WORKING WITH FRED ON ACTION AT A DISTANCE

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Abstract. This paper reviews the work the author carried out with Fred Hoyle on the development of electrodynamics and gravitation as direct particle theories. In this account the author reviews how the work was started, and went through stages of increasing sophistication, e.g., extending the Wheeler-Feynman electrodynamics to curved spacetime, its consequences in different cosmologies, and the issues arising from its quantization. The resolution of ultraviolet divergences in quantum electrodynamics is also briefly discussed. The parallel development of a Machian theory of gravitation followed the lead from electrodynamics. In both theories one sees a strong link between the large scale structure of the universe and local physics, as might be expected from an action-at-a-distance framework. It is recalled why Fred considered this an important aspect of a physical theory.

1. Historical Background

I still vividly recall a wet afternoon in Varenna on Lake Como in northern Italy in the summer of 1961. I was a student participant in one of the annual summer schools held in this scenic resort. Our school was on various aspects of gravitational theories and observations and the lecturers included Bob Dicke, Alfred Schild, Joshua Goldberg, Bruno Bertotti and Fred Hoyle. We also had seminars from a few other scientists who passed by for a short duration. That day it was Hermann Bondi who lectured on in his inimitable style, though hampered by frequent sneezes brought about by hayfever. Nevertheless, his topic was extremely interesting.

What Bondi reported on that day concerned a very unusual aspect of electrodynamics, an aspect that linked it to cosmology. This was a follow up by a Canadian physicist named Jack Hogarth on the work done by John Wheeler and Richard Feynman in the 1940s on action at a distance electrodynamics (Wheeler and Feynman, 1945, 1949). Briefly, the history was as follows.

In fact the story begins a hundred years before the Wheeler-Feynman paper of 1945, with no less a person than Gauss. In a letter to Weber on March 19, 1845, Gauss wrote:

I would doubtless have published my researches long since were it not that at the time I gave them up I had failed to find what I regarded as the keystone, Nil actum reputans si quid superesset agendum, namely, the derivation of the additional forces – to be added to the interaction of electrical charges at rest, when they are both in motion – from an action which is propagated not instantaneously but in time as is the case with light.

Gauss's attempts came some two decades before the Maxwellian field theory and six decades before special relativity. The sucess of these two theories shifted

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the emphasis from action at a distance to fields and it was not until well into the present century that the problem posed by Gauss was solved.

A beginning was made by Schwarzschild (1903), Tetrode (1922), and Fokker (1929a,b, 1932), who independently formulated the concept of delayed action at a distance. The action principle as formulated by Fokker may be written in the following form:

$$J = -\sum_a \int m_a da - \sum_{a<b}\sum \int\int e_a e_b \delta(s_{AB}^2)\eta_{ik} da^i db^k. \tag{1}$$

In the above expression the charged particles are labeled, *a,b,...* with e_a and m_a the charge and mass of particle a. The worldline of a is given by the coordinate functions $a^i(a)$ of the proper time a. The spacetime is Minkowskian, so that

$$da^2 = \eta_{ik} da^i da^k, \tag{2}$$

with η_{ik}= diag $(-1, -1, -1, 1)$. The first term of J therefore discribes the inertial term. The second term describes the electromagnetic interaction between the worldlines of a typical pair of particles a, b. The delta function shows that the interaction is effective only when s_{AB}^2 , the invariant square of distance between typical world points A, B on the worldlines of a and b, vanishes. This implies delayed action: s_{AB}^2= 0 means that world points A and B are connected by a light ray.

Although this formulation met the requirement of relativistic invariance it gave rise to other difficulties. The major difficulty is as follows. For a typical point A on the worldline of a there are two points B_+ and B_- on the world line of b for which $s_{AB}^2 = 0$. The effect of A is felt at B_+ (at a later time) and at B_- (at an earlier time). Similarly, since the action principle guarantees the equality of action and reaction, the reaction from B_+ and B_- is felt at A. Thus there are influences propagating with the speed of light, not only into the future but also into the past. This led to a conflict with the principle of causality, which seems to hold in everday life. The other difficulties were of a less serious nature although not ignorable. For example, there was no ‘self-action’ ($a = b$ is avoided in the double sum) and so there did not appear to be any obvious way of accounting for rediation damping.

These difficulties were removed by Wheeler and Feynman (1945) by bringing into the discussion the important role of the absorber. In our above example, the reactions from B arrive at A instantaneously, whatever the spatial separation of a and b. So it becomes necessary to take into account the reaction from the entire universe to A. Although the remote particles are expected to contribute less, their total number is large enough to make the calculation nontrivial. The essence of the argument given by Wheeler and Feynman is described below.

