X

There are only two charges. We can hit one of them with a non-charged object. We can imagine that there is a guy standing there hitting one of the charges. Of course, if there is no external force, this solution just represents some motion of the charges around each other.

BONDI

The trouble is that some formulations do not allow the "guy"; I do not like such formulations.

X

But if there is no guy, we can set up a problem in which the two charges pass each other close by. At the moment they go past each other and accelerate, they must react back on each other. I have not worked out any "unguyed" theories.

HOGARTH

I do not disagree with anything X has said. In our scheme TH↔RAD↔COS, I was working with the connection between RAD and COS. The thermodynamics, in principle, was not supposed to enter into my discussion. I think that X's criticism is that I need to make certain integrals convergent and that I picked up a little piece of thermodynamics and included it for convenience. My next paper will discuss this in more detail. If we make the assumption that the arrow of time in thermodynamics is determined by the arrow of time in radiation, then we can try consistency tests. We can see that this assumption works for one direction of time but not for the other.

The type of attenuation that was necessary to make these integrals converge was not critical, and there are possibly other ways to make these integrals converge rather than using thermodynamical arguments. If the integrals converge, then the arrow from COS to RAD is completely self-consistent without introducing thermodynamics.

II. Time-Symmetrical Electrodynamics and Cosmology[1]

F. HOYLE AND J. V. NARLIKAR

There are two problems in connection with the scheme TH↔RAD↔COS. First, why is there any arrow at all, and second, why do these three arrows seem to be consistent with each other? We shall consider the second problem to begin with. One point of view that we put forward is that the thermodynamic arrow follows from the first two arrows, the cosmological arrow and the electromagnetic arrow.

If we observe a thermodynamic system over a finite length of time, we can decide which is the initial condition and which is the final condition of the system. This allows us to find a time arrow according to thermodynamics. What is it that makes the system go only one way from the initial to the final state although the laws of physics themselves are invariant under time reversal? The point of view which we have is that the universe causes this anisotropy and acts as a sort of sink with which the system is constantly in interaction. This property is due mainly to the fact that the universe is expanding. Because the universe is expanding, as we know from Olbers' paradox, there is a predominance of matter over radiation; the density of radiation is very small. This condition is maintained by the steady-state cosmology, since as the universe expands, the high entropic energy in the form of radiation is continually lost because of the red shift, and new entropic energy in the form of matter is continually being created. This maintains the universe in a constant state of a sink. The tendency of the system to fill the sink is guaranteed by the presence of purely retarded interactions.

In the "big-bang" cosmology, the universe must start with a marked degree of thermodynamic disequilibrium and must eventually run down, so it just happens that at present radiation is less important.

We wish to connect the electromagnetic arrow of time with the arrow of time given by the expansion of the universe, and in the rest of this paper we should not bring in any thermodynamic consideration. We consider all the continuously expanding cosmological models. In order to connect the electromagnetic arrow of time with the cosmological arrow of time, we follow considerations similar to those given earlier by Hogarth. In this we have a time-symmetrical electrodynamics given by the Wheeler-Feynman theory of direct interparticle action. Two world lines, 1 and 2, of matter interact along each other's null cones, as in Figure II-1. A point P on the world line of particle 1 interacts with particle 2 through an advanced part above and retarded part below. So this problem is entirely time-symmetrical. We can write down the force on the particle P. We want to see how an arrow of time can emerge from this apparently time-symmetrical description. If we consider each separate particle, we give equal importance to advanced and retarded solutions. Where can the asymmetry come from? The asymmetry would normally come in this case from the asymmetry in the universe. The null cone in the future is different from that in the past. The distribution of the particles, the density, and so forth, in the future is in general different from the distribution in the past, and therefore it is not unlikely that when we perform the summation we may find an asymmetry.

