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GRAVITATIONAL SCREENS AND SUPERLUMINAL SEPARATION IN QUASARS

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ABSTRACT

The differential bending of radio waves by a suitably placed "gravitational screen" such as an intervening galaxy can lead to large magnification of the separation velocity of the components in the nuclear region of a quasar. It is suggested that the apparent superluminal separation of such components observed in some quasars by the VLBI techniques could be due to velocity magnifications of this type. The astrophysical feasibility of this explanation is critically examined, and an optical search for objects of large mass-to-light ratio en route to quasars showing superluminal separation is advocated.

Subject headings: quasars — radio sources: general

I. INTRODUCTION

The very long base line interferometric measurements have yielded considerable information about the detailed structure of the extragalactic radio sources. Several radio sources monitored with VLBI over a number of years reveal components in their nuclei which are apparently separating at large multiples of the speed of light, a remarkable and puzzling set of observations for both their astrophysical and cosmological implications.

One of the most straightforward explanations of the apparent superlight motion would be that the sources are local. The observed motions will then turn out to be subluminal if the objects are situated at distances which are much closer than those implied by the Hubble interpretation of their redshifts. Because of the ongoing controversy about the interpretation of the nature of the quasar redshifts and as all but one of the observed superluminal cases are of QSOs, it is not altogether unlikely that the explanations based on the superluminal separation of the components might even be unnecessary. The exceptional case is that of the galaxy 3C 120 whose identification with a galaxy of stars has been questioned (Kellermann 1978).

Another class of explanations invoke so-called illusions, where a contrived set of geometrical conditions create the effect of superluminal expansion even though in reality the motions of separating components are subluminal (McKee, Rees, and Blandford 1977). The first indication for apparent superlight motion came from the rapid intensity fluctuations in quasars, and the possibility of superlight expansion was raised by Rees (1967) who argued that the relativistic expansion of a source of speed v can result in its size increasing at an apparent speed of $v/(1 - v^2/c^2)^{1/2}$ $(\equiv \gamma v)$. The difference in arrival times of the signals from various parts of the source can then cause the apparent size to expand with a transverse velocity $\sim \gamma c (\gamma \gg 1)$. There are, however, difficulties with models which are based on or are some variant of the relativistic motion of the source. In any spherically symmetric expansion the components will be ejected isotropically; thus the expansion cannot accommodate the observed superlight motion which is largely aligned with the extended double components. It has also been realized that the Doppler-modified luminosity of the components in the relativistic models is at variance with their observed intensity ratio (Kellermann 1978).

An explanation based on a lens effect where the light rays from a distant source are gravitationally deflected by compact objects along the line of sight has been invoked to account for properties associated with the QSOs (Gott and Gunn 1974; Barnothy 1965, 1975). Although in the lens models for large gravitational deflections, the components may have comparable intensity, the effect turns out to be so weak that it is difficult to achieve apparent magnification much larger than the linear size of the lens. We propose here a model based on a "gravitational screen" in the form of some intervening clump of matter like a galaxy along the line of sight which progressively bends light rays emanating from the source behind it and in the process produces the appearance of superlight expansion. It should perhaps be stressed here that the differential gravitational bending by a density distribution of matter in the screen is different from the gravitational effects invoked in the lens models mentioned above. The physical idea is based on the fact that, apart from local irregularities, the density of matter in galaxies decreases outward from the center at a slow enough rate, and this leads to the situation where the net gravitational bending is likely to increase roughly at least halfway up to the rim of the galaxy and then to decrease outward. The inner regions of the intervening galaxy are then likely to lead to a substantial magnification of the source-image situated near a conjugate point of the optical system provided by the galaxy. A preliminary look at the problem (Chitre and Narlikar 1979) suggested that for the parameters usually associated with galaxies, the observed velocities of expansion come out to be very much in excess of the speed of light. In this paper we examine this idea in more detail, both in its theoretical foundations and in its astrophysical feasibility.

The remaining part of the paper is organized as follows. In § II we briefly discuss the deflection by gravity in the Newtonian and relativistic theories. Since this calculation is generally found in many textbooks on relativity, we shall limit ourselves to specifying our notation and to the statement of the results which will be needed later. The optics of gravitational bending is considered in § III, and its applications to radio sources are indicated in § IV. Finally, the astrophysical feasibility of the model is discussed in § V.

II. THE BENDING OF LIGHT BY GRAVITY

a) The Newtonian Theory

Following the usual adaptation of Newtonian gravity to include photons, we consider the scenario described in Figure 1. A ray of light Γ starting from a source S reaches the observer O after passing through a spherical mass distribution $\mathscr{G}$. Let M denote the mass of $\mathscr{G}$ and R its radius. Choose polar coordinates (r, ϕ) in the plane of Γ and the center C of $\mathscr{G}$ such that r measures the distance of a typical point P (in this plane) from C and ϕ measures the angle OCP. Let $Mm(r)$ denote the mass of the matter contained in the sphere with center C and radius CP. If $\rho(r)$ is the density distribution in $\mathscr{G}$,

$$m(r) = \int_0^r \frac{4\pi}{M}\rho(x)x^2dx \qquad \text{for} \qquad r \leq R\,, \tag{1}$$

$$m(r) = 1 \qquad \text{for} \qquad r \geq R\,.$$

By the Newtonian law of gravitation, a photon at P is attracted by the mass $Mm(r)$. Let the ray Γ have an apsidal point at (a, ϕ_0), so that the "angular momentum per unit mass" of the photon is given by $h = ac$. Writing $u = 1/r$, the equation of the ray Γ is then given by

$$u(\phi) = \frac{1}{a}\cos(\phi - \phi_0) + \int_{\phi_0}^{\phi} \frac{GM}{a^2c^2} m\left(\frac{1}{u}\right) \sin(\phi - \phi_1)d\phi_1\,. \tag{2}$$

