Primordial power spectrum: a complete analysis with the WMAP nine-year data

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Abstract

We have improved further the error sensitive Richardson-Lucy deconvolution algorithm making it applicable directly on the un-binned measured angular power spectrum of Cosmic Microwave Background observations to reconstruct the form of the primordial power spectrum. This improvement makes the application of the method very much straight forward by cutting some intermediate stages of analysis allowing us to reconstruct the form of the primordial spectrum with higher efficiency and precision and with lower computational expenses. Applying the modified algorithm we have been able to fit the WMAP 9 year data using the optimized reconstructed form of the primordial spectrum with more than 300 improvement in χ^2 with respect to the best fit power-law. This is clearly beyond the reach of other alternative approaches and reflects the efficiency of the proposed method in the reconstruction process and allow us to look for any possible feature in the primordial spectrum projected in the CMB data. Though the proposed method allow us to look at various possibilities for the form of the primordial spectrum, all having good fit to the data, proper error-analysis is needed to test for consistency of theoretical models. Reconstructed error-band for the form of the primordial spectrum using many realizations of the data, all bootstrapped and based on WMAP 9 year data, shows proper consistency of power-law form of the primordial spectrum with the WMAP 9 data at all wave numbers. Including WMAP polarization data in to the analysis have not improved much our results due to its low quality but we expect Planck data will allow us to make a full analysis on CMB observations on both temperature and polarization separately and in combination.

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Contents

1 Introduction

Observations of Cosmic Microwave Background (CMB), in particular Wilkinson Microwave Anisotropy Probe (WMAP) [1] provide us with some of the most promising data to study the signatures of the early universe. The initial seed of the structure formation, sowed at the beginning of inflation by quantum fluctuations, propagated through its evolution during inflationary epoch, leaves its imprints in the angular power spectrum in the temperature anisotropy observed in CMB. The detected temperature and polarization angular power spectrum, contains the information of the background cosmology as well as the initial conditions of the universe. Precision measurements of anisotropies in the cosmic microwave background, and also of the clustering of large scale structure, suggest that the primordial density perturbation is dominantly adiabatic and has a nearly scale invariant spectrum [1, 2, 3, 4, 5, 6]. This is in good agreement with most simple inflationary scenarios which predict power law or scale invariant forms of the primordial perturbation [7, 8]. The data have also been used widely to put constraints on different parametric forms of primordial spectrum, mostly motivated by inflation [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50]. However, despite the strong theoretical appeal and simplicity of a featureless primordial spectrum, it is quite important to determine the form of the primordial power spectrum directly from observations with minimal theoretical bias. Many model independent searches have been made to look for features in the CMB primordial power spectrum [51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70]. The Richardson-Lucy (RL) deconvolution algorithm, was proposed as one of the first approaches to reconstruct the form of the primordial spectrum directly from CMB data and has been extensively used till now applying on different CMB observations [53, 60, 61, 67]. In this paper we modify further the RL deconvolution algorithm and we reconstruct the power spectrum using WMAP 9 year data. Amongst several new features in this paper, in particular we shall use directly the *un-binned* correlated C_ℓ data to reconstruct the power spectrum. We note that all earlier studies have used the uncorrelated binned data to reconstruct the PPS and we show that this new modification improves the efficiency of the method significantly. At first glance though it seem to be a mathematically inappropriate study, since RL needs uncorrelated data to begin with, we shall show that the use of the un-binned data improves the algorithm in various ways for our particular case of study. We show that using this method for the first time we have been able to directly reconstruct a power spectrum which improves the χ^2 fit to the most recent CMB data by about 200-300 better than the best fit power-law form of the PPS. We should mention that it is the improved quality of the present day data that allows us to directly use the un-binned data in a practical way and in modification of RL algorithm we had an eye on the forthcoming Planck [71] data as well. While getting such a huge improvement in the χ^2 fit is exciting and hints towards the fact that we should be able to see any feature in the data, we also discuss on the issue of the consistency check of different models to the data and we show that the power-law form of the primordial spectrum is indeed consistent to the data after a complete error-analysis. So we emphasise to the readers that these two issues, reconstruction and falsification, should not be confused. The paper is organized as the following. First we discuss the RL algorithm and its modification which we have implemented in this work. Then we explain in detail the way to incorporate the binned, un-binned and the combined data and we discuss some technical issues in the analysis. Next, we present the reconstructed results for the primordial spectrum and the corresponding angular power spectrum and at the end we discuss about the method and the results and conclude.

2 Formalism

In this section we shall discuss the RL algorithm. As it has been used for primordial reconstruction before, we would like to refer to the earlier papers [53, 60, 61, 65, 67] for detailed and more complete discussions. The use of the un-binned data and the combined data are the major and new implications in this work and we shall explain our assumptions in subsection 2.2. Apart from obtaining a power spectrum providing a better fit, through error estimation, we shall argue whether power law power spectrum is still allowed by the data within its uncertainty. Finally we shall aim for a power spectrum which is capable of providing a much better fit than the conventional slow roll inflationary scenarios but at the same time does not include too many fluctuations in it so that it can be obtained from inflationary potential with a limited number of parameters.

2.1 The Richardson-Lucy algorithm

In the field of astronomy, RL algorithm is typically used for reconstruction of images [72, 73, 74, 75]. It has been first demonstrated in article [53] that this method can be used to reconstruct the primordial power spectrum from the angular power spectrum using the deconvolution. The observational angular power spectrum $C_\ell^{\rm D}$ can be directly related to the theoretically calculated angular power spectrum $C_\ell^{\rm T}$. Primordial power spectrum, P_k generates the $C_\ell^{\rm T}$ by its convolution with the radiative transport kernel $G_{\ell k}$ through the following relation.

$$C_\ell^{\rm T} = \sum_i G_{\ell k_i} P_{k_i} \tag{1}$$

It should be pointed out, barring the primordial inflationary information from P_k, the $C_\ell^{\rm T}$ *only* depends on $G_{\ell k}$, which in turn is function of the cosmological parameters defining the ingredients and expansion rate of the universe and the history of reionization through the optical depth τ (for more discussion on the radiative transport kernel see ref [53, 60, 61]).

Keeping the parameters mentioned above fixed, obtaining the P_k from the C_ℓ is a deconvolution procedure. Derived from the elementary probability distributions, the RL deconvolution theorem iteratively solves for the primordial power spectrum using the following algorithm.

