

On gravitational charge of stationary spacetimes

Naresh Dadhich*

Inter-University Centre for Astronomy and Astrophysics
Post Bag 4, Ganeshkhind, Pune - 411 007, INDIA

Abstract

We point out that ours is a definition of gravitational charge and not of quasilocal mass [1]. We have defined gravitational charge as the flux of effective gravitational acceleration across a closed 2 - surface. It turns out that tidal force can change its sense from convergence to divergence outside the horizon for suitable range of values of rotation and electric charge parameters of the black hole.

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*E-mail address : naresh@iucaa.ernet.in

In a recent paper, Bergqvist [1] has compared the various definitions of quasilocal mass for event horizons of the Reissner - Nordstrom and the Kerr blackholes and has concluded that no two definitions are equivalent. This is simply the reflection of inherent ambiguity in defining energy in general relativity (GR) arising out of non-localizability of gravitational field energy. One among the expressions compared is our definition of gravitational charge, which is a measure similar to quasilocal mass, but is not the same and is perhaps less ambiguous and manageable (we had initially used the term effective mass [2, 3] but have subsequently called it gravitational charge [4,5]). Gravitational charge can essentially be inferred from looking at the effective acceleration felt by a neutral test particle. It can in principle be computed for an asymptotically non-flat spacetime as well whereas for quasilocal mass asymptotic flatness is a crucial requirement.

The purpose of this note is first to clarify that ours is a definition of gravitational charge and not of quasilocal mass for stationary spacetimes. Second we discuss some features of our expression.

We have essentially adopted the Gauss theorem to stationary spacetime to define gravitational charge. For the gravitational field strength we take red shifted acceleration experienced by a test particle relative to infinity and define [3-5] gravitational charge as

$$M_c = \frac{1}{4\pi} \int \mathbf{g} \cdot d\mathbf{s} \quad (1)$$

where $\mathbf{g} = -\alpha \nabla \ln \alpha$ and is the red shift factor. For stationary spacetime $\alpha = (g_{tt} + \omega g_{t\phi})^{1/2}$ where $\omega = -g_{t\phi}/g_{\phi\phi}$ is the frame dragging velocity (angular velocity of the locally non-rotating or zero angular momentum particle). The integration is taken over a closed 2-surface.

We would like to note the following points :

- (a) The Komar integral [6] defines the conserved mass for an empty spacetime admitting a timelike Killing vector. When either of the condition is relaxed, the integral yields an expression that depends upon the location of the surface over which the integral is performed. We have first defined [2] effective mass (gravitational charge) by generalising the Komar integral for the Kerr spacetime by replacing timelike Killing vector by timelike co-rotating vector to write

$$M_{eff} = \frac{1}{8\pi} \oint \epsilon_{abcd} \nabla^a \eta^b dx^c \wedge dx^d \quad (2)$$

where $\eta = \frac{\partial}{\partial t} + \omega \frac{\partial}{\partial \phi}$ and ϵ_{abcd} is the alternating tensor. This choice of the vector field is the most natural ; it remains timelike throughout $r > r_+$ (unlike the Killing vector $\frac{\partial}{\partial t}$), and at the horizon it coincides with the null generators of the horizon while it represents time translation asymptotically.

The relations (1) and (2) are equivalent [3]. For the static fluid sphere they agree with the Tomann's integral for mass [7,8]. When evaluated on the horizon.

$M_c = KA$, where K and A are respectively surface gravity and area of the horizon. This is because $g(r_+) = K$. What we have proposed is that g is the measure of gravity off the horizon as well as its flux across a closed 2-surface-gives the gravitational charge enclosed by the surface.

The mass of a blackhole is defined as $M = KA + 2\omega_H J + Q\Phi$, where ω_H, J, Q and Φ refer to angular velocity, angular momentum, charge and electric potential of the horizon. Thermodynamically KA represents the internal energy while the other terms represent the work done on the horizon. That means the gravitational charge $M_c = KA = (M^2 - a^2 - Q^2)^{1/2}$, where $a = J/M$, is the measure of internal energy of the hole which produces gravitational field. For the maximal black hole, $M^2 = a^2 + Q^2$, surface gravity or temperature vanishes because gravitational charge enclosed by the horizon vanishes.

- (b) We have computed gravitational charge for black holes immersed in magnetic field [3,5]. There are contributions from both black hole parameters and magnetic field. The spacetime is not asymptotically flat but the relation (1) gives physically satisfactory expression.
- (c) For the Reissner-Nordström spacetime, $M_c = M - Q^2/r$ and $g = M_c/r^2$, the following construction is quite illuminating. To find gravitational potential at some r , we should subtract the electric energy lying exterior to r from the total mass M , i.e. $M - Q^2/2r$. That means potential will go as

$$= -\frac{M - Q^2/2r}{r}$$

and acceleration will go as

$$= -\frac{M - Q^2/r}{r^2} = \frac{M_c}{r^2}.$$

The Penrose quasilocal mass is given by $M - Q^2/2r$ while our gravitational charge is $M - Q^2/r$. This demonstrates the difference between quasilocal mass and gravitational charge.

- (e) In defining gravitational charge, we have used the red shifted acceleration g as the measure of gravity. It essentially measures the tension in the string that suspends a particle onto a blackhole from infinity. This tension should normally go on increasing as the particle comes closer to the hole. Owing to repulsive effects of rotation and charge on the hole g cannot have a monotonic behaviour and hence it would be interesting to find where does it attain maximum value and does it happen outside the horizon ?