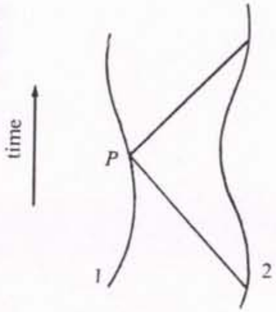

FIGURE II-1

We decided to look for a self-consistent solution. It is important to avoid misunderstanding about the term "self-consistent." When we look for a self-consistent solution with retarded potentials, we are also assuming that F_H, the homogeneous solution that must be added, is zero. We try to find one particular cosmological model in which the F_H is zero. In the Wheeler-Feynman approach, F_H must be zero, since there are no fields without sources.

This restriction makes the problem definite, as can be seen from the following treatment. Suppose we assume that the retarded fields are consistent, and consider a charge "a" with known acceleration $\boldsymbol{u}(t)$. Under the assumption of retarded fields, we know how the field is going to propagate into the future light cone, and we know that the force acting on another particle "b" is given by this retarded field. This force sets the particle in motion, and we consider only that part of the motion of "b" which has been caused by the retarded field of the accelerated charge "a." The principle of direct interparticle action says that, because of the motion of "b," there will be an action back on particle "a." This reaction can be computed, and the summation can be made for all the particles in the universe. For the steady-state model, the assumption of consistent retarded solutions with $F_H = 0$ gives a self-consistent argument. If we consider the corresponding advanced solution, with $F_H = 0$, we do not get a consistent solution for this model. In the case of the Einstein–de Sitter cosmology, the situation is exactly reversed. There is a self-consistent situation for the advanced field and not for the retarded field.

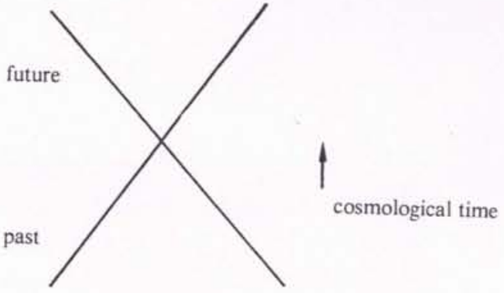

FIGURE II-2

GOLD

Is not "advanced" defined with respect to the motion of the nebulae, so that presumably we would see a contracting universe if we used this solution?

NARLIKAR

Yes. The arrow of time has been fixed beforehand as the one in which the universe expands. Because of the expansion of the universe, there is a difference between the past and the future. An electromagnetic wave which travels in the future is red-shifted and an electromagnetic wave which travels in the past is blue-shifted. This difference may lead to an asymmetry.

To discuss the details of the process, then, we make the assumption of a self-consistent retarded field. As the field propagates into the future, it interacts with other particles, and their motion gives rise to other fields. These new fields modify the original field, with the effect that the medium has a dielectric constant or a refractive index. Some specific assumption must be made about the nature of this refractive index. In order to avoid going into thermodynamics, we decided to stick to pure electrodynamics by assuming that the refractive index arises from the radiative reaction itself. This makes the problem unambiguous. Suppose we must test whether or not F_{ret} is self-consistent. Consider a typical particle and the field nearby it, which is F_{ret}. From F_{ret} we have to subtract the field felt by the test particle, which will be $\frac{1}{2}(F_{\text{ret}}+F_{\text{adv}})$ according to the action principle. So

$$F_{\text{ret}} - \tfrac{1}{2}(F_{\text{ret}} + F_{\text{adv}}) = \tfrac{1}{2}(F_{\text{ret}} - F_{\text{adv}})$$

is the field which acts on the particle itself. The F_{ret} is the field acting on the particle, and $\frac{1}{2}(F_{\text{ret}}-F_{\text{adv}})$ gives the familiar Dirac force of radiative reaction. If, on the other hand, we assume that the fields are everywhere only advanced, then the sign is changed. Therefore we can can give a definite sign for the force of radiative reaction. This follows only from electrodynamic considerations. We can write down the dielectric constant in the form

$$\varepsilon = 1 - \frac{4\pi N e^2}{m\omega^2}\begin{pmatrix} 1 + O(\omega) \\ 1 + O\left(\dfrac{1}{\omega}\right) \end{pmatrix}.$$