$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \left[\sum_\ell \widetilde{G}_{\ell k} \left(\frac{C_\ell^{\rm D} - C_\ell^{{\rm T}(i)}}{C_\ell^{{\rm T}(i)}} \right) \right], \tag{2}$$

Here $P_k^{(i+1)}$ and $P_k^{(i)}$ are the power spectrum evaluated in iterations $i+1$ and i respectively and the quantity $\widetilde{G}_{\ell k}$ is same as $G_{\ell k}$ appearing in eq. 1 normalized to unity in every multipole. Here, we should point out due to noise and the cosmic variance C_ℓ^D has some non-zero error $\sigma_\ell^{\rm D}$. Since eq. 2 do not depend on the errors associated to the data this method gives equal weight to every data point, which leads to a power spectrum fitting possible noise in the data. Hence, in this paper, we shall use and modify the Improved Richardson-Lucy (IRL) algorithm (it has first been implemented in ref [53]) which incorporates the errors in the data and in each iteration the modifications to the primordial power spectrum depend on the noise in the related angular power spectrum. IRL algorithm can be formulated as follows,

$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \left[\sum_\ell \widetilde{G}_{\ell k} \left(\frac{C_\ell^{\rm D} - C_\ell^{{\rm T}(i)}}{C_\ell^{{\rm T}(i)}} \right) \ \tanh^2 \left[\frac{C_\ell^{\rm D} - C_\ell^{{\rm T}(i)}}{\sigma_\ell^{\rm D}} \right]^2 \right], \tag{3}$$

Basically the convergence factor $\tanh^2 \left[\frac{C_\ell^{\rm D} - C_\ell^{{\rm T}(i)}}{\sigma_\ell^{\rm D}} \right]^2$, weighs down the contribution of two types of multipoles towards modification of the primordial spectrum, namely, the $C_\ell^{\rm T}$ which has already closer to $C_\ell^{\rm D}$ *w.r.t* others, and the data with higher errors. Here we should mention that we have scaled the data with a factor such that the primordial power spectrum in each iterations remains COBE normalized.

2.2 Working with the binned, the un-binned and the combined data

As has been indicated in the introduction, the earlier works on the reconstruction of primordial power spectrum have been performed using the binned data from WMAP. Although the comparison with the binned data seems a reasonable approximation, it has limitations. The angular power spectra generated from the reconstructed primordial spectra, though, gives a better fit to the binned data compared to the power law spectrum, it provides a worse fit to the complete data which incorporates un-binned auto and cross correlations of temperature and polarization anisotropies. As the reconstructed power spectrum by definition attempts to fit only the few binned data-points, in between two binned multipole spurious oscillations are imposed which are not expected to agree with the complete datasets and provides a worse likelihood to the total data. Smoothing of these spurious oscillations needs to be implemented in order to average out unwanted features and obtain a better fit to the data [53, 60, 61].

On the other hand, though working with un-binned correlated data does not seem to be a perfect choice *mathematically*, we found that this approach is much better than working with the binned data in various ways. We should remember that we are using IRL method adjusted for our case of study dealing with data with different uncertainties which is a clear physical problem rather than looking for exact solutions for a set of mathematical equations.

We can modify equation. 3 considering the correlated error matrix of the data, henceforth we shall call it MRL (Modified Richardson-Lucy):

$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \left[\sum_\ell \widetilde{G}_{\ell k} \left(\frac{C_\ell^{\rm D} - C_\ell^{\rm T(i)}}{C_\ell^{\rm T(i)}} \right) \ \tanh^2 \left[Q_\ell (C_\ell^{\rm D} - C_\ell^{\rm T(i)}) \right] \right], \tag{4}$$

$$Q_\ell = \sum_{\ell'} (C_{\ell'}^{\rm D} - C_{\ell'}^{\rm T(i)}) COV^{-1}(\ell, \ell') \tag{5}$$

where $COV^{-1}(\ell, \ell')$ is the inverse of the error covariance matrix. In practice, this inverse of the error matrix from WMAP 9 years observations can not be implemented directly due to complication of likelihood estimators. However, still one can use a likelihood function similar to the one used by WMAP team in their first data release to generate a covariance matrix using pseudo C_ℓ's. In this work we show that even using the diagonal terms of the covariance matrix which are given by WMAP 9 year results would still work perfectly fine in our context of study. We should note here that all our results are at the end based on the likelihood codes given by WMAP mission. Looking at fig. 1 and comparing blue and green lines which are the likelihoods given by the WMAP codes and the χ^2 derived by using the diagonal terms of the covariance matrix, we see that they are following each other clearly at all scales and this reflects that we have chosen a reasonable approximation in our reconstruction procedure. The new approach requires much less iterations to reach a reconstructed power spectra providing a better fit. On the other hand the reconstructed power spectra directly provides a much better fit ($\sim 200 - 300$) to the total data set (computed by WMAP likelihood code) compared to the best fit power law case. Finally, unlike the works with the binned datasets, we do not have to smooth the power spectrum in this case to achieve a good likelihood to the whole data. However, we should mention that the method of using the un-binned data has its own disadvantages. After the multipole moment $\ell \sim 900$ the quality of data gets worse as the noise becomes comparable to the signal (or higher as we go to higher ℓ). In the range $\ell = 900 \sim 1200$ one encounters a number of negative C_ℓ which forces the MRL method to set P_k to *zero* at high k region, which is purely unphysical artefact of the negative C_ℓ's. To avoid this problem we have carried out our analysis by setting the negative C_ℓ's to zero. We shall demonstrate the results obtained in the section 3.

We find, despite of setting the negative C_ℓ's to zero (which we consider a crude approximation) the reconstructed power spectra provide a better fit of ~ 200 when compared with the conventional power law primordial spectrum. We note that this is indeed an artefact of the limitations of the high ℓ data towards improvement of fit. Here it is important to emphasize that as we have *neglected* the negative C_ℓ's, the amplitude of power around the third peak is *enhanced*, which, in turn leads to an amplification of scalar power spectrum at large k ($\sim 0.1\text{Mpc}^{-1} - 0.2\text{Mpc}^{-1}$).

Finally to get rid of the *unphysical* amplification described above, we have implemented the MRL algorithm to the combined binned and un-binned temperature data. We have considered the un-binned data till the multipole $\ell = 900$ an the binned data afterwards [1]. With this procedure the MRL algorithm reads as,

$$\begin{aligned} P_k^{(i+1)} - P_k^{(i)} &= P_k^{(i)} \times \left[\sum_{\ell=2}^{\ell=900} \widetilde{G}_{\ell k}^{\rm un-binned} \left\{ \left(\frac{C_\ell^{\rm D} - C_\ell^{\rm T(i)}}{C_\ell^{\rm T(i)}} \right) \ \tanh^2 \left[Q_\ell (C_\ell^{\rm D} - C_\ell^{\rm T(i)}) \right] \right\}_{\rm un-binned} \right. \\ &+ \left. \sum_{\ell_{\rm binned} > 900} \widetilde{G}_{\ell k}^{\rm binned} \left\{ \left(\frac{C_\ell^{\rm D} - C_\ell^{\rm T(i)}}{C_\ell^{\rm T(i)}} \right) \ \tanh^2 \left[\frac{C_\ell^{\rm D} - C_\ell^{\rm T(i)}}{\sigma_\ell^{\rm D}} \right]^2 \right\}_{\rm binned} \right], \end{aligned} \quad (6)$$

where, $\widetilde{G}_{\ell k}^{\rm un-binned}$ and $\widetilde{G}_{\ell k}^{\rm binned}$ correspond to the un-binned and binned kernel respectively, normalized to 1 in every multipole.

