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Evolution of Multipolar Magnetic Field in Isolated Neutron Stars

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ABSTRACT
The evolution of the multipolar structure of the magnetic field of isolated neutron stars is studied assuming the currents to be confined to the crust. We find that except for multipoles of very high order ($l \gtrsim 25$) the evolution is similar to that of a dipole. Therefore no significant evolution is expected in pulse shape of isolated radio pulsars due to the evolution of the multipole structure of the magnetic field.

Key words: magnetic fields: multipole–stars: neutron–pulsars: general

1 INTRODUCTION

Strong multipole components of the magnetic field have long been thought to play an important role in the radio emission from pulsars. Multipole fields have been invoked for the generation of electron positron pairs in the pulsar magnetosphere. For example, Ruderman & Sutherland (1995) model requires that the radius of curvature of the field lines near the stellar surface should be of the order of stellar radius to sustain pair production in long period pulsars. This is much smaller than the expected radius of curvature of the dipole field. Barnard & Arons (1982) showed that such small radius of curvature is only possible if the field structure has at least one dominant higher multipole. e.g. a quadrupole.

Magnetic multipole structure at and near the polar cap is also thought to be responsible for the unique pulse profile of a pulsar (Vivekanand & Radhakrishnan 1980, Krolik 1991, Rankin & Rathnasree 1995). The recent estimates that there should be several tens of sparks populating the polar cap is also best explainable if multipole fields dictate the spark geometry near the surface (Deshpande 1998, Seiradakis 1998). Significant evolution in the structure of the magnetic field during the lifetime of a pulsar may therefore leave observable signatures. If the multipoles grow progressively weaker in comparison to the dipole then one can expect pulse profiles to simplify with age and vice versa.

The evolution of the magnetic fields in neutron stars in general is still a relatively open question. During the last decade, two major alternative scenarios for the field evolution has emerged. One of these assumes that the field of the neutron star permeates the whole star at birth, and its evolution is dictated by the interaction between superfluid vortices (carrying angular momentum) and superconducting fluxoids (carrying magnetic flux) in the stellar interior. As the star spins down, the outgoing vortices may drag and expel the field from the interior leaving it to decay in the crust (Srinivasan 1990). In a related model, plate tectonic motions driven by pulsar spindown drags the magnetic poles together, reducing the magnetic moment (Ruderman 1991a,b,c).

The other scenario assumes that most of the field is generated in the outer crust after the birth of the neutron star (Blandford, Applegate & Hernquist 1983). The later evolution of this field is governed entirely by the ohmic decay of currents in the crustal layers. The evolution of the dipole field carried by such currents has been investigated in some detail in the recent literature (Geppert & Urpin 1994, Urpin & Geppert 1995, 1996, Konar & Bhattacharya 1997, 1998). These studies include field evolution in isolated neutron stars as well as those accreting from their binary companions. The results show interesting agreements with observations lending some credence to the crustal picture.

In this paper, we explore the ohmic evolution of higher order multipoles in isolated neutron stars assuming the currents to be originally confined in the crustal region. Our goal is to find whether there would be any observable effect on the pulse shape of radio emission from isolated pulsars as a result of this evolution. In section 2 we discuss the details of the computation and in section 3 we present our results and discuss the implications.

2 COMPUTATIONS

The evolution of the magnetic field, due to ohmic diffusion, is governed by the equation (Jackson 1975) :

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c^2}{4\pi}\nabla \times (\frac{1}{\sigma} \times \nabla \times \mathbf{B}), \qquad (1)$$

where $\sigma(r,t)$ is the electrical conductivity of the medium. Following Wendell, Van Horn & Sargent (1987) we introduce a vector potential $\mathbf{A} = (0, 0, A_\phi)$ assuming the field to be purely poloidal, such that:

$$S(r,\theta,t) = -r\, sin\theta\, A_\phi(r,\theta,t),$$

where $S(r,\theta,t)$ is the Stokes' stream function. S can be separated in r and θ in the form :

