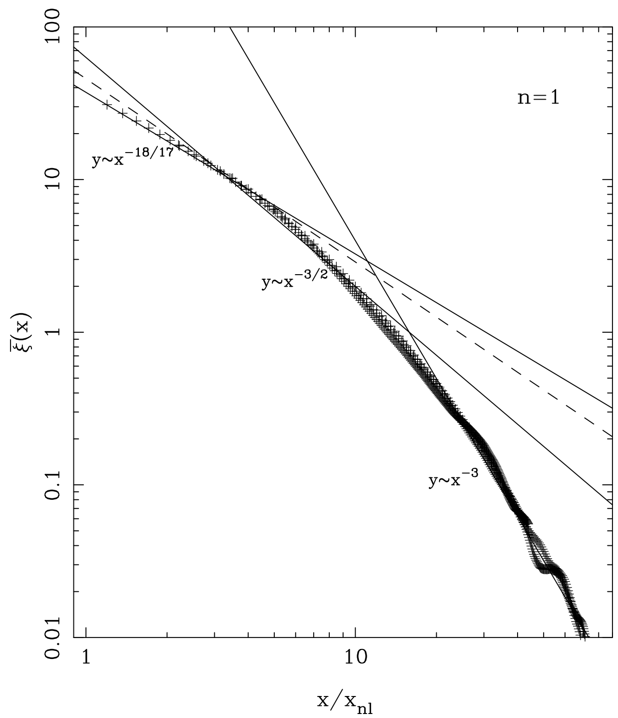

FIG. 2*a*

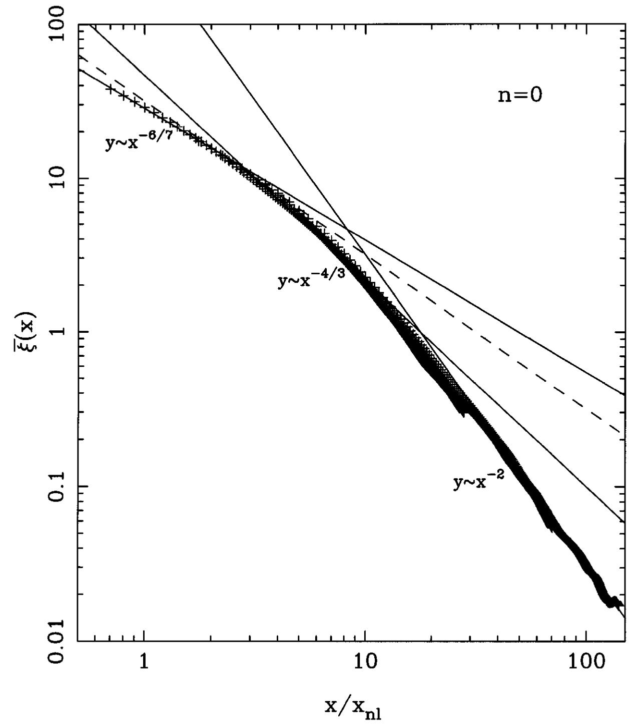

FIG. 2*b*

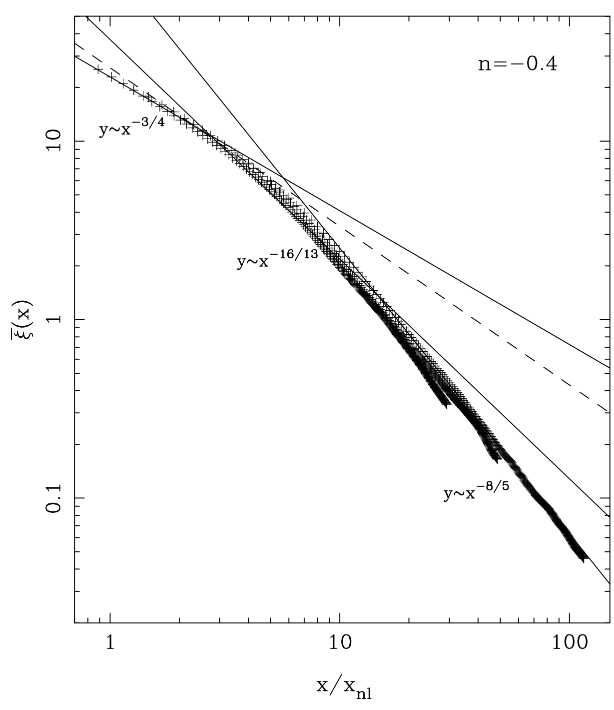

FIG. 2*c*

FIG. 2.—(*a*) Correlation function $\bar{\xi}(x)$ as a function of $x/x_{\rm NL}$ for the $n = 1$ model. Here $x_{\rm NL} \propto a^{-2/(n+2)}$. Thick lines mark slopes expected from the nonlinear scaling relations shown in Fig. 1. The dashed line marks the expected slope of the correlation function in the stable clustering limit. The mismatch between the expected slope and the true slope in the intermediate regime may arise from the fact that the assumption of $\bar{\xi} \gg 1$ used in computing the slope is not valid at the lower end of the regime. (*b*) Same as (*a*), but for $n = 0$. (*c*) Same as (*a*), but for $n = -0.4$.