In an idealistic reconstruction, one should take the polarization data into account. However, the present polarization data from WMAP-9 is not good enough to improve significantly our reconstruction process. For instance a quick glance of the un-binned EE angular power spectrum should reveal many negative points in every cosmological scales which is certainly not physical. Binned EE and TE spectrum can be used for reconstruction as in [67], but we should emphasize that the huge improvement of fit, which is the main crux of this paper comes from the un-binned reconstruction which certainly can not be achieved by taking into account the binned polarization data. With the upcoming results from Planck [71] we hope that we shall have access to a far better polarization data which can be used for a more efficient reconstruction.

Here, we should also discuss briefly the effect of CMB gravitational lensing on the reconstruction procedure. Apart form the underlying cosmological model, the effects of lensing on the angular power spectrum depends mildly on the form of the primordial power spectrum as well. This makes it quite complicated to incorporate the effect of lensing directly in the RL kernel $G_{\ell k}$. However, we find that the effect of lensing for scales probed by WMAP does not differ drastically when we compare between reconstructed power spectrum and power law spectrum. In this paper for simplicity we neglect the effect of lensing in the reconstruction process (though in the likelihood estimation it is considered) since the background model is fixed. For the purpose of cosmological parameter estimation with the reconstructed spectra, which we are presently pursuing, we in fact take into account the effect of lensing through some approximation that would be reported in a forthcoming publication.

2.3 Error-estimation

As has been described in earlier literature [53, 60, 61], after a few iterations, MRL method converges to a reconstructed power spectrum which provides a *huge* improvement of fit when

[1]We note that here too, we neglect the first negative C_ℓ at $\ell = 890$ by setting it to zero. We assume this approximation will have a negligible impact on the total likelihood

compared with the standard power law case. It should be mentioned that, as the MRL method attempts to reconstruct the primordial spectra by fitting the mean values of the observed C_ℓ data, we find the reconstructed C_ℓ goes through the vicinity of most of the data points. Now, as the data has the probability to reside within its observed errors, we should be able to reconstruct a set of primordial spectra which generate C_ℓ's residing within the errors associated with the data. We synthesize 1000 C_ℓ datasets from the original data points with Gaussian random fluctuations with a variance associated to the corresponding 1σ errors to the data-points (in the context of error-estimation also see ref. [67]).

Using the above formalism, we generate 1000 primordial power spectra. To obtain the band associated to the uncertainty, in each mode we identify the most dense region which contains 68.3% and 95.5% (corresponding to the 1σ and 2σ regions respectively) spectra within. This statistics helps us to get rid of the distribution around the mean value and the errorband obtained is expected to purely contain the 1σ and 2σ region of maximum occurrence of events.

2.4 Smoothing algorithm

In this work we have used the un-binned C_ℓ's to reconstruct the primordial power spectrum. It has been indicated in earlier works that working with the binned data requires a post-iteration smoothing [53, 60, 61] to get a power spectrum providing a better fit to the complete CMB data (likelihood obtained using the complete covariance matrix including polarization) *w.r.t* the power law spectrum. Although, the use of the un-binned data privilege us to provide a huge better fit to the data without smoothing, we would like to point out that this reconstructed power spectrum contains unphysical artefacts due to presence of observational and statistical noise in the data.

Smoothing of the reconstructed power spectrum helps us to find out the broad shape of the power spectrum directly from the CMB observation with the averaging out the possible noise effects.

To smooth the spectrum we adopt the following algorithm using Gaussian filters.

$$P_k^{\rm Smooth} = \frac{\sum_{\tilde{k}={\rm k_{min}}}^{\rm k_{max}} P_{\tilde{k}}^{\rm Raw} \times \exp\left[-\left(\frac{\log\tilde{k}-\log k}{\Delta_k}\right)^2\right]}{\sum_{\tilde{k}={\rm k_{min}}}^{\rm k_{max}} \exp\left[-\left(\frac{\log\tilde{k}-\log k}{\Delta_k}\right)^2\right]}, \tag{7}$$

where, Δ_k indicates the width of the Gaussian filter and note that as Δ_k approaches zero, the smooth spectrum becomes identical to the raw one.

In our work we have tested two types of smoothing using the filter width Δ_k. In the first method we choose the width Δ_k to be constant over all scales. We investigate the dependence of the likelihood as a function of the width under different iterations. Now, keeping in mind that the quality of the data is not good at higher multipoles ($\ell > 900$) to provide tight constraints on power spectrum at high k, we use a variable width. Here, we use $\Delta_k \propto k$. This allows us to incorporate the broad oscillations (possible artifacts of inflationary model) at the intermediate cosmological scales and average out the violent oscillations (if any), possibly generated from the noise.

2.5 A few words about the numerical methods adopted

We have used publicly available code CAMB [76, 77] to compute the kernel $G_{\ell k}$. Following the IRL algorithm implemented in earlier works, we have developed a new FORTRAN 90 code. We have used our code as an add-on to CAMB. We have fixed the values of Ω_b, $\Omega_{\rm CDM}$, H_0 and τ to their best fit obtained from the analysis of WMAP nine-year data [1].

For the reconstruction with the binned data we have made use of the WMAP nine-year data and its binning. We bin the kernel $G_{\ell k}$ and the $C_\ell^{\rm T}$'s (appearing in eq. 6) using the same binning as in WMAP nine-year data. We normalize $G_{\ell k}$ to one in every multipole as indicated in earlier works to get $\widetilde{G}_{\ell k}$. For the analysis with the un-binned data we follow the same procedure as above except the binning and for the combined data we follow eq. 6 and use the binned and the un-binned data as required by the algorithm. For the initial guess we have used the best fit primordial power spectrum from WMAP-9. We have used nearly 1700 k-space samples to calculate the C_ℓ's. Using a higher number of k-space points obviously provides a much better fit to the spectra ($\Delta\chi^2 > 300$) as it contains more degrees of freedom, but at the same time contains violent oscillations at higher wavenumbers. If we aim to converge on a power spectrum that can be motivated by an inflationary theory with a few parameters, we should have less fluctuations in the power spectrum and it requires a careful smoothing of the spectra which is of course easier to implement in the former case with less k-space sampling.