These are limiting values for high and low wavelengths. When we take the square root of this quantity, we have to consider the sign of the terms, and this requires that we know the sign of the radiative reaction. A difference between the past and the future appears because ω is different in past and future. As the electromagnetic wave goes into the future, ω becomes small; as the wave goes into the past, ω becomes large. The variety provided by the different cosmologies is due to the fact that N, the number density of charged particles per unit proper volume, can be different for different cosmologies. For instance, in the steady-state cosmology, N is constant in past and future, and ω of course decreases due to the red shift in the future. Therefore as ω tends to zero, the dielectric constant, ε, is less than zero. If ω tends to infinity, the limit of ε is one. This means that in the case of the steady-state universe, we may specify the cosmological arrow by considering the future light cone of a particular particle. The arrow depends on the "ionosphere" in the future; the past is essentially transparent.

On the other hand, consider the Einstein–de Sitter cosmology, using the same formula. In this case, N is proportional to ω^3. This follows from the fact that matter is conserved, and as the universe expands, the proper density decreases. In the past we get involved with the ionosphere, and the future is transparent. In this result we immediately see the difference between the predictions of the steady-state and Einstein-de Sitter cosmologies. We assume a self-consistent solution with $F_{II}=0$ throughout.

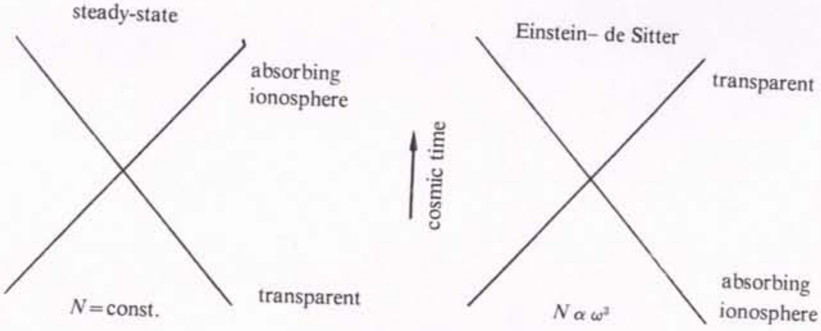

FIGURE II-3

WHEELER

How does the expansion rate come into this treatment? Suppose that the expansion rate eventually diminishes to zero. Then surely some dimensionless factor must enter in order to give a transformation over to the usual situation.

BONDI

I think that this is perhaps the most valuable part of Hogarth's analysis. Solving the equations shows how exceedingly critical these parameters are. As long as one side still acts as an absorber, it does not matter what the other side does, unless it is a perfect absorber. The situation becomes exceedingly critical if the past and future absorbers begin to absorb nearly equally. Here the analysis breaks down.

I have wondered what life would be like if the universe were a lot more opaque than it is. We would not be able to see things, and of course there is a case in which all observers would be within adiabatic enclosures, and nothing could happen at all.

MORRISON

I am worried about that. For the refractive index formula, it is assumed in this treatment that there are no inelastic processes. N refers to a number of charged particles, and the system remains neutral overall. But treating the charged particles separately affects the dispersion relations. Do you consider only the electrons or do you actually consider both electrons and protons?

NARLIKAR

Our case is that of one particle only.

MORRISON

There surely is some difficulty in that, because the other particles are there for scattering.

X

Does matter really become transparent at high frequencies? The photons may make pairs, so the medium may get dirtied. Although the atmosphere is transparent for visible light, gamma rays get absorbed.

GOLD

It makes a difference what frequency range we start with. From the radiation-damping idea alone, it is all very well to say that the blue shift is on one side of the light cone vertex and the red shift on the other for electromagnetic radiation in general. If we start with very hard gamma rays, however, we cannot allow any blue shift, because then the gamma rays are easily absorbed, but we can allow a lot of red shift.

It seems that we could even construct a theory in which the interaction of gamma rays is considered, but where ordinary light is forgotten. The direction of the future might even depend on the choice of frequencies, relative to the transparency of the ionosphere. Why do we fix attention on one part of the spectrum?