To begin with, define the 4-potential at X due to particle b by

$$A_i^{(b)}(X) = e_b \int \delta(s_{XB}^2)\eta_{ik} db^k, \tag{3}$$

and the corresponding direct-particle field by

$$F^{(b)}_{ik} = A^{(b)}_{k;i} - A^{(b)}_{i;k}. \tag{4}$$

A direct particle field is not an ordinary field, because it does not have any independent degrees of freedom. The 4-potential *identically* satisfies the relations

$$A^{(b)k}_{;k} \equiv 0, \quad \Box A^{(b)k} \equiv \eta^{mn} A^{(b)k}{}_{;mn} = 4\pi j^{(b)k}, \tag{5}$$

where $j^{(b)k}(X)$ is the current density vector of the particle b at a typical point X, defined in the usual way. Thus although (5) resembles the Maxwell wave equation (and the guage condition) it represents identities.

The equation of motion of a typical charge a is obtained by varying its worldline and requiring $\delta J = 0$. We get the analogue of the Lorentz force equation in which the charge a is acted on by all other charges in the universe. It appears therefore that an alternative formulation of electrodynamics based on action at a distance has been found. However, this appearance is illusory.

We now turn to the difficulty introduced by the time symmetry of this formulation. Instead of being the retarded solution of (5), (3) is the time-symmetric half-advanced and half-retarded solution. The same applies to the direct-particle fields. Suppressing the indices i, k, we may write

$$F^{(b)} = \frac{1}{2}[F^{(b)}_{\text{ret}} + F^{(b)}_{\text{adv}}]. \tag{6}$$

This field is present in the past as well as the future light cone of B.

Wheeler and Feynman argued in the following way. If we move the charge b, it generates a disturbance that affects all other charges in the universe. Their reaction arrives back instantaneously. They then showed how to calculate such a reaction in a universe of static Minkowski type with a uniform distribution of electric charges. They found that the reaction to the motion of charge b can be calculated in a consistent fashion and comes out to be

$$R^{(b)} = \frac{1}{2}[F^{(b)}_{\text{ret}} - F^{(b))}_{\text{adv}}]. \tag{7}$$

Thus a test particle in the neighbourhood of charge b experiences a net total 'field'

$$F^{(b)}_{\text{tot}} = F^{(b)} + R^{(b)} = F^{(b)}_{\text{ret}}. \tag{8}$$

This is the pure retarded field observed in real life! The self- consistency of the argument follows from the fact that the reaction $R^{(b)}$ has been calculated by adding the $\frac{1}{2}F^{(a)}$ fields of all particles $a \neq b$ that have been excited by this total field $F^{(b)}$. Thus only the future light cone of B comes into play. The reaction from the future cancels the advanced component of $F^{(b)}$ and doubles its retarded component.

Also according to the Lorentz force equation, $R^{(b)}$ is the force arising from all *other* particles in the universe experienced by the particle B. This is nothing but the radiative reaction to the motion of b as obtained earlier by Dirac (1938) on empirical grounds. Earlier, Dirac's rule was difficult to understand within the context of the field theory, although it was known to give the right answer. Here the Dirac formula is understood as the consequence of a response of the universe to the local motion of the charge. Thus the theory not only gets round the problem of causality but it also accounts for the radiation damping formula.

Physically, what happens is the following. To the motion of b the future half of the universe acts as an absorber. It 'absorbs' all the 'energy' radiated by b, and in this process sends the reaction $R^{(b)}$, which does the trick! For this reason Wheeler and Feynman called this theory the *absorber theory of radiation*. The presence of the absorber is essential for the calculation to work. For example, it will not work in an empty universe surrounding the electric charge.

In the above self-consistent derivation there was still one defect: it was not unique. Another self-consistent picture was possible in which the net field near every particle was the pure advanced field and the radiative reaction was of opposite sign to that of (7). The two solutions are compared thus. In one we have the retarded solution while in the other we have the advanced solution. In the former, absorption in the future light cone is responsible while in the latter, it is the absorption in the past that plays the crucial role. The important role of the absorbers is that they convert the time-symmetric situation of an isolated charge, to a time-asymmetric one. It is, however, not possible to distinguish between the two without reference to some other independent time asymmetry.

Wheeler and Feynman realized this and linked the choice to thermodynamics. Given the usual thermodynamic time asymmetry, they argued that the latter situation would be highly unlikely (under the probability arguments of statistical mechanics) and that the usual asymmetry of initial conditions will favour the former (retarded) solution.

It was, however, pointed out by Hogarth (1962) that it is not necessary to bring thermodynamics into the picture at all. If one takes account of the fact that the universe is expanding, its past and future are naturally different. The reaction from the absorbing particles in the future light cone (designated by Hogarth collectively as the *future absorber*) does not automatically come out equal and opposite to the reaction from the past absorber. Thus the two pictures do not always follow in an expanding universe. Hogarth found that for the retarded solution to hold, the future absorber must be perfect and and past absorber imperfect; and vice versa for the advanced solution.