3 Results

In this section we shall demonstrate the results of our analysis. We should point out that, unless otherwise explicitly mentioned, the figures presented in this section are obtained using the combined un-binned and binned data following eq. 6. Following the smoothing algorithm in the subsection 2.4 we first illustrate the effects of a constant smoothing width Δ_k on the likelihood ($\mathcal{L}$). In Fig. 1 we have shown the quantity $-\ln\mathcal{L}$ (or $\chi^2/2$)as a function of the smoothing width. This figure shows the likelihood calculated considering temperature data alone and including the polarization. We would like to point out that smoothing of the primordial spectrum affects the temperature and the polarization spectrum in a similar way and this fact justifies that our method of working with the un-binned data is quite apt. Further the inset of the figure suggests using the combined un-binned and binned data only after 20 iterations we are able to get a better fit of $\sim 70-80$ without any smoothing of the data [2]. Needless to add, as expected we get better fit with the low width of the smoothing filter. However, we would like to add a couple of sentence regarding the increase of χ^2 obtained from highly smoothed spectra as we increase iterations. A quick glance at the WMAP un-binned data reveals for a number of multipoles the theoretically angular power spectrum from power law is unable to fit the data. Basically the tanh convergence factor introduced in the IRL (and MRL) algorithm induces the broad features in the spectrum which helps us to fit the the outliers with low noise in first few iterations. If we allow higher

[2]Note that the inset captures the part of the figure where the smoothing width negligible and as it has been discussed in the preceding section the smoothed spectrum matches the actual spectrum for low value of Δ_k

iterations the it is possible to get a huge improvement of fit ($\sim$ 300) by fitting the possible noise in the data. Now a smoothing of such highly oscillatory spectrum using a filter of uniform width in all scales suppresses the imposed oscillations in uniform manner which leads to a worse fit. Fig. 1 suggests that for $\Delta_k \sim 0.1$ the spectra obtained after 70 and 100 iterations actually fit the data worse than the power law, while note that the smoothed spectra after 20 iterations is indeed providing a better fit. However, with decreasing Δ_k we recover that with higher iterations we get higher better fit to the data. Here, we should also mention that using a different smoothing algorithm it should be possible to get rid of unwanted oscillations in the spectrum. We find smoothing filter with width proportional to the wavenumbers can be more effective in providing a power spectrum containing only large oscillations (expected to be physical artefacts) which also provides a reasonable better fit to the data.

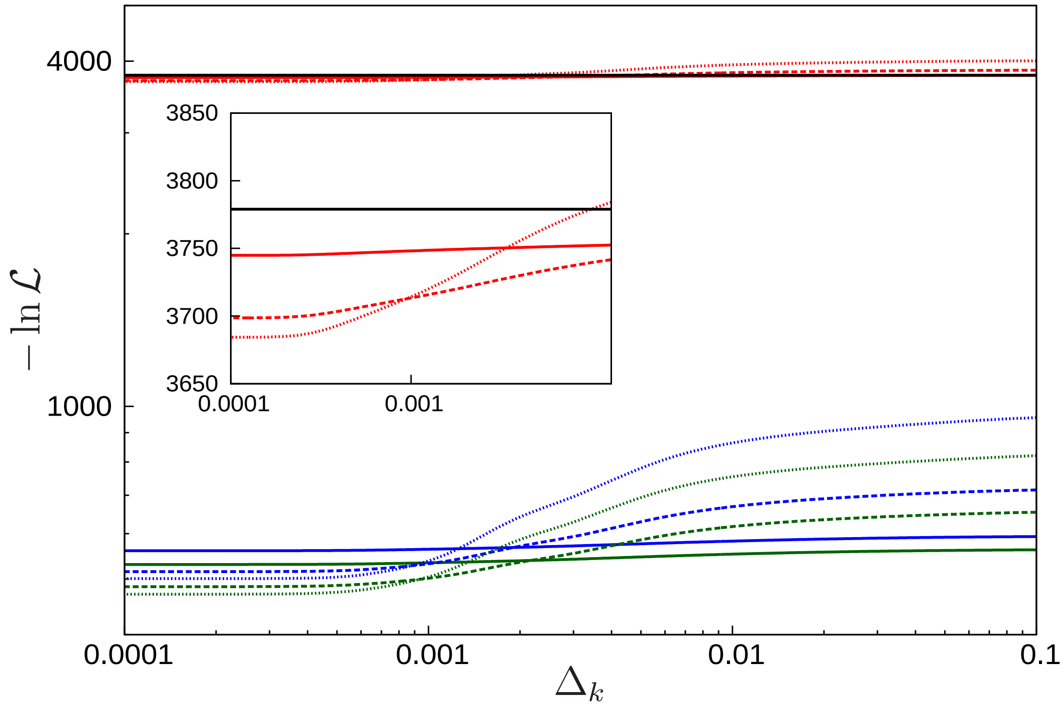

Figure 1: The dependence of the likelihood on the width of the smoothing filter in logarithmic scale. In red we have plotted the $-\ln\mathcal{L}$ of the total data obtained using the WMAP likelihood. Using the diagonal term of the inverse covariance matrix the quality $-\ln\mathcal{L}$ is plotted using simple χ^2 statistic (in green) and using the WMAP likelihood (in blue) for the temperature auto-correlation data only.In black we have indicated the total $-\ln\mathcal{L}$ in the case of power law. Moreover here we illustrate the results of the smoothing after three different iterations. Results for iterations 100 (in dots), 70 (in dashed-lines) and 20 (in solid lines) are shown. The inset provides the total $-\ln\mathcal{L}$ for three different iterations in linear scale. The dotted red line (for 100 iterations) suggests that we are able to achieve a better fit of about 200 *w.r.t.* the power law likelihood.

A few reconstructed primordial power spectra and the corresponding $C_\ell^{\rm TT}$ are plotted in fig. 2. $C_\ell^{\rm TE}$ and $C_\ell^{\rm EE}$ associated with the same set of plots are given in fig. 3. In both the plots black dots with error-bars are the data points while the black straight line on fig. 2 refers to the best fit primordial power spectrum from WMAP-9. Noticeably, the plot suggests that