HOYLE

This choice would not affect the search for the consistent retarded solution, because here we are concerned only with the direction of the future in the case where the frequencies tend to zero. But this does of course raise the question as to what happens on the past half of the light cone.

BONDI

Of course this analysis does not work if the frequency is greater than a certain value. But in my view, it is quite all right to consider the spectrum as a whole, although we must admit that at most frequencies there is absorption before the light plays any part in the analysis. The rest of the frequencies then encounter absorbers which are transparent on one side of the light cone and "ionospheric" on the other.

X

The cross section for absorption of high-frequency gamma rays by atoms approaches a constant. If we have a constant cross section for absorption, and if all the matter of the contracting universe is coming in toward us, as if the thermodynamics goes in the usual way, but the solution of the universe is changed, then I do not understand how the matter can look transparent. By the time a photon reaches a distant nebula, it has become a gamma ray. Does not this imply infinite absorption of such high-frequency gamma rays as they go through the nebulae even if the cross section for the interaction is finite? I am trying to show that everything is still consistent in the case of a contracting universe. The future absorber is also opaque. I do not like making the cross section go down as the frequency goes up.

When there is a binding, then there is an absorption line at that binding energy. The special case of zero-frequency radiation could therefore give trouble. Suppose then that electrons were equipped with a small finite binding, instead of being exactly free. A small binding would enable us to avoid the trouble of waiting for the ultimate absorption at low frequency. Suppose we have an ordinary atom that has some resonance at some finite low frequency. It would then be interesting to integrate the curve of the resonance absorption law from finite frequency, past the bump, down to zero. We can work out the consistent theory of the absorption with that radiation resistance. The only thing that worries me is that by luck all the absorption connected with the index might be at zero frequency. In that case, the frequency could be moved up to a finite value, and then the integral could be calculated to see whether a small number results.

I would not change the assumptions which have been made for this treatment, except to include a binding for charges. I am not arguing about the signs or about thermodynamics. I only suggest changing one little feature before doing the integral. This change is just for technical reasons.

Bergmann

There is also a physical reason. If the charges have no binding, then there is danger of permanent polarization.

Salpeter

It is said that the contracting case is self-consistent if matter is uniform. Has anyone attempted to see how stellar evolution would go? Has anyone looked in detail at how such a cosmology would work with the condensations we do have?

Hoyle

I think the thermodynamics would also go the other way around in this event. We have to think entirely the other way around. Omit for the moment the possibility that we get into Olbers' paradox, as we probably would. I think that we would actually see the universe contracting.

Salpeter

What worries me is that we would still have a heat engine working as usual, and light would be coming in the usual way. But we would see the distant matter coming at us blue-shifted. There would be an enormous Olbers' paradox.

Bondi

Surely the point is that this problem is difficult enough in the world in which we live! If we try to work through any further consequences in a contracting universe, the situation surpasses the imagination.

Salpeter

I do not quite agree. I think that many of our arguments would become clearer if we considered a cosmological model in which there are different arrows going in different directions. We could really follow out some process and see whether in that case the thermodynamic arrow goes with the cosmology or with the radiation.

Narlikar

We now consider the reason for the existence of the time arrow. Here we consider only the case of the steady-state cosmology which has the line element

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

$$R(t) = e^{Ht}$$

$$H = \text{Hubble's constant.}$$

In the steady-state theory, of course, the density is constant. This result follows from the perfect cosmological principle of Bondi and Gold. The same result can be obtained through Einstein's field equations with an extra term on the right-hand side. More generally, we take a solution of the form

$$R^3 = A(1 + \cosh t);$$

$$\rho = \frac{3H^2}{4\pi G}\,\frac{\cosh t}{1 + \cosh t},$$

where t is measured in units of $(3H)^{-1}$.

