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Zero-point length from string fluctuations

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Abstract

One of the leading candidates for quantum gravity, viz. string theory, has the following features incorporated in it. (i) The full spacetime is higher-dimensional, with (possibly) compact extra-dimensions; (ii) there is a natural minimal length below which the concept of continuum spacetime needs to be modified by some deeper concept. On the other hand, the existence of a minimal length (*zero-point length*) in four-dimensional spacetime, with obvious implications as UV regulator, has been often conjectured as a natural aftermath of any correct quantum theory of gravity. We show that one can incorporate the apparently unrelated pieces of information—zero-point length, extra-dimensions, string T-duality—in a consistent framework. This is done in terms of a modified Kaluza–Klein theory that interpolates between (high-energy) string theory and (low-energy) quantum field theory. In this model, the zero-point length in four dimensions is a "virtual memory" of the length scale of compact extra-dimensions. Such a scale turns out to be determined by T-duality inherited from the underlying fundamental string theory. From a low energy perspective short distance infinities are cutoff by a minimal length which is proportional to the square root of the string slope, i.e., $\sqrt{\alpha'}$. Thus, we bridge the gap between the string theory domain and the low energy arena of point-particle quantum field theory.

There are several pieces of evidence that suggests that the three constants of physics c, $\hbar$ and G are likely to be replaced in a more fundamental theory by three other constants c, $\hbar$ and l_0^2, where l_0 is a length scale and l_0^2 has the dimensions of area. It has been conjectured for a long time that a fundamental, zero-point, length of spacetime will arise from the quantum gravity and—in the naive models, $l_0^2 \approx G\hbar/c^3$. (This idea has a long history: see for example, [1–4], etc.; for a review and more references, see [5].) The loop quantum gravity (which works with four dimensions) has the same idea arising as a minimal area [6] in terms of l_0^2. The existence of such a minimal area has immediate implications for the entropy of black holes and a holographic interpretation of gravity (see, e.g., [7]). It might even be possible see the effects of such a minimal length in cosmological observations (for the earliest attempt, see [8]; for a recent one see [9]).

This theoretical conjecture was taken forward in [10] in which an unexpected connection between zero point length and a path integral duality was discovered. It was shown that if the path integral amplitude $\exp[il(x,y)]$ used for the definition of the propagator is modified so that it is invariant under the duality transformation $l(x,y) \to l_0^2/l(x,y)$, then: (i) the propagator becomes UV finite, and (ii) l_0 represents a residual, or zero-point length. This idea was followed up in [11] which showed that concrete computations can be performed in quantum field theory leading to UV finite results. Given the fact that string theory is a leading candidate for quantum gravity (which also has a notion of T-duality) it is natural to ask whether these ideas can be combined in a more formal manner.

In attempting this, we must remember that string theory introduces a fundamental length scale as well as—a less evident, but not less important—a second length scale in the form of a *compactification scale*. This is because, in most models, the extra-dimensions of string target spacetime must be compact

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in order to be unobservable at the present day available energies. In this Letter we propose a single consistent framework for connecting the apparently unrelated pieces of information, i.e., zero-point length, extra-dimensions, string T-duality.

We do this by introducing a low-energy 4D vacuum which keeps the memory of compact extra-dimensions only through topologically nontrivial fluctuations. These virtual processes are sensitive both to the presence of extra-dimensions and to the string excitation spectrum. This leads to a zero-point length which is proportional to the compactification scale. Furthermore, such a scale respects the T-duality inherited from the underlying fundamental string theory. As the spectrum of closed strings cannot distinguish between a compactification radius R and a radius α'/R no physical meaning can be attributed to length scale lower than $\sqrt{\alpha'}$. We conclude that $l_0 \propto \sqrt{\alpha'}$. From a low energy perspective short distance infinities are cutoff by a minimal length, which is proportional to the square root of the string slope defined as $\sqrt{\alpha'}$. Thus, we bridge the gap between the ultra-relativistic, ten-dimensional, string domain and the four-dimensional low energy arena of point-particle quantum field theory.