An absorber is perfect if it entirely absorbs the radiation emitted by a typical charge. In the static universe discussed by Wheeler and Feynman both the past and future absorbers are perfect; and this leads to the ambiguity mentioned earlier. However, Hogarth found that the ambiguity is resolved if the cosmological time asymmetry is taken into account. He found, for example, that in most big-bang models which expand forever, the advanced (and not the retarded) solution is valid. In the steady-state model (Bondi and Gold, 1948; Hoyle, 1948), only the retarded solution holds. In the big-bang models that expand and contract both the absorbers are perfect and the outcome is ambiguous. This was the work that Bondi reported on in Varenna.

2. Follow-up on the Absorber Theory with Fred Hoyle

Fred Hoyle and I were very impressed by this conclusion. Only a few months before we had argued against the claim by Martin Ryle and his group in Cambridge that their observations of radio source counts ruled out the steady state theory. Those arguments involved several details of observational errors and extrapolations of theory. Here, on the contrary was a clear cut conclusion that did not depend on such messy details. Because of its cosmological conclusions, we felt that this approach of action at a distance needed to be pushed further.

Though interesting, Hogarth's work was, however, incomplete in two aspects. First, he had not shown how to generalize the Fokker action to curved spacetimes needed to describe the expanding world models. Second, he had used collisional damping to decide upon the nature of absorbers, past and present; and this brought in thermodynamics by the back door! In fact, Feynman had criticised Hogarth's work on this very ground.

Later Fred and I (Hoyle and Narlikar, 1963) completed the work by first re-writing the Fokker action (1) in curve space as is necessary for any cosmological discussion. We too arrived at conclusions similar to Hogarth's but by using the *radiative damping* for producing absorption. This kept the asymmetry entirely within electrodynamics and cosmology and away from collective phenomena and thermodynamics.

However, a greater challenge lay ahead. If the action at a distance picture was to go further, on parity with the rival field theoretic picture, then it must be quantizable. For, quantum electrodynamics presents a far richer set of phenomena than its classical counterpart. Our next step was to demonstrate that it is indeed possible to extend the entire picture into the quantum domain and it is possible to describe the entire range of phenomena of quantum electrodynamics, such as spontaneous transition of the atomic electron, Compton scattering, pair creation, etc., even the Lamb shift and anomalous magnetic moment of the electron without recourse to field theory (Hoyle and Narlikar, 1969, 1971). This therefore removes any possible objection to the concept of action at a distance in so far as it is applicable to electrodynamics. In fact, the work two decades later, described briefly in Section 6 shows that even the infinities introduced by the so-called 'self interactions' can be eliminated in this formulation.

The crucial role played in the whole calculation is that of the *response of the universe*. In the classical calculation the steady-state universe generates the 'correct' response so that the local electric charges interact through retarded signals. The response from the big-bang models is of the wrong type. We are thus able to distinguish between the different cosmological models and decide on their validity or otherwise on the basis of the Wheeler-Feynman theory. We also see *why* charges interact through retarded signals: they do so because of the response of the universe. In the Maxwell field theory the choice of retarded solutions of Maxwell's equations is by an arbitrary fiat.

In the quantum calculation also it can be shown that an asymmetric phenomenon with respect to time, like the spontaneous downward transition of an atomic electron, is caused by the response of the universe. By contrast, the quantization of the Maxwell electromagnetic field ascribes such an asymmetry to the quantum vacuum and to the rather abstract rules of quantization.

The direct-particle approach therefore achieves for electrodynamics what Mach sought to achieve for inertia. By bringing in the response of the universe to a local experiment in electrodynamics we have essentially incorporated Mach's principle into electromagnetic theory. Given the correct response of the universe, we can almost decouple our local system from it, although strictly speaking the theory would not be possible without the universe.

Can the same prescription be applied to the original Mach's principle *per se*, to inertia and gravitation? We discuss this problem in the following section, for this issue also attracted Fred and me in the 1960s, parallel to our work with electrodynamics.

3. Inertia as a Direct-Particle Field

We now return to the problem of achieving a 'reconciliation' between general relativity and Mach's principle. To this end we shall look for a theory with the following properties:

(a) It has Mach's principle built into one of its postulates.

(b) It is conformally invariant.

(c) It does not have the conceptual difficulties associated with the case of a single particle in an otherwise empty universe.

(d) For a universe containing many particles the theory reduces to general relativity for most physical situations.

