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2. Conformal Invariance in Physics and Cosmology*

Fred Hoyle and J. V. Narlikar

The significance of conformal transformation is discussed in relation to physical theories and cosmology. It is well known that Maxwell's equations and the Dirac equation for a massless particle are conformally invariant, whereas Einstein's gravitational equations, and the Dirac equation for a particle with mass, are not. However, the Dirac equation can be made conformally invariant provided mass transforms as the reciprocal of length. A conformally invariant gravitational theory in which masses transform in this way can also be formulated. The latter theory gives the same results as Einstein's provided

(i) local fields are weak,
(ii) particles are conserved.

For strong local fields the theory is different. The theory also permits non-conservation of particles. It permits creation of matter without recourse to the so-called C-field.

Since all isotropic, homogeneous cosmological models are conformally flat it is possible to discuss all such models in flat space. Masses then change with epoch and the cosmological red-shift is explained by this property.

The role of cosmological boundary conditions is also considered. The Friedmann models and the steady-state model appear in sharp contrast when discussed from this point of view.

I. Units and Dimensions

A survey of scientific literature reveals a chaotic situation concerning units. Although experimentalists understandably prefer to avoid describing measured quantities by very large or very small numbers, this disadvantage appears a small price to pay—especially in an age when much data is analyzed by computer—in order to simplify the present extremely unsatisfactory position. We begin by asking: What is the minimum number of units required to describe physical theories?

Theoretical physicists frequently work with units of length, mass and time. This number can be further reduced if we take account of two major developments of physical thought in the first quarter of the present century. The special theory of relativity introduced a fundamental velocity c, the velocity of light. The square of the proper distance between two world events separated by a coordinate difference (x, y, z, t) is given by

$$s^2 = c^2t^2 - x^2 - y^2 - z^2. \tag{1}$$

The invariance of (1), and the constancy of c, clearly suggests that we reduce the number of unnecessary symbols by setting $c = 1$. This identifies the time unit with the space unit and thereby reduces the number of units to two.

Another constant was introduced into physics by quantum theory. This is Planck's constant h, or more commonly $\hbar = h/2\pi$. What significance can we attach to $\hbar$? The behavior of a classical system can be described by the principle of stationary action

$$\delta S = 0. \tag{2}$$

However, in classical physics the action functional S is not dimensionless. S has the same dimensionality as $\hbar$, so that (2) can be written in dimensionless form

$$\delta(S/\hbar) = 0. \tag{3}$$

Although the insertion of $\hbar$ in (3) is trivial for classical theory it is not trivial in quantum theory. Following Dirac (1958) and Feynman (1948) we interpret

$$\exp(iS/\hbar)$$

as the probability amplitude for a physical system to follow a certain

course specified by S. The total amplitude for all possibilities can be written as

$$\sum \exp(iS/\hbar). \tag{4}$$

Feynman has shown how this sum can be related to the quantum mechanical propagator. It is easy to see how (2) can be obtained from (4) in the classical limit $S \gg \hbar$. By the principle of stationary phase only paths close to that for which (2) is satisfied make a significant contribution to the sum (4) in this limit.

From this discussion it is clear that $\hbar$ has appeared because we have made the 'mistake' of attributing dimensionality to S. With $c = 1$, $\hbar = 1$ we then have only one independent unit. To fix ideas we take this as the unit of length. All physical quantities are now expressed as some power of length. For example

$$\text{mass} \sim L^{-1}, \text{ frequency} \sim L^{-1}, \text{ charge} \sim L^0, \text{ electric field} \sim L^{-2},$$

and so on. The choice of length unit converts all quantities into numbers.