The starting point of our technical analysis is the string Lagrangian in the light-cone gauge [12] (for the sake of simplicity, we shall consider only the case of a bosonic string)

$$L = \frac{\pi\rho}{2}\left(-2\dot{x}_0^+\dot{x}_0^- + \dot{x}_0^i\dot{x}_{0i}\right) + \frac{\rho}{2}\int_0^\pi d\sigma\left[-2\dot{\eta}^+\dot{\eta}^- + \dot{\eta}_i\dot{\eta}^i + 2\eta'^+\eta'^- - \eta'_i\eta'^i\right], \tag{1}$$

where, x_0^μ denotes the center of mass coordinate and η^μ the relative coordinate; $\rho = 1/2\pi\alpha'$ is the string tension; the index i labels $d-1$ "transverse" spacelike directions, for the sake of simplicity we shall consider the simplest case where only one of the transverse dimension is a circle of radius R. Given the Lagrangian, we can write the transition amplitude from an initial to a final configuration for the whole system as a path integral

$$\langle f|i\rangle \equiv \int_{x_{0,i}}^{x_{0,f}}[Dx_0]\int_{\eta_i}^{\eta_f}[D\eta]\exp\left[i\int_0^T d\tau\, L(\dot{x}_0, x_0; \dot{\eta}, \eta)\right], \tag{2}$$

where $x_{0,i}$ and $x_{0,f}$ represent initial and final position of the string center of mass, while $\eta_i \equiv \eta(0,\sigma)$ and $\eta_f \equiv \eta(T,\sigma)$ are initial and final configuration of the fluctuating part of the string. In the low energy limit, described the quantum field theory, we will be interested in the propagator for a particle-like object which will be described by the center of mass of the string. So, what we are really interested in, is the effective propagator for the string center of mass propagating in vacuum in which all possible string fluctuations take place. To account for virtual transitions among string states we need to sum over closed paths in the η configurations space, i.e., with the boundary condition $\eta_i = \eta_f$. This leads to the following expression

$$Z(T) \equiv \oint[D\eta]\exp\left[i\int_0^T d\tau\, L(\dot{\eta}, \eta)\right], \tag{3}$$

where $L(\dot{\eta}, \eta)$ represents the second term on the right-hand side of Eq. (1). Note that Z does *not* describe a physical gas of strings [13], but a mathematical quantity encoding the feature that all kind of virtual transitions take place in the string physical vacuum.

In order to compute Z we first have to remove unphysical modes. This can be done by choosing the *light-cone gauge*, which is a frame where all the oscillations along + direction are turned-off, i.e., $\eta^+ = 0$, $\dot{\eta}^+ = 0$, $\eta'^+ = 0$. Thus, in the light-cone gauge the partition functional reads

$$Z(T) = \oint[D\vec{\eta}]\exp\left[i\frac{\rho}{2}\int_0^T\int_0^\pi d\tau\, d\sigma\left(\dot{\vec{\eta}}^2 - \vec{\eta}'^2\right)\right], \tag{4}$$

where only transverse physical oscillations are summed over. In analogy to the Coulomb gauge in electrodynamics, the light-cone gauge allows to remove both "timelike" η^+ and "longitudinal" η^- components of the η-field.

Using the Fourier expansion for the transverse coordinates we can write $Z(T)$ in the form of a partition functional for two infinite families of transverse harmonic oscillators

$$Z(T) = \oint\prod_{n=1}^\infty[D\vec{x}_n]\prod_{n=1}^\infty[D\tilde{\vec{x}}_n]\exp\left[i\frac{\rho\pi}{4}\int_0^T d\tau \times\sum_{n=1}^\infty\left[\left(\dot{\vec{x}}_n^2 - 4n^2\vec{x}_n^2\right) + \left(\dot{\tilde{\vec{x}}}_n^2 - 4n^2\tilde{\vec{x}}_n^2\right)\right]\right]. \tag{5}$$

Thus, the string partition function turns out to be an infinite product of harmonic oscillator path integrals computed over families of closed paths

$$Z_{\text{ho}}(T) = \sum_{N,\tilde{N}=0}^\infty \exp\left[-iT\left(N + \tilde{N} - \frac{d-1}{12}\right)\right]. \tag{6}$$

But, whenever one (or more) dimension is compact, strings can wrap around it an arbitrary number of times. Accordingly, we have to take into account the contribution from the different winding modes. For the sake of simplicity, let us consider again the case of a single compact dimension. Then, we find

$$Z_{\text{ho}}(T, R) = \sum_{N,\tilde{N},w=0}^\infty \exp\left[-iT\left(N + \tilde{N} - \frac{d-1}{12} + w^2\frac{R^2}{\alpha'}\right)\right]. \tag{7}$$

By including winding modes in Eq. (7) we encode a topological feature which makes the string substantially different from a pure "gas" of pointlike oscillators.