Some discussion is needed as to why the theory should be conformally invariant. The reasons are twofold. First, when we take note of the local Lorentz invariance of special relativity, the natural units to use are those in which the fundamental velocity c=1. The quantum theory, with which our theory should be consistent throws up another fundamental constant, the Planck constant related to the uncertainty principle. Thus it is natural to use units in which $\hbar$=1. This makes the classical action J, for example, dimensionless and the natural unit of mass is the Planck mass which we shall quantify by

$$m_P = \sqrt{\frac{3c\hbar}{4\pi G}}. \tag{9}$$

All masses are then expressible as numbers in the unit of this mass. With our choice of units, only one independent dimension out of the three length, mass and time, remains. Taking it as the dimension of mass, length goes as reciprocal of mass.

However, in a Machian theory, we expect the particle masses to be functions of space and time and as such not necessarily constant. Therefore, the standards of lengths and time intervals also may vary from one point to another. We therefore need laws of physics which are invariant with respect to this variation. Conformal invariance guarantees this.

Our second reason is based on the nature of action at a distance. Given that as in electrodynamics, the interaction propagates principally along null rays, we need an invariance that preserves the global structure of light cones. This again is guaranteed by conformal invariance. Just as Lorentz invariance identifies the light cone as an invariant structure *locally*, so does conformal invariance identify it *globally*.

A theory following these guidelines was developed by Hoyle and me (1964, 1966) and its broad features are described next. We begin by a second look at the Fokker action for electrodynamics, this time rewritten in a curved Riemannian space-time:

$$J = -\sum_a \int m_a da - \sum_{a<b}\sum 4\pi e_a e_b \int\int \bar{G}_{i_A k_B} da^{i_A} db^{k_B}. \tag{10}$$

Here, in going from (1) to (10) the first term of J needs a trivial modification: da is now computed with a Riemannian metric. The modification of the second term of J requires considerable thought. The $\delta(s^2_{AB})\eta_{ik}$ is now replaced by $\bar{G}_{i_A k_B}$, a bivector propagator between A and B. It is the symmetric Green's function for the wave euqation

$$\Box \bar{G}_{ik_B} + R^l_i \bar{G}_{lk_B} = [-\bar{g}(X, B)]^{-1/2} \bar{g}_{ik_B} \delta_4(X, B). \tag{11}$$

Here $\bar{G}_{ik_B}$ behaves as a vector at X and B, respectively, with the indices i and k_B (the subscript X on i is suppressed for the convenience of writing). $\bar{g}_{ik_B}$ is the parallel propagator between X and B [see Synge (1960) for details] and $\bar{g}(X, B)$ its determinant. In the limit $g_{ik} \to \eta_{ik}$, $4\pi \bar{G}_{i_A k_B} \to \delta(s^2_{AB})\eta_{ik}$. The detailed structure of this propagator has been studied by DeWitt and Brehme (1960).

The electromagnetic part of J is conformally invariant but the mechanical part (the first term) is not. We now compare (10) with the action for field theory of Maxwell and for general relativity. This action, denoted by $J^{(F)}$ is given by

$$J^{(F)} = \frac{1}{16\pi G}\int R(-g)^{1/2}d^4x - \sum_a \int m_a da -$$

$$\frac{1}{16\pi}\int F^{lm}F_{lm}(-g)^{1/2}d^4x - \sum_a \int A_i da^i. \tag{12}$$

The third and fourth term of $J^{(F)}$ represent, respectively, the free-field term and the field – particle interaction term. In the direct-particle theory the second term of J replaces these two terms of the field theory. The fields as such lose their independent status and are replaced by propagators connecting particle world lines. What can we do about the first two terms of (12)? The second term already exists in (10) and it is tempting to simply insert the first term into (10) as representing gravity.

This procedure, however, is contrary to the spirit of the direct-particle picture. The first term of (12), although containing geometrical information, has also the character of a field. Hence it is out of place.

The clue to the correct procedure that needs to be adopted is provided by a comparision of the last term of (12) with the second term of (10). If in the former we replace the potential A_i by a sum over the direct-particle potentials defined by a relation analogous to (3) for a curved space, we shall recover something that looks like the latter! In the same way we now replace the masses m_a by direct-particle fields defined in the following manner:

$$m^{(b)}(X) = \int \lambda_b G(X, B)db, \quad \lambda_b = \text{a coupling constant.} \tag{13}$$

$$m_a(A) = \lambda_a \sum_{b\neq a} m^{(b)}(A), \quad \lambda_a = \text{a coupling constant.} \tag{14}$$

The propagator $G(X, B)$ has to be biscalar since masses are scalars and we wish to preserve a symmetry between X and B. The action (10) is now changed to

$$J = -\sum\sum_{a<b} \int\int \lambda_a\lambda_b G(A, B)dadb$$

$$-\sum\sum_{a<b} 4\pi e_a e_b \int\int \bar{G}_{i_A k_B} da^{i_A} db^{k_B}. \tag{15}$$

What should be the exact form of $G(A, B)$? Taking a clue from electromagnetism, we expect it to be a symmetric Green's function of a scalar wave equation.