Define r_p for a particle P of mass m_p by

$$r_p = Gm_p.$$

It is convenient to choose a standard reference particle and to take r_p as the unit of length. When this is done for the proton we obtain the numbers set out in the following table:

Lengths Relative to r_p for the Proton

Hubble radius H^{-1}	$\sim 10^{80}$
Electron radius $4\pi e^2/m_e$	$\sim 3\times10^{40}$
Compton wavelength of electron $1/m_e$	$\sim 3\times10^{41}$
Compton wavelength of proton $1/m_p$	$\sim 10^{38}$
Fermi	$\sim 10^{39}$

We also note that $G \sim L^2$ and is numerically equal (with this choice of unit) to the Compton wavelength of the proton. Another quantity of interest, the cosmological mass density, $\sim L^{-4}$ and is numerically $\sim 10^{-200}$. The fact that all lengths of importance in atomic and nuclear physics fall close to $(r_pH^{-1})^{1/2}$ suggests the existence of important relationships between local and cosmological phenomena.

Having now reduced the number of units to one, we turn next to a mathematical transformation that is closely related to the measurement of this remaining unit.

II. Conformal Transformations: Should Physical Theories Be Conformally Invariant?

In early discussions of the general theory of relativity considerable attention was paid to the measurement of length. The assumption of a line element

$$ds^2 = g_{ik}\,dx^i\,dx^k \tag{5}$$

implies that the square of length between two neighboring points separated by a coordinate displacement dx^i is given by ds^2. However, suppose we have a new line element

$$ds^{*2} = g^*_{ik}\,dx^i\,dx^k, \qquad g^*_{ik} = \Omega^2 g_{ik}, \tag{6}$$

where $\Omega(x^i)$ is a real non-zero function of position. Since in physics we are concerned ultimately with dimensionless numbers, and since these are obtained by appropriate combinations of quantities with dimensionality L^n, $n = 0, \pm 1, \ldots$, it follows that provided such quantities are all taken at the same point (5) and (6) would be physically undistinguishable. But if significant dimensionless numbers are built from quantities not taken at the same point (5) and (6) will in general be distinguishable. The transformation (5) → (6) is known as a conformal transformation. Physical theories that are invariant under this transformation are said to be conformally invariant.

We now ask: Should physical theories be conformally invariant? For a physical theory not to be conformally invariant it is necessary that there be some form of propagation from a point, A say, to another point, B say, whereby information concerning the length unit at A is carried to B. Then the length units at A and B can be compared and (5) and (6) can be distinguished.

Next we notice that not all points B can be reached by physical propagation from A. Thus B must lie in or on the future light cone of A. Now the light cone is conformally invariant since $ds^* = 0$ if $ds = 0$. Hence the set of points B that can be reached by physical propagation from A is conformally invariant. This important property of physical theories suggests we take the view that all other properties of physical theories should also be conformally invariant.

Which of the existing theories are conformally invariant? It is well known that Maxwell's equations

$$F^{ik}_{\ ;k} = 4\pi j^i, \tag{7}$$

are conformally invariant. That is, the form (7) remains unchanged using

$$F^*_{ik} = F_{ik}, \qquad j^{*i} = \Omega^{-4} j^i, \tag{8}$$

and going from (5) to (6). [Note that $F^{*ik} = \Omega^{-4}F^{ik}$.] The Dirac equation for a massless particle is also conformally invariant. But the Dirac equation for a particle of mass m,

$$(\not{\nabla} + im)\psi = 0 \tag{9}$$

is not. However, (9) can be made conformally invariant by requiring that

$$m^* = m\Omega^{-1}, \qquad \psi^* = \psi\Omega^{-3/2} \tag{10}$$

The transformation of ψ is that which we expect from the probability interpretation of $\psi^*\psi$, but the transformation of the mass is new. On the other hand we have already remarked that with $c = 1$, $\hbar = 1$, mass has dimension L^{-1} so that the transformation of m is in accordance with our dimensional interpretation. Or we can look on the transformation of m in another way. The statement $c = 1$ is conformally invariant because null geodesics are unaffected by a conformal transformation. The adoption of $m^* = m\Omega^{-1}$ makes the statement $\hbar = 1$ also conformally invariant.