We can now put all the results together and give a more definite meaning to the center of mass kernel in the vacuum which is filled up with both Kaluza–Klein type fluctuations and the new kind of virtual processes brought in by the string excitation modes

$$K(x_{0,f}-x_{0,i};T) = \sum_{N,\tilde{N},w=0}^{\infty} \exp\left[-iT\left(N+\tilde{N}-\frac{d-1}{12}+w^2\frac{R^2}{\alpha'}\right)\right] \times \int_{x_0(0)=x_{0,i}}^{x_0(T)=x_{0,f}} [Dx_0] \exp\left[i\int_0^T d\tau\, L(\dot{x}_0,x_0)\right]. \quad (8)$$

This path integral can be computed by weighting each path by its canonical action in phase-space

$$S=\int_0^T d\tau \left[P_+\dot{x}^+ + P_-\dot{x}^- + P_j\dot{x}^j + P_d\dot{x}^d - \frac{1}{2\pi\rho}\left(2P_+P_- + P_jP^j + P_d^2\right)\right]. \quad (9)$$

Trajectories along the compact dimensions must satisfy periodic boundary conditions, i.e., $x^d(T)=x^d(0)+nl_0$ where $l_0=2\pi R$. Integration over center of mass degrees of freedom gives (see [14,15] for details)

$$K_{\text{reg}}(x_f-x_i;T) = \left(\frac{1}{4i\pi\alpha' T}\right)^{\frac{d-1}{2}} \sum_{N=0,\,w,n=1}^{\infty} \exp\left[-\frac{(x_f-x_i)^2+n^2l_0^2}{4i\alpha' T}\right] \times \exp\left[-iT\left(2N+nw-\frac{d-1}{12}+\frac{w^2R^2}{\alpha'}\right)\right], \quad (10)$$

where we have taken into account the level matching condition $\tilde{N}-N=nw$ and dropped out the zero-modes $n=0$ and $w=0$. The rationale behind this subtraction is discussed in detail in [14,15] and will not be repeated here. Further comments about this technical step can be found at the end of this Letter. From (10) it is possible to obtain the Green function by integration over the unmeasurable lapse of time T as follows:

$$\begin{aligned} G(x_f-x_i) &\equiv (2\pi)^{\frac{d-1}{2}} \int_0^\infty dT\, e^{-i2\alpha' m_0^2 T} K_{\text{reg}}(x_f-x_i) \\ &= \left(\frac{1}{2i\alpha'}\right)^{\frac{d-1}{2}} \sum_{N=0,\,w,n=1}^{\infty} \int_0^\infty dT\, T^{-\frac{d-1}{2}} \\ &\quad\times \exp\left[-\frac{(x_f-x_i)^2+n^2l_0^2}{4i\alpha' T}\right] \\ &\quad\times \exp\left[-iT\left(2\alpha' m_0^2+2N+nw-\frac{d-1}{12}+\frac{w^2R^2}{\alpha'}\right)\right], \end{aligned} \quad (11)$$

where the mass m_0 can be zero or nonvanishing and has been introduced to account for low energy effects, e.g., spontaneous symmetry breaking. In order to evaluate the short distance behavior of the Green function (11) it is useful to Fourier transform it

$$G(p)=\sum_{N=0,\,w,n=1}^{\infty} \frac{nl_0}{\sqrt{p^2+M^2_{N,w,n}}} K_1\left(nl_0\sqrt{p^2+M^2_{N,w,n}}\right). \quad (12)$$

The mass term $M^2_{N,w,n}$ is defined as

$$M^2_{N,w,n} \equiv \frac{1}{\alpha'}\left(2N+nw-\frac{d-1}{2}+\frac{w^2R^2}{\alpha'}+2\alpha' m_0^2\right). \quad (13)$$