This tends to the steady-state solution as t tends to $\pm\infty$, and, as you see, is time-symmetrical. We have chosen the constant of integration so that the origin is at $t=0$. For $t<0$ there is contraction and continuous annihilation of matter, and for $t>0$ there is expansion and continuous creation. As $t\to-\infty$, the solution tends to the steady state with continuous annihilation of matter; at $t\to+\infty$, it tends to steady state with continuous creation. Since Einstein's field equation is time-symmetrical, any terms that can be introduced are also invariant under time reversal because in any case we find a time-symmetrical solution. If t is changed to $-t$, everything is still seen the same way. Now an interesting question arises. Consider what is seen by an observer picked at random in space-time. If he is at $t=+\infty$, he observes expansion, continuous creation, and retarded potentials. If we take the view that the thermodynamic arrow follows the electromagnetic one, then the observer's own thermodynamic development is compatible with this view. If the observer is at the other end of the time axis, toward $t=-\infty$, he sees a similar situation. The thermodynamic development of the universe in this view would again be in the same direction as that in which the universe is expanding. This direction is obtained by changing the direction of t. This observer also sees matter being created as if the direction of time were reversed.

There is no quantum field theory of the continuous matter creation in the steady-state universe, so we may say for heuristic reasons, but without particular justification, that "matter" is created in the part $t>0$, and that "antimatter" is created in the part $t<0$, so there is a matter-antimatter symmetry. The terms are only conventions, of course.

In the region about $t=0$, observers see a mixture of advanced and retarded fields.

Morrison

A photon that goes into one direction covers essentially all frequencies from the starting frequency. This seems to involve the sum-rule integral, provided that we include the resonance. The implication probably is that there is a range of frequencies for which it makes no difference whether the photon goes into past or future, as far as the cross section is concerned, and that there is a range of frequencies in which this direction does make a difference. On the other hand, I do not know whether the density alone will be enough to give results which are independent of the dispersion. I suspect that the dispersion results are really not important except in determining whether some frequency is above or below the resonance. We cannot include charges initially to produce scattering, and then avoid putting their interaction into the balance.

Hoyle

We of course have the conviction that most of the atoms are ionized, but I agree that in principle one must include this interaction.

Bondi

There are two points in this connection, surely. One is that in the case of something like the Einstein–de Sitter universe, we must allow for a time variation in the ionization, and the other is that in carrying out a calculation, we must make sure, as Gold said earlier, that no process is temporarily reversed.

Gold

Let us consider the "zig" and the "zag." The lines are the world lines of particles; the solid line stands for a particle in an excited state and the dotted line represents its de-excited state. The slanted line is a light signal. The "zig" is what we see between world lines of particles in the real world.

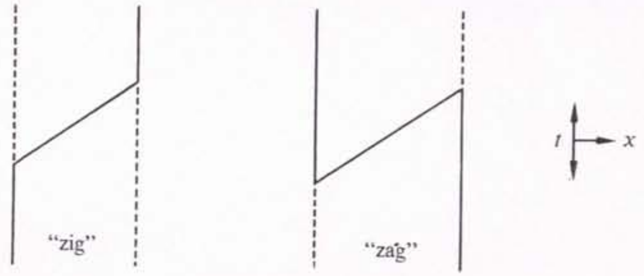

Figure II-4

Of course, if we sum the energy during the "zag," we find more than we should have. But the point is surely that *both* the "zig" and the "zag" are completely time-symmetrical.

We can ask why "zigs" occur and "zags" do not. That is one kind of a problem. But the answer to that does *not* tell us where the time asymmetry comes from.

Bondi

The asymmetry is quite clear. To use terms invented earlier, we distinguish between "guyed" charges and "guyless" charges. "Guyed" charges involve information outside the electromagnetic field, such as X's chap who hits the charges; "guyless" charges are the ones that simply follow the field. Then it becomes crystal clear that one of these is hit and the other follows.

Gold

That is absolute nonsense. We are considering a fundamental process in which an atom spontaneously de-excites itself by emitting a photon which is spontaneously absorbed by another atom. This process is absolutely symmetrical.