This nonlinear integral equation can be solved approximately in the case of weak gravity. The solution is

$$u(\phi) = \frac{1}{a}\cos(\phi - \phi_0) + \frac{GM}{a^2c^2}\int_0^{\phi-\phi_0} m(a\sec\phi_1)\sin(\phi - \phi_0 - \phi_1)d\phi_1\,. \tag{3}$$

If the endpoints of Γ have the coordinates, $O:(d, 0)$ and $S:(\mathrm{L}, \alpha)$, we have two equations from (3) to determine

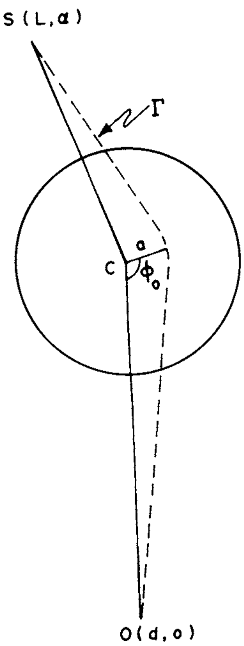

FIG. 1.—The gravitational bending of a ray of light Γ from a source S reaching the observer O after the passage through a spherical mass-distribution $\mathscr{G}$ with the center of mass C. The light ray has an apsidal point given by the polar coordinates (a, ϕ_0), and the endpoints of the ray have coordinates, $O:(d, 0)$ and $S:(L, \alpha)$.

the two parameters (a, ϕ_0) which characterize Γ. It is easily verified that for $d \gg a$, $L \gg a$, the net bending of Γ is given by

$$\Delta_N \approx \frac{2GM}{ac^2} \int_0^{\pi/2} m(a \sec \phi) \cos \phi d\phi \,. \tag{4}$$

For a ray lying entirely outside $\mathcal{G}$, $m(a \sec \phi) = 1$ and we recover the familiar value of $2GM/ac^2$.

b) *General Relativity*

The Newtonian theory has been superseded by the general theory of relativity, and it may therefore appear that the above Newtonian analysis is not relevant to the present discussion. Such a conclusion is, however, not correct in the context of weak gravity. The so-called PPN formalism (cf. Misner, Thorne, and Wheeler 1973) can be used to show that the relativistic bending Δ_R is twice the Newtonian bending Δ_N as derived above. Keeping this difference in mind, it is possible to use the Newtonian framework. Since the formula (3) will be used in subsequent work, it is desirable to establish a direct correspondence between the (more complicated) relativistic framework and the Newtonian framework. We therefore give below the relativistic expressions for the bending of light.

As before, we assume $\mathcal{G}$ to be a spherically symmetric distribution of matter. Using the coordinates (r, θ, ϕ, t), we write the spacetime metric in the form

$$ds^2 = c^2 e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2(d\theta + \sin^2 \theta d\phi^2) \,. \tag{5}$$

Outside $\mathcal{G}$, i.e., for $r > R$, we have

$$e^{\nu} = e^{-\lambda} = 1 - 2GM/c^2 r \,. \tag{6}$$

Inside $\mathcal{G}$, i.e., for $r \leq R$, we need to know the energy momentum tensor of matter in $\mathcal{G}$. In terms of $u = 1/r$ and ϕ, the light track Γ satisfies the differential equation

$$\frac{d^2 u}{d\phi^2} + u = F'(u) \,, \tag{7}$$

where

$$F(u) = \frac{GM}{c^2} m u^3 + \frac{1}{2} A^2 e^{-(\lambda+\nu)} \,, \qquad A = \text{constant} \,. \tag{8}$$

In a problem involving strong gravitational fields, the field equations of relativity must be solved to determine λ and ν and then the equation (7) solved. In a weak field situation, with which we will be concerned here, the above problem is simplified. The weak field situation is characterized by these assumptions: (i) the geometry is non-Euclidean to only first order, (ii) the bending of Γ is small, and (iii) $p \ll \rho c^2$, where p and ρc^2 are the pressure and energy density inside $\mathcal{G}$.

It is then easy to verify that $A = 1/a$, and

$$F'(u) \approx \frac{4\pi G}{uc^2} \left(\bar{\rho} - \rho + \frac{\rho}{a^2 u^2} \right) \cdot \tag{9}$$

Here $\bar{\rho}$ is the "mean density" out to radial coordinate r:

$$Mm(r) = \frac{4\pi}{3} \bar{\rho}(r) r^3 \,. \tag{10}$$

The relativistic bending is given by

$$\Delta_R \approx \frac{8\pi G a^2}{c^2} \int_0^{\pi/2} (\bar{\rho} + \rho \tan^2 \phi) d\phi \,. \tag{11}$$

In this notation the Newtonian bending (4) may be written as

$$\Delta_N \approx \frac{4\pi G a^2}{3c^2} \int_0^{\pi/2} \bar{\rho} \sec^2 \phi d\phi \,. \tag{12}$$

A straightforward analysis will show that

$$\Delta_R = 2\Delta_N \,. \tag{13}$$

Thus, whatever the density distribution inside $\mathcal{G}$, provided it meets the above weak field criteria, we may use the Newtonian theory to calculate the bending produced and double the answer. Although this conclusion is to be expected from the general considerations of the PPN formalism, we have described it here in detail because usually the PPN formalism is applied to the bending produced outside $\mathscr{G}$. Also, in working from first principles it becomes clearer as to which terms are important and which are not.