$$S(r,\theta,t) = \sum_{l \geq 1} R_l(r,t)\, sin\theta\, P_l^1(cos\theta),$$

where $P_l^1(cos(\theta))$ is the associated Legendre polynomial of degree one and R_l is the multipole radial function. From equation (1) we obtain :

$$\frac{\partial^2 R_l}{\partial x^2} - \frac{l(l+1)}{x^2} R_l = \frac{4\pi R_*^2 \sigma}{c^2} \frac{\partial R_l}{\partial t} \qquad (2)$$

where $x \equiv r/R_*$ is the fractional radius in terms of the stellar radius R_*. The solution of this equation with the boundary conditions :

$$\begin{aligned} \frac{\partial R_l}{\partial x} + \frac{l}{x} R_l &= 0, \ \text{as } x \to 1 \\ R_l &= 0, \ \text{at } x = x_c \end{aligned} \qquad (3)$$

for a particular value of l gives the time-evolution of the multipole of order l. Here, the first condition matches the correct multipole field in vacuum at the stellar surface and the second condition makes the field vanish at the core-crust boundary (where $r = r_c$, the radius of the core) to keep the field confined to the crust. We assume that the field does not penetrate the core in the course of evolution, as the core is likely to be superconducting.

2.1 Crustal Physics

The rate of ohmic diffusion is determined mainly by the electrical conductivity of the crust. The conductivity of the solid crust is given by

$$\frac{1}{\sigma} = \frac{1}{\sigma_{\rm ph}} + \frac{1}{\sigma_{\rm imp}}$$

where $\sigma_{\rm ph}$ is the phonon scattering conductivity, which we obtain from Itoh *et al.* (1984) as a function of density and temperature, and the impurity scattering conductivity $\sigma_{\rm imp}$ is obtained from the expressions given by Yakovlev & Urpin (1980).

We construct the density profile of the neutron star in question using the equation of state of Wiringa, Fiks & Fabrocini (1988) matched to Negele & Vautherin (1973) and Baym, Pethick & Sutherland (1971) for an assumed mass of 1.4 $M_\odot$. As conductivity is a steeply increasing function of density and since the density in the crust spans eight orders of magnitude the conductivity changes sharply as a function of depth from the neutron star surface. Thus the deeper the location of the current distribution, the slower is the decay.

Another important factor in determining the conductivity is the temperature of the crust. In absence of impurities the scattering of crustal electrons come entirely from the phonons in the lattice (Yakovlev & Urpin 1980) and the number density of phonons increases steeply with temperature. The cooling of an isolated neutron star brings the surface temperature down to $\sim 10^{4.5}$ K in about 10^7 yrs (van Riper 1991) with an attendant interior temperature of the nearly isothermal core of the order of 10^7 K. We assume an isothermal neutron star crust following Gudmundsson, Pethick & Epstein (1983). This is a fair assumption as excepting the initial phase ($\lesssim 10^3$ year) the non-uniformity in temperature is small over the main fraction of the crust.

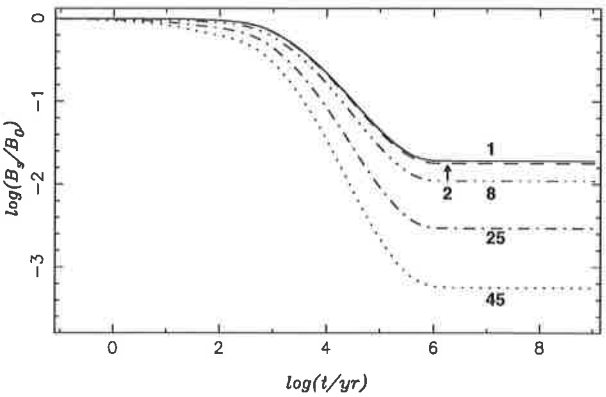

Figure 1. The evolution of the surface magnetic field for various multipoles due to pure diffusion. The numbers next to the curves correspond to respective orders of multipole. All the curves correspond to $Q = 0.0$ and a depth of current concentration at $x_o = 0.98$ i.e., a density of $\rho = 10^{11}$ g cm^{-3}.