However, we also want the equation to be conformally invariant. These two requirements fix the form of the scalar propagator uniquely to within a multiplicative factor. We shall take $G(A, B)$ to satisfy the scalar wave equation

$$\Box G(X, B) + \frac{1}{6}R(X)G(X, B) = [-\bar{g}(X, B)]^{-1/2}\delta_4(X, B). \tag{16}$$

The wave operator is uniquely fixed by the requirement of conformal invariance.

Turning from these purely formal aspects to those of interpretation we note that (13) and (14) are essentially Machian ideas on inertia expressed mathematically. The mass of a particle a at its world point A is the sum of the contributions of all other particles in the universe. Thus requirement (a) has been met. Requirement (c) is also met, because for a single particle in an otherwise empty universe there is no action! The minimum number of particles required to define J is two. Thus for each of the two particles the other provides the 'background' in the Machian sense. The requirement of conformal invariance is also met by our choice of the propagator. It therefore remains to examine requirement (d).

So far we have concentrated on inertia and ignored gravity. The action (15) does not contain the gravitational term

$$\frac{1}{16\pi G}\int R(-g)^{1/2}d^4x$$

explicitly. Yet, as we shall see in the following section, the theory is fully capable of describing gravitational phenomena.

4. Conformal Gravity

Returning to the action (12) we note that when we try to derive the Einstein field equations by the Hilbert action principle, we get the Einstein tensor from the first term. This term does not exist any more in the direct particle action (15). Shall we get any gravitational term at all from (15) if we sought to perform the metric veriation $g_{ik} \to g_{ik} + \delta g_{ik}$? A look at the electromagnetic part of (12) does not inspire confidence that the answer to this question should be in the affirmative. There it is the third rather than the fourth term that contributes the energy tensor of electrodynamics, and it is the fourth term that was used in going over to (15). Nevertheless, a closer examination shows that the terms in (15) do give nontrivial answers when the metric variation is performed.

The reason for this is understood as follows. Consider the electromagnetic propagator $\bar{G}_{i_A k_B}$ connecting A and B respectively, on the world lines of a and b. Suppose we perform a variation in the space-time metric of a compact region Ω. Since the propagator is a global property of space-time strcture, it will change because of this change in structure of Ω. The change in the propagator is therefore expressible, in a first order calculation, as a functional of δg_{ik} over the volume Ω.

In the electromagnetic case the answer may be expressed in the following form

$$-\delta \sum_{a<b}\sum 4\pi e_a e_b \int\int \bar{G}_{i_A k_B} da^{i_A} db^{k_B} = -\frac{1}{2}\int T^{ik}\delta g_{ik}(-g)^{1/2} d^4x, \qquad (17)$$

where

$$T^{ik} = \frac{1}{8\pi}\sum_{a<b}\sum [\frac{1}{2} g^{ik} F^{(a)mn}_{\text{ret}} F^{(b)}_{mn\ \text{adv}} - F^{(a)i}_{l\ \text{ret}} F^{(b)kl}_{\text{adv}} - F^{(b)i}_{l\ \text{ret}} F^{(a)kl}_{\text{adv}}]. \qquad (18)$$

The details of this derivation are given by Narlikar (1974).

It is interesting to note in passing that this derivation resolves an ambiguity about the energy tensors of direct-particle electrodynamics. Wheeler and Feynman (1949) had discussed two tensors for this theory. Of these one was the canonical tensor given above by (18) and the other was the Frenkel tensor whose form differs from that given in (18) in having all the direct particle fields $F^{(a)lm}$ as the symmetric half-advanced-plus -half-retarded fields. Wheeler and Feynman had concluded:

From the standpoint of pure electrodynamics it is not possible to choose between the two tensors. The difference is of course significant for the general theory of relativity, where energy has associated with it a gravitational mass. So far we have not attempted to discriminate between the two possibilities by way of this higher standard.

As mentioned above, the usual prescription of metric variation uniquely yields the canonical tensor. The fact that we could get a nontrivial answer to the variational problem and that this resolves a long-standing ambiguity, reinforces our belief that we are proceeding along the correct path toward a theory of gravitation.

We now consider the variation of the first term of (15) as $g_{ik} \rightarrow g_{ik} + \delta g_{ik}$. We shall ignore the second term and concentrate on gravitation alone. Also for simplicity we begin by putting $\lambda_a = 1$ for all a. This does not alter the essential features of the theory.