In § V we will use the Newtonian formula to calculate Δ_N and Δ_R for the various density profiles of $\mathscr{G}$. We next consider the types of images to be expected by this process. It will then be possible to apply these results to the problems of quasars.

III. THE OPTICS OF GRAVITATIONAL BENDING

a) Focusing

Is it possible for the above system to focus rays from S at O? An affirmative answer requires a bunch of rays in the neighborhood of Γ to start from S and end at O. Compared to Γ, a neighboring ray would have different values of a and ϕ_0. However, since a and ϕ_0 are not independent, once the endpoints are specified, we may express the above requirement by demanding that a small variation of (3) under $a \to a + \delta a$, $\phi_0 \to \phi_0 + \delta\phi_0$ should still make it possible to satisfy the relation at S and O.

To express this condition mathematically, we apply (3) at S and O. It is convenient to define, as in Figure 2,

$$\phi_0 = \tfrac{1}{2}\pi + \beta\,, \qquad \alpha = \pi + \gamma\,, \tag{14}$$

where $\beta > 0$, $\gamma > 0$ are expected to be small for the systems under consideration. With suitable approximations we therefore get the following relations when (3) is applied respectively at S and O:

$$\gamma - \beta = -\frac{a}{L} + \frac{GM}{ac^2}\int_0^{\pi/2} m(a \sec\phi)\cos\phi d\phi\,, \tag{15}$$

$$\beta = -\frac{a}{d} + \frac{GM}{ac^2}\int_0^{\pi/2} m(a \sec\phi)\cos\phi d\phi\,. \tag{16}$$

Eliminating β, we get

$$\gamma = \frac{2GM}{ac^2}\int_0^{\pi/2} m(a \sec\phi)\cos\phi d\phi - a\left(\frac{1}{L} + \frac{1}{d}\right) \cdot \tag{17}$$

If we let L, $d \to \infty$, we recover the relation (4) for Δ_N. We now consider variation of (17) for a small change in the apsidal distance a, keeping γ, L, and d fixed:

$$0 = -\frac{2GM}{a^2c^2}\int_0^{\pi/2} m(a \sec\phi)\cos\phi d\phi - \left(\frac{1}{L} + \frac{1}{d}\right) + \frac{2GM}{c^2a}\int_0^{\pi/2} m'(a \sec\phi)d\phi\,.$$

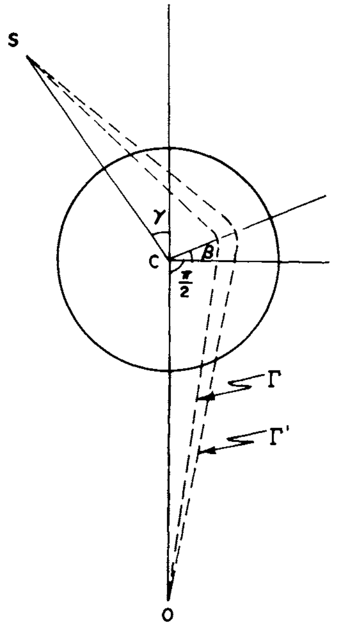

FIG. 2.—Two neighboring rays Γ and Γ' from the source S arriving at the observer O. The apsidal point of Γ is denoted by $(a, \pi/2 + \beta)$, and the coordinates of S are taken as $(L, \pi + \gamma)$.

Using ρ and $\bar{\rho}$ from (10) and (12), this relation becomes

$$\frac{1}{L}+\frac{1}{d}=\frac{8\pi Ga}{c^2}\int_0^{\pi/2}(\rho-\tfrac{1}{3}\bar{\rho})\sec^2\phi d\phi \equiv \Delta_N'(a)\,. \tag{18}$$

This is the condition for focusing, i.e., for O and S to be conjugate points for the optical system $\mathscr{G}$.

Notice first that for rays lying entirely outside $\mathscr{G}$, focusing is impossible. For we have $\rho = 0$, $\bar{\rho} > 0$, and the right-hand side is negative. Indeed, from (18) we see that focusing is possible only for rays going through $\mathscr{G}$ and for which the integral is positive.

If near the center of $\mathscr{G}$, $\rho(r)$ varies as

$$\rho(r) > r^{-n}\,, \tag{19}$$

then we see that $\rho > \frac{1}{3}\bar{\rho}$ requires

$$n < 2\,. \tag{20}$$

We shall return to this point in § V.