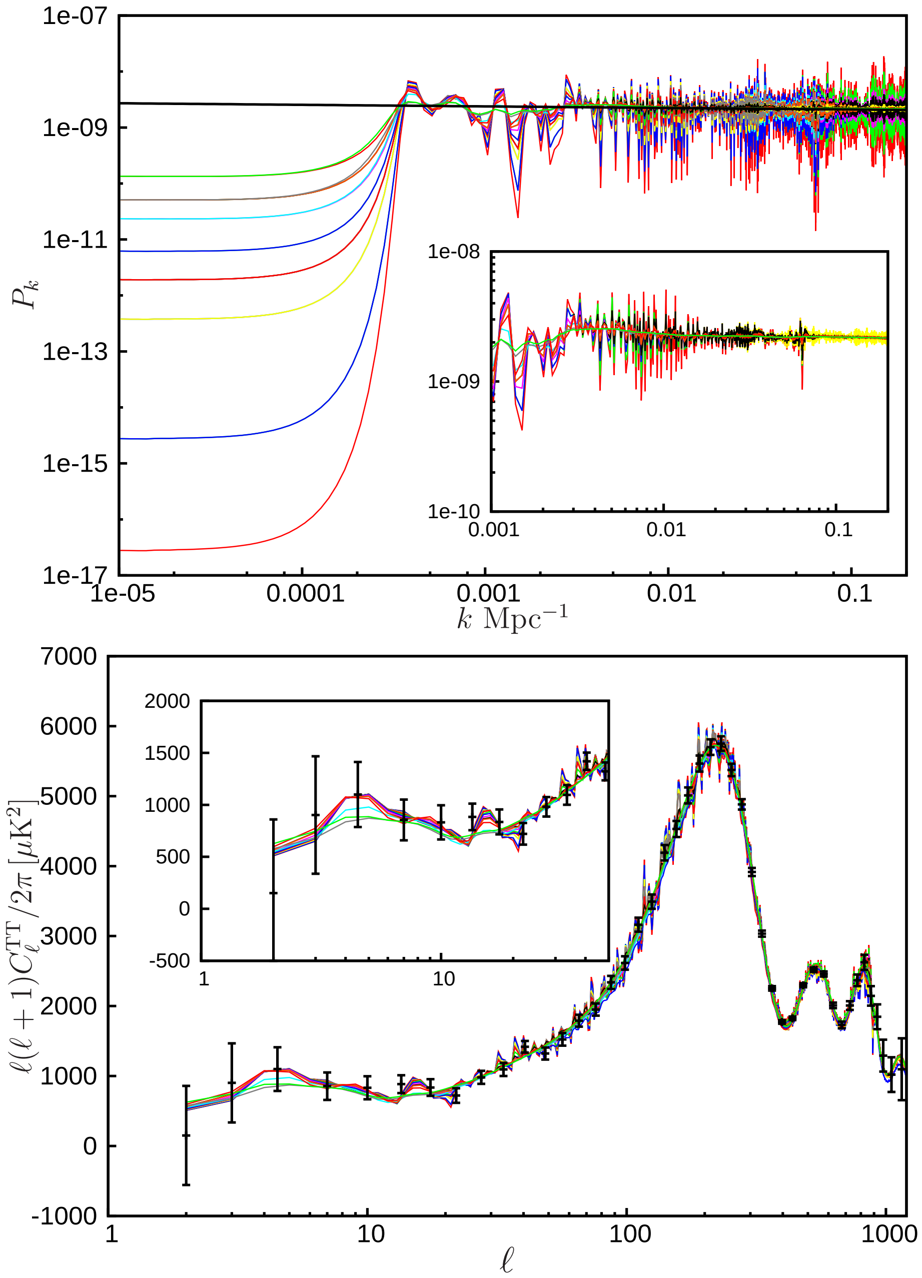

Figure 2: The reconstructed primordial power-spectra (on top) and the corresponding angular power spectra (at the bottom) that provide a better fit ranging from $\sim 2-300$ compared to the power law model. We have plotted 20 sample power spectra (for different iterations and different smoothing width) and the corresponding angular power spectrum along with the data with error-bars (in black). As expected the reconstructed power spectrum can address all the outliers that can not be fit be the power law model. The inset on the top figure contains 10 sample power spectrum where a using a high width of smoothing we have been able to average out the oscillations responsible for fitting the possible noise in the data. Note that at high k the smooth spectrum is nearly scale invariant and is in good agreement with the power law (in black). The Sachs-Wolfe Plateau region is highlighted in the inset of the second figure which illustrates the fitting of the low-ℓ outliers by the features in the reconstructed power spectrum.

there are no '*outliers*' to the reconstructed spectrum. To arrive at these figures we have used the MRL algorithm with the combined un-binned and binned data. As it has been indicated in the formalism section, due to the bad quality of the data at high ℓ ranges, we encounter several negative C_ℓ's in high multipoles. In fact since the radiative transport kernel $G_{\ell k}$ is a positive definite matrix and P_k are positive, the C_ℓ's also should be positive. We have found that MRL method forces the P_k to be zero to high wavenumbers to match the unphysical and negative data points evident leads to wrong reconstruction. To avoid the negative C_ℓ's we have imposed zero values to the C_ℓ's to the multipoles where the data goes negative. While this assumption stops P_k to go to zero at at high k, we get an overall amplification of power at high k as imposing the negative C_ℓ's to zero introduces an average enhancement of power at high ℓ's. However, we would like to mention that, despite of this amplification we get a better fit of about ~ 200 as the high ℓ data has relatively low contribution on the overall likelihood. The combination of un-binned and the binned data on the other hand works better in this cases. As described in eq. 6 we have used the un-binned data till $\ell = 900$ and binned data thereafter. We encounter only one negative C_ℓ's at $\ell = 900$ and we set it to zero as has been indicated earlier. We find that this method is the optimum choice as the reconstructed P_k using combined data provides a better likelihood compared to the P_k obtained using the binned data and the power spectrum is completely free from the unphysical amplification at high k.

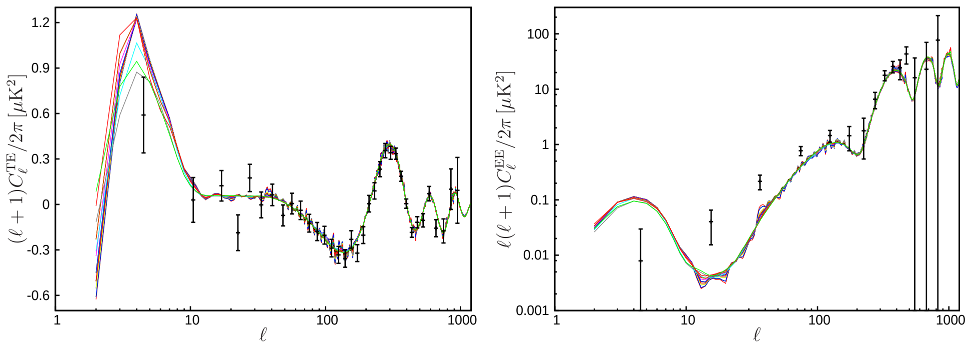

Figure 3: The C_ℓ^{TE} and C_ℓ^{EE} obtained from the 20 reconstructed power spectrum as shown in the fig. 2. The data and the error bars are plotted in black. It should be highlighted that while the theoretical C_ℓ's are largely in agreement with the TE data, for EE data there are a few outliers as well.