At high energy (momentum) the asymptotic behavior of the propagator (12) is essentially determined by the lowest energy level $n=w=1$

$$G(p) \approx \frac{l_0}{\sqrt{p^2+M^2_{0,1,1}}} K_1\left(l_0\sqrt{p^2+M^2_{0,1,1}}\right) \approx \frac{\sqrt{l_0}}{(p^2)^{3/4}} \exp\left(-l_0\sqrt{p^2}\right). \quad (14)$$

As closed strings cannot probe compactification scales lower than $\sqrt{\alpha'}$, then we can replace l_0 in (14) with $2\pi\sqrt{\alpha'}$. Thus, it becomes manifest as UV divergences are exponentially suppressed at string energy scale. Generalization to an hyper-torus with more than one compact dimensions is straightforward. The standard Minkowski vacuum, with its pathological short-distance behavior, can be recovered in the limit $l_0 \to 0$. It is worth observing that in our formulation the limit $l_0 \to 0$ is equivalent to the infinite tension limit, i.e., $\alpha' \to 0$, where strings shrink to structureless points and the point-particle picture of matter is recovered.

Let us take stock of the result from a wider perspective. String theory uses $(4+D)$ dimensions of which D are compact. The path integral in Eq. (2) represents the transition amplitude in the full theory. On the other hand, the low energy quantum field theory uses only 4 dimensions and the theory is described by a propagator $G(x,y)$. To get $G(x,y)$ from the full theory, it is appropriate to identify the center of mass of the string as representing the particle of the quantum field theory. But then the propagator will be affected by the virtual fluctuations in the string vacuum. In particular, when $(x-y)^2$ is smaller than the size of the compact dimensions, these fluctuations will lead to corrections to the propagator. These vacuum fluctuations can be divided into two sets: the topologically trivial zero-modes which do not probe the internal dimensions (the $n=0$ modes) and the topologically nontrivial ones ($n \neq 0$). We have shown that when the latter ones are retained, the following results are obtained. (a) The propagator picks up corrections which essentially involves replacing $(x-y)^2$ by $(x-y)^2+l_0^2$ introducing a zero point length. (b) This is identical in form to the results obtained earlier in [10] and shows that the T-duality does lead to the path integral duality. (c) It provides a prescription for incorporating the "stringy" effects in the standard quantum field theory and the theory is now UV-finite [11].

Once we select the histories which are closed along the extra-dimensions for evaluating the path integral, the full path integral factorizes into the product of the four-dimensional path integral for the propagator times the vacuum partition functional accounting for virtual fluctuations along extra-dimensions. Our result arises from dropping the zero-modes, which describe topologically trivial fluctuations; i.e., fluctuations described by paths which can be continuously shrunk to a point. With hindsight, it is clear how zero-modes bring ultraviolet divergences in, as they are "blind" to the extra-dimensions and can probe arbitrary short distance. The need of a "by hand" subtraction of the zero-mode follows from the choice of the simplest toroidal compactification. Hopefully, in a more sophisticated compactification scheme, or when we understand the structure of some guiding principle behind the theory, the zero-mode will be absent from the spectrum from the very beginning. This could be an effective criterion for selecting the appropriate kind of compact dimension(s) among many possible topologies.

Thus, zero-point length in four-dimensional spacetime can be seen as the *virtual memory* of the presence of compact extra-dimensions which can occur even much below the threshold energy needed to produce real Kaluza–Klein particles. Within the Kaluza–Klein quantum field theory picture, the actual value of l_0 remains undetermined. In the more general framework provided by string theory, T-duality selects the *unique* self-dual value for the compactification scale, and, accordingly determines $l_0 = 2\pi\sqrt{\alpha'}$. One could, therefore, expect to see deviations from the theoretical predictions of standard quantum field theory due to the presence of the modified Feynman propagator (12) at an intermediate energy regime much below the string scale. In this respect, the most optimistic scenario is offered by the TeV scale unification models, where the string scale is lowered down to a few TeV [16,17]. Such a "low-energy" unification can be realized provided the extra-dimensions are compactified to a "large" radius of some fraction of millimeter. In this case $l_0 \approx 10^{-17}$ cm and its presence would be detectable in the high-energy scattering experiments [18] planned for the next generation of particle accelerators.

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