Turning now to gravitation, Einstein's equations are obtained from

$$S = \frac{1}{16\pi G}\int R\sqrt{(-g)}\,d^4x - \sum_a m_a \int da, \tag{11}$$

the volume integral being over some assigned 4-dimensional region and the line integrals being over the portions of the world lines of the particles that fall within this volume. The classical $\delta S = 0$ is used, the variation δS being computed for the most general infinitesimal changes of g_{ik}. By slightly rewriting the second term of (11)

$$-\sum_a \int m_a\,da \tag{12}$$

we can make this term conformally invariant. Thus da is the element of length along the world line of a, so that $da^* = \Omega\,da$. With $m^*_a = m_a\Omega^{-1}$ we therefore have $m^*_a\,da^* = m_a\,da$. Proceeding in the same way for the first term of (11) we first write

$$\frac{1}{16\pi}\int \frac{R\sqrt{(-g)}}{G}\,d^4x \tag{13}$$

and consider $G^* = \Omega^2 G$, since G has dimensionality L^2. But the device does not work this time because the scalar curvature R does not transform in accordance with a simple power of Ω. Derivatives of Ω appear in the relation of R^* and R. Hence Einstein's theory cannot be made conformally invariant in the simple way that the Dirac theory can, or as (12) can. We are therefore led to ask: Is Einstein's theory correct? In particular, is the first term of (11) correct?

We shall proceed by showing that a gravitational theory can be obtained from (12) alone, and that this theory agrees with Einstein's in all cases of practical interest for weak fields. The new theory differs from Einstein's, however, for strong fields.

The transformation $m_a^* = m_a\Omega^{-1}$ demands a physical interpretation of mass. We can no longer regard the mass of a particle as a fixed quantity that we assign to the particle. It becomes necessary to take a Machian point of view concerning the nature of mass. According to this point of view m_a arises from the rest of the particles in the universe. Let each particle $b \neq a$ give rise to a 'mass field'. Denote this field at a general point x by $m^{(b)}(x)$. At any point A on the world line of a we therefore have $m^{(b)}(A)$ as the contribution of particle b to the mass of a at A. Summing for all $b \neq a$ gives

$$m_a = \sum_{b \neq a} m^{(b)}(A). \tag{14}$$

Our expression for the action S can therefore be written as

$$S = -\sum_a \int m_a \, da = -\sum_a \sum_{b \neq a} \int m^{(b)}(A) \, da. \tag{15}$$

In order that (15) shall have symmetry with respect to every pair of particles a, b it is necessary that $m^{(b)}(x)$ be of the form

$$m^{(b)}(x) = \int P(x, B) \, db, \tag{16}$$

where $P(x, B)$ is a biscalar, symmetric with respect to x and B. It is then easy to see that (15) can be rewritten as

$$S = -\sum_{a \neq b}\sum \iint P(A, B) \, da \, db, \tag{17}$$

from which it is clear that S is conformally invariant provided

$$P^*(A, B) = \Omega(A)^{-1}\Omega(B)^{-1}P(A, B). \tag{18}$$

The function $P(x, B)$ propagates the mass field of particle b and we expect it to satisfy a wave equation of the usual kind. It turns out that for (18) to hold it is only possible to choose the wave equation in one way, namely

$$\Box_x P(x, B) + \tfrac{1}{6}R(x)P(x, B) = [-g(B)]^{-1/2}\delta^{(4)}(x, B), \tag{19}$$

where $\delta^{(4)}(x, B)$ is the 4-dimensional delta-function representing a source of unit strength at point B.

The gravitational equations are now calculated from $\delta S = 0$, using (17) for S, and making general infinitesimal changes of the components of the tensor g_{ik}. The calculation is rather long and we have obtained (Hoyle and Narlikar 1964a, 1966) the following gravitational equations

$$F(R_{ik} - \tfrac{1}{2}g_{ik}R) = -3(T_{ik} + \Phi_{ik}) + (g_{ik}\Box F - F_{;ik}), \tag{20}$$

where

$$F = \tfrac{1}{2}\sum_{a \neq b}\sum m^{(a)}m^{(b)} \tag{21}$$

$$\Phi_{ik} = -\tfrac{1}{2}\sum_{a \neq b}\sum [m_i^{(a)}m_k^{(b)} + m_k^{(a)}m_i^{(b)} - g_{ik}m_l^{(a)}m^{(b)l}], \tag{22}$$

and T_{ik} is the same matter tensor as in the usual theory. All quantities in (20) are evaluated at a general field point x.