For the Charged black hole, g is maximum at $r \equiv r_0 = 3Q^2/2M$ which will be $\geq M + (M^2 - Q^2)^{1/2}$ for $Q^2/M^2 \geq 8/9$. The tidal force is proportional to g' and hence it will change its sense (from convergence to divergence) at r_0 . For $Q^2/M^2 \geq 8/9$, this radius is accessible for observation. That is it would be easier to keep a particle suspended at $r_+ < r < r_0$ than at r_0 . The timelike geodesic congruence will experience deceleration in convergence for $r < r_0$.

For the rotating black hole g is given by

$$g = \frac{M}{\Sigma \rho^2} [(r^2 + a^2)^2 \rho^2 - 4M^2 r^3 a^2 \sin^2 \theta] \quad (3)$$

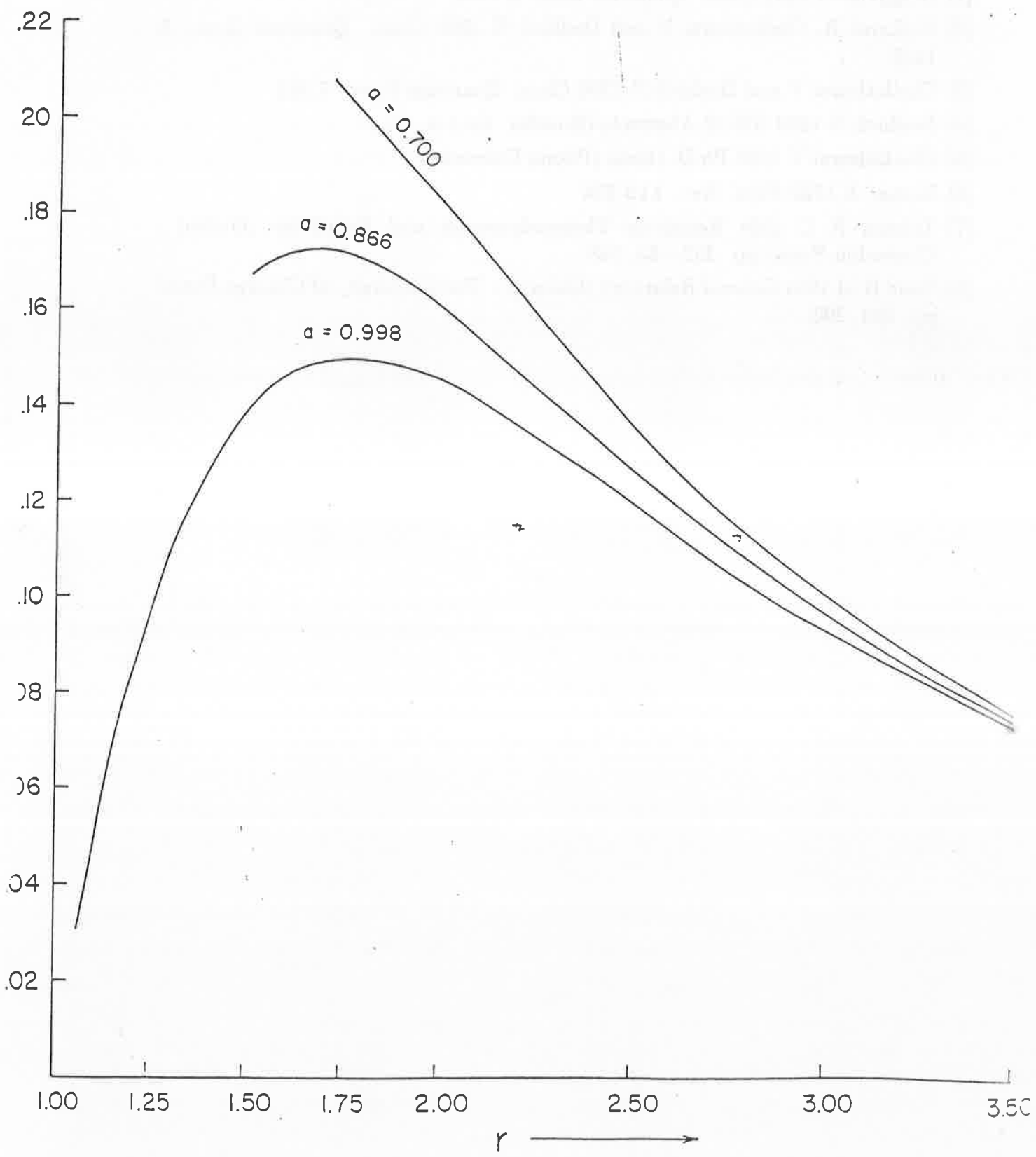
where

$$\begin{aligned} \Sigma &= (r^2 + a^2)^2 \rho^2 + 2Mra^2 \sin^2 \theta \\ \rho^2 &= r^2 + a^2 \cos^2 \theta. \end{aligned}$$

Along the axis $\theta = 0$, g will attain maximum at $r \equiv r_0 = \sqrt{3} a$ and for $r_0 \geq M + (M^2 - a^2)^{1/2}$ will require $a/M \geq \sqrt{3/2} \approx 0.866$. This is quite a canonical value for the Kerr spacetime occurring in various situations. For the equatorial plane $\theta = \pi/2$, g is plotted against r in the Fig. It shows that g attains maximum for slightly lower a/M than the above value.

Finally we wish to reemphasize that ours is a definition of gravitational charge (and not of quasilocal mass) arising out of a natural definition of effective gravitational acceleration (field strength) and then applying the Gauss theorem. Due to the repulsive effects of rotation and charge, the effective acceleration can attain the maximum value outside the horizon for suitable values of charge and rotation parameters, indicating the change of sense (from convergence to divergence) of the tidal force.

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