The method is similar to that adopted for electromagnetism. We compute the change in the propagator $G(A, B)$ as the geometry changes in any compact region Ω. The details of this somewhat lengthy calculation are given elsewhere [see Hoyle and Narlikar (1974)]. We simply quote the result. The field equations turn out to be

$$\frac{1}{2}\phi(R_{ik} - \frac{1}{2}g_{ik}R) =$$

$$-T_{ik} + \frac{1}{6}[g_{ik}\Box\phi - \phi_{;ik}] + \frac{1}{2}[m_i^{\text{ret}} m_k^{\text{adv}} + m_k^{\text{ret}} m_i^{\text{adv}} - g_{ik} m^{l\ \text{ret}} m_l^{\text{adv}}], \qquad (19)$$

where,

$$m(X) = \sum_a \int G(X, A) da, \quad m_i = \partial m / \partial x^i, \qquad (20)$$

$$\phi(X) = m^{\mathrm{adv}}(X)m^{\mathrm{ret}}(X), \tag{21}$$

and m^{ret} and m^{adv} denote twice the retarded and advanced parts of $m(X)$, respectively. The energy tensor T_{ik} is the familar energy tensor for a system of particles $a, b, \ldots$ with masses as defined by the Machian presciption (13) and (14). Note that the masses are time symmetric. The function $m(X)$ satisfies the conformally invariant wave equation

$$\Box m + \frac{1}{6}Rm = N, \tag{22}$$

where

$$N(X) = \sum_a \int \delta_4(X, A)[-\bar{g}(X, A)]^{-1/2} da \tag{23}$$

is the invariant particle number density.

There are 10 equations in (19) and one equation (22) for the 11 unknowns g_{ik} and m. However, the divergence and trace of (19) identically vanish, showing that there are in fact five fewer independent equations. This is hardly surprising since four of these five are due to the general coordinate invariance (as in general relativity) while the fifth identity (the vanishing of trace) is due to conformal invariance. It is easy to verify that if $[g_{ik}, m]$ is a solution of these equations then so is $[\zeta^2 g_{ik}, \zeta^{-1}m]$ for an arbitrary well-behaved (i.e. of type C^2) nonvanishing finite function ζ of space and time. This arbitrary function is nothing but the expression of the arbitrariness of mass-dependent units discussed in Section 3.

Suppose now that it is possible to choose ζ such that

$$m^{ret}\zeta^{-1} = m_0 = \text{constant}. \tag{24}$$

Suppose also that the response of the universe is such as to cancel all advanced components and double the retarded ones so that the effective mass function is m^{ret}. Then the field equations are simplified to

$$R_{ik} - \frac{1}{2}g_{ik}R = -\kappa T_{ik}, \quad \kappa \equiv \frac{8\pi G}{c^4} = 6/m_0^2. \tag{25}$$

We shall later identify m_0 with the mass of the Planck particle. However, as seen above,we have arrived at the familiar equations of general relativity! The conformal frame for which (24) and (25) hold will be called the *Einstein frame*. We have thus completed the remaining part of the programme outlined at the beginning of Section 3.

The following points are worth emphasizing in the above derivation of Einstein's equations, which is so radically different from the standard ones (used for example, by Einstein in 1915 and by Hilbert later the same year).

(I) The approach to Einstein's equations is via the wider framework of a conformally invariant gravitation theory. Only in the limit of many particles in a suitably responding universe do we arrive at Einstein's equations. In the other limit of zero or no particles there is no theory! Thus it brings out the reason why the Machian paradox of one particle in an empty universe is not valid in the context of Einstein's equations. This reason does not emerge in the standard derivations of Einstein's equations.

(II) It is significant that the coupling constant κ is positive in this approach. This conclusion is unaffected by the change of sign of the coupling constants λ_a, λ_b, etc. (taken here as unity); nor is it affected by the choice of signature (i.e $---+$ instead of $+++-$) of the spacetime metric. The choice of the conformally invariant scalar propagator leads to the coupling constant being positive, i.e. to gravity being 'attractive'. In the standard derivation the coupling constant is fixed (in sign as well as magnitude) by a comparison with Newtonian gravity.

(III) A considerable discussion has gone on regarding the admissibility of the so-called λ-term in Einstein's equations. This is because this term could be accmmodated in Einstein's heuristic derivation or in Hilbert's action principle. It is worth emphasizing that the direct-particle approach to gravity given so far does not permit the λ-term. As we shall see later, this term does arise in a Machian way in the direct particle theory, *provided* we allow the wave equation (22) for inertia to be nonlinear. The present cosmological observations generally seem to require the cosmological constant (Bagla et al., 1996).