What is the significance of (18)? In optical jargon, a real image of S is formed at O. If, on the other hand, an observer at O is viewing the source S, the apparent image of S seen by O is enormously magnified. We will make use of this property in the explanation for the apparent superluminal separation.

b) *Virtual Images*

The formation of virtual images is illustrated in Figure 3. Taking $OS' = \rho$ and the angle $SOS' = \psi$, where S' is the virtual image of S as seen from O, we get

$$\tan\psi = u\left.\frac{d\phi}{du}\right|_0, \qquad \rho = \frac{du}{d\psi}\sin\psi|_0\,. \tag{21}$$

The astronomer is usually able to measure only ψ directly, and hence we will not calculate ρ. For weak gravitational fields ψ is expected to be small so that we may set $\psi \approx \tan\psi$ in (21). The use of (3) then gives

$$\psi \approx \frac{a}{d}\left[1-\frac{GM}{ac^2}\int_0^{\pi/2} m(a\sec\phi)\sin\phi d\phi\right]^{-1}\,. \tag{22}$$

What is the value of a to be used in (22)? It is given by (17). In the case of external bending (i.e., Γ always outside $\mathscr{G}$) it is easy to see that in general there are two images, for from (17) we have to solve the quadratic equation for a:

$$\gamma = \frac{2GM}{ac^2} - a\left(\frac{1}{L}+\frac{1}{d}\right)\,. \tag{23}$$

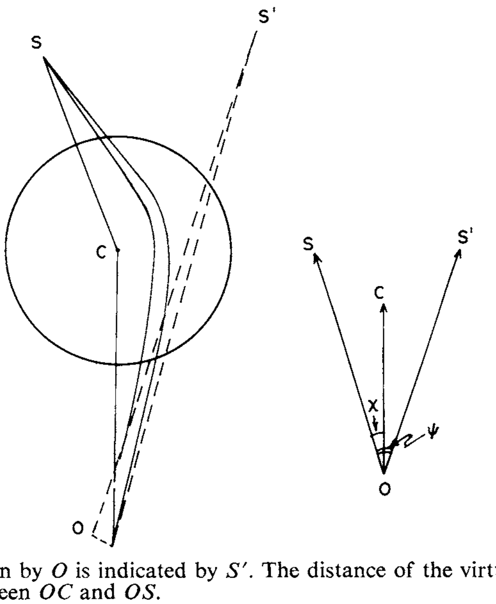

FIG. 3.—The virtual image of S as seen by O is indicated by S'. The distance of the virtual image from the observer, $OS' = \rho$ and $SOS' = \psi$, while χ is the angle between OC and OS.

This has two real roots for a, one positive, one negative. The negative root may be interpreted as giving an image "on the other side" of S. It is of course essential in this case for both roots to lie outside $\mathscr{G}$, i.e., to have magnitudes not less than R. For symmetrically located images ($\gamma = 0$) this condition is

$$R \leq \left[\frac{2GMLd}{c^2(L+d)}\right]^{1/2} . \tag{24}$$

We will return to this condition in § V. (For the relativistic case, replace M by $2M$ in [24], as the bending is double.)

For internal bending the number of images cannot be determined unless we know the form of $m(r)$. If very near the center of $\mathscr{G}$, $\rho(r) \approx A - Br$, where A, B are constants, then the integral in (17) is of the form $pa^3 - qa^4$, where p, q are constants, and we get a *cubic* in a. Thus three images may be possible in this case.

c) *Magnification*

In Figure 3, let χ denote the angle SOC. We then have from (17)

$$\chi = \frac{\gamma L}{L+d} = \frac{\Delta L}{L+d} - \frac{a}{d} , \tag{25}$$

where Δ is given by (4) in the Newtonian case and by twice its value in the relativistic case. If now S is displaced slightly toward the right, χ *decreases* slightly. The image S' is also displaced slightly to the right, thus *increasing* ψ slightly. The relative displacement of the image and the source may be termed "magnification," and this is given by

$$f = \left|\frac{d\psi}{d\chi}\right| = \left[1 - \frac{Ld}{L+d}\Delta'(a)\right]^{-1} . \tag{26}$$

Here we have neglected the integral in (22) in comparison with unity. Notice that $f \gg 1$ if the expression in the square brackets vanishes. But this is precisely the condition for the points O and S to be conjugate, as obtained in § III*a*.

IV. APPLICATIONS TO RADIO SOURCES

We now consider the application of these results to radio sources as indicated in the Introduction. In a typical situation envisaged there, the two components A, B of a double radio source are found to separate from each other at a superluminal speed. Here we consider three different scenarios, in each of which the gravitational bending of radio waves from A and B produced by some intervening clump of matter like a galaxy creates the *impression* of superluminal separation whereas in reality the separation is subluminal. The various astrophysical issues relating to these scenarios will be discussed in the following section where we argue for and against the feasibility of these models.

a) *The Separation of Components in a Stationary Source*

Here we imagine the separation of the components A and B at a subluminal speed v (i.e., $v < c$) but with the central nucleus having no transverse motion relative to the observer O. Figure 4 illustrates the situation where we have taken the center C of the intervening clump of matter $\mathscr{G}$ to be in the plane OAB. In § V we will investigate the consequence of C lying outside the plane OAB. The bending produced by $\mathscr{G}$ causes the images of A and B to be formed at A' and B', and these images move as A and B move away from each other. What is observed is the separation of A' and B', not of A and B; and we now show that large magnifications can raise the observed value of separation speed by a substantial factor.