In the actual universe there is a large number of particles, so that the double summations in (21) and (22) contain very many terms. All mass fields from particles at cosmological distances from x have the same sign (cf. footnote on p. 24), and all mass fields from particles local to x also have the same sign provided local gravitational fields are weak, i.e., provided the $(1/6)R$ term in (19) is small. Changes of sign are possible, however, when fields are strong. This can be seen from the study of the one-particle problem (Hoyle and Narlikar 1966). Excluding the case of strong local fields it follows that since F is quadratic in the mass fields all terms contributing to F must be positive and F cannot be zero.

Evidently for a conformal transformation we have

$$F^* = \Omega^{-2}F. \tag{23}$$

Suppose now that F has been determined for some given g_{ik}. In general F will be dependent on x. But by making an appropriate conformal transformation we can then obtain F^* independent of x.

Hence by fixing the conformal frame in a suitable way we can simplify (20). In fact, dropping the star notation, we can write

$$R_{ik} - \tfrac{1}{2}g_{ik}R = -8\pi G(T_{ik} + \Phi_{ik}), \tag{24}$$

where

$$8\pi G = \frac{3}{F} > 0. \tag{25}$$

Except for the Φ_{ik} term we now have Einstein's equations. The Φ_{ik} term is indeed of exactly the form we have previously attributed to a C-field. Formerly we had

$$-\tfrac{1}{2}f\sum_{a\neq b}\sum [C_i^{(a)}C_k^{(b)} + C_k^{(a)}C_i^{(b)} - g_{ik}C_l^{(a)}C^{(b)l}], \qquad f > 0, \tag{26}$$

in place of Φ_{ik}. This is not only similar in form but similar in sign.

The C-field was used to permit the creation of matter. Does the Φ_{ik} term in (24) permit the creation of matter? We shall show in section IV that the answer to this question is affirmative. Indeed both the Friedmann cosmologies and the steady-state cosmologies are solutions of (24). The Friedmann cosmologies are obtained when we postulate no creation of matter and the steady-state cosmology is obtained when we permit creation of matter.

When a large number of terms contribute mass fields all of the same sign we can write

$$F = \tfrac{1}{2}\sum_{a\neq b}\sum m^{(a)}m^{(b)} \approx \tfrac{1}{2}m^2, \tag{27}$$

where

$$m = \sum_a m^{(a)}, \tag{28}$$

the summation in (28) being over all particles. The approximation in (27) consists in ignoring $\sum_a [m^{(a)}]^2$ in comparison with $\sum_{a\neq b}\sum m^{(a)}m^{(b)}$. When there are N terms, $N \gg 1$, in (28) the double sum is greater by a factor of order N than the single sum. To the same approximation we can write

$$\Phi_{ik} = -\{[\sum_a m_i^{(a)}][\sum_b m_k^{(b)}] - \tfrac{1}{2}g_{ik}[\sum_a m_l^{(a)}][\sum_b m^{(b)l}]\}. \tag{29}$$

If there is no creation of matter

$$\sum_a m_i^{(a)} = [\sum_a m^{(a)}]_i = m_i, \tag{30}$$

in which case

$$\Phi_{ik} = -(m_i m_k - \tfrac{1}{2}g_{ik}m_l m^l). \tag{31}$$

But F = constant requires m = constant, so that (31) vanishes and (24) reduces to Einstein's equations.

Hence we obtain Einstein's equations when

(*i*) local gravitational fields are weak,

(*ii*) particles are conserved.

The vanishing of Φ_{ik} when (*ii*) is satisfied is again similar to the behavior of (26). The C-field was considered to arise from the ends of broken world lines—i.e., from particle creation, so that when there was no particle creation there was no C-field. Although there are thus considerable similarities between Φ_{ik} and our former use of a C-field, it is important that our present theory is conformally invariant whereas the C-field theory was not.