The error estimation is an integrated part of the study of the power spectrum reconstruction. We have shown that the MRL method using the the combined data can reconstruct a power spectrum capable of providing a huge fit to the data. However only reconstruction of a spectrum does not guarantee its evidence. It is expected that the observed data is likely to reside within its errors. At this stage it is important to address what is the range of uncertainty in the primordial power spectrum corresponding to the error bars associated to the observed data and whether the power law power spectrum falls within the uncertainty band. As discussed in the previous section, we have generated 1000 realizations from the actual data using random noise with Gaussian distributions of variance associated to the errors of the data. Using MRL algorithm we find 1000 reconstructed power spectrum from which, in each k, we have chosen the most concentrated region containing 68.3% and 95.5%

spectra corresponding to 1σ and 2σ error bands respectively. We do not use distributions around the mean value as we think it might be biased towards the spectrum providing better fit only. We have implemented this statistic to obtain the error bands for the case of reconstruction with the un-binned and binned data (see, fig.4) and the combined data (see, fig.5). These 3 figures contain the 1σ and 2σ errors associated to the power spectra indicated by blue and cyan region respectively. Notice that the left panel of fig. 4, which illustrates the result of the un-binned data indicates an overall enhancement of power at high k values ($k \sim 0.1$ Mpc^{-1}). We would like to mention that this is simply the artifact of imposing zero values for the negative C_ℓ's. However, the reconstruction with the binned data does not contain any such amplifications for obvious reasons. It should be emphasized once again that binned reconstruction does not *directly* provide a better fit to the complete dataset.

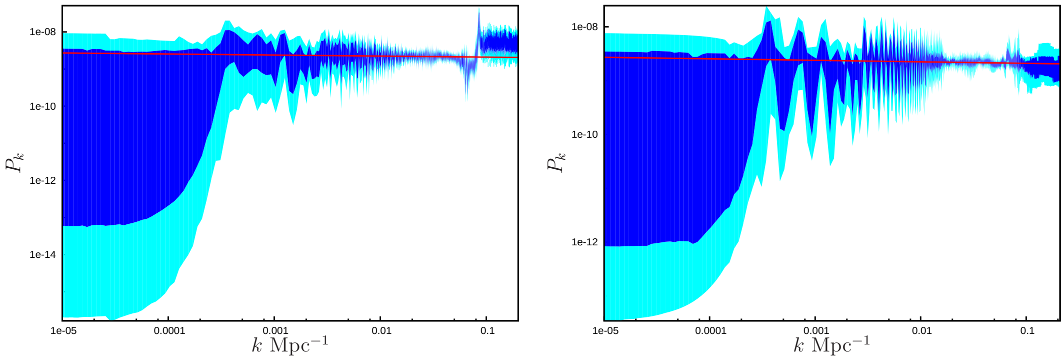

Figure 4: The 1-σ (blue) and 2-σ (cyan) estimated error-band associated to reconstructed primordial power spectrum. On the left we have plotted the results obtained from the analysis of the un-binned data only and on to the right we have plotted the corresponding results from the binned reconstruction. The red line is the best fit power law primordial power spectrum from WMAP-9.

The red line in the 3 plots in figures 4 and 5 describes the best fit power law primordial spectrum. As it is evident from these 3 figures, we would like to report that in all the cases we find that power law spectrum lies *very much* within the 1σ errors of the data. Now, as we have argued, we find our result using the combined data is free from the unwanted low scale amplification of power and it directly provides a better likelihood to the total data. It is expected with the upcoming results from Planck we shall have access to tightly constrained C_ℓ's. It will be exciting to examine whether we can achieve a highly constrained primordial power spectrum using similar formalism. Further as Planck data will provide the C_ℓ's till $\ell \sim 2000$ (considerably higher than WMAP), it will certainly allow the reconstruction for wider cosmological scales.

Finally we should mention that, we have incorporated the polarization data by considering the combination of $C_\ell^{\rm TT} + 2C_\ell^{\rm TE} + C_\ell^{\rm EE}$ following the similar reconstruction procedure. We find the reconstructed power spectrum is almost similar to the spectrum reconstructed from the TT data alone without significant improvement in the fit which simply reflects the low quality of the current polarization data for the purpose of PPS reconstruction.

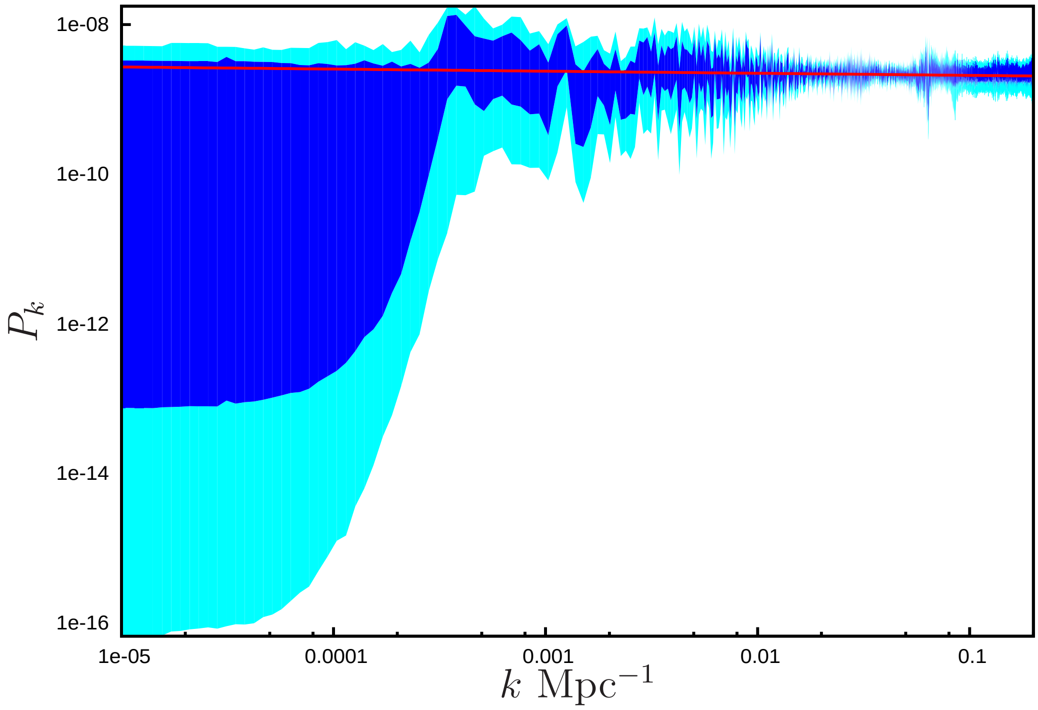

Figure 5: The 1-σ (blue) and 2-σ (cyan) estimated error-band associated to reconstructed primordial power spectrum obtained using the combined data, which we find to be the optimum choice for the power spectrum reconstruction. Like the previous figure, the red line stands for the best fit power law primordial spectrum, indicating that it is still well within the above said 1σ errorband.