(IV) The condition (24) that leads to Einstein's equations needs to be reexamined carefully under two special circumstances. Near a typical particle a, we expect the mass function $m^{(a)}(X)$ to 'blow up' so that $m(x) \to \infty$, as $X \to A$, on the worldline of a. In order to make $\zeta^{-1}m(X)$ finite at A, we therefore require $\zeta \to \infty$,as $X \to A$. However, we have already ruled out such conformal functions by restricting ζ to finite values. Thus the transition to Einstein's equations is not valid as we tend to any typical source particle. The nature of the equations and their solutions near a particle in this theory have been discussed by Hoyle and Narlikar (1966) and by Islam (1968). The other aspect arises if there exist $m = 0$ hypersurfaces. Clearly we cannot choose a finite $\zeta(\neq 0)$ to make $m =$ constant > 0 on these hypersurfaces. If we insist on driving such solutions into the Einstein frame, we end up having spacetime singularity there. This has been pointed out by Kembhavi (1978).

5. Cosmological Constant and the Creation of Matter

In recent years, this theory has been further generalized and applied to cosmology to include the cosmological constant, as well as explicit description of creation of matter (see Hoyle et al., 1995).

Taking the cosmological constant into account first, we may ask whether (22) is the most general conformally invariant wave equation satisfied by a scalar function $m(X)$. The answer is *No!* The most general such equation is

$$\Box m + \frac{1}{6}Rm + \Lambda m^3 = N, \tag{26}$$

where Λ is a constant. This of course makes the scalar Machian interaction non-linear and more difficult to handle. However, it very naturally leads to the cosmological constant of the right magnitude at the present epoch. For, if we assume that for a single particle, the value of this constant is unity, then in the sum (20) leading to $m(X)$, because of the presence of a large number $\mathcal{N}$ of particles within the cosmological horizon, the cube term is less effective by the factor $\mathcal{N}^{-2}$. Thus, the factor Λ in (26) is of this order. With the identification of m_0, with the only possible fundamental quantity with the dimension of mass, viz, the Planck mass

$$m_0 = \sqrt{\frac{3\hbar c}{4\pi G}}, \tag{27}$$

one can write the familiar cosmological constant in the Einstein equations as

$$\lambda = -3\Lambda m_0^2. \tag{28}$$

With around 2.10^{60} Planck particles in the horizon, we get the value of $\lambda \sim -2.10^{-56}\text{cm}^{-2}$. It is of the right order, but *negative*. However, it leads to interesting and physically relevant cosmological models.

Let us consider matter creation next. The standard relativity theory starts with the assumption that matter can neither be created nor destroyed, that is, the worldlines of particles are endless. However, in the big bang singularity, all worldlines are incomplete, thus leading to a contradiction with the basic assumption. Indeed the presence of the singularity signifies the breakdown of the basic rules such as the action principle from which the equations of general relativity are obtained.

To account for the creation of matter in a nonsingular fashion, we introduce the additional input into the theory that the particle worldlines are with ends. The same action as before then describes this theory but the endpoints generate extra terms in the field equations.

For details of this work see the paper Hoyle et al(1995). The field equations in the 'constant mass' conformal frame then take the form:

$$R_{ik} - \frac{1}{2}g_{ik}R + \lambda g_{ik} = -\kappa[T_{ik} - \frac{2}{3}(c_i c_k - \frac{1}{4}g_{ik}c^l c_l)]. \tag{29}$$

The scalar c-field arises from the contribution to inertia from ends of particle worldlines. These are the contributions of the Planck particles created but which last a very short time scale$\sim 10^{-43}$ second.

Sachs et al. (1996) have solved these field equations and obtained a series of cosmological models which are a combination of two kinds:(i) models with creation of matter and (ii) models without creation of matter. The generic solution is known as the *Quasi Steady State Cosmology*. It is a nonsingular model which has a de Sitter type expansion with short term oscillations superposed on it. The former represents the creative and the latter the noncreative mode. The QSSC is being proposed as an alternative to the standard hot big bang cosmology (Hoyle et al., 2000).

6. Finite resolution of the Self-Energy Problem

After this excursion into gravitation, let me return to electrodynamics. In a recent paper Hoyle and Narlikar (1993) had shown that with suitable cosmological boundary conditions, like the de Sitter horizon, there is a cut off on high frequencies that otherwise lead to divergent integrals in the standard electromagnetic field theory. Thus the electron self-energy problem and the various radiative corrections of quantum electrodynamics can be handled without subtraction of one infinity from another. In this sense the direct particle theory fares better than the field theory. This effect comes about in the following way.