X

Absolutely, except for one little thing—the crowd of other atoms!

Gold

And the absorption of the photon in the crowd of the other atoms is described only by statistics! The light cannot be thought to go in predetermined fashion from the emitting atom to a select absorbing atom in the crowd. We think rather of the emitter making a puff of light that might involve any atom in the crowd. So there we must worry about statistics.

X

Just put in the other world lines, please. Now, indicate that the other lines sometimes also get excited. Now let's see this reverse!

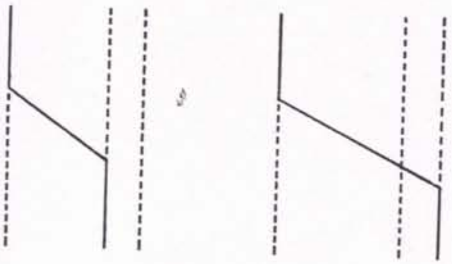

Figure II-5

Gold

That is exactly the point. The asymmetry is only in the statistics.

X

There is another point. Under ordinary circumstances in electrodynamics we have to draw a diagram for each case, starting with the case in which the light is not absorbed anywhere, apart from those for populations of one atom, two atoms, and so on.

Gold

If the light is not absorbed anywhere, then we have a transparent universe; this is where the cosmology enters the analysis.

Sciama

In the case of a static world model the situation is symmetrical, as in Einstein's model of 1917. In an expanding universe, however, the particle world lines which we draw would not be parallel. They diverge because of the expansion. Therefore the time asymmetry arises since there is more energy at emission, say, than at reception.

Gold

The time asymmetry is in the cosmology and it is in the statistics, but it is not in the electrodynamics. All we need as regards electrodynamics is to know the rule that disallows the "zags." We all agree that this can be done locally anyhow. The mixed "zigs" and "zags" then do not give trouble. The statistics ought to be correlated to the cosmology.

Hoyle

Does not this follow from the assumption of time-symmetrical electrodynamics? The drawing of the "zig" and "zag" implies that there is a cancellation due to emission by the particle. This cancellation is the radiation damping, and this automatically forbids the "zags." This cancellation gives the asymmetry of the damping. It is not asymmetry which tells what parts of the lines are solid; it is the sign of the damping terms. In the time-symmetrical electrodynamics, there is just one allowable one-to-one connection between the sign of the self-consistent solution and the sign of the damping term. This particular connection permits only the "zig."

Gold

You are saying essentially that I should draw little arrows on the photon lines, so that the incoming ones can be separated from the outgoing ones. Your discussion is not independent of the Wheeler-

Feynman treatment. Any arrow which can be drawn in a way such that the bookkeeping is consistent in the whole diagram does not affect the reversibility of the diagram.

ROSENFELD

The selection of retarded potentials and the use of such figures as these are justified only if the system which we are describing is within a larger absorbing box. Otherwise this argument is not justifiable.

HOYLE

How do we arrive at the "zig" and "zag" diagrams in the first place? Do we not just make the assumption of time-symmetrical electrodynamics from which comes the sign of the radiation damping as a simple matter of consistency? Do we not define the time arrow, the mathematical time, in such a way that a particular field is the retarded one?

X

The sign of damping is determined by the thermodynamic character of the matter in the rest of the universe.

BERGMANN

I believe that we can derive the sign of the radiation-damping term uniquely if we start out with a system which has only the degrees of freedom of a finite number of point charges and of the free Maxwell field. There is no absorber. We write down first the variational principle, which is manifestly Lorentz-covariant. Now we take the total field and dissect it into two parts, one being some sort of Green's functions associated with the particles, the other the remainder, which we may call the "free field," "external field," or whatever we called the F_H. Since we are performing only a mathematical decomposition, we have a free choice, and it does not spoil the physics. It does not matter that the Green's functions are nonsymmetric with respect to the time. We can choose either an asymmetric or a symmetric form. The character of F_H depends, of course, on how we do the splitting; for different choices of Green's functions we will not get the same F_H, no matter what mathematical symmetry is implied. The total set of Lagrange equations for which we want to solve must be equivalent.