4 Discussion

In this paper we have made a new modification to the Richardson-Lucy deconvolution algorithm [53, 60, 61, 65, 78] by using the whole un-binned correlated CMB data directly in the reconstruction of the primordial power spectrum. We have shown that the new modification can improve the efficiency of the method significantly in gaining better fit to the data and reducing the computational expenses. We have applied the method on the most recent CMB data from WMAP 9 years observations and we managed to get a better fit of more than 300 in χ^2 with respect to the best fit power-law. We have demonstrated, that unlike the binned reconstruction, to get a betterment of fit to the whole CMB data using the likelihood code provided by WMAP team we do not have to smooth the reconstructed results. However, due to the presence of some negative and unphysical C_l's in the un-binned data the reconstructed power spectra have been contaminated with an unphysical enhancement in power at small scales. This is due to the fact that RL method is applicable only on positive definite matrices and both C_l and $P(k)$ should be positive. We have solved this enhancement problem by combining the un-binned and binned data. We have used the un-binned data till the multipole where the quality of the data is good, basically for $l < 900$ and have used the binned data for the multipoles after it. We have shown that using the MRL (Modified Richardson-Lucy) algorithm with the combined data is an optimum choice for reconstruction of the primordial spectrum. We have also performed smoothing on the reconstructed results to get variety of the cases with different fluctuations that could rise to the high likelihood to the data. We have presented a few samples of reconstructed smooth primordial power spectrum and the corresponding angular power spectra which provide better likelihoods w.r.t. the power law spectrum. Here we should also mention that we have implemented the smoothing to obtain a primordial power spectrum that can be generated with only a handful of parameters such that this can be motivated from an inflationary scenarios. After comparing the effect of the width of smoothing filter on the total likelihood and the likelihood obtained from the diagonal term of the inverse covariance matrix we argue that our method of using the combined data (unbinned and binned) in MRL algorithm is a robust assumption. Finally to obtain the error band for the reconstructed primordial spectrum associated to the uncertainty of the observational data we generated 1000 datasets bootstrapped from the original C_l's. We find that though the features provide a huge better fit to the data, all these features could be presented because of random fluctuations and noise and in fact power law is still very much consistent to the CMB observations. We should emphasis here that there is a delicate difference between reconstruction and falsification. While doing falsification we test possibility of the observed data given the model, in the reconstruction approach we try to look at all phenomenological possibilities. In this work we have done both of these tasks and while we see that power-law is consistent to the data, we present handful of cases and forms for the primordial spectrum that all can give very good fit to the data. With the upcoming data from Planck we expect that it will be possible to use un-binned data up to higher range of multiples and with our proposed procedure we will be able to probe smaller scales of primordial power spectrum. Further, we also expect to get a tighter errorband on the primordial spectrum than what we have achieved with WMAP-9 using high quality of Planck polarization data. In fact the quality of the WMAP polarization data was very much inferior to its temperature data and we could not achieve any significant better fit to the whole data by incorporation of WMAP polarization data. While in this paper we proposed a new and easy to use approach for the reconstruction of the primordial spectrum, in a companion paper we perform the important task of cosmological parameter estimation allowing the free form of the PPS [79].

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References

[1] G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett, J. Dunkley, M. R. Nolta and M. Halpern *et al.*, arXiv:1212.5226 [astro-ph.CO].

[2] J. Hamann, A. Shafieloo and T. Souradeep, JCAP **1004** (2010) 010 [arXiv:0912.2728 [astro-ph.CO]].

[3] H. V. Peiris and L. Verde, Phys. Rev. D **81** (2010) 021302 [arXiv:0912.0268 [astro-ph.CO]].

[4] L. Samushia, B. A. Reid, M. White, W. J. Percival, A. J. Cuesta, L. Lombriser, M. Manera and R. C. Nichol *et al.*, Mon. Not. Roy. Astron. Soc. **429** (2013) 1514 [arXiv:1206.5309 [astro-ph.CO]].

[5] B. A. Reid, L. Samushia, M. White, W. J. Percival, M. Manera, N. Padmanabhan, A. J. Ross and A. G. Sanchez *et al.*, arXiv:1203.6641 [astro-ph.CO].

[6] C. Blake, S. Brough, M. Colless, C. Contreras, W. Couch, S. Croom, D. Croton and T. Davis *et al.*, Mon. Not. Roy. Astron. Soc. **425** (2012) 405 [arXiv:1204.3674 [astro-ph.CO]].

[7] A. A. Starobinsky, Phys. Lett. B **117** (1982) 175.

[8] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49** (1982) 1110.

[9] L. A. Kofman and D. Y. .Pogosian, Phys. Lett. B **214** (1988) 508.

[10] D. S. Salopek, J. R. Bond and J. M. Bardeen, Phys. Rev. D **40** (1989) 1753.

[11] A. A. Starobinsky, JETP Lett. **55** (1992) 489 [Pisma Zh. Eksp. Teor. Fiz. **55** (1992) 477].

[12] D. Polarski and A. A. Starobinsky, Nucl. Phys. B **385** (1992) 623.

[13] J. A. Adams, G. G. Ross and S. Sarkar, Nucl. Phys. B **503** (1997) 405 [hep-ph/9704286].

[14] D. J. H. Chung, E. W. Kolb, A. Riotto and I. I. Tkachev, Phys. Rev. D **62** (2000) 043508 [hep-ph/9910437].

[15] J. Martin, A. Riazuelo and M. Sakellariadou, Phys. Rev. D **61** (2000) 083518 [astro-ph/9904167].

[16] J. Martin and R. H. Brandenberger, Phys. Rev. D **63** (2001) 123501 [hep-th/0005209].

[17] J. Barriga, E. Gaztanaga, M. G. Santos and S. Sarkar, Mon. Not. Roy. Astron. Soc. **324** (2001) 977 [astro-ph/0011398].

[18] J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64** (2001) 123514 [astro-ph/0102236].

[19] U. H. Danielsson, Phys. Rev. D **66** (2002) 023511 [hep-th/0203198].

[20] C. R. Contaldi, M. Peloso, L. Kofman and A. D. Linde, JCAP **0307** (2003) 002 [astro-ph/0303636].

[21] O. Elgaroy, S. Hannestad and T. Haugboelle, JCAP **0309** (2003) 008 [astro-ph/0306229].

[22] N. Kaloper and M. Kaplinghat, Phys. Rev. D **68** (2003) 123522 [hep-th/0307016].

[23] H. V. Peiris *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **148** (2003) 213 [astro-ph/0302225].

[24] J. Martin and C. Ringeval, Phys. Rev. D **69** (2004) 083515 [astro-ph/0310382].

[25] J. Martin and C. Ringeval, Phys. Rev. D **69** (2004) 127303 [astro-ph/0402609].

[26] J. Martin and C. Ringeval, JCAP **0501** (2005) 007 [hep-ph/0405249].

[27] P. Hunt and S. Sarkar, Phys. Rev. D **70** (2004) 103518 [astro-ph/0408138].