How are the high-frequency contributions to various integrals of quantum electrodynamics which diverge in normal field theory calculations, get ‘damped’ in the absorber theory? The de Sitter line element which is used in the steady state theory, (or its asymptotic form in the quasi-steady state cosmology),

$$ds^2 = c^2dt^2 - \exp(2Ht)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{30}$$

can be written in a manifestly conformally flat form

$$ds^2 = (1 - H\tau)^{-2}\left[c^2d\tau^2 - dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{31}$$

by a time-transformation

$$H\tau = 1 - \exp(-Ht) \tag{32}$$

where H is the Hubble constant. The fact that $\tau < H^{-1}$, suggests that there is an event horizon in the future absorber. This property in turn tells us that the response of the future absorber is cut off at a frequency

$$k_{\max} = \omega_{\text{eff}}/HT, \tag{33}$$

where, ω_{eff} is the effective frequency of future absorber and T is the time duration of the local process. For a medium with N as the number density of charged particles in the future absorber, ν the typical plasma frequency and m the electron mass, one has

$$\omega_{\rm eff} = \left[2\nu Ne^2/mH\right]^{1/2} \tag{34}$$

where ν is defined in terms of the physical condition of the plasma. For a free electron, one may take $T = \hbar/mc^2$.

A cutoff at $k_{\rm max}$ results in a finite value for the observed mass of an electron in terms of its bare mass:

$$m_{\rm obs} = m + \Delta m, \quad \Delta m = m \times \left[1 + (3e^2/2\pi)\ln(\hbar k_{\rm max}/mc^2)\right]. \tag{35}$$

For a free electron, the choice of $T \sim 10^{-21}$s, $H \sim 3.10^{-18}\text{s}^{-1}$ and $\omega_{\rm eff} \sim 80\ \text{s}^{-1}$, yields $\Delta m \sim 0.15m$. *The important result is that radiative corrections are finite* and no subtraction of one infinity from another is required. This cutoff necessarily results from the 'local-distant' interaction that is characteristic of action at a distance.

7. Concluding Remarks

We thus have both in the case of electrodynamics and gravity, a close connection between the local and the distant parts of the universe. The connection is established through the Green's functions G, that is, the propagators of the interaction. These Green's functions establish the connection along the light cone and as such the action at a distance is not instantaneous. However, when one examines the details of the absorber theory of radiation in the electromagnetic case, it tells us that a signal from a local source at $r = 0$, sent out at $t = 0$, reaches a distant absorber particle at distance $r = R$ at $t = R/c$. The response of the absorber travels *backwards* in time and so reaches the original source at $t = R/c - R/c = 0$, i.e., instantaneously ! This mixture of advanced and retarded signals serves to create an instantaneous effect without violating relativistic invariance.

Does this, however, violate causality ? The answer is 'yes', but at a very minute level. As was discussed by Dirac (1938), the radiation reaction formula has an advanced component, and this may be interpreted as generating 'pre-acceleration'. Since in this framework, one is looking at a self-consistent solution, this acausal effect is not disturbing. At the quantum level, it may be more interesting as was pointed out by Hoyle and Narlikar (1995).

The quantum version of the Wheeler-Feynman theory involves an influence functional through which the local system interacts with the large scale cosmological boundary conditions. This local+cosmological interaction appears as transition probability for a local system, wherein all cosmological variables are integrated out. Phenomena like spontaneous transition or a collapse of the wavefunction are seen to arise from this interaction. This suggests that, the attempts to explain some of these pheneomena through local hidden variables having failed, the real clue to the mystery may lie in the response of the universe in the above fashion.

Experiments by Aspect et al. (1982a,b) inspired by Bell's inequality (1966) have generated considerable discussion on nonlocality of hidden variables, and apparent acausal effects across spacelike separations. The response of the universe provides an additional factor which has been so far ignored in such discussions.

To summarize, the ideas which go under the name Mach's Principle are capable of wider applications than thought earlier by Ernst Mach. One can use the Machian concept in electrodynamics where the response of the universe can play a key role in both classical and quantum electrodynamics. The action at a distance framework used here is consistant with special relativity as well as with causality. The formalism can be used to give an expression to inertia as a direct long range effect from the distant parts of the universe. From inertia, one can arrive at a theory of gravity which is wider in its applications than general relativity. The theory can be extended to incorporate the cosmological constant and the concept of creation of matter without spacetime singularity. It leads to a viable cosmological model, known as the quasi-steady state cosmological model.

Finally, the as yet unchartered territory for this framework lies in the direction of epistemological aspects of quantum mechanics, such as understanding the rationale behind the collapse of a wavefunction and the correlations found across spacelike separations by Aspect-type experiments.

These local $\leftrightarrow$ cosmological interactions occupied Fred Hoyle's interest right to the end. Although the bulk of our work on action at a distance was done in the 1960s, Fred liked to return to the topic from time to time, vide our work on electron self energy problem in 1992–93 and the genesis of the quasi-steady state cosmology in 1993–95.

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