A third parameter that should be considered in determining conductivity is the impurity concentration. The effect of impurities on the conductivity is usually parametrised by a quantity Q, defined as $Q = \frac{1}{n}\sum_i n_i(Z - Z_i)^2$, where n is the total ion density, n_i is the density of impurity species i with charge Z_i, and Z is the ionic charge in the pure lattice (Yakovlev & Urpin 1980). In the literature Q is assumed to lie in the range 0.0 - 0.1. But statistical analyses indicate that the magnetic field of isolated pulsars do not undergo significant decay during the radio pulsar life time (Bhattacharya *et al.* 1992, Hartman *et al.* 1997, Mukherjee & Kembhavi 1997). It has been shown (Konar 1997) that to be consistent with this impurity values in excess of 0.01 are not allowed in the crustal model.

2.2 Numerical Scheme

To solve equation (2) we assume the multipole radial profile used by Bhattacharya & Datta (1996, see also Konar & Bhattacharya 1997). This profile contains the depth and the width of the current configuration as input parameters and we vary them to check the sensitivity of the result to these. We solve equation (2) numerically using the Crank-Nicholson method of differencing. We have modified the numerical code developed by Konar (1997) and used by Konar & Bhattacharya (1997) to compute the evolution of multipolar magnetic fields satisfying the appropriate boundary conditions given by equation (3).

3 RESULTS AND DISCUSSION

In figures [1] and [2] we plot the evolution of the various multipole components of the magnetic field, assuming the same initial strength for all, with time due to pure diffusion in an isolated neutron star. It is evident from the figures that except for very high multipole orders ($l \gtrsim 25$) the reduction in the field strength is very similar to that of the dipole component. For a multipole of order l there would be 2^l reversals across the stellar surface. For typical spin-periods the size of the polar cap bounded by the base of the open field lines is $\sim 0.01\%$ of the total surface area. To contribute to the

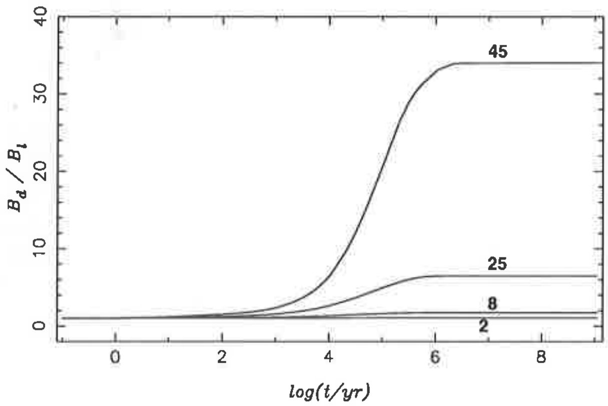

Figure 2. The ratio of the dipole surface field to the multipole field is plotted as a function of age. The numbers next to the curves correspond to respective orders of multipole. All the curves correspond to $Q = 0.0$ and a depth of current concentration at $x = 0.98$ i.e., a density of $\rho = 10^{11}\ \mathrm{g\,cm^{-3}}$.

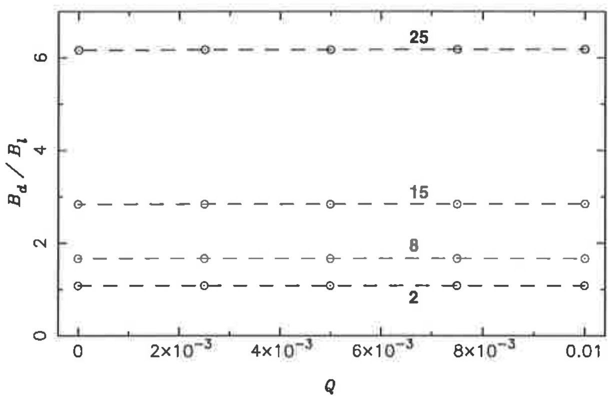

Figure 3. The ratio of the dipole surface field to that of the multipoles at 10^7 years as a function of Q. The numbers next to the curves correspond to respective orders of multipole. All curves correspond to a depth of $x = 0.98$ corresponding to a density of $\rho = 10^{11}\ \mathrm{g\,cm^{-3}}$, at which the initial current is concentrated.