The present derivation of Einstein's equations, subject to (*i*) and (*ii*), gives us additional information not present in the usual theory. We have deduced that $G > 0$, so we deduce that weak gravitational fields must be attractive. In Einstein's theory we might equally well take G in (11) to be negative and this would lead to fields being repulsive. So $G > 0$ is an assumption in the usual theory, not a deduction.

We also see that the 'cosmical term' λg_{ik}, sometimes introduced into Einstein's equations, must be absent. The requirement of conformal invariance removes this term, although the g_{ik} term in Φ_{ik} has some similarities of behavior to the cosmical term. Practical details of the Lemaître cosmology, which depends on the λg_{ik} term, are in a number of respects very similar to the practical details of the steady state model. Mathematically this can be traced to the similar effects of λg_{ik} and of the g_{ik} term in Φ_{ik}.

We have sometimes heard physicists argue that the cosmical term λg_{ik} should be present in order that the gravitational equations take their most general form. To dispose of this argument it is only necessary to remark that a similar argument used in the electromagnetic theory leads incorrectly to the Proca equations for the 4-potential instead of to the usual correct equations.

III. Conformally Invariant Cosmological Models

When we turn to cosmology there are two directions we can follow. If we decide that particles must be conserved our gravitational equations reduce to Einstein's in the conformal frame in which F = constant. Since, moreover, masses are constant in this frame we have the usual Friedmann models. Or we can decide to permit creation of matter, in which case the Φ_{ik} term in (24) is important and leads to the steady state model, as we shall verify in the following section.

All these models belong to the class of isotropic homogeneous cosmologies described by the Robertson-Walker line element

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\Theta^2 + \sin^2\Theta\, d\varphi^2)\right],$$
$$k = 0, \pm 1. \quad (32)$$

It is well known that coordinate transformations $\tau = \tau(t, r)$, $\rho = \rho(t, r)$ can be found that permit (32) to be expressed in the form

$$ds^2 = e^{2\zeta}[d\tau^2 - d\rho^2 - \rho^2(d\Theta^2 + \sin^2\Theta\, d\varphi^2)], \quad (33)$$

where ζ is a function of τ, ρ depending on the form of the expansion factor $S(t)$ and on the choice of k. Hence this whole class of cosmologies is conformal to flat space. Because Einstein's theory is not conformally invariant it is not possible to take advantage of this geometrical simplification. But now that we have shown that all the cosmologies in question can be obtained from a conformally invariant gravitational theory, we can make the transformation $\Omega = e^{-\zeta}$, in which case we have $m^* = me^\zeta$. The situation is that we can work either with constant masses (F = constant) and with the line element (32) or equivalently with

$$ds^{*2} = d\tau^2 - d\rho^2 - \rho^2(d\Theta^2 + \sin^2\Theta\, d\varphi^2), \quad (34)$$
$$m^* = me^\zeta. \quad (35)$$

Here $m = \sum_a m^{(a)}$ is the total mass, but all individual mass fields $m^{(b)}(x)$ are also changed by e^ζ. In this conformal frame the wave equation (19) has the flat space 'elementary solution'

$$P^*(x, B) = \frac{1}{4\pi}\,\delta(s^{*2}_{xB}). \quad (36)$$

All individual mass fields are seen to be positive.†

At first sight it might seem as if a different theory can be obtained by changing the sign of the right-hand side of (19), but this is not so. The mass fields are then systematically negative, but F, being quadratic in the mass fields, is unchanged, as is Φ_{ik}. T_{ik} is linear in the mass fields, but the sign of the T_{ik} term in (20) is also changed, to $+3T_{ik}$.

† Local contributions to the mass field can be negative because of the $(1/6)R$ term in (19), but not unless such fields are strong.