[28] M. Bridges, A. N. Lasenby and M. P. Hobson, Mon. Not. Roy. Astron. Soc. **369** (2006) 1123 [astro-ph/0511573].

[29] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. **97** (2006) 191304 [hep-ph/0605035].

[30] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen and A. Mazumdar, JCAP **0706** (2007) 019 [hep-ph/0610134].

[31] J. C. Bueno Sanchez, K. Dimopoulos and D. H. Lyth, JCAP **0701** (2007) 015 [hep-ph/0608299].

[32] L. Covi, J. Hamann, A. Melchiorri, A. Slosar and I. Sorbera, Phys. Rev. D **74** (2006) 083509 [astro-ph/0606452].

[33] J. M. Cline and L. Hoi, JCAP **0606** (2006) 007 [astro-ph/0603403].

[34] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **170** (2007) 377 [astro-ph/0603449].

[35] P. Hunt and S. Sarkar, Phys. Rev. D **76** (2007) 123504 [arXiv:0706.2443 [astro-ph]].

[36] J. Hamann, L. Covi, A. Melchiorri and A. Slosar, Phys. Rev. D **76** (2007) 023503 [astro-ph/0701380].

[37] M. Joy, V. Sahni and A. A. Starobinsky, Phys. Rev. D **77** (2008) 023514 [arXiv:0711.1585 [astro-ph]].

[38] M. Joy, A. Shafieloo, V. Sahni and A. A. Starobinsky, JCAP **0906** (2009) 028 [arXiv:0807.3334 [astro-ph]].

[39] R. K. Jain, P. Chingangbam, J. -O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901** (2009) 009 [arXiv:0809.3915 [astro-ph]].

[40] C. Pahud, M. Kamionkowski and A. RLiddle, Phys. Rev. D **79** (2009) 083503 [arXiv:0807.0322 [astro-ph]].

[41] R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, JCAP **1006** (2010) 009 [arXiv:0907.2916 [hep-th]].

[42] M. J. Mortonson, C. Dvorkin, H. V. Peiris and W. Hu, Phys. Rev. D **79** (2009) 103519 [arXiv:0903.4920 [astro-ph.CO]].

[43] R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82** (2010) 023509 [arXiv:0904.2518 [astro-ph.CO]].

[44] C. Dvorkin and W. Hu, Phys. Rev. D **81** (2010) 023518 [arXiv:0910.2237 [astro-ph.CO]].

[45] N. Barnaby and Z. Huang, Phys. Rev. D **80** (2009) 126018 [arXiv:0909.0751 [astro-ph.CO]].

[46] K. Ichiki, R. Nagata and J. 'i. Yokoyama, Phys. Rev. D **81** (2010) 083010 [arXiv:0911.5108 [astro-ph.CO]].

[47] D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010** (2010) 008 [arXiv:1005.2175 [astro-ph.CO]].

[48] W. Hu, Phys. Rev. D **84** (2011) 027303 [arXiv:1104.4500 [astro-ph.CO]].

[49] M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798 [astro-ph.CO].

[50] D. K. Hazra, JCAP **03** (2013) 003 [arXiv:1210.7170 [astro-ph.CO]].

[51] S. Hannestad, Phys. Rev. D **63** (2001) 043009 [astro-ph/0009296].

[52] M. Tegmark and M. Zaldarriaga, Phys. Rev. D **66** (2002) 103508 [astro-ph/0207047].

[53] A. Shafieloo and T. Souradeep, Phys. Rev. D **70** (2004) 043523 [astro-ph/0312174].

[54] S. L. Bridle, A. M. Lewis, J. Weller and G. Efstathiou, Mon. Not. Roy. Astron. Soc. **342** (2003) L72 [astro-ph/0302306].

[55] P. Mukherjee and Y. Wang, Astrophys. J. **599** (2003) 1 [astro-ph/0303211].

[56] S. Hannestad, JCAP **0404** (2004) 002 [astro-ph/0311491].

[57] D. Tocchini-Valentini, Y. Hoffman and J. Silk, Mon. Not. Roy. Astron. Soc. **367** (2006) 1095 [astro-ph/0509478].

[58] N. Kogo, M. Sasaki and J. 'i. Yokoyama, Prog. Theor. Phys. **114** (2005) 555 [astro-ph/0504471].

[59] S. M. Leach, Mon. Not. Roy. Astron. Soc. **372** (2006) 646 [astro-ph/0506390].

[60] A. Shafieloo, T. Souradeep, P. Manimaran, P. K. Panigrahi and R. Rangarajan, Phys. Rev. D **75** (2007) 123502 [astro-ph/0611352].

[61] A. Shafieloo and T. Souradeep, Phys. Rev. D **78** (2008) 023511 [arXiv:0709.1944 [astro-ph]].

[62] R. Nagata and J. 'i. Yokoyama, Phys. Rev. D **78** (2008) 123002 [arXiv:0809.4537 [astro-ph]].

[63] R. Nagata and J. 'i. Yokoyama, Phys. Rev. D **79** (2009) 043010 [arXiv:0812.4585 [astro-ph]].

[64] K. Ichiki and R. Nagata, Phys. Rev. D **80** (2009) 083002.

[65] A. Shafieloo and T. Souradeep, New J. Phys. **13** (2011) 103024 [arXiv:0901.0716 [astro-ph.CO]].

[66] P. Paykari and A. H. Jaffe, Astrophys. J. **711** (2010) 1 [arXiv:0902.4399 [astro-ph.CO]].

[67] G. Nicholson and C. R. Contaldi, JCAP **0907** (2009) 011 [arXiv:0903.1106 [astro-ph.CO]].

[68] G. Nicholson, C. R. Contaldi and P. Paykari, JCAP **1001** (2010) 016 [arXiv:0909.5092 [astro-ph.CO]].

[69] M. Bridges et al, Mon. Not. R. Astron. Soc. **400** 1075B (2009)

[70] C. Gauthier and M. Bucher, JCAP **1210** (2012) 050 [arXiv:1209.2147 [astro-ph.CO]].

[71] See, `http://www.sciops.esa.int/PLANCK/`.

[72] B. H. Richardson, J. Opt. Soc. Am. **62** 55 (1972)

[73] L. B. Lucy, Astron. J. **79** 6 (1974)

[74] C. M. Baugh and G. Efstathiou, Mon. Not. R. Astron. Soc. **265** 145 (1993)

[75] C. M. Baugh and G. Efstathiou, Mon. Not. R. Astron. Soc. **267** 323 (1994)

[76] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. **538** (2000) 473 [astro-ph/9911177].

[77] See, `http://camb.info/`.

[78] T. Souradeep and A. Shafieloo, Prog. Theor. Phys. Suppl. **172** (2008) 156.

[79] D. K. Hazra, A. Shafieloo and T. Souradeep (In prepration)