Obviously, at one stage of the game we have to do a classical renormalization because the total field at the location of each particle is infinite. This renormalization is straightforward. If we perform it in any manifestly Lorentz-covariant fashion, the sign of the radiation damping comes out uniquely and automatically, depending of course on whether we started by specifying advanced or retarded potentials. If we use the half-advanced and half-retarded potentials, we get zero radiation damping. Since we started with the manifestly Lorentz-covariant variational principle, the Noether theorem automatically gives conservation laws of linear momentum, energy, and angular momentum—all the basic conservation laws. Every type of formalism that we derive will automatically lead to the same conservation laws.

There is one more remark to be made. Since the free field, the F_H, is itself a vacuum solution of the Maxwell equations, no matter how the original formal split was made, it already satisfies the conservation laws. Suppose we had started with the field only, without particles. The field is also Lorentz-covariant and, as we know, also satisfies the conservation laws. If we consider as meaningful only such solutions in which the external field is not excited, then the remainder still has to satisfy conservation laws. This excludes the "zags," provided we have made no mistakes.

X

You have made the mistake of assuming the energy in the field is positive in the "zag" line. If the field energy is negative, the law of energy conservation certainly allows the "zag."

BERGMANN

But I assumed that in fact the field line made no contribution whatever to the energy. The expression for the energy which results from this treatment naturally involves the kinetic energy of the particles, together with an interaction term which is given by the incident field at the location of the particle. The field can be incident from the future or from the past, or from both. There are in any case no electromagnetic degrees of freedom left, since we specifically assumed $F_H = 0$. Since the photon lines are all internal, since they all originate and terminate inside the diagram, they do not contribute to the total energy. Consider this situation (Figure II-6). If we make a determination of the energy at times 1, 2, and 3, then the contributions are as follows: at

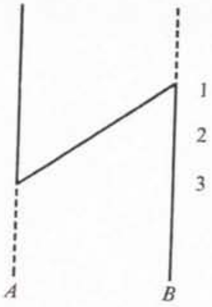

FIGURE II-6

times 1 and 3, the only contributions come from the excited particle; at time 2, the expression for the energy contains a product involving the behavior of the particle itself, its four-velocity, as well as the light cone on which the other particle is located.

X

But then how do you show that this "zag" is impossible?

Bergmann

By looking in detail at the expressions. We get contributions from particle A and from particle B. The way we calculate, there is no contribution from radiation.

Schiff

The argument of the particle energy changes from line A to line B!

Bergmann

Ah, yes!

X

I still think that the analysis which Bergmann made is very nice. This argument shows further how the thermodynamics is connected to the electrodynamics. If we have been calculating with a Hamiltonian and so forth, then all we have to do is add one more thing to the analysis. What we add is the sort of procedure which is done in statistical mechanics, that of assuming that there is some state which is somehow more ordered than another at a certain time. The resultant system behaves just like those treated by a Boltzmann description; it makes no difference what is assumed about the field. The light is irrelevant, so to speak, since it is "correlated out," and the statistical results do not depend on it. So thermodynamics and light really "lock together" in any situation where they are intimate enough. The only difficulty occurs when the light can be emitted to infinity, or when it escapes. The treatment will always work in the case of a closed box. The thermodynamics and retarded potentials will work together. If we work everything out with advanced potentials, we get the same motions, the same apparent phenomena. We do not necessarily have to assume that the walls of the box are absorbers. The treatment would still work if, say, some of the wall were not.

Bergmann has especially indicated that the field should not be given a status which is in principle different from that of the particles. If we want to understand thermodynamics, we cannot expect to construct a system in which both the light and the particles are to be causally organized. Any kind of organization in particles or fields must be treated *à la* Boltzmann, since it is a Liouville mechanical system. I think that Bergmann has made the connection between the radiation arrow and the thermodynamic arrow. The thermodynamic arrow is the real one, the other is apparent, under Bergmann's assumptions.