In the remainder of the present section we consider two examples. First, the simplest Friedmann model, the Einstein de Sitter case, $k = 0$, for which

$$ds^2 = dt^2 - (\tfrac{3}{2}Ht)^{4/3}[dr^2 + r^2(d\Theta^2 + \sin^2\Theta\, d\varphi^2)]. \quad (37)$$

The length H^{-1} is not a physical constant here but is defined by

$$H^{-1} = \tfrac{3}{2}t_0, \quad (38)$$

where t_0 is the present epoch. The time transformation

$$H\tau = 2(\tfrac{3}{2}Ht)^{1/3} \quad (39)$$

changes (37) to

$$ds^2 = (\tfrac{1}{2}H\tau)^4[d\tau^2 - dr^2 - r^2(d\Theta^2 + \sin^2\Theta\, d\varphi^2)]. \quad (40)$$

The required conformal transformation is evidently

$$\Omega = (\tfrac{1}{2}H\tau)^{-2}, \quad (41)$$

and

$$m^* = (\tfrac{1}{2}H\tau)^2 m. \quad (42)$$

We have thus achieved the required transformation without needing to change the r coordinate, which must be done in more complicated Friedmann models ($k = \pm 1$). The range of the coordinates is

$$0 \le \tau < \infty, \quad 0 \le r < \infty, \quad -\pi \le \Theta \le \pi, \quad 0 \le \varphi \le 2\pi, \quad (43)$$

and the present epoch on the τ scale is $2H^{-1}$.

The result (42) shows that masses increase with epoch. The frequencies of radiation from atomic transitions also increase with epoch—in the same way as masses. This leads to the cosmological red-shift effect. In fact, it is easy to verify that the flat space form of the theory gives exactly the same red-shift effect as the curved space form.

Next we consider the steady state model. The curved space form of the line element is

$$ds^2 = dt^2 - \exp 2Ht[dr^2 + r^2(d\Theta^2 + \sin^2\Theta\, d\varphi^2)] \quad (44)$$

in which H^{-1} is now a physical constant related to the creation process (cf. the following section). Once again we do not need to transform the r coordinate, since

$$H\tau = -e^{-Ht} \quad (45)$$

transforms (44) to

$$ds^{*2} = \left(-\frac{1}{H\tau}\right)^2 [d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)]. \qquad (46)$$

The required conformal transformation is now

$$\Omega = -H\tau, \qquad (47)$$

and

$$m^* = -m/H\tau. \qquad (48)$$

The ranges of the variables are

$$-\infty < \tau \leq 0, \qquad 0 \leq r < \infty, \qquad -\pi \leq \theta \leq \pi, \qquad 0 \leq \varphi \leq 2\pi, \qquad (49)$$

and the present epoch has been taken as $t = 0$, $\tau = -H^{-1}$. Once again masses increase with time and lead to exactly the same red-shift effect as in the form (44).

In their flat space forms all cosmological models are geometrically the same. The differences now arise from the physical dependence of mass on epoch. From many points of view it is easier to work with the flat space form since our intuitive geometrical perceptions then become applicable.

The flat space forms of the Einstein de Sitter and of the steady state models are strikingly similar. A comparison of the coordinate ranges (43) and (49) shows that if we were to write $-\tau$ for τ in (49) the ranges would be identical. But we have arranged (49) so that τ increases with t, as it does in (43). This means that in (43) the universe evolves 'away' from $\tau = 0$, whereas in (49) it evolves 'toward' $\tau = 0$. Here we choose the sense of evolution to agree with the sense in which masses increase, this being susceptible to observation from the red-shift effect.

The difference we have just noted is important in the absorber theory of radiation (Hogarth 1962, Hoyle and Narlikar 1964b). The consistency of retarded electromagnetic solutions in the steady state model arises because matter acts as a perfect absorber as $\tau \rightarrow 0$, i.e., in the future. Matter also acts as a perfect absorber as $\tau \rightarrow 0$ in the Einstein de Sitter, but this now represents the beginning of the universe and leads to advanced electromagnetic solutions being self-consistent.