Let l be the actual distance AB, and let $\Delta\chi$ be the angle subtended by AB at O. In our earlier notation we have

$$\Delta\chi(d+L) \approx l_\perp , \tag{27}$$

where $l_\perp$ is the projection of l perpendicular to OC. We similarly write the transverse speed as

$$v_\perp = \dot{l}_\perp , \tag{28}$$

where the dot denotes differentiation with respect to time. (Strictly speaking, we should make a difference between the time at the source and the time at the observer. This subtlety, however, turns out to be unnecessary to illustrate our present point.)

The estimated observed separation is, on the other hand,

$$l' = \Delta\psi(d+L) = f\Delta\chi(d+L) = fl_\perp , \tag{29}$$

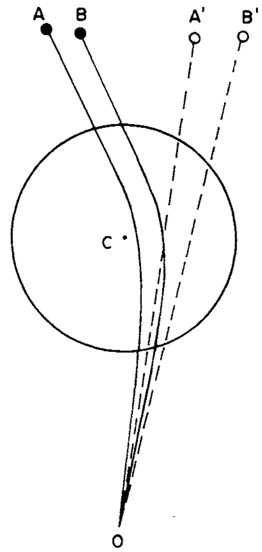

FIG. 4.—The images of two components A, B produced by the gravitating clump of matter $\mathscr{G}$ are formed at A' and B', respectively. Here the center C of $\mathscr{G}$ is taken to lie in the plane OAB.

where f is given by (26). Hence the separation speed estimated on the basis of actual observations is

$$v' \equiv \frac{dl'}{dt} = fv_\perp = \left[1 - \frac{Ld}{L + d}\Delta'(a)\right]^{-1} v_\perp . \tag{30}$$

Thus near a conjugate point we expect a large magnification of the speed of separation. In § V we will estimate this with numbers.

b) *Stationary Components in a Transversely Moving Source*

This situation is different from the previous case in that the center of the source moves transversely, i.e., perpendicular to the line OC, but the actual separation between A and B is not taking place. Even in this case the images A', B' appear to separate because $\Delta'(a)$ changes across $\mathscr{G}$.

To estimate this effect, differentiate (29) with respect to t, keeping $l_\perp$ constant. We get

$$v' \equiv \frac{dl'}{dt} = l_\perp \frac{df}{dt} = l_\perp f^2 \Delta''(a)\frac{Ld}{L + d}\frac{da}{dt} \cdot \tag{31}$$

To estimate da/dt, we differentiate (25), noting that $(L + d)\dot{\chi}$ gives $-V_\perp$, where $V_\perp$ is the transverse speed of the source as a whole. We then get

$$\frac{da}{dt} = f\frac{d}{L + d}V_\perp . \tag{32}$$

Substitution of (32) into (31) gives the required answer:

$$v' = f^3\left(\frac{d}{L + d}\right)^2 l_\perp L\Delta''(a)V_\perp . \tag{33}$$

Notice that whereas in case (a) we had the effect due to *differential* bending, here we have the effect arising from *the second derivative* of the bending, $\Delta(a)$.

c) "*In Situ*" *Bending*

It is believed that the central regions of radio sources are locations of energy machines for generating fast particles. These regions are expected to be dense, so it is tempting to suggest that a substantial bending of radio waves from A and B takes place *in situ.* In Figure 5 we illustrate this situation with A, B lying *inside* $\mathscr{G}$ and the midpoint of AB coinciding with C, the center of mass of $\mathscr{G}$.

In this case we expect α to be of the same order as ϕ_0, i.e., $|\alpha - \phi_0| \ll \pi/2$. The relation (3) then gives $L \approx a$, where now $2L$ = length of AB. We therefore have

$$|\chi| \approx a/d , \tag{34}$$

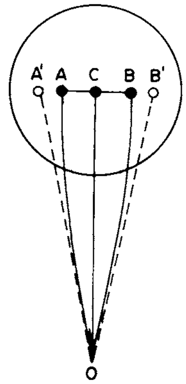

FIG. 5.—The images A' and B' produced by the *in situ* bending of light rays form A and B, where the midpoint of AB coincides with the center C of $\mathscr{G}$.

while $|\psi|$ is given by (22), i.e., by

$$|\psi| \approx \frac{a}{d}(1 - \xi)^{-1}, \tag{35}$$

where

$$\xi = \frac{GM}{ac^2}\int_0^{\pi/2} m(a \sec \phi) \sin \phi d\phi . \tag{36}$$

The velocity magnification is therefore negligible unless $\xi \approx 1$. In cases (a) and (b) we ignored the factor $(1 - \xi)^{-1}$ on the assumption that $\xi \ll 1$. However, in the dense mass concentrations near C, we expect that this factor may be very high.