A significant source of errors in large simulations is the addition of a small displacement in each step (fraction of a grid length) to a large position (up to 2048 grid lengths). We avoid this problem by using net displacement for internal storage.

We will show the correlation function and the pair velocity only for length scales larger than four grid lengths. We do this to avoid error due to shot noise and other artifacts introduced by various effects at smaller scales. This ensures that errors in our results are acceptably small. (Variations between different realizations give a dispersion of less than 10% in the correlation function.)

In Figure 1 we have plotted the nonlinear correlation function $\bar{\xi}(x)$ as a function of the linearly extrapolated correlation function $\bar{\xi}_L(l)$. Here the scales x and l are related by $x^2(1+\bar{\xi}) = l^2$. Data for $n = 1$ is represented by circles, that for $n = 0$ by stars and a plus sign marks the points for $n = -0.4$. Clearly, there are no systematic differences between the three models, and the data points trace out a simple curve with three distinct slopes. (We have also marked the 2 σ errors calculated by averaging over several data sets. The error bars are plotted away from the NSR plot, for visibility and clarity.) The NSR, shown as thick lines, is

$$\bar{\xi}(a, x) = \begin{cases} \bar{\xi}_L(a, l) , & \bar{\xi}_L(l) \le 0.5;\ \bar{\xi}(x) \le 0.5 , \\ 2\bar{\xi}_L(a, l)^2 , & 0.5 \le \bar{\xi}_L(l) \le 2;\ 0.5 \le \bar{\xi}(x) \le 8 , \\ 4.7\bar{\xi}_L(a, l)^{3/4} , & 2 \le \bar{\xi}_L(l);\ 8 \le \bar{\xi}(x) . \end{cases} \quad (3)$$

The slope in the intermediate regime is as expected. The asymptotic regime has a different slope than that predicted by stable clustering, which is shown as a dashed line. Unlike the observed relations for clustering in three dimensions, the coefficient for the intermediate regime is large. This has important implications for the critical index.

Panels of Figure 2 show $\bar{\xi}(x)$ as a function of $x/x_{\rm NL}$ for the three models. These confirm that the slope of $\bar{\xi}(x)$ is consistent with the NSR shown in Figure 1. In each of these panels, the slope expected in the stable clustering limit is shown as a dashed line.

As mentioned above, the existence of the NSR (eq. [3]) implies that the slope of the correlation function will depend on the initial spectral index. To this extent, gravitational clustering does not erase the memory of initial conditions. However, the differences of slope are significantly reduced by nonlinear evolution.

3. CONCLUSIONS

Our conclusions can be summarized as follows:

1. We have verified that the NSR for the correlation function exists for clustering in two dimensions in all three regimes, just as in three dimensions. This NSR is independent of the power-law index, at least for the three indices studied here

2. In the intermediate regime, the NSR in the form of equation (3) can be understood in terms of radial infall around peaks. Our simulations verify the predictions (Padmanabhan 1996b) for this regime.

3. In the asymptotic regime, our results do *not* agree with the stable clustering hypothesis. The slope of the NSR in the asymptotic regime in Figure 1 implies $h =$ constant. We find that, in this regime, $h \simeq \frac{3}{4}$ for all the models studied here.

4. The existence of the NSR implies that the asymptotic slope of the correlation function depends on the initial slope. However, this is strictly true only for pure power-law models; for other models it is possible for the spectra to be driven to a universal form.

The NSR in the asymptotic regime seems to be linked to the logarithmic nature of the potential. Issues relating to theoretical modeling of this regime will be addressed in a future publication.

While this paper was in preparation, a preprint (Munshi et al. 1997) that discusses similar issues appeared on SISSA archives. However our results are different from theirs in several aspects: (*a*) we find a model-independent NSR with an asymptotic slope of $\frac{3}{4}$, whereas Munshi et al. (1997) only report deviations from stable clustering. (*b*) We do *not* find that $h(n+2) =$ constant is a good fit to our data. They seem to conclude differently, even though their Figure 2 shows a large scatter. Their fit to $h(n+2) =$ constant is also not good, and the omission of the first point will make their fit consistent with a constant asymptotic value for h around 0.5–0.75. (*c*) Last, a comparison of our Figure 1 with the top panel of Figure 1 in their paper shows that whereas we get the same transition points between the three regimes for all the models, the transition points deduced by them tend to vary between models. The differences can possibly be understood as arising from lower resolution and inadequate levels of nonlinearity in their simulations.

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