IV. The Steady State Model

In this section we verify that the steady state model with creation of matter satisfies the field equation (24). Returning to the action (15) we first note that a world line in the steady state model has a beginning. Thus when we vary S with respect to the world line a the following relation must be satisfied at the beginning:

$$m_a \frac{da^i}{da} = 0. \qquad (50)$$

This leads to $m_a = 0$. To understand the implication of this result we have to assume that the elementary interaction $P(A, B)$ does not act at endpoints of world lines. The line integrals (17) are therefore over open rather than closed intervals.

Since the ends of world lines do not contribute to the mass interaction we can no longer write

$$\sum_a m^{(a)}_{;i} = [\sum_a m^{(a)}]_{;i} \qquad (51)$$

as in (30). Thus there are additional terms besides φ_{ik}. We again have

$$m = \sum_a m^{(a)} = \text{constant}, \qquad (52)$$

but now new contributions appear continually in $\sum_a$. In a steady state situation

$$\frac{dm}{dt} - \sum_a \dot{m}^{(a)} \qquad (53)$$

must be independent of time. Writing the difference (53) as λm, where λ is a constant, we get

$$0 = \frac{dm}{dt} = \sum_a \dot{m}^{(a)} + \lambda m. \qquad (54)$$

Thus we have

$$\varphi^1_1 = \varphi^2_2 = \varphi^3_3 = \tfrac{1}{2}\lambda^2 m^2 = \lambda^2 F, \qquad \varphi^4_4 = -\lambda^2 F. \qquad (55)$$

We also have non-zero contributions from the tensor

$$\psi_{ik} = \tfrac{1}{2} \sum_{a \neq b}\sum \{g_{ik}\Box m^{(a)} m^{(b)} - (m^{(a)} m^{(b)})_{;ik}\}. \qquad (56)$$

A simple calculation using (54), and the line element (44) gives

$$\psi^1_1 = \psi^2_2 = \psi^3_3 = \tfrac{8}{3}(\lambda^2 - H^2)F, \qquad \psi^4_4 = -2(\lambda^2 + 2H^2)F. \qquad (57)$$

The field equations at a point other than on a particle ($T_{ik} = 0$) are

$$F(R^i_k - \tfrac{1}{2}g^i_k R) = \psi^i_k - 3\varphi^i_k. \tag{58}$$

It is easily verified that these are satisfied if we set

$$\lambda^2 = H^2, \qquad \text{i.e., } \lambda = H. \tag{59}$$

The equation (54) now becomes

$$\sum_a \dot{m}^{(a)} = -\lambda m = -H \sum_a m^{(a)}. \tag{60}$$

This is consistent with our interpretation of mass as the reciprocal of length. A quantity which behaves as a reciprocal of length changes logarithmically with time according to (60) in the steady state universe.

The above solution does not tell us the density of the matter. To obtain this we have to consider a region which includes particles. We can then define average density ρ from the equation

$$\Box m + \tfrac{1}{6}Rm = \sum_a N^{(a)} \tag{61}$$

For m = constant, we get from above

$$\rho = Nm = \frac{3H^2}{2\pi G}. \tag{62}$$

Notice that $\Box m$ behaves peculiarly; it has singularity on a particle but is non-zero elsewhere. Its average over a region including particles is zero.

This completes the verification.

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3. Cosmology and Microwave Astronomy

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The Discovery of the Microwave Background

The technique of measuring the effective noise temperature of the sky by comparing the noise obtained at the terminals of an antenna with that of a resistor is as old as radio astronomy itself. Carl Jansky used this technique in his original measurements at 14.6 m. The great result of his work was the determination of the angular variation of the noise he had discovered rather than its absolute intensity. He noted its sidereal variation and placed the maximum at "a right ascension of 18 hours and a declination of −10 degrees," and correctly attributed its origin to the galactic center (Jansky 1933).

Because of our location in the galaxy we encounter galactic radiation from all directions although in varying amounts, owing to the geometry of the disc. The size of the effect falls off strongly with decreasing wavelength, so that galactic measurements at wavelengths shorter than one meter are quite difficult over much of the sky. (The shortest wavelength at which an absolute temperature map of the sky has been made is 74 cm, with the lowest equivalent