V. DISCUSSION OF THE FEASIBILITY OF THE MODEL

a) *Density Distributions*

We shall now consider the astrophysical feasibility of the proposed model by computing the differential gravitational bending produced by a variety of density distributions in the gravitational screen and estimate the resultant magnification of the source image. To fix ideas, we choose the following density profiles which have been suggested to represent possible matter distributions in various types of galaxies (cf. King 1958; de Vaucouleurs 1959):

$$\begin{aligned}
&(\mathrm{A}) \qquad \rho(r) = \rho(0) \exp(-4r/R) ,\\
&(\mathrm{B}) \qquad \rho(r) = \rho(0)[1 - r^2/R^2]^{1/2} ,\\
&(\mathrm{C}) \qquad \rho(r) = \rho(0)(1 - r^2/R^2) ,\\
&(\mathrm{D}) \qquad \rho(r) = \rho(0)[1 + r^2/R^2]^{-1} ,\\
&(\mathrm{E}) \qquad \rho(r) = \rho(0)[1 + r^2/R^2]^{-5/2} .
\end{aligned} \tag{37}$$

The net Newtonian gravitational bending seen by a distant observer, $\Delta(a)$, in units of $\Delta_0 = 2GM/c^2R$, is shown in Figure 6 corresponding to the various density profiles. Here M is the mass of the intervening matter distribution $\mathscr{G}$ contained within the radial distance R.

It is clear from the plots that $\Delta(a)$ increases for $a/R \lesssim 0.75$ and then decreases slowly in the outer regions of $\mathscr{G}$, for all profiles except (A). In the case of (A) because of the steep density decline with radial distance, the maximum of $\Delta(a)/\Delta_0$ is attained closer to the center, at $a/R \approx 0.57$. However, in all cases $\Delta'(a) > 0$ for a substantial part of the interior of $\mathscr{G}$, a condition necessary for large magnifications. For example we have seen in § III that the focusing of light rays from the source S at the observer O is possible only if the light rays pass through the regions of $\mathscr{G}$ where the density decreases outward relatively slowly so that $\Delta'(a) > 0$. In each of the typical profiles described above, the density decreases outward more slowly than $1/r^2$ to imply $\rho > \frac{1}{3}\bar{\rho}$ in equation (18), in the inner regions of $\mathscr{G}$.

Let us first consider the situation where the two components have been ejected from the central nucleus and are moving out along a preferential direction. Assume that the central nucleus itself has no transverse motion relative to the observer. Then it follows from equation (30) that the apparent speed of separation is given by

$$v' = \left[1 - \frac{Ld}{L + d}\Delta'(a)\right]^{-1} v_\perp \equiv f v_\perp .$$

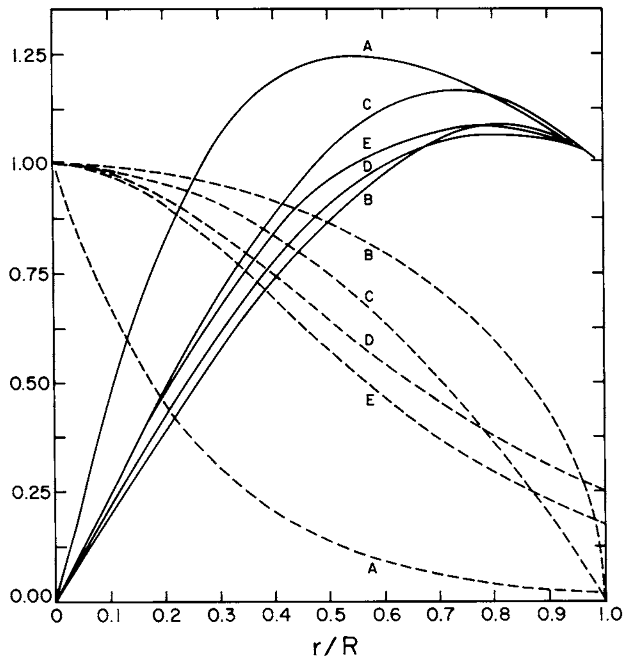

FIG. 6.—The net gravitational bending $\Delta(a)$ in units of $\Delta_0 = 2GM/c^2R$ produced at different radial distances from the center are indicated by full lines, and the corresponding distribution of density in units of $\rho(0)$ are shown by dashed lines. The labels A, B, C, D, E refer to the various density-profiles described in the text.

For rays passing through the inner regions of $\mathscr{G}$ the possibility therefore exists for $Ld\Delta'(a) \approx (L + d)$ and as a consequence for f to be $\gg 1$. Specifically, if z is the redshift of the source, we may set the distance to the source $(L + d) \lesssim 2 \times 10^{28} z$ cm, corresponding to Hubble constant of ~ 50 km s^{-1} Mpc^{-1}. Let $d = \eta(L + d)$, with $0 < \eta < 1$. For the various density profiles adopted above, we have typically $0.5 \lesssim R\Delta'(r)/\Delta_0 \lesssim 2$ over the range $0 \leq r/R \leq \frac{1}{2}$. We choose R as the characteristic dimension of $\mathscr{G}$ within which most of the gravitating mass is concentrated. Thus, for an elliptical galaxy, R is expected to be considerably less than the optical radius of the galaxy. Consequently, we have

$$\frac{Ld}{L + d} \Delta'(a) = \alpha z \left(\frac{M}{10^{11} M_\odot}\right) \left(\frac{3 \text{ kpc}}{R}\right)^2 , \tag{38}$$

where α is a fraction comparable to unity. It is thus possible to have the right-hand side of (38) close to unity as required by $f \gg 1$. Hence the case for apparent superluminal separation along the lines of § IV*a* seems feasible; i.e., it is possible to have $f \gg 1$, thus converting a real subluminal speed $v_\perp$ into an apparent superluminal speed v'.