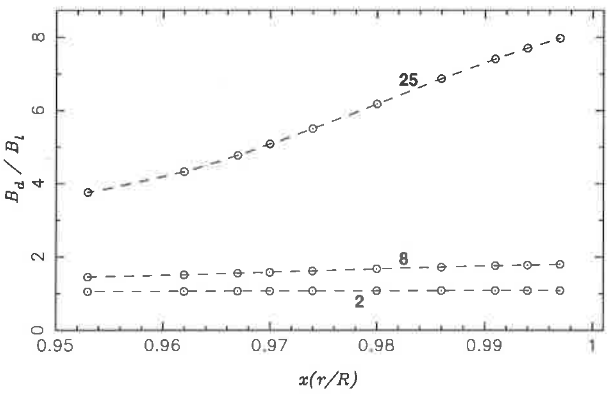

Figure 4. The ratio of the dipole surface field to that of the multipoles at 10^7 years as a function of depth. The points marked in the plots here correspond to densities $\rho = 10^{13.5}, 10^{13}, 10^{12.5}, 10^{12}, 10^{11.5}, 10^{11}, 10^{10.5}, 10^{10}, 10^{9.5}, 10^{9}\ \mathrm{g\,cm^{-3}}$. The numbers next to the curves correspond to respective orders of multipole. All curves correspond to $Q = 0$.

substructure of the pulse therefore the required multipoles must have a few reversals in the polar cap which demands that the multipole order must be five or more. On the other hand if the multipole order is very large ($l > l_{\max} \sim 20$) the fine structure would be so small that it would be lost in the finite time resolution of observations. Therefore, l values in the range 5 to $l_{\max}$ would be the major contributors to the observed structure of the pulse profile. However, as seen from figures [1] and [2] multipoles of such orders evolve similarly to the dipole. Therefore no significant evolution is expected in the pulse shape due to the evolution of the multipole structure of the magnetic field. As discussed before multipole orders contributing to the required field line curvature for pair-production are low, most prominently a quadrupole. As the evolution of these low orders are also very close to the dipole the radii of curvature of the field lines on the polar cap are not expected to change significantly in the lifetime of a radio pulsar.

To test the sensitivity of these results on the impurity concentration of the crust and the density at which the initial current is concentrated we have evolved models with various values of these parameters. The results are displayed in figures [3] and [4] where we plot the ratio of the dipole to higher multipoles at an age of 10^7 years. It is seen that the results are insensitive to these parameters, particularly for low orders of multipoles of interest.

Krolik (1991) and Arons (1993) conjectured that except for multipoles of order $l \gtrsim R_*/\Delta r$ the decay rates would be similar due to the finite thickness Δr of the crust over which the current is confined. The evolution plotted in figure [1] assumes that $\Delta r = 1.2$ km for which $R_*/\Delta r \sim 8$. However it is seen from figures [1] and [2] that significant decay occurs only for $l \gtrsim 25$, much greater than $R_*/\Delta r$. This is most likely caused by steep increase in conductivity towards the interior.

In conclusion, our results indicate that for a crustal model of the neutron star magnetic field there would be no significant change in the multipolar structure with age. This fact seems to be corroborated by observations: studies identifying multiple components in pulse profiles (Kramer *et al.*, 1994) show that the number of components does not vary with the age of the pulsar. Thus the evolution of the multipolar structure of the magnetic field is unlikely to leave any observable signature on pulsar emission. This is in contrast with the predictions from the plate-tectonics model of Ruderman (1991a,b,c) which suggests a major change in the field structure with pulsar spin evolution.

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