Next we consider the case § IV*b* wherein there is no actual separation of components relative to the nucleus of the source but the source *as a whole* has a transverse velocity $V_\perp$ relative to the observer. We now need $\Delta''(a)$ instead of $\Delta'(a)$, and our numerical estimates in general give $\Delta''(a) \approx 2GM/c^2a^3$ with $a \sim R$, the characteristic dimension of $\mathscr{G}$. The apparent separation velocity of the components is then

$$v' \approx 10^{-3} f^3 \beta z \left(\frac{l_\perp}{5 \text{ pc}}\right) \left(\frac{M}{10^{11} M_\odot}\right) \left(\frac{3 \text{ kpc}}{R}\right)^3 V_\perp , \tag{39}$$

where β is a numerical factor of order unity. It is clear that the small factor 10^{-3} in the right-hand side of (39) has to be compensated by a large magnification factor. The possibility of large separation velocities, in particular of $v' > c$, seems difficult unless $\Delta''(a)$ is considerably larger than estimated above. This may happen in the regions very close to the center of $\mathscr{G}$. Nevertheless, it is worth noting that this particular effect points out that, with a gravitational screen around, even small separation velocities—though observed—may not be real!

There is always matter in the immediate vicinity of the source which can give rise to gravitational bending of light rays, especially from the dense central regions of the radio source where the components are observed to be located. However, as is evident from (36) insofar as weak gravity is concerned, the effect is too small for direct relevance to the superlight motion problem. The effect of large *in situ* bending is relevant only if the parameter

$$\xi \sim 10^{-6} \left(\frac{M}{10^{11} M_\odot}\right) \left(\frac{3 \text{ kpc}}{a}\right) \int_0^{\pi/2} m(a \sec \phi) \sin \phi d\phi \tag{40}$$

is of the order of unity. In this case we shall have to demand a large density of matter in the nuclear region. Perhaps a cluster of neutron stars or black holes may fill the bill. We do not propose to discuss the strong field effects in this paper. As another esoteric possibility of achieving superlight expansions in strong gravitational fields we briefly mention the white hole model of Narlikar and Kapoor (1978). In this model, however, impossibly large masses ($\gtrsim 10^{14}\ M_\odot$) are required for these effects to be seen over a span of several years.

b) Probability Considerations

Of the three scenarios discussed above, we find the case of § IV*a* astrophysically the most attractive in the present context. Apart from the question of whether large magnifications are possible (which we have attempted to answer above in the affirmative), the probability of occurrence of a gravitational screen en route must also be taken into account.

The suggestion that intervening galaxies provide absorption line redshifts of QSOs has been advocated by many theoreticians. The location of more than one absorbing systems en route has sometimes been suggested in order to explain multiple absorption line systems in the spectra of QSOs. Our suggestion of a gravitational screen en route cannot therefore be considered as being too contrived.

Bahcall and Spitzer (1969) have estimated the number of galaxies between a QSO of redshift z and the observer as being given approximately by

$$N \approx 0.006\left(\frac{R}{3\ \mathrm{kpc}}\right)^2\left(\frac{N_g}{0.1\ \mathrm{galaxy\ Mpc^{-3}}}\right)[(1+z)^{3/2}-1]. \tag{41}$$

Here R = typical radius of the intervening galaxy and N_g = number density of galaxies at the present epoch. The chance of the presence of an intervening galaxy along the line of sight is clearly not negligible. However, the chance of $\mathscr{G}$ to be so placed as a gravitational lens that O and S of Figure 3 are near-conjugate points must be considerably smaller than this. There are, however, two considerations which go in the opposite direction and make the effect more likely.

The first is a selection effect which operates in any radio survey limited to a given flux-density lower limit. When we consider the intensities of the images formed by a gravitational lens (cf. Press and Gunn 1973), the primary image (with which we are concerned here) is brighter and the secondary image(s) fainter. We are therefore more likely to pick up cases with $f \gg 1$ than cases with $f \lesssim 1$ in a typical flux-density-limited survey.

The second point is that the intervening matter may not be a galaxy of stars. It can be any gravitating system, not necessarily a typical galaxy. Since there is a gap between the density of matter in the form of galaxies and the density of the universe predicted by many viable cosmological models, we cannot rule out the existence of as yet undetected faint but massive objects in intergalactic space. For this reason, any probability estimates obtained on the basis of galaxies alone—e.g., from (41)—have to be looked upon as lower limits.

Although of the total number of quasars examined by VLBI the fraction showing superluminal separation speeds is fairly large, the total number is still small enough to make a realistic assessment of the probability of the scenario proposed here. We suggest, therefore, that an optical search be made for gravitating systems en route to the handful of cases showing superluminal separation of components. Since these systems may well have larger mass-to-light ratios than even the elliptical galaxies, the Space Telescope may be useful for their detection.

c) Intensity Variation

The intensity of the image will depend on the magnification factor f; and as f changes, the observed intensity should change. From (26) we get

$$\dot{f} = f^2 \frac{Ld}{L+d}\Delta''(a)\dot{a}\,. \tag{42}$$

Using $\Delta''(a) \sim 2GM/c^2a^3$ and $Ld/(L+d) \sim 10^{28}\gamma z$ cm, with γ of the order of unity, we have

$$\frac{\dot{f}}{f} \sim 3f\gamma z \times 10^{-4}\left(\frac{M}{10^{11}\ M_\odot}\right)\left(\frac{\dot{a}}{c}\right)\left(\frac{a}{3\ \mathrm{kpc}}\right)^{-3}\mathrm{yr}^{-1}\,. \tag{43}$$

From this we see that the change in intensity of the source over a period of a few years is not expected to be striking unless $f \gtrsim 10^4$.

Since the intensity increases with f, we should see, according to this model, a correlation between the observed separation velocity and intensity. Such correlation studies need to be made in order to test the viability of this model.

d) Effects of Matter in $\mathscr{G}$

Although we have been mainly concerned with the overall gravitational effects of $\mathscr{G}$, it is worth pointing out a few side effects.

For example, will the intervening matter in $\mathscr{G}$ seriously block the passage of radio waves from the source? The effective optical depth of the absorbing material in the screen turns out to be small enough not to cause any significant attenuation of radio waves. The interception of a radio source by a gravitational screen can in fact provide information about the density of intervening matter. If there are significant density fluctuations within the intervening matter distribution, these will be reflected in the fluctuations of the magnification factor f. The result will be a somewhat fluctuating curve for the angular separation of the two components plotted as a function of time. Such a "wobble" has indeed been noticed in the VLBI measurements. In the kinematic models of illusory superluminal separation, the apparent separation velocity is expected to be a smooth function of time. Large forces will be needed to produce the wobble in such models, forces large enough to disrupt the entire radio source.

The presence of an intervening galaxy $\mathscr{G}$ along the line of sight is expected to cause a certain amount of depolarization of the electromagnetic waves originating at the individual components. Following the arguments of Burn (1966), it is straightforward to verify that it is extremely unlikely that there will be any significant depolarization either in our own Galaxy or in any intervening clump of matter en route. This is a consequence of the consideration that the degree of polarization at wavelength λ is given by

$$p(\lambda^2) = p_i \exp\left[-2K^2(n_e B_\parallel)^2 l^2 \left(\frac{L + d}{h}\right)\lambda^4\right] \equiv p_i \exp(-s\lambda^4)\,. \tag{44}$$

Here p_i is the intrinsic polarization, $B_\parallel$ the component of the magnetic field along the line of sight, l the transverse dimension of the component, h the thickness, and n_e the mean electron density of the intercepting galaxy $\mathscr{G}$, and $K = 2.62 \times 10^{-17}$ in cgs units. For the choices of the parameters adopted for our model, the factor $s\lambda^4$ comes out to be very small. If any polarization measurements are made of the individual components, clearly the presence of a gravitational screen would hardly make any significant contribution to the depolarization.

e) Alignment of Components

There is an important question concerning the alignment of the components. In the observed cases of superluminal separation the separating components in the nuclear regions are very often found to be aligned with the extended double components of the radio source. The objection that the images produced by a gravitational lens are not in general expected to account for the alignment of VLBI structures is not applicable in the present context. Our model relies on the magnification of the velocity of separating components which are ejected from the nuclear regions, and successive outbursts should be aligned if they represent expansion along a preferred direction like, for example, the rotation axis.

In Figure 4 we have considered the situation where the plane OAB passes through the center C of the lens galaxy $\mathscr{G}$. If the center C should not lie in the plane OAB, then the misalignment of the components from the axis of the extended doubles is by an amount $\sim(x/a)L\Delta/D$. Here D is the separation between the extended double components, x the perpendicular distance of C from the plane formed by the observer O with the extended doubles, a the apsidal distance of the light ray Γ, and Δ the gravitational bending. The misalignment turns out to be small whenever $x \lesssim a$, which is again consistent with the demand for the rays to pass through the inner regions of the lens galaxy $\mathscr{G}$.

VI. CONCLUDING REMARKS

The apparent superlight motion is explained on the basis of a model which makes use of differential gravitational bending of light rays by a suitably positioned "gravitational screen" such as a galaxy along the line of sight. Such an interception by intervening galaxies has been invoked to account for multiple absorption redshifts of QSOs. Our model makes use of the optical consideration that the inner regions of the intervening galaxy produces a substantial magnification of the source image situated near a conjugate point of the lens galaxy. The resulting velocities of expansion of the separating components in a stationary radio source then turn out to be very much in excess of the speed of light.

The chance of the special placement of a gravitational lens with the source and the observer as conjugate points is in general small. However, a selection effect operating the other way makes the component images so formed appear brighter in intensity and hence more probable to be detected in a survey down to a given flux density. Unlike the kinematic models of superluminal expansion, the present model accommodates the scatter in the separation-velocity plot against the epoch as arising from density inhomogeneities in the gravitational screen. It is estimated that there will not be any significant amount of attenuation of radio waves by the absorbing material en route, nor will there be much if any depolarization.

The two crucial issues which will decide whether such an idea is tenable are the statistics of the fraction (of the total number of quasars) which show superluminal separation of inner components and of the alignment of these components with respect to the outer ones. At present the number of quasars studied by VLBI is not large enough to justify confidence in the statistics of either of the two observations (i.e., number of cases of $v > c$ and number of well aligned sources). Finally, as a direct test of the idea, we advocate a systematic optical search for objects of large mass-to-light ratio along the line of sight to the quasars showing superluminal separation of the nuclear components.

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